where K_p is the adjustable parameter. In state-space notation,

$$\dot{x}_p = \begin{bmatrix} 0 & 1 \\ -a_0 - K_p b_p & -a_1 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ K_p b_p \end{bmatrix} u \quad . \quad . \quad (13)$$

With a similar reference model, where only $b_m \neq b_p$ and $c_1 = b_m - K_p b_p$, from eqn. 6,

$$Dz = \begin{bmatrix} 0 & 0 & 0 \\ c_1 & 0 & c_1 \end{bmatrix} z = \begin{bmatrix} 0 \\ x_{p1} + u \end{bmatrix} c_1 = Mc_1 \quad . \quad . \quad . \quad (14)$$

Substituting M in eqn. 7b with $\gamma_1 = \phi_1 = 0$ yields the adaptive law

$$\dot{c}_1 = -\lambda_1 \rho_1 = -\lambda_1 (x_{p1} + u)(e_1 p_{12} + e_2 p_{22} + \varepsilon_2) \quad . \quad . \quad (15)$$

Because $\dot{c}_1 = -\dot{K}_p$, since b_m is time-invariant and b_p is slowly time varying, with eqn. 15 an adaptive law is given for the adjustment of K_p, which cannot be obtained when the usual Lyapunov approach is followed.

Conclusions: A design method according to Lyapunov's second method is presented for multivariable m.r.a.c. systems in which the adjustable plant parameters appear in more than one element of the plant matrices. An assumption is that the decomposition carried out in eqn. 6 is realisable. The resulting adaptive laws are of a proportional-plus-integral-plus-differential form and include a parameter misalignment function. The adaptive laws can be regarded as a generalisation of the results previously obtained by Hang.[6]

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ACTIVE *RC* BANDPASS AND LOWPASS FILTERS USING THE D.V.C.C.S./D.V.C.V.S.

Indexing terms: Active filters, Bandpass filters

A novel configuration using the differential voltage-controlled current source, differential voltage-controlled voltage source (d.v.c.c.s./d.v.c.v.s.) as the active building block is introduced. The realisations of bandpass and lowpass filters are given as special cases; both filters are canonic, and a moderate pole Q-factor can be obtained in each case.

Recently, a new versatile linear active element acting simultaneously as a differential voltage-controlled current source (d.v.c.c.s.) and a differential voltage-controlled voltage source (d.v.c.v.s.) was introduced.[1,2] The d.v.c.c.s./d.v.c.v.s. has infinite input impedance at both inputs and at output 1, whereas the output-2 impedance is zero. The device is defined by the relations

$$I_0 = G(V_- - V_+) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$V_0 = \alpha V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The possibility of connecting external elements to the output 1 of the device makes it more versatile than the conventional operational amplifier.[2]

General configuration: The basic structure is shown in Fig. 1, where N represents an RC 1-pole 3-port network. By direct analysis, the voltage transfer function of the network is

$$G(s) \equiv \frac{V_0}{V_1} = \frac{T(s)T_{12}(s)}{1 - T(s)T_{32}(s)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$T(s) = \frac{V_0}{V_+}, \quad T_{12} = \left.\frac{V_+}{V_1}\right|_{V_3=0}, \quad T_{32} = \left.\frac{V_+}{V_3}\right|_{V_1=0} \quad . \quad . \quad (4)$$

From the circuit, and using eqns. 1 and 2,

$$T(s) = \frac{\alpha G}{(G_1 + G) + sG_1} = \frac{K}{s + a} \quad . \quad . \quad . \quad . \quad (5)$$

where

$$K = \frac{\alpha G}{C_1} \qquad a = \frac{G + G_1}{C_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Bandpass filter: The basic network is capable of realising a canonic 2nd-order bandpass filter. The network N in this case will be chosen as in Fig. 2a. In this case,

$$T_{12}(s) = \frac{h_1 s}{s + b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$T_{32}(s) = \frac{h_2 s}{s + b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where

$$\left.\begin{aligned} b &= GG_4/Cg \\ G &= G_2 + G_3 \qquad g = G + G_4 \\ h_1 &= G_2/g \qquad h_2 = G_3/g \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (9)$$

From eqns. 5, 7 and 8 in eqn. 3:

$$G(s) = \frac{Kh_1 s}{s^2 + (a + b - Kh_2)s + ab} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

which realises a bandpass transfer function having

$$\omega_0 = \sqrt{(ab)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$Q = \frac{\sqrt{(ab)}}{a + b + Kh_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

To ensure the stability of the active filter, the circuit components should be chosen such that

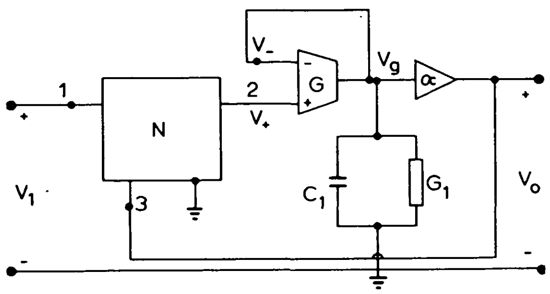

Fig. 1 *General filter configuration*

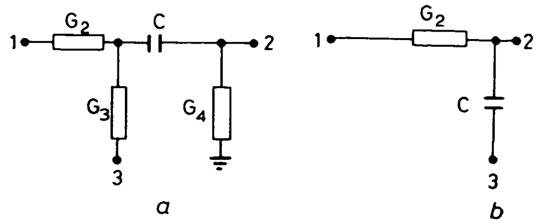

Fig. 2
a Network N for bandpass realisation
b Network N for lowpass realisation

$$G_1/G \geqslant \alpha - 1 \qquad (13)$$

Lowpass filter: The network N in this case is chosen as shown in Fig. 2*b*. Here

$$T_{12}(s) = \frac{d}{s+d} \qquad (14)$$

$$T_{32}(s) = \frac{s}{s+d} \qquad (15)$$

where

$$d = G_2/C \qquad (16)$$

From eqns. 5, 14 and 15 in eqn. 3:

$$G(s) = \frac{Kd}{s^2 + (a+d-K) + ad} \qquad (17)$$

which is the characteristic of a lowpass 2nd-order filter having

$$\omega_0 = \sqrt{(ad)} \qquad (18)$$

$$Q = \frac{\sqrt{(ad)}}{a+d-K} \qquad (19)$$

Again, the stability of the filter will be guaranteed if

$$G_1/G \geqslant \alpha - 1 \qquad (20)$$

If a unity d.c. gain is desired, G_1/G should be chosen to be equal to $\alpha - 1$, and, in this case,

$$\omega_0 = \frac{\sqrt{\{(G+G_1)G_2\}}}{CC_1} \qquad Q = \frac{\sqrt{\{C(G+G_1)\}}}{C_1 G_2} \qquad (21)$$

Note that the passive RC network N involved in the realisation of the lowpass characteristic is obtained from that required to realise the bandpass filter. More specifically, the circuit of Fig. 2*b* is obtained from that in Fig. 2*a* by setting $G_4 = 0$ and $G_3 = \infty$ and interchanging ports 2 and 3.

Conclusions: The use of the d.v.c.c.s./d.v.c.v.s. as the active building block in the realisation of canonic bandpass and lowpass filters has been illustrated by two networks. Both networks are generated from a new general configuration.

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MAXIMUM Q-FACTORS FOR SYMMETRICAL BANDPASS FILTERS

Indexing terms: Bandpass filters, Q-factor

The letter presents curves for the estimation of the maximum pole Q-factors for geometrically symmetrical Butterworth, Chebŷshev and elliptic bandpass filters.

Introduction: In Reference 1, Holt and Attikiouzel presented three graphs for the estimation of the maximum Q-factors for the geometrically symmetrical Butterworth and Chebŷshev bandpass filters and their application in the cascade active filter design. However, for the following reasons, their graphs are of limited use. First, large interpolation is usually required in the estimation, and, secondly, only the cases corresponding to a passband ripple of 0·5 and 1·0 dB are included for the Chebŷshev filters. In this letter, a comprehensive estimation is given for the Butterworth and Chebŷshev bandpass filters of even degree 4 to 20, or, equivalently, for the corresponding lowpass prototype of degree 2 to 10. In addition, with the use of a multiplying factor K, the estimation of the maximum Q-factor for the elliptic-type bandpass filter is also included.

Procedure: The following parameters are needed (Fig. 1):

(*a*) The number of 2nd-order sections n, or the degree of the l.p. prototype.

(*b*) The peak-to-peak passband ripple A_{max} in decibels.

(*c*) The centre frequency f_0.

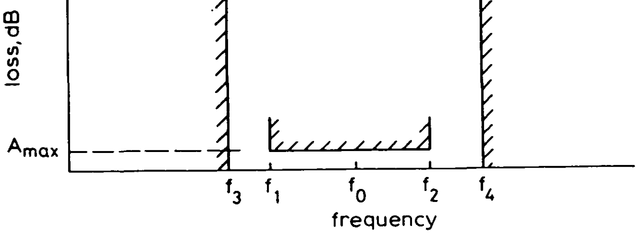

Fig. 1 *Representation of geometrically symmetrical bandpass filters*

(*d*) The passband bandwidth $f_2 - f_1$.

and, for elliptic b.p. filters,

(*e*) The transition ratio $\Omega_s = (f_4 - f_3)/(f_2 - f_1)$.

Note that, for geometrically symmetrical b.p. filters,

$$f_1 f_2 = f_3 f_4 = f_0^2$$

The maximum Q-factor Q_{max} can be obtained from Fig. 2* for Butterworth b.p. filters and from Fig. 3* for Chebŷshev and elliptic b.p. filters.

Accuracy: The estimates improve as the ratio $f_0/(f_2 - f_1)$ increases. For Butterworth and Chebŷshev filters, the estimate is usually within 2% for $f_0/(f_2 - f_1) = 3$ and within 5% for $f_0/(f_2 - f_1) = 2$. For elliptic filters, the estimate is usually within 15% for $f_0/(f_2 - f_1) \geqslant 2$.

Example: Referring to Fig. 1, let the parameters of an elliptic b.p. filter be lowpass prototype of degree 3, centre frequency $f_0 = 2805$ Hz, passband bandwidth $f_2 - f_1 = 90$ Hz, passband ripple $A_{max} = 0{\cdot}1$ dB and stopband bandwidth $f_4 - f_3 = 225$ Hz. Hence $\Omega_s = 225/90 = 2{\cdot}5$ and

$$f_0/(f_2 - f_1) = 2805/90.$$

From Fig. 3,* we have

$$Q_{max} \bigg/ \left(\frac{f_0 K}{f_2 - f_1}\right) = 2{\cdot}05 \quad \text{and} \quad K = 1{\cdot}1825$$

Therefore the estimate of $Q_{max} = 75{\cdot}6$ is within 2% of the actual value of 74·1.

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