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LOW-VOLTAGE MOS CHAOTIC OSCILLATOR BASED ON THE NONLINEARITY OF G_m

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This paper presents a chaotic oscillator based on the nonlinearity of the typical transconductance (G_m). This chaotic circuit only consists of 13 MOS transistors and three grounded capacitors, which is one of the smallest chaotic oscillators. This circuit operates on low voltage supply (±1.5 V). The dimensionless form of the circuit is also introduced to confirm the circuit simulation.

Keywords:

1. Introduction

With the advances in the very large scale integration (VLSI) technology and the demand for portable electronic products, there came a need for low voltage supply circuits (lower than 3 V) and the use of current-mode signal processing techniques.[1] The chaotic circuits are very important in different fields such as communications and medicine.[2]

The design of chaotic oscillators are very powerful area of research.[3,4] Many chaotic oscillators are observer in the last few years.[1−11] The chaotic oscillator presented is based on MOS transistors and grounded capacitors without the use of a large block such as operational amplifier, current feedback operational amplifier, . . . , etc.[5,8] This gives the advantage of a very small area in fabrication. In addition the grounded capacitors give the freedom of using an external capacitance.

The chaotic equations are essentially of third-order differential equation or three first-order differential equation and must contain a nonlinear term. The realization of nonlinear term can take different methods such as translinear circuit, diode[9] or transistors,[9] but this paper discusses a chaotic circuit based on the use of G_m–C integrators using the nonlinearity of the transconductor as its nonlinear elements. The mathematical dimensionless form are also included.

In Sec. 2, the typical transconductor will be presented based on two MOS transistors acting as an inverter.[7] This transconductor was used to generate a G_m–C integrator which will be one of the basic elements of the chaotic oscillator.

In Sec. 3, two G_m–C integrators were used with an inverting circuit to develop a sinusoidal oscillator.[7] This sinusoidal oscillator that generates two quadrature signals lacks a control signal, so a modified oscillator will be introduced.

In Sec. 4, the modified oscillator is realized by adding a new element to the previous oscillator producing a bias signal that will be used later to generate the chaotic oscillator. Two modified oscillators were presented, the first generates different waveforms (square waves, triangular waves and sinusoidal-like waves) and the second generates only sinusoidal-like waveforms.

In Sec. 5, the chaotic oscillator is presented by adding a new G_m–C integrator, a transistor that acts as a diode to control the bias signal in the previous modified oscillator circuit and with small changes in the circuit parameters, the chaotic oscillations are produced.

For each section PSpice simulations and numerical simulations of the mathematical models are given to confirm the circuit behavior. The PSpice simulations are performed using Mietec 0.5 micron technology and the numerical simulations are done using Stiff method.

2. The Typical Transconductor

A typical transconductor consisting of only two transistors and operating on a supply ±1.5 V is presented in this section.[7] This transconductor consists of two transistors M_1 and M_2 (simple inverter) that are operating in the saturation region. From Fig. 1

$$I_0 = g_m V_I \,, \tag{1a}$$

$$g_m = -2K(V_{DD} - V_T) \,, \tag{1b}$$

where $K = k(W/L)$, k is the process conductance parameter, (W/L) is the transistor aspect ratio. V_T is the threshold voltage of the transistors.[7]

By the addition of a capacitor at the output terminal voltage integrators were obtained where the relation between the output voltage and the input voltage becomes

$$\overset{\bullet}{V_0} = a_1 V_I \,, \tag{2a}$$

$$a_1 = -\frac{g_m}{C}\,. \tag{2b}$$

In order for the transconductor circuit to work properly, it must satisfy the following conditions

$$K_1 = K_2 = K\,, \tag{3a}$$

$$V_{TN} = |V_{TP}| = V_T\,. \tag{3b}$$

PSpice simulation when the input signal is square waveform showing the output signal is triangle waveform as in Eq. (2a), is shown in Fig. 2. The signal swing can be controlled by changing the value of K.

3. The Oscillator

The possible chaotic nature of conventional sinusoidal oscillators has become one of the important domains of research in the generation of chaos.[6] The simplest equation that gives simple harmonic motion is

$$\ddot{V}_X + \omega_0^2 V_X = 0\,. \tag{4}$$

The block diagram of the well-known oscillator representing this equation is shown in Fig. 3 that consists of two G_m–C integrators and an inverting voltage circuit to produce a sinusoidal waveform.[7]

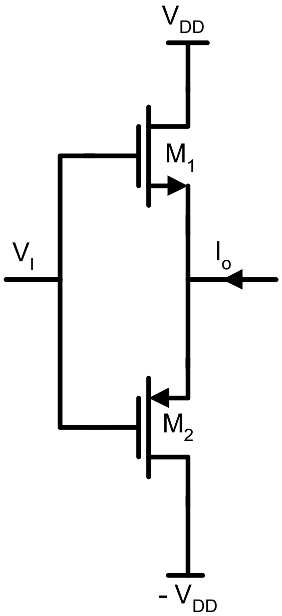

Fig. 1. The typical transconductor.

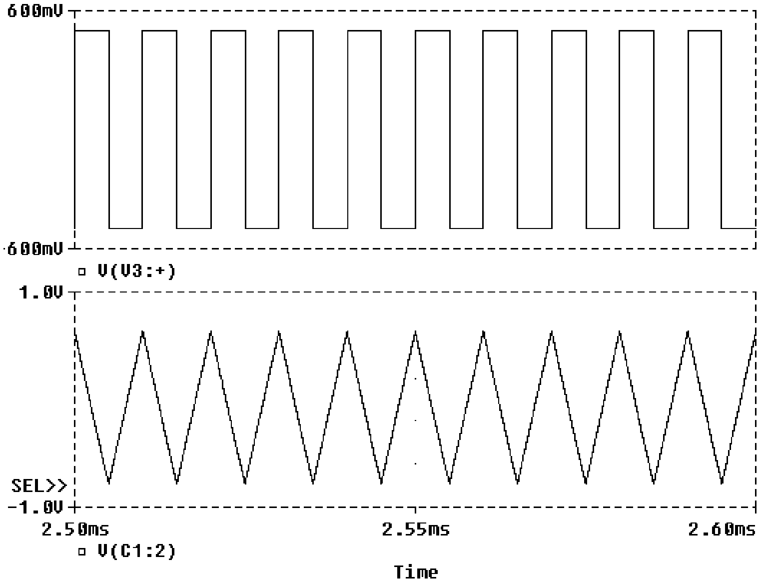

Fig. 2. PSpice simulation of the G_m–C integrator when the input is square wave.

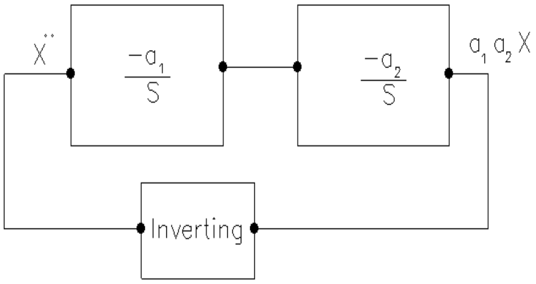

Fig. 3. The block diagram of the typical oscillator.

3.1. *Circuit description*

The first G_m–C integrator consists of transistors M_4, M_9 and a capacitor C_1. The second G_m–C integrator consists of transistors M_5, M_8 and a capacitor C_2 as shown in Fig. 4. The voltage inversion circuit is achieved by the transistors M_2, M_3, M_{10} and M_{11} as shown in Fig. 4, with the following conditions

$$K_2 = K_{11} \quad \text{and} \quad K_3 = K_{10}\,. \tag{5a}$$

Then $I_{\mathrm{S}3} = I_{\mathrm{D}10}$ and $I_{\mathrm{S}2} = I_{\mathrm{D}11}$ through the negative feedback which ensures that I_X = zero (approximately). Due to both transistors M_3 and M_{10} work in the

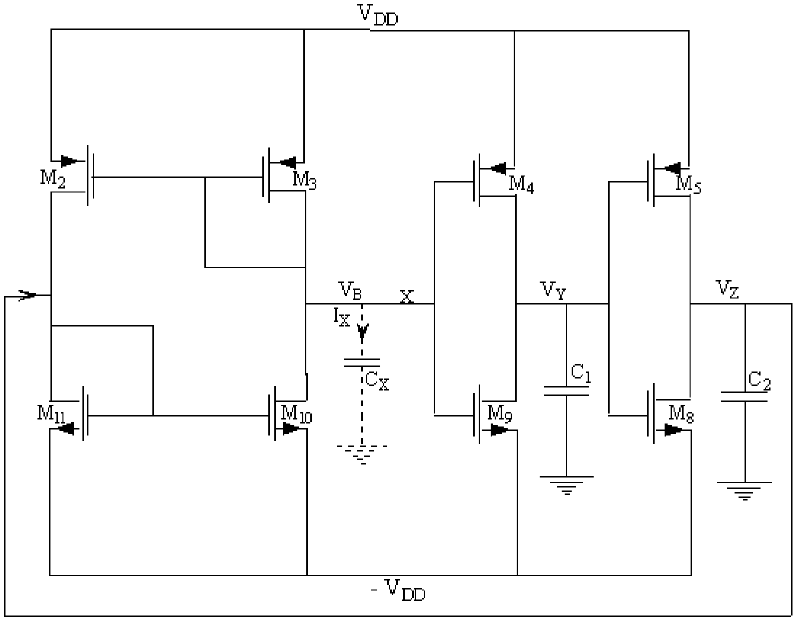

Fig. 4. Circuit realization of the typical oscillator.

saturation region so

$$V_{\mathrm{SG3}} = V_{\mathrm{GS10}}\,, \tag{5b}$$

$$V_{DD} - V_X = V_Z + V_{DD}\,. \tag{5c}$$

Therefore

$$V_X = -V_Z\,. \tag{5d}$$

So the circuit is a sinusoidal oscillator. If the G_m–C constants are equal $a_1 = a_2 = a$, then the radial frequency

$$\omega_0 = \sqrt{a} \tag{6}$$

From the study of the circuit the conditions for oscillation were found to be:

$$K_4 = K_9\,, \tag{7a}$$

$$K_5 = K_8\,, \tag{7b}$$

$$\frac{K_2}{K_3} = \frac{K_{11}}{K_{10}}\,, \tag{7c}$$

All transistors work in the saturation region, this is done by controlling the aspect ratio (W/L) of the transistors. (7d)

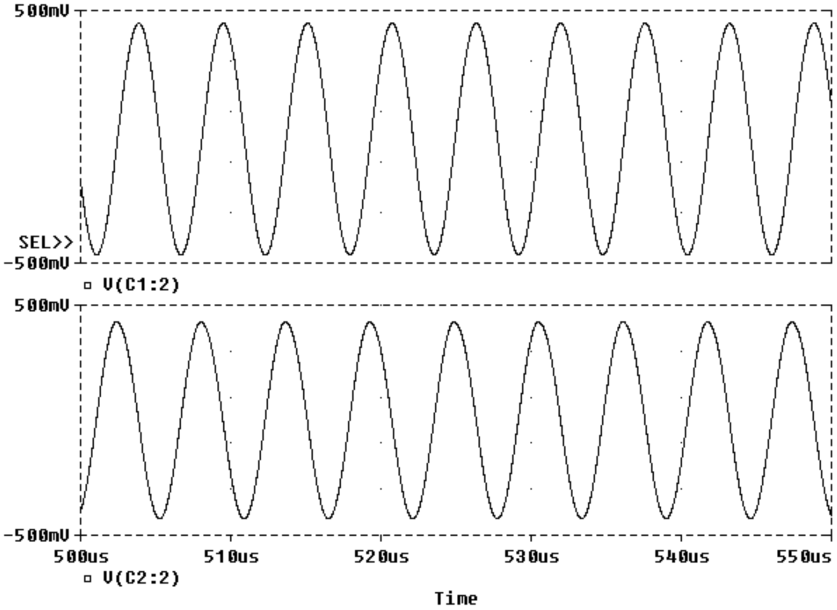

Fig. 5. PSpice simulation of the typical oscillator.

PSpice simulation results for the voltage V_Y and V_Z waveforms are shown in Fig. 5.

4. The Modified Oscillator

In the main oscillator the function of M_2 and M_3 is to ensure that $V_X = -V_Z$ and that both of them work in saturation based on a current mirror principle. As can be noticed the main oscillator has no control signal that may be used to generate chaos.

In order to overcome this problem the connection between G–D of M_3 was removed and a new transistor M_1 was introduced, that is controlled by an external source V_B that is connected to both transistors M_2 and M_3 so that they remain working as a current mirror. In such case $V_X \neq -V_Z$ as shown in Fig. 6.

In the main oscillator the connections M_{10} and M_{11} as well as M_2 and M_3 stabilize the current through this loop so that $I_X =$ zero, and I_{S3} always equals I_{D10} so that no current is allowed to pass through the parasitic capacitance C_X (the summation of all the capacitance observed at point X).

In the modified oscillator it cannot be guaranteed that I_X will always equal zero, which makes it impossible to neglect the effect of this current, in which case it is a must to take into account the parasitic capacitance C_X, which can be obtained from $C_X \approx W_{\mathrm{M4}}L_{\mathrm{M4}}C_{\mathrm{JP}} + W_{\mathrm{M9}}L_{\mathrm{M9}}C_{\mathrm{JN}}$, where C_{JP} is the gate capacitance of the PMOS transistor, C_{JN} is the gate capacitance of the NMOS transistor, W is the width of the transistor and L is the length.

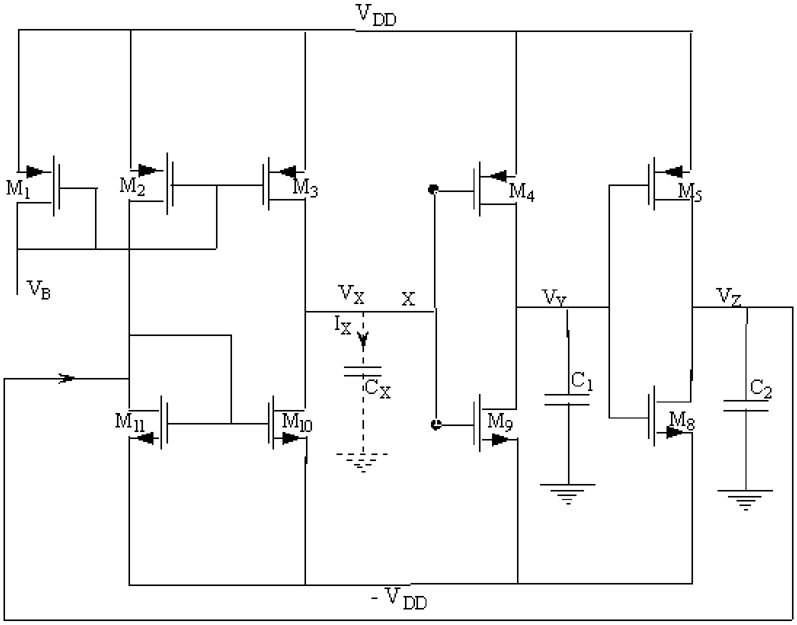

Fig. 6. Circuit realization of the modified oscillator.

4.1. *Circuit description*

The operation of the modified oscillator are:

(1) Assume V_Z will increase ($V_Z > -V_B$) then I_X will consequently increase resulting in an increase in I_{CX} that leads to a drop in V_X until it approximately approaches $-V_{DD}$. In this case M_9 will be off and M_4 will work in the saturation region acting as a constant current source that passes through C_1 causing V_Y to linearly increase. V_Z will appear to be convex parabolic since V_Z is the negative integration of V_Y, at the same time I_X will remain in the same direction. V_Z and I_X increase initially and then gradually decrease till I_X reaches zero and at the same instant $V_Z = -V_B$.

(2) As I_X reverses its direction V_X will increase to approach V_{DD} as a result M_4 is turned off, M_9 is forced to work in the saturation region (acting as a constant current source) and V_Y is forced to decrease linearly. V_Z will appear to be concave parabolic since V_Z is the negative integration of V_Y. At the same time I_X will remain in the same direction. V_Z and I_X initially decrease and then gradually increase till I_X reaches zero and at the same instant $V_Z = -V_B$. The cycle will repeat itself.

After describing the above circuit, the following conclusions were obtained:

(1) V_X will be a square wave generator.

(2) V_Y will be a triangular wave generator.
(3) V_Z will act almost like a sinusoidal generator.

4.2. *The mathematical model*

The mathematical model is very essential for circuit verification. The first step is to generate the circuit equations in a dimension form with voltages and currents. The second step is to transform these equations into dimensionless form which is suitable for mathematics, solving it by numerical techniques, showing the same results of circuit simulations.

4.2.1. *Circuit equations*

From the circuit

(1) M_4, M_9 and C_1 form a G_m–C integrator, then

$$C_1 \overset{\bullet}{V}_Y = -2K_4(V_{DD} - V_T)V_X \,. \tag{8}$$

(2) Because I_X is significant so its effect cannot be neglected. M_{11} always works in the saturation region because of the effect of the *G–D* connection. M_2 can operate either in the nonlinear region or the saturation region. Note that M_5 and M_8 work together as a transconductor circuit, so

$$C_2 \overset{\bullet}{V}_Z = -2K_5(V_{DD} - V_T)V_Y + I_{\mathrm{S2}} - 0.5K_{11}(V_Z + V_{DD} - V_T)^2 \,, \tag{9a}$$

$$I_{\mathrm{S2}} = \left\{ \begin{array}{ll} \dfrac{K_2}{2}(V_{DD} - V_B - V_T)^2 & V_Z - V_B \leq 0.6 \\ \dfrac{K_2}{2}(V_{DD} - V_Z)(V_{DD} - 2V_T - 2V_B + V_Z) & V_Z - V_B > 0.6 \end{array} \right\} . \tag{9b}$$

(3) Due to the large swing of the signal, each of M_3 and M_{10} operate in either the nonlinear region or the saturation region, then

$$C_X \overset{\bullet}{V}_X = I_{\mathrm{S3}} - I_{\mathrm{D10}} \,, \tag{10a}$$

$$I_{\mathrm{S3}} = \left\{ \begin{array}{ll} \dfrac{K_3}{2}(V_{DD} - V_B - V_T)^2 & V_X - V_B \leq 0.6 \\ \dfrac{K_3}{2}(V_{DD} - V_X)(V_{DD} - 2V_T - 2V_B + V_X) & V_X - V_B > 0.6 \end{array} \right\} , \tag{10b}$$

$$I_{\mathrm{D10}} = \left\{ \begin{array}{ll} \dfrac{K_{10}}{2}(V_Z + V_{DD} - V_T)^2 & V_Z - V_X \leq 0.6 \\ \dfrac{K_{10}}{2}(V_{DD} + V_Z)(V_{DD} - 2V_T + 2V_Z + V_X) & V_Z - V_X > 0.6 \end{array} \right\} . \tag{10c}$$

4.2.2. *The dimensionless form*

In order to undergo a mathematical simulation it was necessary to transform the previous equations into a dimensionless form, so the following new dimensionless parameters are defined

$$X = \frac{V_X}{V_{DD}}\,, \qquad X_1 = \frac{V_Y}{V_{DD}}\,, \qquad X_2 = \frac{V_Z}{V_{DD}}\,, \qquad X_B = \frac{V_B}{V_{DD}}\,,$$

$$\tau = \frac{K_R V_{DD}}{C_R} t\,, \qquad \beta_1 = \frac{C_R}{C_1}\,, \qquad \beta_2 = \frac{C_R}{C_2}\,, \qquad \beta_X = \frac{C_R}{C_X}\,, \tag{11}$$

$$\alpha_1 = \frac{K_I}{K_R}\,, \ \text{ for all transistors}\,.$$

The new equations will be as follows:

$$X_1^{\bullet} = -1.2\beta_1\alpha_4 X\,, \tag{12}$$

$$X_2^{\bullet} = \beta_2[-1.2\alpha_5 X_1 + XI_2 - 0.5\alpha_{11}(X_2 + 0.6)^2]\,, \tag{13a}$$

$$XI_2 = \left\{ \begin{array}{ll} \frac{\alpha_2}{2}(0.6 - X_B)^2 & X_2 - X_B \le 0.4 \\ \frac{\alpha_2}{2}(1 - X_2)(0.2 - 2X_B + X_2) & X_2 - X_B > 0.4 \end{array} \right\}, \tag{13b}$$

$$X^{\bullet} = \beta_X[XI_3 - XI_{10}]\,, \tag{14a}$$

$$XI_3 = \left\{ \begin{array}{ll} \frac{\alpha_3}{2}(0.6 - X_B)^2 & X - X_B \le 0.4 \\ \frac{\alpha_3}{2}(1 - X)(0.2 + 2X_B + X) & X - X_B > 0.4 \end{array} \right\}, \tag{14b}$$

$$XI_{10} = \left\{ \begin{array}{ll} \frac{\alpha_{10}}{2}(X_2 + 0.6)^2 & X_2 - X \le 0.4 \\ \frac{\alpha_{10}}{2}(1 + X_2)(0.2 + 2X_2 - X) & X_2 - X > 0.4 \end{array} \right\}. \tag{14c}$$

4.3. ***Simulation results***

The transistor aspect ratios are shown in Table 1. The PSpice simulation waveforms of V_X, V_Y and V_Z are shown in Fig. 7. The numerical mathematical dimensionless model of the modified oscillator circuit was done using Stiff method with time step = 0.005, $K_R = 100 \times 10^{-6}$ A/V^2 and $C_R = 10^{-11}$ F, and the parameters are shown in Table 2. The supply $V_{DD} = 1.5$ V, $C_1 = C_2 = 50$ pF, C_X was calculated as 1.233 pF. The waveforms of X, X_1 and X_2 are shown in Fig. 8.

Table 1. Transistor aspect ratios for the modified oscillator circuit.

Transistor	W/L (μm/μm)
M_2, M_4, M_5	20/15
M_8, M_9, M_{11}	20/65.5
M_3	100/5
M_{10}	100/22

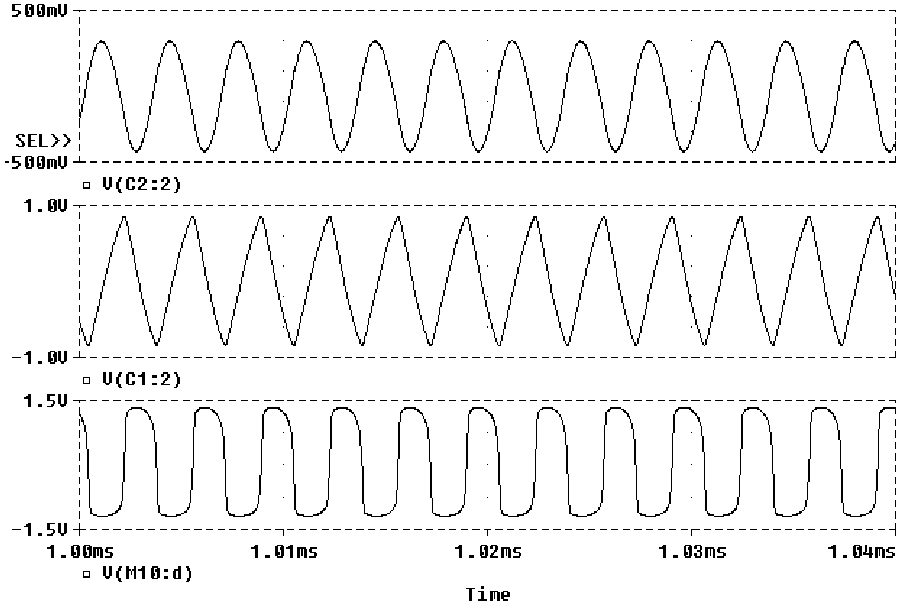

Fig. 7. PSpice waveforms of the modified oscillator.

Table 2. Dimensionless parameters for the modified oscillator model.

Parameter	Value
β_1, β_2	0.2
β_X	8.115
$\alpha_2, \alpha_4, \alpha_5, \alpha_{11}$	0.305
α_3, α_{10}	4.545

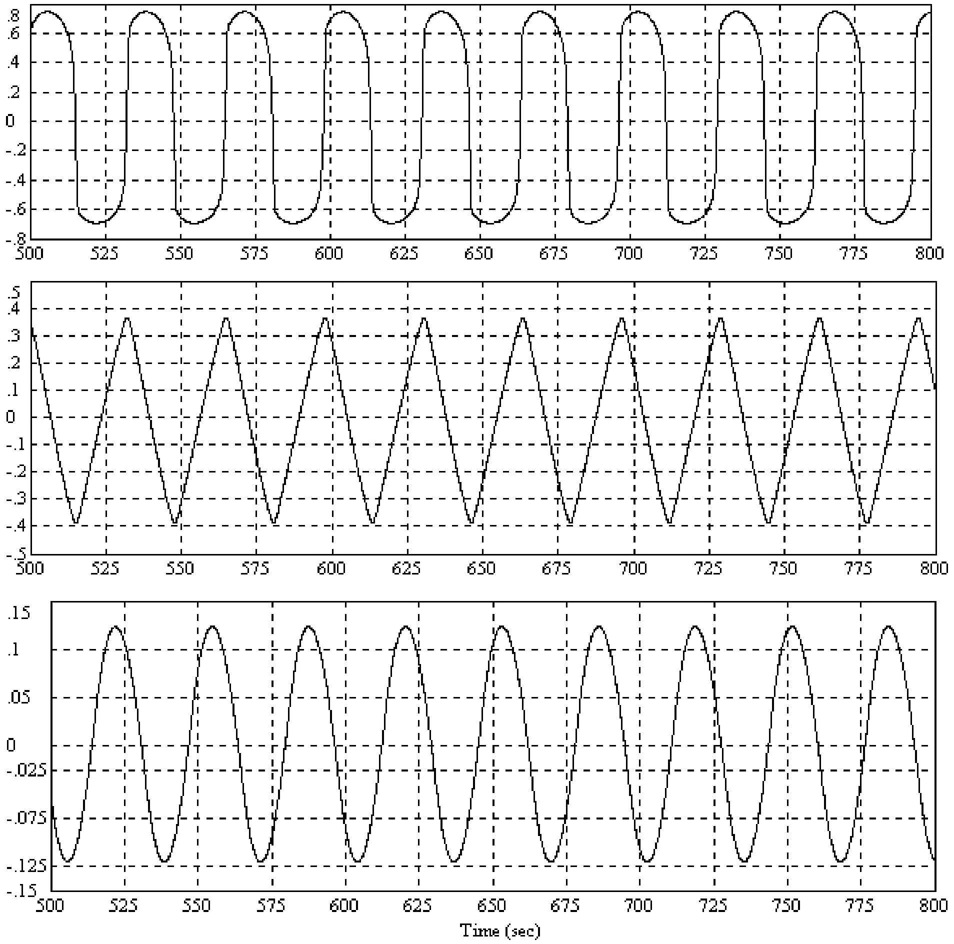

Fig. 8. Numerical simulation of the dimensionless form of the modified oscillator.

4.4. *Sinusoidal-like oscillator*

It is possible to generate a sinusoidal-like waveform from the previous modified oscillator by decreasing the aspect ratios of transistors M_3, M_{10} and increasing the aspect ratios of M_2, M_{11}. By these new modifications V_X will change gradually and not abruptly, producing a sinusoidal-like waveform consequently V_Y and V_Z will also produce a sinusoidal-like waveform.

The PSpice simulation of this new modified oscillator was done using the transistor aspect ratios shown in Table 3. The PSpice simulation waveforms of V_X, V_Y and V_z waveforms are shown in Fig. 9. The numerical solution of the dimensionless form using the values of the parameters is stated in Table 4. The X, X_1 and X_2 waveforms are shown in Fig. 10.

Table 3. Transistor aspect ratios for sinusoidal-like-oscillator circuit.

Transistor	W/L (μm/μm)
M_4, M_5	20/15
M_8, M_9	20/65.5
M_2	20/1.5
M_3	7/15
M_{10}	7/65.5
M_{11}	20/7

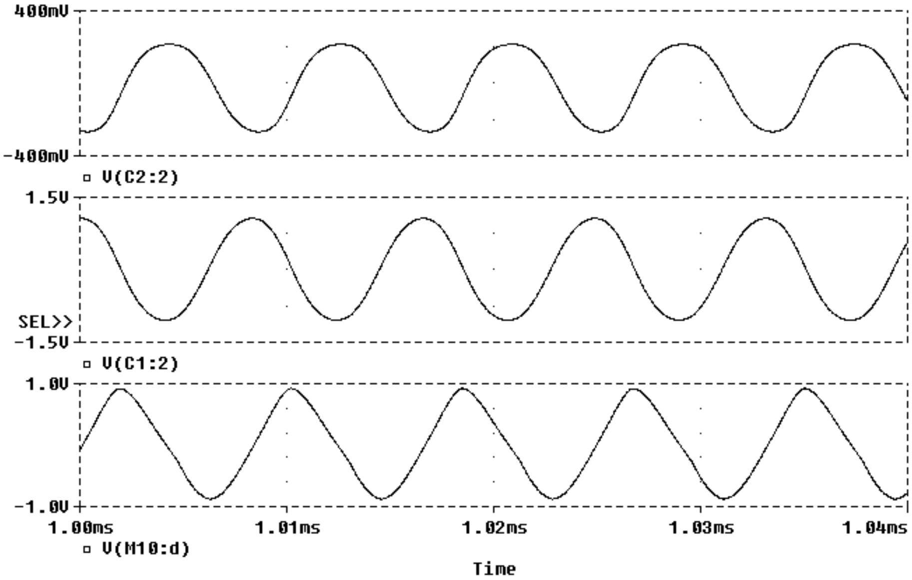

Fig. 9. PSpice waveforms of the sinusoidal-like oscillator.

Table 4. Dimensionless parameters for sinusoidal-like-oscillator model.

Parameter	Value
β_1, β_2	0.2
β_X	8.115
α_4, α_5	0.305
α_2, α_{11}	2.85714
α_3, α_{10}	0.1068

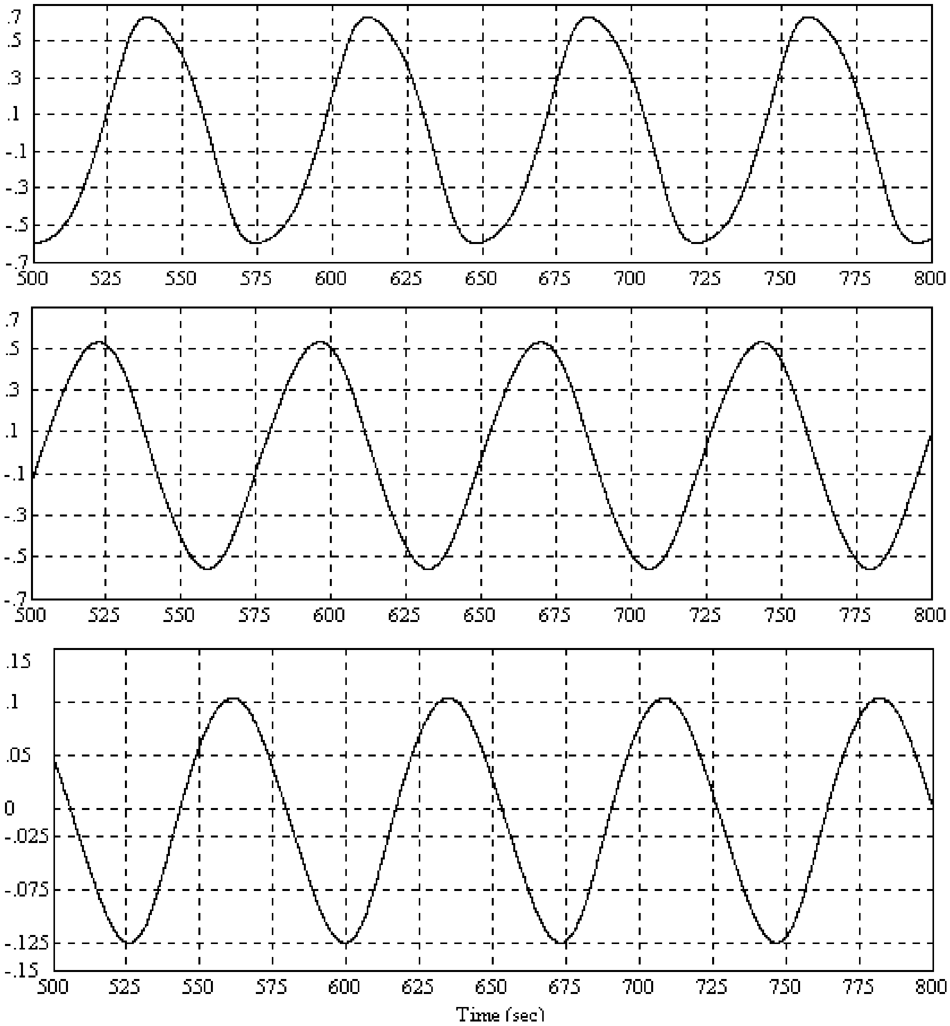

Fig. 10. Numerical simulation of the dimensionless form of the sinusoidal-like oscillator.

5. The Proposed Chaotic Circuit

In order to generate a chaotic circuit from the previous modified oscillator there was a need to add an integrator through connecting new elements which are transistors M_6, M_7 and a capacitor C_3. A feedback connection was added from V_W to control V_B through adding the two elements M_{12} and M_{13} as shown in Fig. 11.

It is clear that the previous modified oscillator circuit does not show a change in its functionality. So its equations remain almost the same. However there are two equations added to those modified oscillator in order to generate chaos. The first one concerning the current flow through the capacitor C_B (the sum of all the

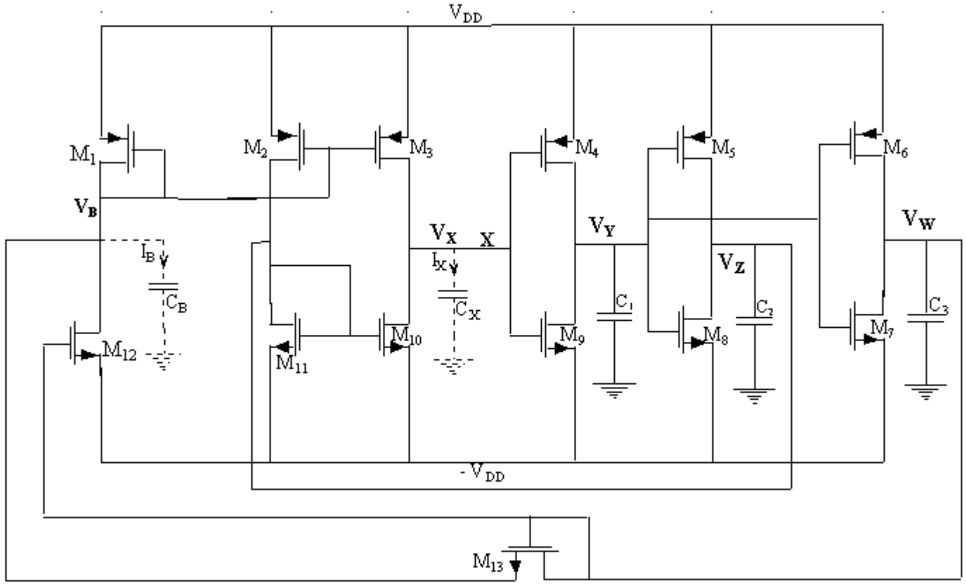

Fig. 11. Circuit realization of the proposed chaotic circuit.

capacitance observed at this point B) and the second concerning the newly-added G_m–C part. For some small changes of M_2 chaos will be generated.

It is clear that the chaotic response depends on the nonlinearity inside the G_m itself and some parasitic capacitance related to the design.

5.1. *The mathematical models*

5.1.1. *The mathematical equations*

In addition to the equations in the modified oscillator circuit (8)–(10), the following two equations are added

(1) M_1 always operates in saturation, either M_{12} operates in saturation when M_{13} is off or M_{12} operates in triode region when M_{13} operates in saturation, then

$$C_B\overset{\bullet}{V_B} = \left\{\begin{array}{ll} I_{S1} - \dfrac{K_{12}}{2}(V_W - V_B - V_T)^2 & V_W - V_B \le 0.6 \\ I_{S1} - \dfrac{K_{12}}{2}(V_B + V_{DD})(2V_W - V_B + V_{DD} - 2V_T)^2 & \\ \quad + \dfrac{K_{13}}{2}(V_W - V_B - V_T)^2 & V_W - V_B > 0.6 \end{array}\right\}, \tag{15a}$$

$$I_{S1} = \frac{K_1}{2}(V_{DD} - V_B - V_T)^2\,. \tag{15b}$$

(2) M_6 and M_7 with C_3 acts as a G_m–C integrator and M_{13} operates in saturation or cutoff regions, then

$$C_3\overset{\bullet}{V}_W = \left\{\begin{matrix} -2K_6(V_{DD} - V_T)V_Y - \dfrac{K_{13}}{2}(V_W - V_B - V_T)^2 & \qquad V_W - V_B \geq 0.6 \\[2ex] -2K_6(V_{DD} - V_T)V_Y & \qquad V_W - V_B < 0.6 \end{matrix}\right\}. \tag{16}$$

5.1.2. *The dimensionless form*

The two added equations will be transformed into the dimensionless form using the same basis as the modified oscillator. The added parameters are as follows

$$X_3 = \frac{V_W}{V_{DD}}\,, \qquad \beta_3 = \frac{C_R}{C_3}\,, \qquad \beta_B = \frac{C_R}{C_B}\,. \tag{17}$$

The dimensionless form of these two equations are

$$\overset{\bullet}{X}_B = \left\{\begin{matrix} I_{\mathrm{S1}} - \dfrac{\alpha_{12}}{2}(0.6 - X_3)^2] & \qquad X_3 - X_B \leq 0.4 \\[2ex] I_{\mathrm{S1}} - \dfrac{\alpha_{12}}{2}(X_B + 1)(2X_3 - X_B + 0.2)^2 & \\[2ex] \qquad + \dfrac{\alpha_{14}}{2}(X_3 - X_B - 0.4)^2] & \qquad X_3 - X_B > 0.4 \end{matrix}\right\}, \tag{18a}$$

$$I_{\mathrm{S1}} = \beta_B[\frac{\alpha_1}{2}(0.6 - X_B)^2\,, \tag{18b}$$

$$\overset{\bullet}{X}_3 = \left\{\begin{matrix} \beta_3\left[-1.2\alpha_6 X_1 - \dfrac{\alpha_{14}}{2}(X_3 - X_B - 0.2)^2\right] & \quad X_3 - X_B \geq 0.4 \\[2ex] \beta_3[-1.2\alpha_6 X_1] & \quad X_3 - X_B < 0.4 \end{matrix}\right\}. \tag{19}$$

5.2. *Simulation results*

The transistor aspect ratios are shown in Table 5 taking into consideration increasing the aspect ratio of the transistor M_2. The PSpice simulation waveforms of V_Y, V_Z and V_W are shown in Fig. 12. The V_Z–V_Y and V_Z–V_W projections are shown in Fig. 13.

The numerical mathematical dimensionless model of the modified oscillator circuit was done using Stiff method with time step = 0.05, $K_R = 100 \times 10^{-6}$ A/V^2 and $C_R = 10^{-11}$ F, and the parameters are shown in Table 6. The supply $V_{DD} = 1.5$ V. The X_2–X_1 and X_2–X_3 projections are shown in Figs. 14 and 15.

The chaotic behaviors of the presented circuit are very clear from the mathematical and circuit simulation. The largest Lyapunov exponent of the dimensionless form is calculated to be approximately 4.1 using nonlinear time series analysis.

Table 5. Transistor aspect ratios for the proposed chaotic circuit.

Transistor	W/L (μm/μm)
M_1	5/15
M_3, M_4, M_6	20/15
M_2	26/15
M_7, M_9, M_{10}, M_{11}	20/65.5
M_5	40/15
M_8	40/65.5
M_{12}	1/5
M_{13}	5/2.5

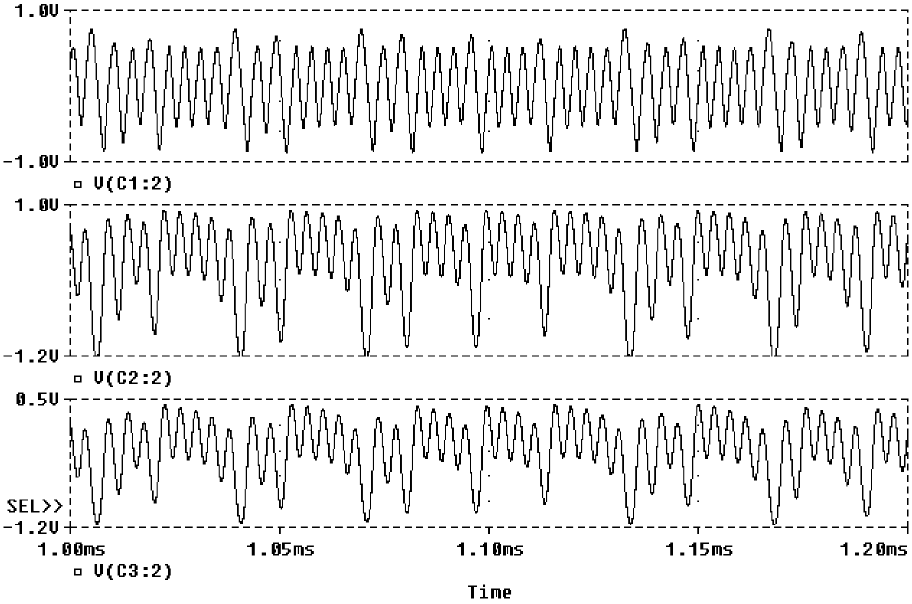

Fig. 12. PSpice waveforms of the chaotic circuit.

6. Conclusion

The proposed chaotic circuit is considered one of the smallest chaotic circuits which operate on low voltage supply (±1.5 V) using only thirteen transistors and three grounded capacitors. This circuit depends on the nonlinearity of the typical transconductor. Two modified oscillators are introduced in the paper. The presented circuit is easily used for fabrication in portable devices. The dimensionless form of the novel circuit is also obtained to ensure the same characteristics of the circuit simulation.

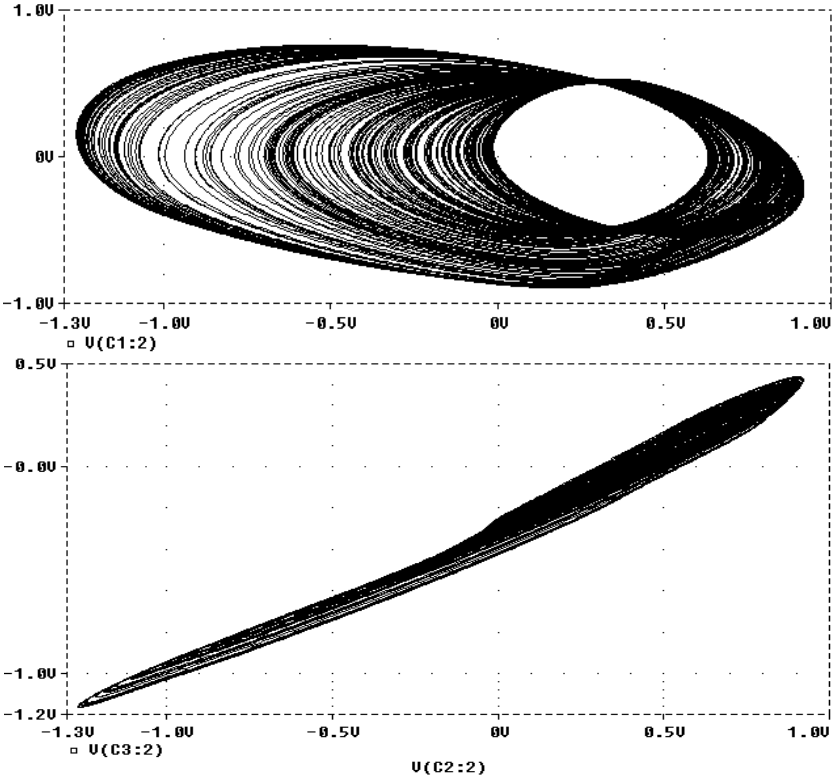

Fig. 13. V_Z–V_Y and V_Z–V_W PSpice projections of the chaotic circuit.

Table 6. Dimensionless parameters for the proposed chaotic model.

Parameter	Value
$\beta_1, \beta_2, \beta_3$	0.2
β_X	8.115
β_B	156
$\alpha_3, \alpha_4, \alpha_6, \alpha_{10}, \alpha_{11}$	2
α_1	0.5
α_{12}	1
α_{13}	12
α_5	4
α_2	2.245

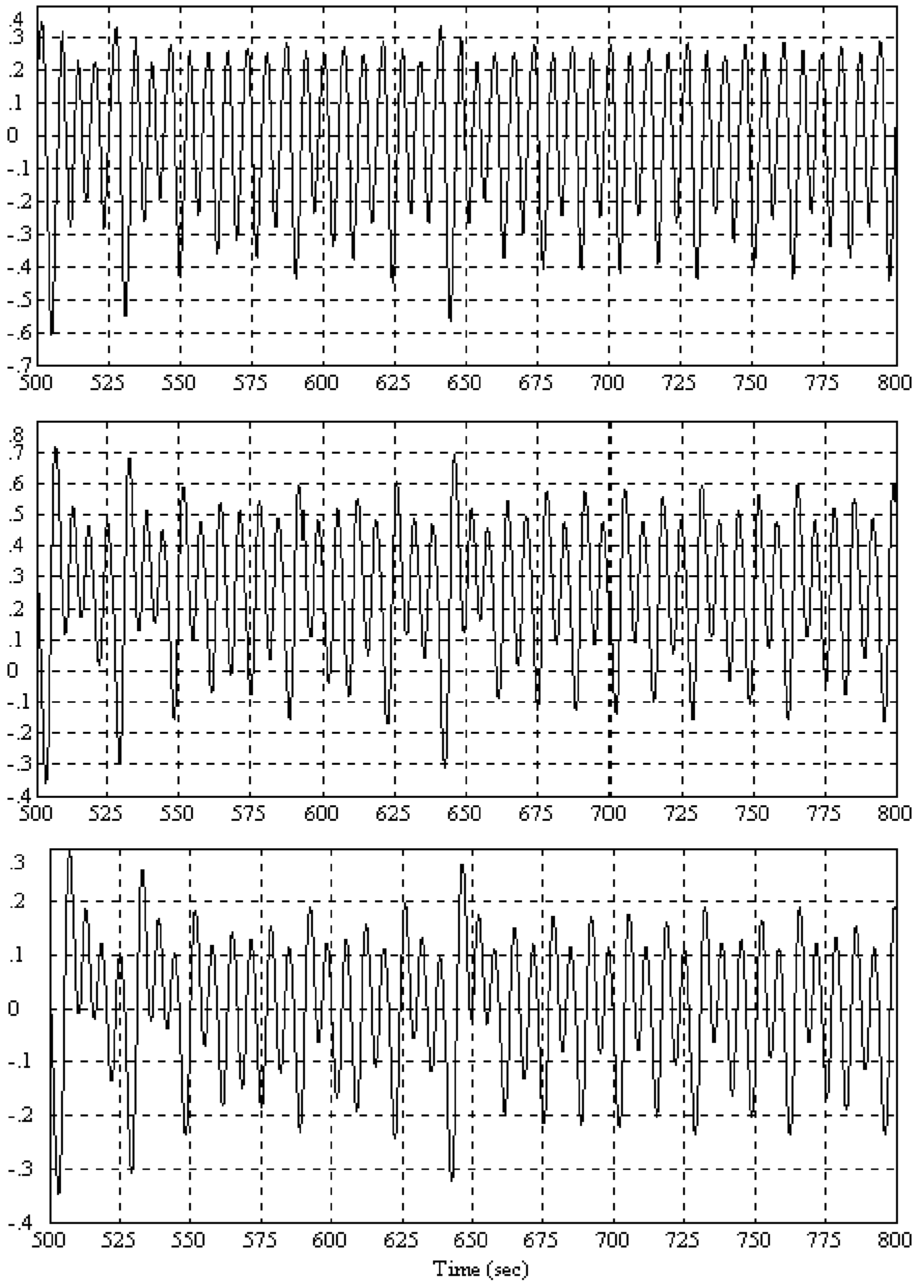

Fig. 14. Numerical waveforms of the chaotic dimensionless form.

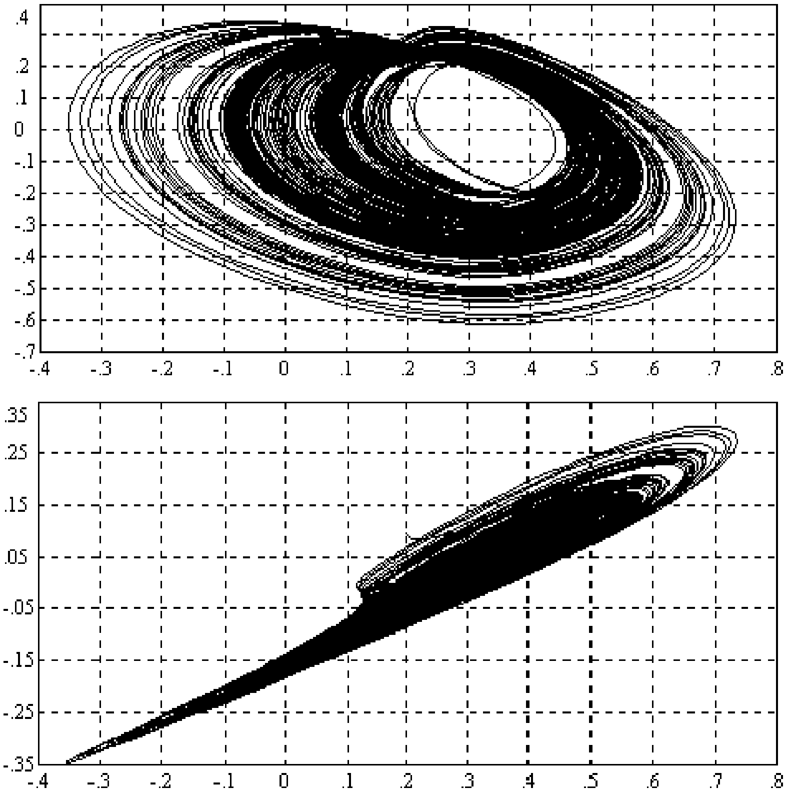

Fig. 15. X_2–X_1 and X_2–X_3 numerical projection of the chaotic dimensionless form.

References

1. D. Manners and T. Makimoto, *Living with the Chip* (Chapman and Hall, London, 1995).
2. J. W. Freeman, Searching for signal and noise in the chaos of brain waves, *The Ubiquity of Chaos* (1990), pp. 47–55.
3. A. S. Elwakil and A. M. Soliman, High-frequency Wien-type chaotic oscillator, *Electron. Lett.* **34** (1998) 1161–1162.
4. A. S. Elwakil and A. M. Soliman, Current conveyor chaos generators, *IEEE Trans. Circuits Syst. I* **46** (1999) 393–398.
5. A. S. Elwakil and A. M. Soliman, A family of Wien-type oscillators modified for chaos, *Int. J. Circuit Theor. Appl.* **25** (1997) 561–579.
6. A. S. Elwakil, A new approach for realizing electronic chaos generators, Master thesis, Cairo University (1997).
7. A. S. Sedra and K. C. Smith, *Microelectronic circuits* (Oxford University Press, Inc., New York, 1998).
8. A. S. Elwakil and A. M. Soliman, Two twin-*T* based op amp oscillators modified for chaos, *J. Franklin Inst.* **335B** (1998) 771–787.

9. A. S. Elwakil and M. P. Kennedy, Construction of circuit-independent chaotic oscillators using passive-only nonlinear devices, *IEEE Trans. CAS-I* **48** (2001) 289–307.
10. M. D. Restituo, M. Linan and A. R. Vazquez, CMOS 2.4 μm chaotic oscillators: Experimental verification of chaotic encryption of audio, *Electron. Lett.* **32**(9) (1996) 795–796.
11. M. Kataoka and T. Saito, A 2-port VCCS chaotic oscillator and quad screw attractor, *IEEE Trans. CAS-I* **48**(2) (2001) 221–225.