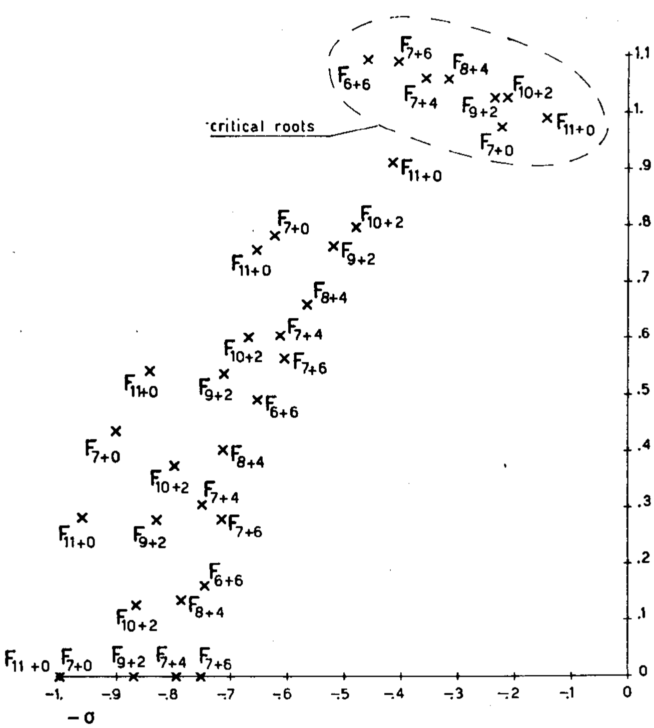

Fig. 1. Root location of some MUCROMAF polynomials.

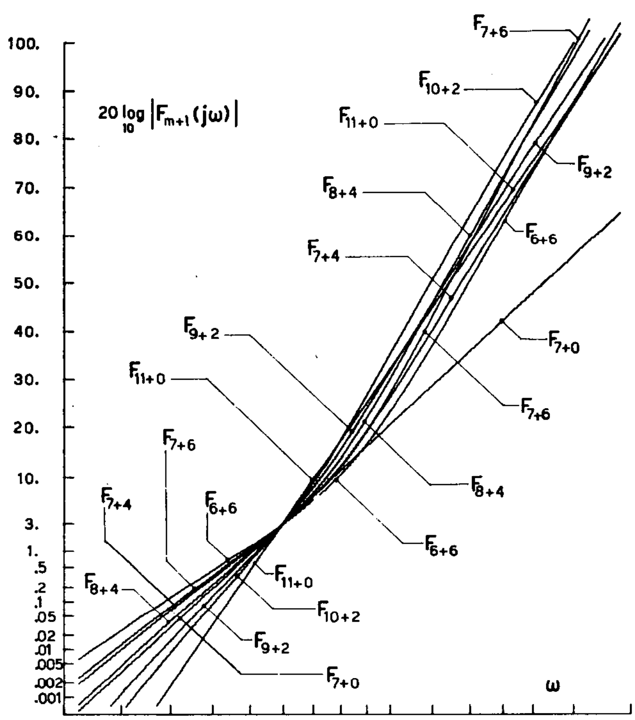

Fig. 2. Attenuation plots of some MUCROMAF polynomials.

Now let us compare some different MUCROMAF polynomials, suitably chosen to point out their properties. Table II shows the roots (multiplicity, modulus, and Q-factor) of these polynomials. In Fig. 1 their relative locations are outlined. It is possible to observe that a higher value of δ corresponds to a smaller modulus for the noncritical roots and to a larger modulus for the critical root pair, while the Q-factors are lower.

Finally the attenuation diagrams are plotted in Fig. 2. For greater convenience the linear abscissa and ordinate are $\log_{10}\omega$ and $10\cdot\log_{10}(|F(j\omega)|^2-1)$, respectively, while the values of ω and of $10\cdot\log_{10}|F(j\omega)|^2$, respectively, are marked.

It is possible to point out some considerations about these attenuation plots.

For a given m, if δ is higher, the attenuation slope at $\omega=1$ is lower, but the attenuation for ω increasing becomes higher (compare F_{7+0}, F_{7+4}, F_{7+6}). For a given n, if δ is higher, the attenuation more rapidly increases for ω great enough (compare F_{7+4}, F_{11+0}). In the end by comparison of F_{7+0}, F_{7+4}, F_{7+6}, and F_{11+0}, F_{9+2}, F_{7+4}, it is possible to observe that the attenuation decreases if m is higher and if δ is lower for $\omega<1$.

In conclusion, we can observe that the MUCROMAF polynomials have lower or higher attenuation than the Butterworth polynomials, depending on the distance from $\omega=1$. This is not an inconvenience because the values near $\omega=1$ are generally considered as a transition band.

Acknowledgment

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Inductorless Realization of an All-Pass Transfer Function Using the Current Conveyor

AHMED M. SOLIMAN

***Abstract*—The use of the second-generation current conveyor as the active element in realizing all-pass transfer functions is illustrated by two circuits.**

Active RC realizations of an all-pass transfer function using the operational amplifier have been described by several authors [1]–[5]. The purpose of this short paper is to demonstrate the use of the second-generation current conveyor [6] in realizing phase-shift networks, which maintain constant amplitude with respect to phase and adjust the phase of the input signal from 0 to 2π (or from $-\pi$ to $+\pi$). Consider the circuit in Fig. 1, with the second-generation current conveyor defined by the instantaneous terminal relationship [6]

$$\begin{bmatrix} i_b \\ v_a \\ i_c \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_b \\ i_a \\ v_c \end{bmatrix}. \quad (1)$$

Analysis of the circuit results in

$$\frac{V_2}{V_1} = \frac{Z_3 - \dfrac{Z_4 Z_2}{2Z_1}}{Z_3 + Z_2}. \quad (2)$$

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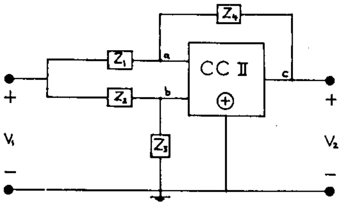

Fig. 1.

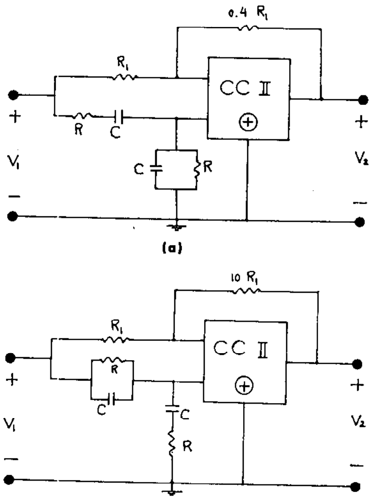

Fig. 2.

Now if

$$Z_1 = R_1$$

$$Z_2 = R + \frac{1}{jwC}$$

$$Z_3 = \frac{R}{1 + jwCR}$$

and

$$Z_4 = R_4 \tag{3}$$

$$\frac{V_2}{V_1} = \frac{jwCR\left(1 - \frac{R_4}{R_1}\right) - (1 - w^2C^2R^2)\frac{R_4}{2R_1}}{3jwCR + (1 - w^2C^2R^2)}. \tag{4}$$

All-pass characteristics would result if $R_4 = 0.4R_1$, and the circuit is shown in Fig. 2(a), for which

$$\frac{V_2}{V_1} = \frac{1}{5}\cdot\frac{1 - j\frac{X}{3}}{1 + j\frac{X}{3}} \tag{5}$$

where

$$X = wCR - \frac{1}{wCR}. \tag{6}$$

At all values of X, $|(V_2/V_1)| = \frac{1}{5}$, and the phase shift can be adjusted from $-\pi$ to $+\pi$ by changing C or R. It is seen that this circuit provides the same characteristics as that given by Genin [2], Bhattacharyya [3], and Aronhime and Budak [4]. It is also noted that the input impedance at port 1 is the same as that in [2], as was derived by Iñigo [5], and is equal to

$$Z_{11} = \frac{R_1(Z_2 + Z_3)}{R_1 + Z_2}. \tag{7}$$

The circuit in Fig. 2(b) also provides all-pass characteristics, and here $|(V_2/V_1)| = 1$, and the phase shift can be adjusted from 0 to 2π [1].

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An Analysis of Lagrangian Nonenergic Network Elements

RICHARD LEE REINHART AND RICHARD A. RIKOSKI

***Abstract*—The properties of traditors are examined and a classification system for traditor types is established. It is demonstrated than an *n*-degree basic traditor can be defined by 2^n Lagrangians which yield $2n$ different traditor types.**

I. Introduction

Within the domain of time-independent passive linear networks, certain network elements form a complete set, i.e., the resistor, capacitor, inductor, transformer, and gyrator. To extend this set of elements to make possible the synthesis of time-independent locally active nonlinear networks, a class of multiport elements can be defined, called traditors, such that they are characterized by nonenergic behavior and based on the Lagrangian dynamical equations. Such a class of multiport elements was originally defined by Duinker [1]–[3] as part of his development of a complete set of basic circuit elements. Here we will attempt to extend certain aspects of Duinker's work with the traditor in more general terms [4].

Duinker began his analysis by defining a class of new nonlinear network elements that is characterized by the property of being nonenergic [5], that is, having the property of neither dissipating nor storing energy but only of transferring energy. It is from this property he chose to give the class the generic name "ideal traditors."

Further, he recognized that a nondissipative system might be considered a dynamical system; thus it is possible to describe the behavior of electrical systems by means of Lagrange's generalized dynamical equations. This idea did not originate with him, but rather had been employed by Maxwell [6] in dealing with linear systems prior to the development of modern network theory. More recently, Millar [7] and Cherry [8] extended this method to include nonlinear networks. It is on this method of generalized dynamics that this work will be based.

II. Traditors of Various Degrees

In its most general form, the Lagrangian expression takes the form

$$\mathcal{L} = f(x_1, \cdots, x_n; \dot{x}_1, \cdots, \dot{x}_n). \tag{1}$$

However, to satisfy the requirement $\Sigma ie = 0$, the definition is narrowed to the form

$$\mathcal{L} = S = A\dot{x}_n f(x_1, \cdots, x_n) \tag{2}$$

and further narrowed to

$$\mathcal{L} = A\dot{x}_n \prod_{i=1}^{n-1} x_i \tag{3}$$

in order to define only traditors of the basic types.

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