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MOS REALIZATION OF THE CONJECTURED SIMPLEST CHAOTIC EQUATION*

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Abstract. This paper presents a general block diagram of a third-order linear differential equation using current mode techniques. The realization of the conjectured simplest chaotic equation of Elwakil and Kennedy based on $G_m - C$ technology is given. The metal oxide semiconductor circuit is composed of 20 transistors and three grounded capacitors, can operate from a supply voltage as low as ± 1.5 V, and covers a very wide range of frequencies. PSpice simulation results using 0.5 μm Mietec technology are given. A numerical solution is also included to verify the circuit operation.
Key words: MOS circuits, chaos, $G_m - C$.

1. Introduction

The study of chaotic phenomena is a very important issue in recent research. The applications of chaotic circuits are always used in areas such as modulation, security, control, encryption, and medicine. Many designs for chaotic oscillators have been introduced, starting from the use of a coil in Chua's circuit [3] to the use of large blocks such as operational amplifiers [1], [2]. In both cases the fabrication area is very large. These designs require the use of a high-voltage supply that is not suitable for portable devices. There are many chaotic equations based on a third-order linear differential equation with controlling coefficients [1], [2]. We present here a general block diagram for a third-order linear differential equation based on the use of $G_m - C$ integrators. The realization of the most important chaotic equation introduced in [1], called the *conjectured simplest chaotic equation*, is also given. The complete realization of the conjectured simplest chaotic equation is presented using metal oxide semiconductor (MOS) transistors and three grounded capacitors, which is the minimum requirement for the implementation of a chaotic oscillator. This circuit operates from a low supply voltage of ± 1.5 V; in this sense,

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the presented circuit overcomes the previously mentioned drawbacks and can be used in the manufacture of portable devices. The $G_m - C$ technology is a very interesting area of research [4], [5]. This paper presents a general block diagram of the simplest chaotic equation based on the use of transconductors (G_m) without the need for any resistors as in [1].

2. General linear third-order differential equation

The general form of a third-order differential equation of autonomous type can be described as follows:

$$\dddot{X} = a\ddot{X} + b\dot{X} + eX. \tag{1}$$

The parameters a, b, and e are constants. This equation can be realized using $G_m - C$ integrators, as shown Section 3. However, equation (1) is in the dimensionless form, which is not suitable for circuit realization. Therefore, it is necessary to make a transformation to a voltage mode equation using the following relations.

The variable X can be transformed into a voltage signal V_X through the equation $X = V_X/V_R$, where V_R is a reference voltage, and the dimensionless time τ can be transformed into the time t with dimension [sec] through the equation

$$\frac{d}{d\tau} = \frac{C}{g} \cdot \frac{d}{dt} \tag{2}$$

where g is a transconductance and C is a capacitance. Applying these transformations to equation (1), the resulting voltage mode third-order differential equation becomes

$$\dddot{V}_X = a\frac{g}{C}\ddot{V}_X + b\left(\frac{g}{C}\right)^2\dot{V}_X + e\left(\frac{g}{C}\right)^3 V_X . \tag{3}$$

3. Synthesis using cascaded current integrators

The current mode techniques are very powerful in low-voltage applications. The use of $G_m - C$ integrators is one of the most popular methods for realizing differential equations [5]. A cascade of three simple current integrators based on the use of $Gm - C$ is shown in Figure 1. The currents I_1 through I_4 are given by

$$I_4 = g_3 V_X \tag{4a}$$

$$I_3 = -C_3\dot{V}_X = g_2 V_Y \tag{4b}$$

$$I_2 = -C_2\dot{V}_Y = g_1 V_Z = \frac{C_2 C_3}{g_2}\ddot{V}_X \tag{4c}$$

$$I_1 = C_1\dot{V}_Z . \tag{4d}$$

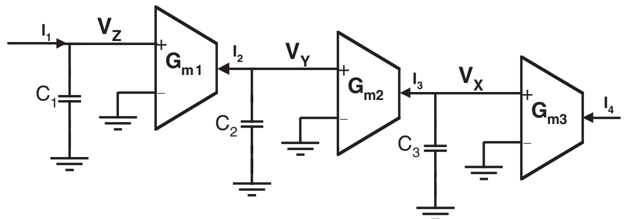


Figure 1. Three cascaded current integrators.

Taking $g_1 = g_2 = g_3 = g$, the following equation is obtained:

$$\dddot{V}_X = \frac{g^2}{C_1 C_2 C_3} I_1. \tag{5}$$

Substituting from equations (4a) and (4b) into equation (3), the following current relation is obtained:

$$I_1 = \left(\frac{a\,C_1}{C}\right) I_2 - \left(\frac{b\,C_1\,C_2}{C^2}\right) I_3 + \left(\frac{e\;C_1\,C_2 C_3}{C^3}\right) I_4. \tag{6}$$

It is clear that one can control the relation between the currents as needed by choosing appropriate values of the grounded capacitors. To simplify this relation, it is preferable to choose the coefficients to be unity, so the relation between the capacitors can be found as follows:

$$C_1 = \frac{1}{a} C, \qquad C_2 = \frac{a}{b} C, \qquad C_3 = \frac{b}{e} C. \tag{7}$$

Applying this relation to equation (6), the current relation is obtained as

$$I_1 = I_2 - I_3 + I_4. \tag{8}$$

The realization of this equation is simple, as it needs only current mirrors for implementation.

4. The conjectured simplest chaotic equation

Elwakil and Kennedy [1] proposed the conjecture that the simplest chaotic dynamics of an autonomous continuous-time chaotic oscillator can be expressed as follows:

$$\dddot{X} = -\ddot{X} - 5k\,\dot{X} - X, \tag{9a}$$

$$\text{where } k = \left\{ \begin{array}{ll} 1, & \dot{X} \geqslant 1 \\ 0, & \dot{X} < 1 \end{array} \right\}. \tag{9b}$$

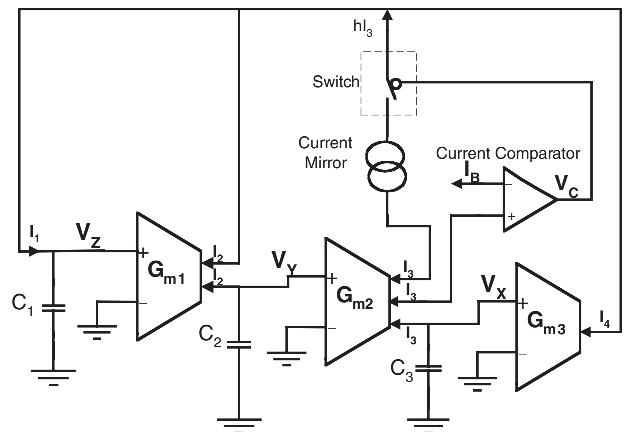


Figure 2. General block diagram of the simplest chaotic equation.

The eigenvalues at the equilibrium point are (−0.207, −0.397 ± j2.163) at $k = 1$ and (−1.446, 0.233 ±j0.796) at $k = 0$ [1]. Comparing equation (6) with equation (1), it is seen that $a = 1$, $b = 5$, and $e = 1$ (the negative signs are taken into effect in the current relation). In this case, the corresponding current relation obtained is given by

$$I_1 = -I_2 + h\, I_3 - I_4 \tag{10}$$

where h is the effect of the switching. Transforming equation (9b) into a voltage mode dimension form is done as follows:

$$h = \left\{ \begin{array}{ll} 1, & I_3 \leqslant I_B \\ 0, & I_3 > I_B \end{array} \right\}, \tag{11}$$

where $I_B = gV_R$ is the bias current of the current comparator. The general block diagram of the overall circuit is shown in Figure 2. This block diagram consists of three transconductors with the same gain and multiple outputs, a current comparator, a bias current (I_B), a current mirror, and a switch.

5. Circuit implementation

The transconductor (Gm) used in this realization is the simplest (only two transistors) version, as shown in Figure 3a. The output current is given by $I_o = g_m V$, where $g_m = 2K(V_{DD} - V_T)$. This relation is obtained assuming the supply voltage is balanced (±1.5 V). The relation also depends on the threshold voltage being approximately equal, $V_{Tn} = |V_{Tp}|$, from the data obtained from Mietec 0.5 μm technology. Finally, $K_n = K_p = K$, which is the condition for proper operation of the Gm.

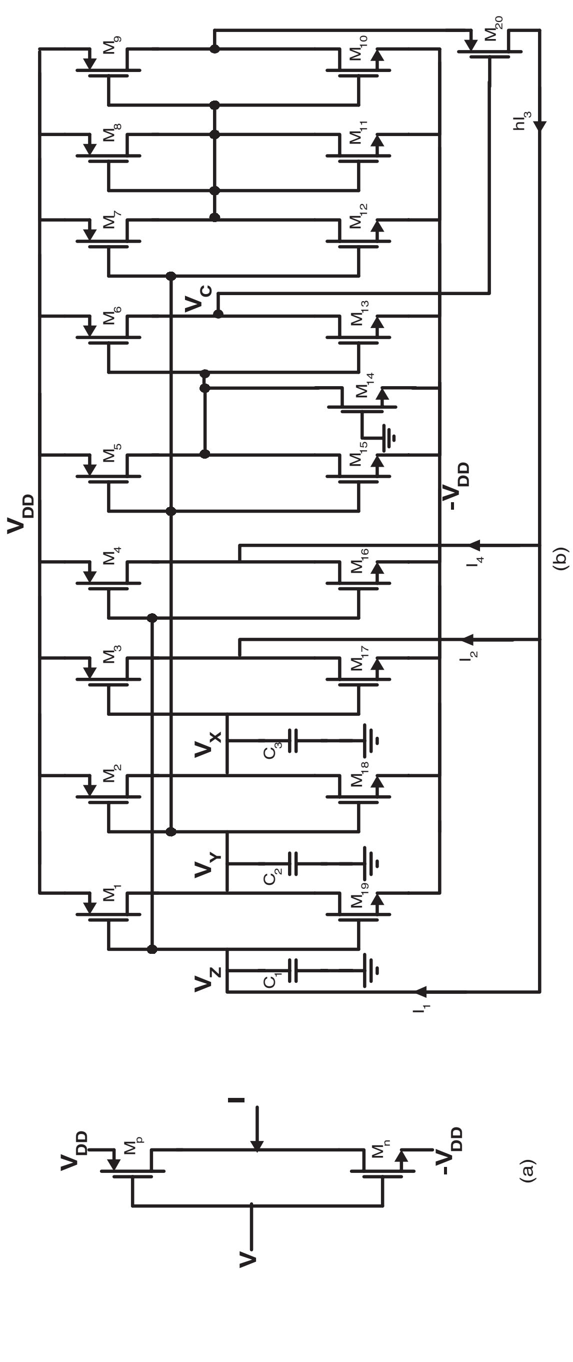


Figure 3. (a) The typical transconductor circuit (b) MOS realization of the conjectured simplest chaotic equation.

Table 1. Transistor aspect ratios of the conjectured simplest chaotic circuit.

Transistor	Aspect ratio (Wμm/Lμm)
M_1, M_2, M_3, M_4, M_5, M_7, M_8, M_9	20/15
M10, M11, M_{12}, M_{15}, M_{16}, M_{17}, M_{18}, M_{19}	20/65.5
M_6	80/15
M_{13}	80/65.5
M_{14}	15/20
M_{20}	40/1

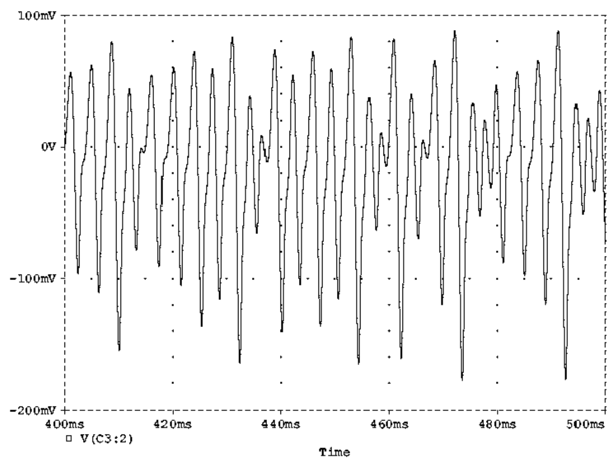


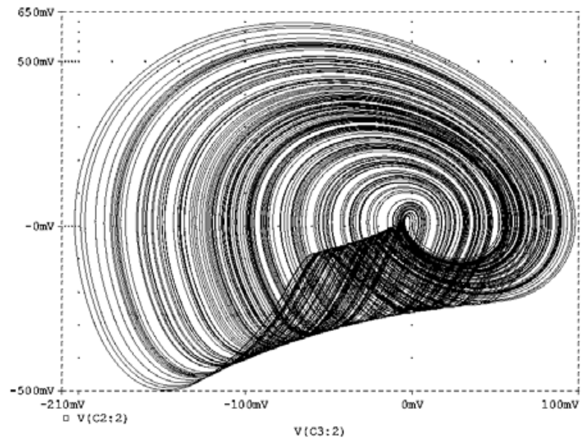
Figure 4. V_X chaotic waveform of PSpice circuit simulation.

The whole circuit realization shown in Figure 3b consists of MOS transistors and three capacitors. The current mirror is simply done through transistors M_7 through M_{12}. The bias current I_B can be achieved by the transistor M_{14} and by changing the aspect ratio to the appropriate value. The current comparator is simply mapped by an inverter, as shown by M_6 and M_{13}. The output of the current comparator V_C is the control signal for the switching element which is realized by M_{20}. The value of V_C (digital output) depends on the following relation:

$$V_C = \begin{cases} \text{High}, & I_3 \geqslant I_B \\ \text{Low}, & I_3 < I_B \end{cases} . \tag{12}$$

Transistors M_4, M_{16} and M_5, M_{15} are other outputs of I_2 and I_3, respectively. The aspect ratio of the transistors is designed to ensure that the input voltage of the transconductor is in the proper range.

(a)



(b)

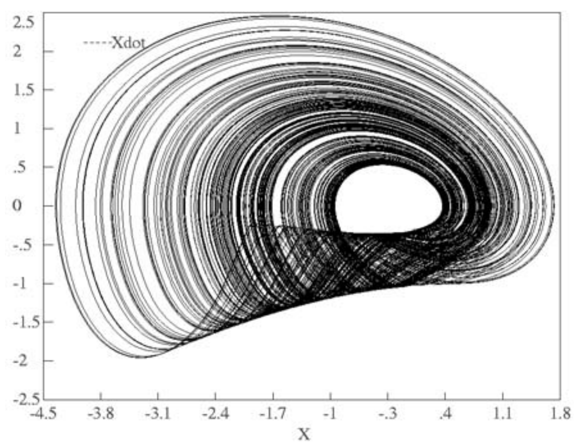


Figure 5. (a) V_X versus V_Y projection of PSpice circuit simulation (b). X versus $(-\dot{X})$ projection of the numerical simulation.

6. Simulation results

The MOS circuit was simulated using PSpice simulation and using the model of Mietec 0.5 μm technology. The numerical analysis is given using a fourth-order Runge–Kutta algorithm with a 0.005 time step [1]. The transistor aspect ratios are given in Table 1, and $C = 50$ nF. The PSpice simulation of V_X is shown in Figure 4. The V_X versus V_Y projection for the circuit realization is shown in

(a)

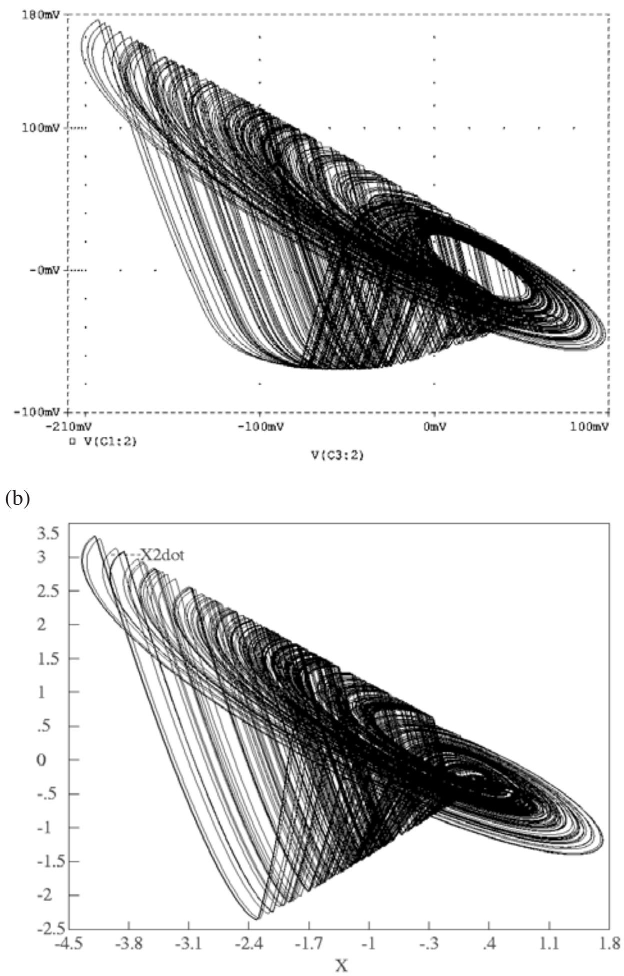


Figure 6. (a) V_X versus V_Z projection of PSpice circuit simulation (b) X versus $\ddot{X}$ projection of the numerical simulation.

Figure 5a. The X versus $(-\dot{X})$ projection of the numerical solution is shown in Figure 5b (the negative sign is due to equation 4b). The V_X versus V_Z projection for the circuit realization is shown in Figure 6a. The X versus $\ddot{X}$ projection of the numerical solution is shown in Figure 6b.

7. Conclusions

This paper presents the general block diagram of any third-order linear differential equation and also the conjectured simplest chaotic equation. The proposed realization is based on using $G_m - C$ circuits. The presented circuit uses the minimum number of MOS transistors and three grounded capacitors, which is the least requirement for any chaotic system. The circuit operates from a supply voltage as low as ± 1.5 V. The frequency response of the output signal can be scaled by changing either C or g or both. PSpice simulations and numerical results are also given.

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