

# Wide Band Squaring Circuit with Application to Exponential V-I Converter

## Breitbandiger Quadrier-Schaltkreis mit Anwendung bei U-I-Konvertern

### Abstract

In this paper, a modification of the squaring circuit introduced in [1] is presented, the modification targets extending the range of operation of the circuit. Local feedback technique has been used to extend the differential input range to any useful required value by tuning some circuit parameters. The paper also presents a solution to extend the common mode range of the circuit. Also another special case solution for band extension is presented if the circuit required to have only one input terminal. The modified circuit has been used to generate exponential V-I converter using a novel approximation function obtained by curve fitting. Pspice simulations are provided based on Mietec 0.5 µm CMOS technology and ±1.5 V supply voltages.

### Übersicht

In diesem Aufsatz wird eine Modifizierung eines in [1] eingeführten Quadrierschaltkreises vorgestellt. Die Änderung betrifft die Erweiterung des Arbeitsbereichs der Schaltung. Lokale Rückkopplungstechnik dehnt den differentiellen Eingangsbereich durch Abstimmung einiger Schaltungsparameter auf jeden sinnvoll nutzbaren Wert aus. Die Arbeit macht auch einen Vorschlag, den „common mode“ der Schaltung zu erweitern. Für den Sonderfall der Forderung nach nur einem einzigen Eingangsterminal wird eine Lösung für die Banderweiterung gegeben. Die modifizierte Schaltung wurde unter Verwendung neuartiger angepaßter Näherungsfunktionen zur Realisierung exponentieller U-I-Konverter genutzt. Simulationsergebnisse mit Pspice basierend auf einer Mietec-0,5 µm-CMOS-Technologie mit Versorgungsspannungen von ± 1,5 V werden vorgestellt.

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Für die Dokumentation  
CMOS / Quadrier-Schaltung / U-I-Konverter

## 1. Introduction

Working with analog signal processing circuits and systems, one finds that many building blocks are needed for building up a high performance system. One of the important signal processing circuits is the squaring circuit. Squaring circuits are very important for realizing multipliers and other signal conditioning systems [1-3].

Many realizations of squaring circuits are reported in literature, with demanding specifications include differential inputs, high linearity and wide range of operation.

One of the promising circuits that has been reported is that presented in [1], which is based on cross-coupling two unbalanced differential pairs to realize the squaring function. Although the circuit exhibits an excellent performance for small differential input range, it suffers from the common problem of the differential pairs namely the trade-off between the differential input range and the common mode input range.

A thorough analysis of the circuit is performed and a number of solutions are proposed to extend the region of operation of the circuit.

## 2. Proposed idea for increasing the differential input range

The circuit introduced in [1] is shown in Fig. 1, where the transistors M2 and M4 are matched and have aspect ratios  $k$  times that of the matched transistors M1 and M3, where  $k$  is assumed to have any value greater than unity.

All the transistors are assumed to be in the saturation region for the following mathematical analysis to be valid. And the transistors are long enough so that the channel length modulation is neglected in the following analysis.

Complete analysis of the circuit gives:

$$I_1 = \frac{1}{k+1} I_B + aV_{in}^2 + bV_{in} \quad (1)$$

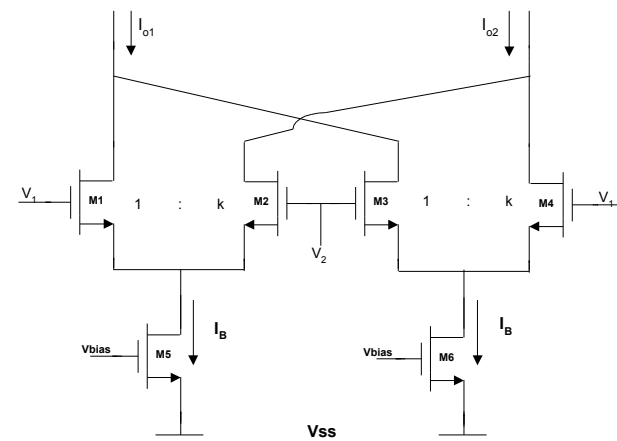


Fig. 1: Squaring circuit introduced in [1]

$$I_2 = \frac{k}{k+1} I_B - aV_{in}^2 - bV_{in} \quad (2)$$

$$I_3 = \frac{1}{k+1} I_B + aV_{in}^2 - bV_{in} \quad (3)$$

$$I_4 = \frac{k}{k+1} I_B - aV_{in}^2 + bV_{in} \quad (4)$$

where

$$a = \frac{K_1 k (k-1)}{2(1+k)^2} \quad (5)$$

$$b = \frac{1}{2(1+k)^2} \sqrt{8(1+k)kK_1I_B - 4kK_1^2V_{in}^2} \quad (6)$$

$$V_{in} = V_1 - V_2 \quad (7)$$

Therefore

$$I_{01} = I_1 + I_3 = \frac{2}{k+1} I_B + 2aV_{in}^2 \quad (8)$$

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$$I_{o2} = I_2 + I_4 = \frac{2k}{k+1} I_B - 2aV_{in}^2 \quad (9)$$

$$I_{out} = kI_{o1} - I_{o2} = 2a(k+1)V_{in}^2 = \frac{K_1 k(k-1)}{1+k} V_{in}^2 \quad (10)$$

The last equation shows that  $I_{out}$  is a scaled version of  $V_{in}^2$ .  
Recalling back that  $k$  is greater than one yield that the linearity range of the circuit is limited to:

$$-\sqrt{\frac{2I_B}{K_1}} < V_{in} < \sqrt{\frac{2I_B}{K_1}} \quad (11)$$

However trying to extend the linearity range of the circuit by increasing  $I_B$  or decreasing  $K_1$  will counter affect the common mode signal range

$$V_{ic} > V_{bias} + \sqrt{\frac{2I_B}{K_1}} \quad (12)$$

However a good solution is to try current feedback [2] to extend the differential mode range.

The circuit in Fig. 2 is a proposed solution for increasing the differential input range where  $I_{o1}$  is fed back to add to  $I_B$  so that the total bias current becomes

$$I_B + mI_{o1}$$

where  $m$  is a scaling factor defined as:

$$m = \frac{K_{10}}{K_9} = \frac{K_{11}}{K_9} \quad (13)$$

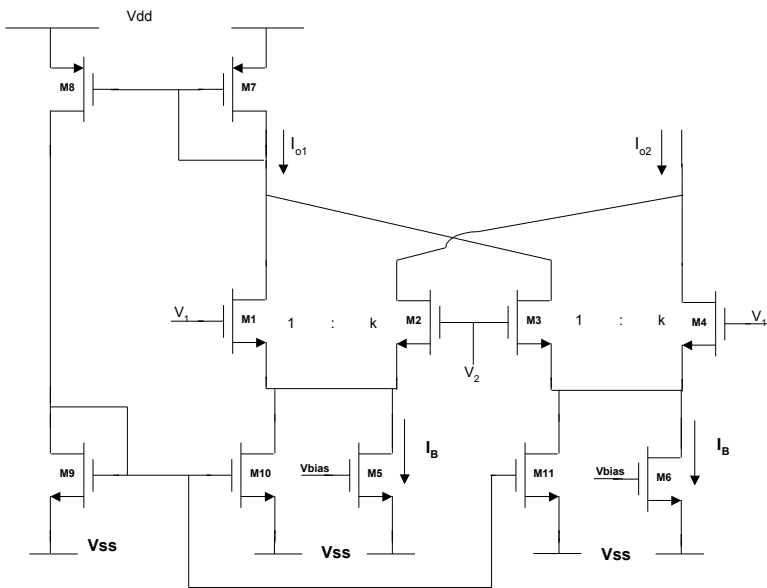


Fig. 2: The proposed solution to increase the differential input range

Table 1: Aspect ratios of the transistors in the circuit of Figure 2.

Transistors	Aspect ratios (W/L)
M1, M3	6/25
M2, M4	18/25
M5, M6	50/4
M7, M8	10/10
M9, M10, M11	25/5

Therefore, with the introduced feedback, the biasing current to the unbalanced differential pairs has changed to a new value given by

$$I_{BIAS} = mI_{o1} + I_B, \quad (14)$$

where,  $I_{BIAS}$  is the new biasing current.

Therefore

$$I_{o1} = \frac{2}{k+1} I_{BIAS} + 2aV_{in}^2 = \frac{2}{k+1} (mI_{o1} + I_B) + 2aV_{in}^2 \quad (15)$$

$$I_{o1} = \frac{2}{k+1-2m} I_B + 2a \left( \frac{k+1}{k+1-2m} \right) V_{in}^2, \quad (16)$$

and the following relation gives the new linearity range:

$$I_B + \frac{2}{k+1-2m} I_B + 2am \left( \frac{k+1}{k+1-2m} \right) V_{in}^2 > \frac{K_1}{2} V_{in}^2 \quad (17)$$

leading to:

$$-\alpha \sqrt{\frac{2I_B}{K_1}} < V_{in} < \alpha \sqrt{\frac{2I_B}{K_1}} \quad (18)$$

Therefore the linear range is extended by a factor  $\alpha$ , where

$$\alpha = \sqrt{\frac{(1+k)^2}{(1+k)^2 - 2m(1+k^2)}} \quad (19)$$

Although that the differential input range can be extended as much as we can by selecting particular values of  $m$  and  $k$ , the minimum allowable common mode input signal presents another constraint for which a number of solutions is proposed in the next section.

Fig. 3 shows the output currents of both the original circuit of Fig. 1 and the proposed circuit of Fig. 2. Common mode Dc signal of 0.6 V is used for both circuits to avoid common mode problems. The parameters of equation (19) are taken as  $k = 3$  and  $m = 0.6$  and supply voltages of  $\pm 1.5$  V are used.

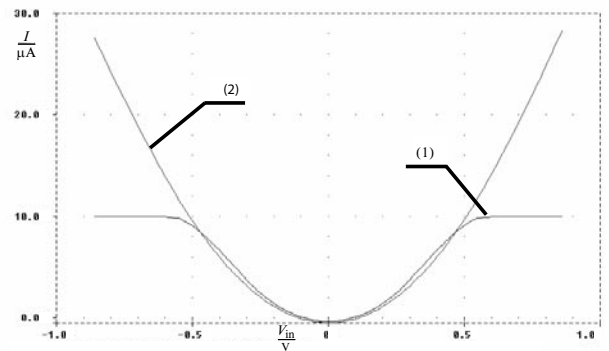


Fig. 3: Simulation results of original circuit (Figure 1) and proposed circuit (Figure 2)

The figure shows that the differential mode range is doubled for the new circuit.

### 3. Proposed idea for increasing the common mode input range

Different solutions are available depending on the application in which the circuit will be used.

**The first solution**

It is clear from (12) that there exists a lower limit on the allowable common mode input range, and there is no such upper limit. However if it is not necessary to have a differential input signal for a certain application, we can make use of the proposed block diagram of Fig. 4 where a single input squaring function is adopted (i.e.  $V_1 = V_{in}$ ,  $V_2 = 0$ ).

The absolute value circuit makes the available differential input signal ( $V_1 - V_2$ ) always positive whatever the value of  $V_1$ , this makes the common mode signal always greater than zero, which means proper operation.

A simple absolute value circuit is given in Fig. 5 where transistors M1 and M2 have their bulks connected to the sources, this would require P-Well technology to be implemented, which is available in Mietec 0.5 technology which is used for simulation purposes.

P-Spice simulation results are given in Fig. 6

The second solution assumes that both inputs are needed for use, which means differential input signal is available.

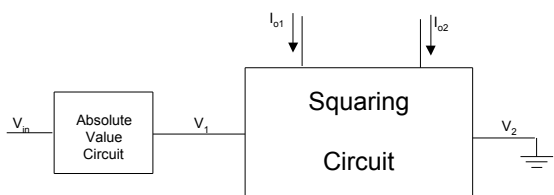


Fig. 4: The first proposed idea to extend Common mode range

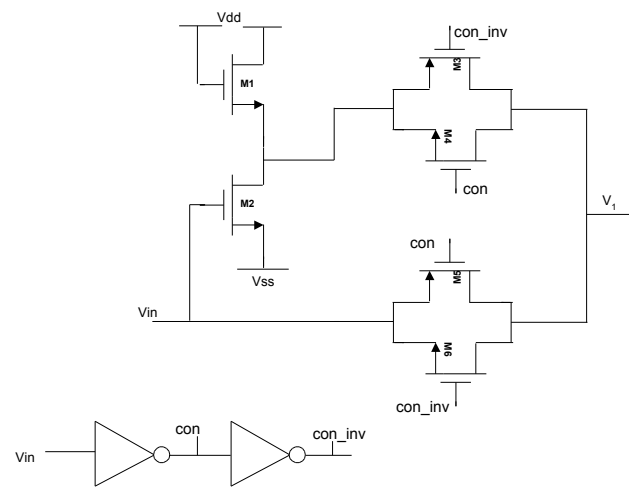


Fig. 5: A simple realization of the absolute value circuit

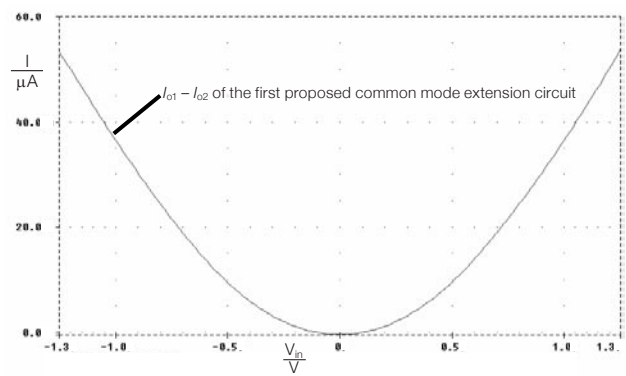


Fig. 6: DC simulation of the first proposed method for common mode extension

Fig. 7 shows the proposed circuit of Fig. 2 together with common mode circuitry. Circuit operation depends on sensing the drain voltages of the biasing transistors (M5, M6) via two PMOS transistors (M12, M13). Therefore as the input common mode level decreases the drain voltages of both M5, M6 will decrease resulting in an increase in M14 drain current which is used to control two level shifters (M17, M19) and (M18, M20). As the current in the shifters increases both  $V+$  and  $V-$  will increase while maintaining a difference equal to the original input signals ( $V_1$ ,  $V_2$ ) difference.

For proper operation of the circuit the difference between  $V+$  and  $V-$  should be precisely equal to the differential input signal ( $V_1 - V_2$ ). This means that bulks of both M19 and M20 should be tied to their sources, which requires NMOS technology to be implemented, available in Mietec 0.5 technology which is used for simulation.

DC simulation results are given in Figure 8.

**4. Exponential V-I Converter**

One of the possible uses of the squaring circuit is to generate an exponential V-I converter which is needed to build a variable gain amplifier (VGA) with exponential gain control [5, 6], considering the following defining equation for the variable gain amplifier

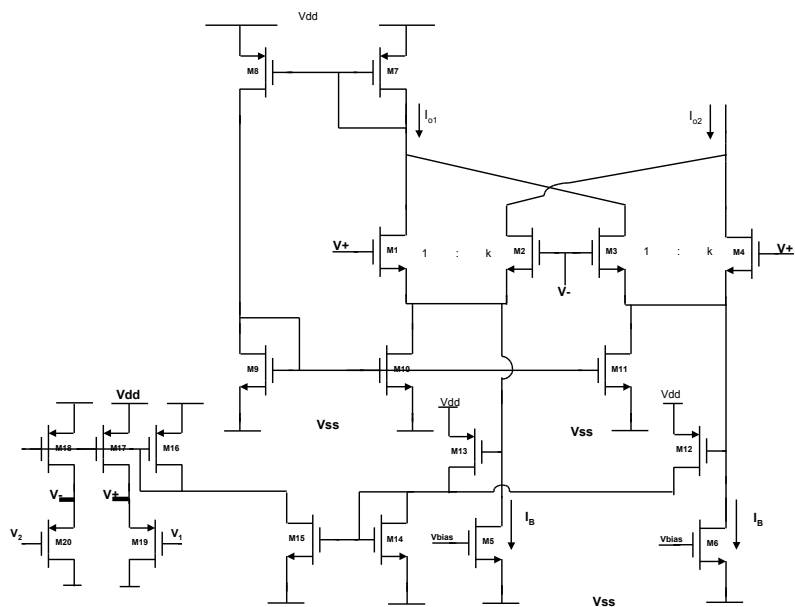


Fig. 7: The second proposed idea to extend common mode range

Table 2: Aspect ratios of the transistors of the circuit of Figure 7.

Transistors	Aspect ratios (W/L)
M1, M3	6/25
M2, M4	18/25
M5, M6	50/4
M7, M8	10/10
M9, M10, M11	25/5
M12, M13	5/4
M14, M15, M16	20/5
M17, M18	40/5
M19, M20	10/1

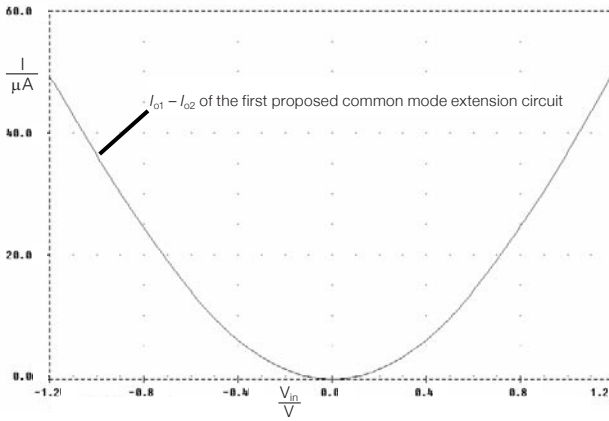


Fig. 8: DC simulation result of the circuit of Figure 7

$$V_{out} = GV_{in} \quad (20)$$

wher

$$G = \text{Exp}(\alpha X) \quad (21)$$

where  $\alpha$  is a constant and  $X$  is the control signal.

One of the ways to realize the exponential relation in CMOS technology is to develop the exponential function with its Taylor series, i. e.:

$$\text{Exp}(X) = 1 + X + \frac{X^2}{2} + \dots, \quad (22)$$

And then truncating the expansion finding a compromise between the required accuracy and the complexity of the hardware.

A common function for integrated circuit realization using the last equation up to the second order term is given by

$$F_1(X) = 1 + X + \frac{X^2}{2}. \quad (23)$$

However, when  $X > 1$ , the error due to the neglected terms begins to be of significant value which causes a very poor quality for higher values of  $X$ .

This problem has been treated in this paper, where a mathematical curve fitting of the exponential function on the general quadratic relation of the following form has been made

$$F_2(X) = 1 + a_1X + a_2X^2 \quad (24)$$

Curve fitting are based on the least squares method and gives the following results:

$$a_1 = 1.5 \quad (25)$$

$$a_2 = 11.25. \quad (26)$$

The above results makes the function  $F_2(X)$  the best quadratic relation, that fits the exponential function of equation (22) with  $\alpha = 3$ .

For fair comparison, equation (23) is modified to

$$F_1^*(X) = 1 + 3X + \frac{(3X)^2}{2}. \quad (27)$$

A plot of the function  $F_1^*(X)$ , the new function  $F_2(X)$  and the exponential function of equation (21) taking  $\alpha = 3$  is given in Fig. 9. It is clear from the figure that  $F_2(X)$  is much better to represent  $\text{Exp}(3X)$  over a range from 0 to 1.2 of  $X$ .

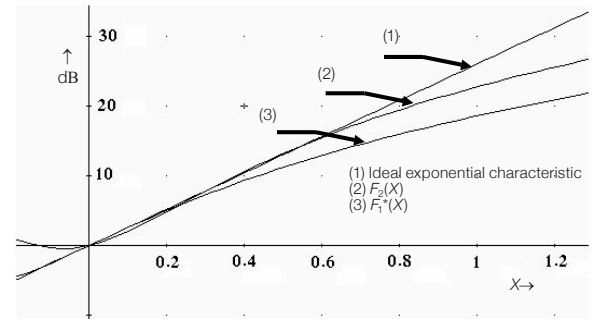


Fig. 9:  $F_1^*(X)$  and  $F_2(X)$  and the exponential function of (21) versus  $X$  on a dB scale

Two squaring circuits have been used to realize  $F_2(X)$  with a differential input signal of the first circuit equal  $V_i - V_{r1}$  and for the second circuit  $V_i - V_{r2}$ , where  $V_i$  is control signal of the approximated exponential characteristic, and  $V_{r1}$  and  $V_{r2}$  are two control voltages used to realize  $F_2(X)$ .

The two circuits are adjusted such that the parameters  $k$  and  $m$  of equation (19) have the values three and one, respectively.

Therefore

$$I_{o1} = I_B + \frac{3}{8}K_1(V_i - V_{r1})^2 \quad (28)$$

$$I_{o2} = I_B + \frac{3}{8}K_1(V_i - V_{r2})^2. \quad (29)$$

And the total output current ( $I_{out}$ ) is chosen such that

$$I_{out} = 2I_{o1} - I_{o2}. \quad (30)$$

It has been shown by simulations of Fig. 10 that the above equation is a good approximation to the exponential characteristic in the range from 0.6 V to 1.5 V of the control voltage  $V_i$  achieving a range 0 to 20 dB with an error less than 3 dB

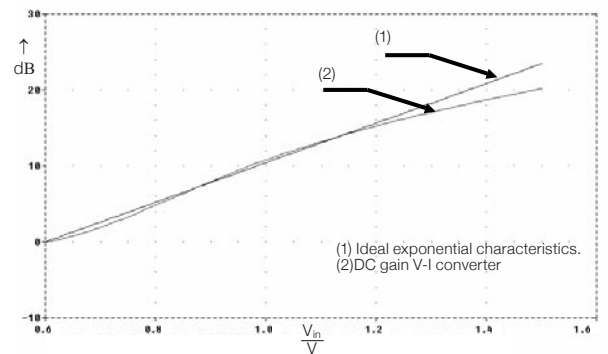


Fig. 10: DC simulation of the exponential V-I converter and the ideal exponential function versus input voltage  $V_1$  on a dB scale

## 5. Conclusion

A proposed modification for an existing squaring circuit is presented to increase the differential input range. Two methods are proposed to increase the common mode input range depending on whether the required squaring function should have single or differential input signals. Simulation results show a wider dynamic range of the squaring circuit over that reported in [1]. Also, a new approximation function to the exponential relation usually used in VGA is proposed and two squaring circuits are used to obtain an exponential V-I converter. P-Spice simulation gives 20 dB dynamic range of the realized converter.

## References

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