Two Twin-T Based Op Amp Oscillators Modified for Chaos

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ABSTRACT: Two complementary sinusoidal oscillator circuits based on the Twin-T network are modified for chaos using a single discrete nonlinear device with antisymmetrical current-voltage characteristics, namely a junction field effect transistor (JFET) operating in its triode region. The two oscillators' design equations are used as a starting point for chaos modification. Mathematical models that describe the observed behaviour in both circuits are derived. Experimental results, PSpice circuit simulations and numerical simulations of the mathematical models agree well and are included. © 1997 The Franklin Institute. Published by Elsevier Science Ltd

Introduction

Since its appearance, Chua's circuit (1) has been the main source for studying chaos in electronic systems. The circuit, which is a paradigm for chaos (2), has been extensively studied (3-5), and different realizations, notably inductorless realizations, that facilitate its implementation have been introduced in the literature (6).

Recently, some well known conventional sinusoidal oscillators have been shown to exhibit chaotic performance. In Refs (7, 8), the Colpitts oscillator was shown to have a chaotic nature. Unlike the nonlinearity of Chua's circuit, which possesses an odd symmetry and is usually represented by piecewise linear characteristics, the nonlinearity in the Colpitts oscillator, introduced by a bipolar transistor operating as a voltage controlled resistor, is antisymmetrical and modelled either by an antisymmetric cubic polynomial (7) or two segment piecewise linear characteristics (8). By coupling the well known Wien-bridge oscillator with a Chua diode, a simple RC chaos generator was proposed in Ref. (9). A simpler chaos generator was obtained by intentionally modifying the Wien-bridge structure by adding a nonlinear element and a single capacitor (10). Two more oscillator configurations were modified for chaos using an antisymmetrical nonlinearity in Ref. (11), leading to very simple RC chaos generators that are tunable by a single grounded resistor. In general, RC chaos generators are easy to construct, simple to tune and produce signals with a large voltage swing.

In this work, two Twin-T based sinusoidal oscillators are modified for chaos by
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direct replacement of one of the linear resistors with a JFET operating in its triode region and performing as a voltage controlled resistor. JFETs are well known for their special suitability to operate as voltage controlled resistors (12). Both oscillators employ the voltage op amp as the active element while the passive Twin-T structure in one circuit is the complement of the other, allowing for the nonlinear element to be grounded in one circuit and floating in the other. It was evident that the internal op amp pole contributes significantly to the chaotic nature of one of the proposed circuits. This was verified mathematically by including a first-order model of the op amp in analysis in order to obtain numerical results identical to those observed experimentally and with PSpice. The TL081 op amp was used for both PSpice simulations and experimental work.

The First Twin-T Configuration

Figure 1(a) represents the first oscillator configuration with the passive Twin-T network shown in Fig. 1(c). The gain (K) is given by:

\[ K = 1 + \frac{R_B}{R_A}. \] (1)

Considering the case where \( K = \infty \) (infinite gain), which implies that the positive op amp terminal is grounded (\( R_B = \infty \) and \( R_A = 0 \)), the condition of oscillation is given by:

\[ R_I + C_1 + C_2 = R_x(R_1 + R_2)C_3 \] (2a)

and the radian frequency of oscillation is given by:

\[ \omega_0 = \frac{1}{\sqrt{(R_1 + R_2)R_3C_1C_2}}. \] (2b)

For the special case of \( R_1 = R_2 = R \), \( R_3 = R/2 \), \( C_1 = C_2 = C \) and \( C_3 = 2C \), the well known symmetrical Twin-T oscillator (13) is obtained and the radian frequency of oscillation simplifies to:

\[ \omega_0 = \frac{1}{RC}. \] (3)

The modified-for-chaos circuit

Figure 2(a) represents the modified-for-chaos circuit where the modification implies the replacement of the linear resistor \( R_3 \) with a JFET having its gate and drain terminals shorted and its source grounded, thus operating in the triode region. A JFET of the type J2N4338 was chosen and used in all simulations and experimental work. This JFET has a small signal resistance at the operating point around 750 Ω and is especially suitable for operation as a voltage controlled resistor. The current–voltage characteristics of this JFET during circuit operation are plotted in Fig. 3. Considering the special case of a symmetrical Twin-T, and since \( R_3 \) is replaced with this JFET, then \( R_1 \) and \( R_2 \) should be taken to be around 1.5 kΩ. The values of the circuit capacitors are
Fig. 1. The two oscillator configurations and the Twin-T network (N).
arbitrary chosen to define the frequency band of interest, which can be estimated from Eq. (3). Tunability in order to change the observed dynamics is achieved through the grounded resistor \( R_g \) or the floating resistor \( R_b \) where, upon tuning, a period doubling route to chaos was observed.

It was evident that the internal op amp pole contributes significantly to the chaotic nature of the circuit. This was verified using PSpice by altering the op amp model used in simulations to disclude the internal compensating capacitor. Using the single op amp pole model given in Ref. (14) and the approximate two segment piecewise linear JFET model of Ref. (10), the circuit is described by the following set of ordinary differential equations:
Fig. 3. The nonlinearity introduced by the JFET during circuit operation.
where,

\[ \begin{align*}
C_1 \dot{V}_{c1} &= I - C_2 \dot{V}_{c2} \\
R_2 C_2 \dot{V}_{c2} &= \dot{V}_{c1} - \dot{V}_{c2} + \dot{V}_{c3} + \alpha \left[ (1 - \frac{1}{K}) (V_{c3} - R_2 C_2 \dot{V}_{c2}) + \frac{(V_{c2} - V_{c1})}{K} \right] \\
R_1 C_3 \dot{V}_{c3} &= V_{c1} - V_{c2} - (R_1 + R_2) C_2 \dot{V}_{c2}
\end{align*} \tag{4} \]

\[ I = \begin{cases} \\
\frac{V_{c3} - V_{c2} - R_2 C_2 \dot{V}_{c2}}{R_{j1}} & (V_{c3} - V_{c2} - R_2 C_2 \dot{V}_{c2}) \geq V_T \\
\frac{V_T}{R_{j1}} + \frac{V_{c3} - V_{c2} - R_2 C_2 \dot{V}_{c2} - V_T}{R_{j2}} & (V_{c3} - V_{c2} - R_2 C_2 \dot{V}_{c2}) < V_T
\end{cases} \]

\(\omega_1\) is the op amp gain-bandwidth product and \(V_T\) is the JFET threshold voltage. For the J2N4338 JFET, the parameters \(R_{j1}, R_{j2}\) and \(V_T\) are approximately equal to 750 \(\Omega\), 150 k\(\Omega\) and \(-0.66\) V, respectively, while for the TL081 op amp, \(\omega_1\) is given as \(2\pi f_T\) with \(f_T\) equal to 3 MHz.

**Mathematical model, PSpice simulations and experimental results**

In order to mathematically verify the circuit behaviour, the equation set Eq. (4) are transformed into a dimensionless form convenient for numerical simulation. For the special case of \(C_1 = C_2 = \bar{C}, C_3 = 2C, R_1 = R_2 = \bar{R}\) and by introducing the following dimensionless quantities:

\[ \begin{align*}
\frac{V_{c1}}{V_T} &= \bar{X}, \quad \frac{V_{c2}}{V_T} = \bar{Y}, \quad \frac{V_{c3}}{V_T} = \bar{Z}, \quad t = \frac{t}{RC}, \quad \alpha = \frac{R}{R_{j1}}, \quad \beta = \frac{R}{R_{j2}}
\end{align*} \]

equation set Eq. (4) becomes:

\[ \begin{align*}
\dot{\bar{X}} &= -\bar{Y} + \begin{cases} \\
\alpha (Z - Y - \bar{Y}) & (Z - Y - \bar{Y}) \leq 1 \\
\alpha + \beta (Z - Y - \bar{Y} - 1) & (Z - Y - \bar{Y}) > 1
\end{cases} \\
\dot{\bar{Y}} &= \bar{X} - \bar{Y} + \bar{Z} + a(Z - \bar{Y}) + b(Y - X) \\
\dot{\bar{Z}} &= \frac{X - Y}{2} - \bar{Y}
\end{align*} \tag{5} \]

where \(a = \omega_1 (1 - 1/K)\) and \(b = \omega_1/K\).

Numerical integration of the equation set Eq. (5) was carried out using a fourth-order Runge-Kutta algorithm (15) with a (0.005) step size. Chaotic behaviour was observed for a wide range of parameter values and the period doubling sequence was verified. Figure 4(a) represents the \(Z-Y\) phase space trajectory given for \(\alpha = 2, \beta = 0.01, a = 22\) and \(b = 6\) while Fig. 4(b) is the corresponding frequency spectrum demonstrating a typical broadband chaotic spectrum.

PSpice simulations of the circuit were carried out for \(R = 1.5\) k\(\Omega\), \(C = 100\) nF, \(R_4 = 10\) k\(\Omega\) and \(R_B = 84.5\) k\(\Omega\). Different capacitor values were also tried. A sample of the op amp’s output voltage waveform is shown in Fig. 4(c).

The circuit was experimentally tested with \(R_4 = 1\) k\(\Omega\), \(R_B = 50\) k\(\Omega\) pot,
Fig. 4. (a) Numerical simulation representing the Z-Y trajectory of the first Twin-T chaos generator.
Fig. 4(b) The frequency spectrum of the $Z$ and $Y$ signals.
Fig. 4(c) A PSpice simulation of the op amp output waveform for the first Twin-T chaos generator given for $C = 100 \text{nF}$ and $R = 1.5 \text{kΩ}$. 
Identical results to those indicated by numerical simulation and PSpice were observed. It is worth noting that the mismatch in the values of $R_1$ and $R_2$ does not critically affect circuit performance and that Eq. (2a) can be used to choose other design equations different from that of the symmetrical Twin-T.

**The Second Twin-T Configuration**

Figure 1(b) represents the second oscillator configuration which employs the op amp as a noninverting voltage controlled voltage source (VCVS) \(16\). For the special case of a symmetrical Twin-T, the condition of oscillation is given by:

\[ K = 1. \]  

That is, the op amp is used in this case as a voltage follower and the radian frequency of oscillation is given by Eq. (3).

**The modified-for-chaos circuit**

Figure 2(b) represents the modified circuit where the floating resistor $R_3$ has been replaced with the JFET. It was evident that the internal op amp pole does not contribute to the chaotic behaviour of the circuit, and thus the following analysis assumes an ideal op amp. Using the approximate JFET model of Ref. \(10\), the circuit is described by the following set of differential equations:

\[
\begin{align*}
C_1 \dot{V}_{C1} &= I + C_2 \dot{V}_{C2} \\
R_2 C_2 \dot{V}_{C2} &= (K-1)(V_{C1} + V_{C2}) - V_{C3} \\
R_1 C_3 \dot{V}_{C3} &= K(V_{C1} + V_{C2}) - V_{C3} + R_1 C_2 \dot{V}_{C2}
\end{align*}
\]  

where

\[
I = \begin{cases} 
\frac{(K-1)V_{C1} + KV_{C2}}{R_{J1}} & \text{if } [(K-1)V_{C1} + KV_{C2}] \geq V_T \\
\frac{V_T}{R_{J1}} + \frac{(K-1)V_{C1} + KV_{C2} - V_T}{R_{J2}} & \text{if } [(K-1)V_{C1} + KV_{C2}] < V_T
\end{cases}
\]

and $K = 1 + \frac{R_B}{R_A}$.

In order to change the circuit dynamics, the gain $K$ must be slightly increased from the nominal value given by Eq. (6).

**Mathematical model, PSpice simulations and experimental results**

For the special case of $C_1 = C_2 = C$, $C_3 = 2C$, $R_1 = R_2 = R$ and with the same settings as for the first Twin-T configuration, the equation set Eq. (7) is transformed to the following dimensionless form:
Fig. 5(a).
Fig. 5. Numerical simulations of the $x$, $y$ and $z$ trajectories for the second Twin-T chaos generator.
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Fig. 6.
Fig. 6. PSpice simulations of the $V_c1 - V_c2$ and $V_c1 - V_c3$ trajectories for the second Twin-T chaos generator given for $C = 1\ \text{nF}$ and $R = 1.592\ \text{k}\Omega$. 
\[
\begin{align*}
\dot{X} &= \dot{Y} + \begin{cases} 
2[(K-1)X + KY] & [(K-1)X + KY] \leq 1 \\
\alpha + \beta[(K-1)X + KY - 1] & [(K-1)X + KY] > 1 
\end{cases} \\
\dot{Y} &= (K-1)(X + Y) - Z \\
\dot{Z} &= \frac{1}{2}[K(X + Y) + \dot{Y} - Z]. 
\end{align*}
\]

Numercial integration of equation set Eq. (8) was carried out using a Runge-Kutta fourth-order algorithm (15) with a (0.005) step size. Figure 5(a) and (b) represent the \(X-Y\) and \(X-Z\) trajectories given for \(\alpha = 2.13\), \(\beta = 0.011\) and \(K = 1.24\). Figure 6(a)
and (b) are PSpice simulations demonstrating the same trajectories and given for \( R = 1.592 \, \text{k}\Omega, C = 1 \, \text{nF}, R_A = 90 \, \text{k}\Omega \) and \( R_B = 9.8 \, \text{k}\Omega \). Experimental observations are identical to simulations. Demonstrative oscilloscope printouts of periodic and chaotic signals are shown in Fig. 7(a) and (b).

**Conclusion**

Two Twin-T based oscillators have been modified for chaos using a JFET. It has been shown that the design equations of both circuits as sinusoidal oscillators are useful as a starting point for chaos modification. The two circuits are additions to recent research concerned with chaos investigation in conventional oscillators. Mathematical models for both circuits have been derived. Experimental results, numerical and PSpice simulations have been included.

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**References**
