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A NEW ACTIVE RC CONFIGURATION FOR REALIZING NONMINIMUM PHASE TRANSFER FUNCTIONS

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SUMMARY

A new active RC configuration for realizing a second order nonminimum phase transfer function having a unity gain factor is given. The circuit has the advantages of being canonic, is always stable and is capable of realizing a high pole Q. The ω_0 and the pole Q sensitivities to all passive and active circuit components are derived. The effect of the limited frequency response of the OA is examined.

INTRODUCTION

The realization of second order all-pass phase shifters having complex poles and zeros have received a great deal of attention recently.[1-6] Most of these realizations are limited to low-Q applications. Another problem which exists in Moschytz[4] noncanonic parallel loading circuit, Deliyannis[3] and Teramato[2] networks is that the gain factor is less than unity and depends on the pole Q of the circuit.

In this paper a new configuration for realizing a second order nonminimum phase transfer function having a constant gain factor of unity is described. The circuit uses a minimum number of capacitors namely two, and is capable of realizing a high pole Q. Design equations for realizing all-pass phase shifters and notch filters are given as special cases.

THE BASIC CONFIGURATION

The circuit shown in Figure 1 consists of a three terminal passive RC network N which is excited at terminal 1 by a noninverting voltage controlled voltage source of gain a to control the zeros of the overall voltage transfer function, terminal 2 is connected to the inverting terminal of an OA with gain A (ideally infinite). The OA activates N at terminal 3 in order to make the circuit capable of realizing complex poles.

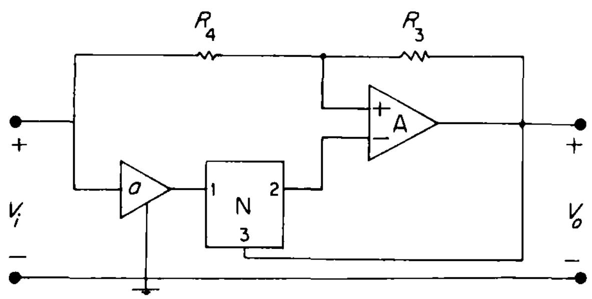

Figure 1. General configuration

The basic topology in Figure 1 can be converted to the well known Dutta Roy's configuration[7] which is restricted to the realization of poles on the real axis ($Q_p < 0{\cdot}5$) by setting $a = 1$, interchanging the OA polarity and connecting terminal 3 to ground instead of the output.

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Analysis of the circuit taking the effect of the OA gain into consideration leads to the following overall voltage transfer function:

$$G(s) \equiv \frac{V_o}{V_i} = \frac{m - aT(s)}{m - T(s) + \frac{1}{A}} \tag{1}$$

where

$$m = \frac{R_3}{R_3 + R_4}$$

and

$$T(s) = \frac{Z_{21}}{Z_{11}},$$

when the network N is considered as an unloaded two port.

Using a passive RC bandpass network for N results in:

$$T(s) = \frac{K\omega_p s}{s^2 + (\omega_p/q_p)s + \omega_p^2} \tag{2}$$

where

$$0 < q_p < 0{\cdot}5 \quad \text{and} \quad Kq_p \leqslant 1$$

Substituting (2) in (1) gives:

$$G(s) = \frac{1}{1 + \frac{1}{mA}} \cdot \frac{s^2 - \omega_p s\left(\frac{aK}{m} - \frac{1}{q_p}\right) + \omega_p^2}{s^2 + \omega_p s\left(\frac{1}{q_p} - \frac{K}{m(1 + 1/mA)}\right) + \omega_p^2} \tag{3}$$

As A approaches infinity, the above equation becomes:

$$G(s) = \frac{s^2 - \omega_p s\left(\frac{aK}{m} - \frac{1}{q_p}\right) + \omega_p^2}{s^2 + \omega_p s\left(\frac{1}{q_p} - \frac{K}{m}\right) + \omega_p^2} \tag{4}$$

Thus the circuit realizes a nonminimum phase transfer function of unity gain factor and having:

$$\omega_o = \omega_p \tag{5}$$

$$Q_o = \frac{q_p}{aKq_p/m - 1} \tag{6}$$

$$Q_p = \frac{q_p}{1 - Kq_p/m} \tag{7}$$

It is seen that the passive RC network N controls both ω_o and Q_p and for absolute stability of the network; Kq_p must be less than m. The VCVS controls the position of the zeros, and for a notch filter:

$$a = \frac{m}{Kq_p} = \frac{1}{1 - q_p/Q_p} \tag{8}$$

For an all-pass transfer characteristics:

$$a = \frac{2m - Kq_p}{Kq_p} = \frac{1 + q_p/Q_p}{1 - q_p/Q_p} \tag{9}$$

In general for a specified Q_0 and Q_p,

$$a = \frac{1 + q_p/Q_o}{Kq_p/m} = \frac{1 + q_p/Q_o}{1 - q_p/Q_p} \tag{10}$$

From the above equation and since N is chosen such that $Kq_p < m$ for stability, it follows that $a > 1$, and thus it can be realized using a single OA[8], as shown in Figure 3.

EFFECT OF THE NONIDEAL OAs

The active element sensitivities

The ω_o and the Q_p sensitivities with respect to A are derived next. From (3) and assuming the OA gain to be real and equal to A_o, the actual values of ω_o and Q_p are given by:

$$\omega_{oa} = \omega_o \tag{11}$$

$$Q_{p_a} = Q_p \frac{1 + 1/mA_o}{1 + Q_p/mq_pA_o} \simeq \frac{Q_p}{1 + Q_p/mq_pA_o} \text{ for } A_0 \gg \frac{Q_p}{mq_p} \gg \frac{1}{m} \tag{12}$$

Thus:

$$S_{A_o}^{\omega_{oa}} = 0 \tag{13}$$

That is the network is ω_o invariant to the OA gain only as long as the bandwidth of the OA can be considered as infinite.

$$S_{A_o}^{Q_{pa}} \simeq \frac{Q_p/mq_pA_o}{1 + Q_p/mq_pA_o} \simeq \frac{Q_p}{mq_pA_o} \text{ for } A_o \gg \frac{Q_p}{mq_p} \gg \frac{1}{mq_p} \tag{14}$$

Next the Q_0 sensitivity with respect to A_1 is derived. Assuming $A_1 = A_{oi}$, thus the actual value of a is given by:

$$a = \frac{a_o}{1 + a_o/A_{o1}} \tag{15}$$

where

$$a_o = 1 + \frac{R_6}{R_5} \tag{16}$$

$$S_{A_{o1}}^{Q_o} = S_a^{Q_o} \cdot S_{A_{o1}}^{a} = -\frac{a^2KQ_o}{mA_{o_1}} \tag{17}$$

Effect of the rolloff of the OA gain

It has been recognized recently that the OA gainbandwidth product is likely to be a major limiting factor in the performance of active filters.[9,10]

Here the frequency limitation equations of the circuit are given based on the one-pole rolloff model of the OA, which is characterized by:

$$A = \frac{A_o\omega_1}{s + \omega_1} \simeq \frac{GB}{s} \tag{18}$$

where

A_o is the open loop dc gain of the OA,

ω_1 is the open loop 3-dB bandwidth, and

$GB = A_o\omega_1$ is the gain-bandwidth product.

When (18) is substituted in (3), the denominator of $G(s)$ becomes:

$$D(s) = \left(s^2+\frac{\omega_o}{Q_p}s+\omega_o^2\right)+\frac{s}{m\,.\,GB}\left(s^2+\frac{\omega_o}{q_p}s+\omega_o^2\right) \tag{19}$$

Following Budak-Petrela analysis,[9] it follows that the relative change in ω_o and Q_p due to the limited frequency response of the OA are given by:

$$\frac{\Delta\omega_o}{\omega_o} = -\frac{1}{2m}\left(\frac{1}{q_p}-\frac{1}{Q_p}\right)\frac{\omega_o}{GB} \simeq -\frac{\omega_o}{2m\,.\,q_p\,.\,GB} \quad \text{for } Q_p \gg q_p \tag{20}$$

$$\frac{\Delta Q_p}{Q_p} = \frac{1}{2m}\left(\frac{1}{q_p}-\frac{1}{Q_p}\right)\frac{\omega_o}{GB} \simeq \frac{\omega_o}{2m\,.\,q_p\,.\,GB} \quad \text{for } Q_p \gg q_p \tag{21}$$

That is if ω_o and Q_p are not to change by more than X per cent from their nominal values and for the case of interest namely $Q_p \gg q_p$; the frequency limitation of the network is:

$$\omega_{o\max} = \frac{m\,.\,X}{50}\,.\,q_p.GB \tag{22}$$

From (19) and using Routh's criterion, it is seen that using the approximation in (18) and for $m > Kq_p$, the network will be stable. From equations (14) and (22) it is seen that the network N plays an important role in controlling the pole Q sensitivity to the OA gain as well as the maximum frequency set for operating the network. Therefore, to make the network suitable for high Q factor applications at high frequencies the network N should be chosen such that q_p approaches its maximum value of 0·5. The parameter m must be greater than Kq_p for the absolute stability of the active structure.

THE RC NETWORK N

Figure 2 represents a minimal realization for the network N. The open circuit voltage transfer function is given by:

$$T(s) \equiv \frac{V_{23}}{V_{13}} = \frac{sC_1R_1}{s^2C_1C_2R_1R_2+s(C_1R_1+C_2R_2+C_1R_2)+1} \tag{23}$$

Comparing with (2), thus:

$$\omega_p = \frac{1}{\sqrt{(C_1C_2R_1R_2)}}, \qquad q_p = \frac{\sqrt{(C_1C_2R_1R_2)}}{C_1R_1+C_2R_2+C_1R_2}, \qquad K = \sqrt{\left(\frac{C_1R_1}{C_2R_2}\right)} \tag{24}$$

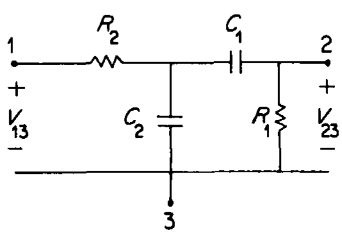

Figure 2. A minimal realization for N

From (5), (6) and (7) it follows that:

$$\omega_o = \frac{1}{\sqrt{(C_1C_2R_1R_2)}} \tag{25}$$

$$Q_o = \frac{\sqrt{(C_1C_2R_1R_2)}}{C_1R_1(a/m-1)-C_2R_2-C_1R_2} \tag{26}$$

$$Q_p = \frac{\sqrt{(C_1C_2R_1R_2)}}{C_2R_2+C_1R_2-C_1R_1(1/m-1)} \tag{27}$$

To insure stability of the active structure the parameter m should be chosen such that:

$$m > \frac{C_1R_1}{C_1R_1+C_2R_2+C_1R_2} \tag{28}$$

PASSIVE SENSITIVITIES

From (25) and (27) the ω_o and Q_p sensitivities to all passive circuit components are given by:

$$S^{\omega_o}_{C_1} = S^{\omega_o}_{C_2} = R^{\omega_o}_{R_1} = S^{\omega_o}_{R_2} = -\tfrac{1}{2} \tag{29}$$

$$\begin{aligned} S^{Q_p}_{R_1} &= -S^{Q_p}_{R_2} = \tfrac{1}{2}+\frac{C_1R_1(1/m-1)}{C_1R_2+C_2R_2-C_1R_1(1/m-1)} \\ S^{Q_p}_{C_1} &= -S^{Q}_{2} = \tfrac{1}{2}+\frac{C_1R_1(1/m-1)-C_1R_2}{C_1R_2+C_2R_2-C_1R_1(1/m-1)} \\ S^{Q_p}_{R_3} &= -S^{Q_p}_{R_4} = -\frac{C_1R_1(1/m-1)}{C_1R_2+C_2R_2-C_1R_1(1/m-1)} \end{aligned} \tag{30}$$

Next the Q_o sensitivities with respect to R_5 and R_6 are given by:

$$S^{Q_o}_{R_5} = -S^{Q_o}_{R_6} = \frac{C_1R_1(a_0-1)}{m[C_1R_1(a_o/m-1)-C_2R_2-C_1R_2]} \tag{31}$$

DESIGN EQUATIONS

It is clear that the network N and the ratio m control the performance of the circuit. Here two cases are considered, the first has low active sensitivities, and high passive sensitivities and the second has very low passive sensitivities and higher active sensitivities.

Case 1

In order to make the network suitable to high Q factor applications at high frequencies, it is necessary to choose the circuit components of N such that q_p approaches 0·5. For the network in Figure 2 this is possible by choosing:

$$R_2 = R, \qquad C_2 = C, \qquad C_1 = bC, \qquad R_1 = R/b \tag{32}$$

From (24) and (27) thus:

$$\omega_0 = \frac{1}{CR}, \qquad q_p = \frac{1}{2+b}, \qquad K = 1 \tag{33}$$

$$Q_p = \frac{1}{2+b-1/m} \tag{34}$$

To insure stability and to realize a high Q_p, choose $m = \frac{1}{2}$, therefore,

$$Q_p = \frac{1}{b} \tag{35}$$

$$q_p \simeq \tfrac{1}{2} \qquad \text{for } Q_p \gg \tfrac{1}{2} \tag{36}$$

One possible set of design equations is as follows:

For $\omega_0 = 1$

$$\begin{aligned} R &= 1 \\ C &= 1 \\ b &= \frac{1}{Q_p} \\ R_2 &= R_4 = 1 \end{aligned} \tag{37}$$

For an all-pass phase shifter:

$$a = 1 + \frac{1}{Q_p} \tag{38}$$

Figure 3 represents the OA realization for the all-pass resonator after normalizing for $\omega_0 = 1$, and using the RC network in Figure 2 for N.

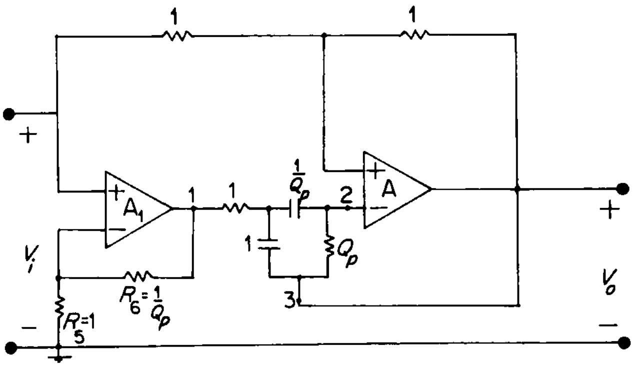

Figure 3. A double OA canonic realization of an all-pass phase shifter

It is noted that the resistor R_6 controls the type of the filter, for example setting $R_6 = 1/2Q_p$ a notch filter results.

It follows from (14) and (36) that:

$$A_o S_{Ao}^{Q_{pa}} \simeq 4Q_p \qquad \text{for } A_o \gg 4Q_p \gg 2 \tag{39}$$

which is approximately half the active sensitivity of the two OA circuit, recently described,[6] allowing a higher realizable pole Q for a specified sensitivity.

From (20) and (21) the sensitivities to the rolloff of the OA gain are:

$$\frac{\Delta\omega_o}{\omega_o} \simeq -\frac{2\omega_o}{GB} \qquad \text{for } Q_p \gg \tfrac{1}{2} \tag{40}$$

$$\frac{\Delta Q_p}{Q_p} \simeq \frac{2\omega_o}{GB} \qquad \text{for } Q_p \gg \tfrac{1}{2} \tag{41}$$

As an example, using the $\mu A741$ OA having $GB = 2\pi \times 10^6$ radians/sec and for $f_o = 1$ kHz, and $Q_p \gg \frac{1}{2}$, the deviation in $\omega_o = -0{\cdot}2$ per cent and the deviation in $Q_p = +0{\cdot}2$ per cent.

From (17), and for an all-pass network;

$$S^{Q_o}_{A_{o_1}} \simeq -\frac{2Q_p}{A_{o_1}} \qquad \text{for } Q_p \gg \tfrac{1}{2} \tag{42}$$

From equations (33) and (35), the ω_o and Q_p sensitivities to the passive circuit component are all $\leqslant 1$. To take the effect of imperfect tracking, the passive Q_p sensitivities are obtainable from equation (30) and are given by:

$$\begin{aligned} S^{Q_p}_{R_1} &= -S^{Q_p}_{R_2} = Q_p + \tfrac{1}{2} \\ S^{Q_p}_{C_1} &= -S^{Q_p}_{C_2} = Q_p - \tfrac{1}{2} \\ S^{Q_p}_{R_3} &= -S^{Q_p}_{R_4} = -Q_p \end{aligned} \tag{43}$$

For an all-pass phase shifter, the Q_o sensitivities to R_5 and R_6 are obtainable from (31) and are given by:

$$S^{Q_o}_{R_5} = -S^{Q_o}_{R_6} = 2. \tag{44}$$

The Q_p sensitivities to the passive circuit components are high and are proportional to Q_p as is the case with other good high frequency performance networks[10,11] and the circuit in this case belongs to class 1 filters as was classified recently by Faulkner and Grimbleby.[10] That is choosing the circuit components to minimize the sensitivities to the active elements, results in a high passive sensitivity.[12,13]

It is clear that the network in this case will perform satisfactorily only when realized in the hybrid-integrated circuit (HIC) technology (consisting of tantalum thin film RC combination and active silicon integrated circuit OAs),[14] for which it is intended.

Case 2

In order to be able to realize the circuit using discrete linear networks, the ratio m and the circuit components of N must be chosen to achieve a very low passive Q_p sensitivities.

From (30), it is clear that to minimize the passive Q_p sensitivities, choose:

$$\begin{aligned} m &= 1 \\ C_1 &= C_2 = C \\ R_2 &= R, R_1 = bR \end{aligned} \tag{45}$$

In this case:

$$\begin{aligned} S^{Q_p}_{R_1} &= -S^{Q_p}_{R_2} = \tfrac{1}{2} \\ S^{Q_p}_{C_1} &= S^{Q_p}_{C_2} = 0. \end{aligned} \tag{46}$$

From (24), (27) thus:

$$\omega_o = \frac{1}{CR\sqrt{b}}, \qquad q_p = \frac{\sqrt{b}}{b+2}, \qquad K = \sqrt{b} \tag{47}$$

$$Q_p = \frac{\sqrt{b}}{2} \tag{48}$$

$$q_p \simeq \frac{1}{2Q_p} \qquad \text{for } Q_p \gg \tfrac{1}{2} \tag{49}$$

The design equations for $\omega_o = 1$, are:

$$R_2 = 1, \qquad R_1 = 4Q_p^2 \qquad C_1 = C_2 = 1 \tag{50}$$

For an all-pass phase shifter:

$$a = 1 + \frac{1}{Q_p^2}, \tag{51}$$

and

$$S_{R_5}^{Q_o} = -S_{R_6}^{Q_o} = 2 \tag{52}$$

It is clear that the structure in this case is similar to the single OA network described before,[6] except that terminal 1 of the network N is excited by a voltage aV_i instead of $V_i (a > 1)$, and the positive terminal of the OA is connected directly to the input instead of being connected to a potential divider from the input. The network in this case belongs to Faulkner and Grimbleby[10] class 2 filters, and is less well suited to high Q factor applications at high frequencies than the realization in case 1, as is seen from Table I.

Table I

	Circuit components				Active sensitivities		Effect of roll off of the OA gain		Passive sensitivities	
	OAs	R	C	Gain factor	$A_o S_{A_o}^{\omega_o}$	$A_o S_{A_o}^{Q_p}$	$GB\left(\frac{\Delta\omega_0}{\omega_o}\right)$	$GB\left(\frac{\Delta Q_p}{Q_p}\right)$	$\lvert S_R^{Q_p}\rvert$	$\lvert S_C^{Q_p}\rvert$
Single OA realization[6]	1	4	2	<1	0	$2Q_p^2$	$-Q_p\omega_0$	$Q_p\omega_o$	$\leqslant\frac{1}{2}$	0
Double OA realization[6]	2	7	3	1	0	$<8Q_p$	$-4\omega_o$	$4\omega_o$	$\propto Q_p$	$\propto Q_p$
New realization Case 1	2	6	2	1	0	$4Q_p$	$-2\omega_o$	$2\omega_o$	$\simeq Q_p$	$\simeq Q_p$
Case 2	2	4	2	1	0	$2Q_p^2$	$-Q_p\omega_o$	$Q_p\omega_o$	$\leqslant\frac{1}{2}$	0

CONCLUSIONS

A new active RC configuration for realizing nonminimum phase transfer functions is given. The network has the advantage of being canonic—is always stable—is capable of realizing a high pole Q, and has a constant gain factor of unity. The active sensitivities and the effect of the rolloff of the OA gain are examined in a simplified way. It is noted that although the circuit is basically offered to realize all-pass phase shifters having complex poles and zeros, it is still capable of realizing an all-pass characteristics having real axis poles and zeros and having a unity gain factor.

Two cases are described, the first is well suited to high Q applications at high frequencies, and should be realized in the HIC technology. The second has very low passive sensitivities, and higher active sensitivities, and is less well suited to high Q factor applications at high frequencies. Finally it is noted that the new basic

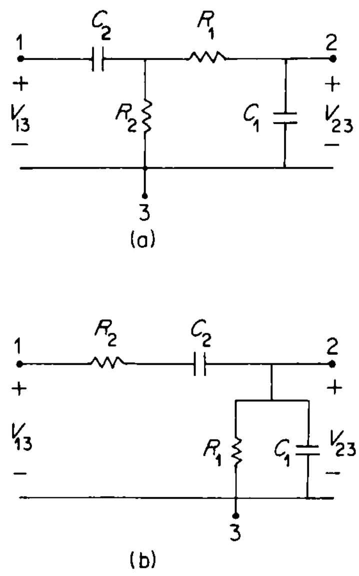

Figure 4. Two other minimal realizations for *N*

structure given in Figure 1 can employ other passive RC networks for *N* different from that in Figure 2. (Two other minimal realizations for *N* are shown in Figure 4†).

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† The use of the Wien network in Figure 4(b) for *N* was suggested by one of the reviewers.