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A new approach for using the pathological mirror elements in the ideal representation of active devices

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SUMMARY

This paper is adopting a new approach to investigate the capabilities of pathological mirror elements in the ideal representation of active building-blocks and shows that the voltage mirror (VM) and current mirror (CM) are the basic pathological elements. The descriptions for the floating mirror elements in the nodal admittance matrix (NAM), using infinity-variables, are derived. The descriptions for nullator and norator using infinity-variables in the NAM are shown to represent special cases from the derived descriptions of the floating VM and the CM, respectively. Hence, new representations for the nullator and norator in terms of the floating VM and CM, respectively, are obtained. A systematic procedure for the derivation of pathological configurations to ideally represent various analog signal-processing properties featured by active building-blocks is presented. This systematic approach became plausible by virtue of the versatility offered by the NAM descriptions of floating mirror elements. Novel pathological configurations ideally describing most popular signal-processing properties that involve differential or multiple single-ended signals; like conversion between differential and single-ended voltages, differential voltage conveying, current differencing, differential current conveying, and inverting current replication; are derived systematically using this procedure. The resulting pathological configurations are shown to be constructed mainly using mirror elements and hence the capabilities of the mirrors as basic pathological elements are further demonstrated. Pathological representations for some active building-blocks, using the derived pathological sections, are presented as application examples. Copyright © 2008 John Wiley & Sons, Ltd.

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KEY WORDS: active building blocks; pathological elements; nullor; voltage mirror; current mirror; systematic synthesis

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1. INTRODUCTION

The pathological elements are ideal network elements of theoretical existence, and have been found useful in solving circuit analysis and design problems. Several authors have investigated the use of methods or computational algorithms based on pathological elements for the synthesis and description of analog active circuits [1–5]. The pathological elements are specified according to the constraints they impose on their terminal voltages and currents. The first introduction of the pathological elements was through nullators and norators shown in Figure 1 [6, 7]. For the nullator $V = I = 0$, while the norator imposes no constraints on its voltage and current. A nullator–norator pair constitutes a universal active two-port network element [8] called the nullor, shown in Figure 2, and hence nullator and norator are also called nullor elements. The attractive feature of the two nullor elements is their ability to model active circuits independently of the particular realization of the active devices with the possibility of generating a number of ideally equivalent circuits from which the best practical ones can thereafter be selected [9, 10].

Despite the ability of nullor elements to describe many active building-blocks, they fail to represent devices like the positive type second-generation current conveyor (CCII+) introduced in [11]. Other passive elements like resistors are combined with nullators and norators in order to obtain the nullor representation of the CCII+ [10, 12]. In order to avoid the use of passive elements in the ideal representation of any building-block, additional pathological elements called mirror elements shown in Figure 3 were introduced in [13] to describe the voltage and current reversing actions. The voltage mirror (VM), shown in Figure 3(a), is a lossless two-port network

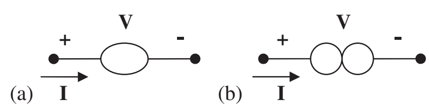

Figure 1. Nullor elements: (a) nullator and (b) norator.

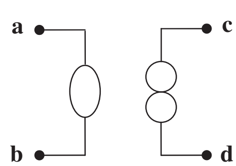

Figure 2. Two-port nullor with a nullator connected between nodes a and b and a norator connected between nodes c and d.

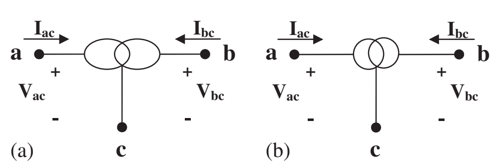

Figure 3. Mirror elements: (a) voltage mirror and (b) current mirror.

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element used to represent an ideal voltage reversing action and it is described by

$$V_{ac} = -V_{bc} \tag{1a}$$

$$I_{ac} = I_{bc} = 0 \tag{1b}$$

Terminal c is the reference terminal for the VM.

The current mirror (CM), shown in Figure 3(b), is a two-port network element used to represent an ideal current reversing action and it is described by

$$\begin{aligned} &V_{ac} \text{ and } V_{bc} \text{ are arbitrary} \\ &I_{ac} = I_{bc} \text{ and they are also arbitrary} \end{aligned} \tag{2}$$

Terminal c is the reference terminal for the CM. Although the CM element shown in Figure 3(b) has the same symbol as the regular CM, it is a bi-directional element and has a theoretical existence [14]. It is worth noting that the reference terminal for each of the VM and the CM symbols shown in Figure 3 can be set to ground and in this case these elements are called grounded mirror elements and used as two terminal elements with the reference node unused [14].

Recently, a symbolic framework for systematic synthesis of linear active circuits based on admittance matrix expansion has been presented in [15–18], in which a $p \times p$ port admittance matrix describing a certain circuit function is expanded to a port-equivalent $n \times n$ nodal admittance matrix (NAM) describing the synthesized circuit, such that $n > p$. The systematic synthesis approach employs nullators and norators in the admittance matrix expansion and the resulting nullor pairs are ideally representing the active elements in the synthesized circuits. In this framework, nullators and norators are described within admittance matrices using unbounded elements called infinity-variables. This approach has been extended in [19, 20] to employ the grounded mirror elements in the admittance matrix expansion and the ideal representation of the active devices in the synthesized circuits. However, a description for floating mirror elements in the NAM is still needed so that to investigate their use in the matrix expansion process and to obtain NAM descriptions for active devices that are ideally represented using floating mirror elements.

In this paper, a new approach is adopted to investigate the capabilities of the pathological mirror elements in the ideal representation of active devices. The use of pathological elements in the ideal representation of active devices, which has been limited to nullor and grounded mirror elements throughout the literature, will be extended to include the floating mirror elements. The paper starts by the derivation of the NAM descriptions, using infinity-variables, for the floating mirror elements. The derived NAM descriptions are shown to have general forms that imply many special cases, each of which feature a certain ideal signal-processing property. Thus, the floating mirror elements are shown to be versatile pathological elements with general terminal characteristics that can describe various signal-processing features. The descriptions for the nullator and norator in the NAM are also shown to represent special cases for the derived descriptions of the floating VM and the CM, respectively. Hence, new representations for the nullator and norator in terms of the floating VM and CM, respectively, will be obtained. A systematic approach for the derivation of pathological configurations to ideally represent various analog signal-processing properties featured by active building-blocks is presented. The flexibility of the proposed systematic procedure is owing to the versatility offered by the NAM descriptions of floating mirror elements. Pathological configurations ideally describing most popular signal-processing properties that involve differential or multiple single-ended signals, like conversion between differential and single-ended voltages,

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differential voltage conveying, current differencing, differential current conveying, and inverting current replication, are derived systematically to demonstrate the proposed procedure. Ideal representations, based on pathological elements for some active building-blocks from the state-of-the-art that possess these analog signal-processing features are presented as examples.

2. FLOATING MIRROR ELEMENTS IN THE NAM

It is known that for physically realizable circuits, all the voltages and currents are always uniquely and definitely determined. This in turn implies that in the ideal representation of a physically realizable circuit, nullators (or VMs) and norators (or CMs) must occur in a pair [16, 21]. Since the representation of active devices using pairs of pathological elements necessarily imposes ideal description, the gain of any dependent source used to model the relation between signals at two ports within a pair of pathological elements (nullor elements, mirror elements, or a combination of them) is always taken as infinity [15, 16]. Hence, the two-port nullor in Figure 2 can be derived as a limit of any one of the four types of dependent sources (VCVS, CCCS, VCCS, and CCVS) when its gain tends to infinity [16].

2.1. General NAM stamp for floating mirror elements

Consider the general four-port floating VM–CM pair in Figure 4, consisting of a floating VM whose terminals are connected to nodes a, b, and c and a floating CM whose terminals are connected to nodes d, e, and f and defined by

$$V_{ac} = -V_{bc} \tag{3a}$$

$$I_{ac} = I_{bc} = 0 \tag{3b}$$

$$\left.\begin{array}{c} V_{df} \text{ and } V_{ef} \text{ are arbitrary} \\ I_{df} = I_{ef} \text{ and they are also arbitrary} \end{array}\right\} \tag{3c}$$

Nodes c and f are the reference nodes for the VM and CM, respectively. This four-port floating VM–CM pair can be represented using four dependent sources when their gains tend to infinity, with every dependent source describing the relation between the signal at one-port in one of the mirror elements and the signal at one-port in the other mirror element. Since the considered synthesis framework uses the admittance matrices and the voltage-controlled current source (VCCS) is the only dependent source that possesses an admittance matrix [16], the floating mirror pair in Figure 4

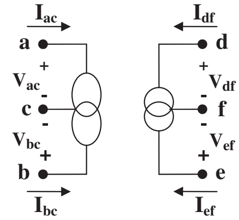

Figure 4. Four-port floating VM–CM pair.

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should be described in terms of VCCSs for which the transconductance gains tend to infinity. Hence, the admittance matrix stamp for the representation of a four-port floating VM–CM pair whose terminals are connected as in Figure 4 can be considered as that for the VCCSs-based ideal model in Figure 5, with the transconductance gain of every VCCS is G_{mi} and is taken to a limit of infinity. This VCCSs-based ideal model can be entered into the NAM in the following form:

$$\begin{array}{c} \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} G_{mi} & G_{mi} & -2G_{mi} \\ G_{mi} & G_{mi} & -2G_{mi} \\ -2G_{mi} & -2G_{mi} & 4G_{mi} \end{bmatrix} \end{array} \tag{4}$$

where G_{mi} is taken to a limit of infinity. Then, rows d, e, and f of the NAM equation set will have the form

$$\begin{bmatrix} I_d \\ I_e \\ I_f \end{bmatrix} = \begin{bmatrix} G_{mi} & G_{mi} & -2G_{mi} \\ G_{mi} & G_{mi} & -2G_{mi} \\ -2G_{mi} & -2G_{mi} & 4G_{mi} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} + \begin{bmatrix} \text{finite terms} \\ \text{finite terms} \\ \text{finite terms} \end{bmatrix} \tag{5}$$

where $G_{mi} \to \infty$. It is known that in order to preserve the finiteness of an equation containing a parameter that tends to infinity, the whole equation must be divided by this parameter before taking the limit. This is provided that the limit applies to the whole equation and not to that parameter individually. In order to apply this principle to the above equations corresponding to rows d, e, and f of the NAM equation set, these equations are divided by G_{mi} and limit of G_{mi} when it tends to infinity is taken for both sides of each row. Thus, rows d, e, and f are described by

$$\begin{bmatrix} \dfrac{I_d}{G_{mi}} \\ \dfrac{I_e}{G_{mi}} \\ \dfrac{I_f}{G_{mi}} \end{bmatrix} = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 4 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} + \begin{bmatrix} \dfrac{\text{finite terms}}{G_{mi}} \\ \dfrac{\text{finite terms}}{G_{mi}} \\ \dfrac{\text{finite terms}}{G_{mi}} \end{bmatrix} \tag{6}$$

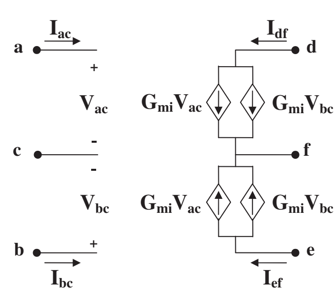

Figure 5. Voltage-controlled current sources based model for the floating VM–CM pair in Figure 4.

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where $G_{mi}\to\infty$. When the limit is taken, dependent current terms on the RHS and finite terms on the LHS will vanish as described by

$$\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}1 & 1 & -2\\ 1 & 1 & -2\\ -2 & -2 & 4\end{bmatrix}\begin{bmatrix}V_a\\V_b\\V_c\end{bmatrix}+\begin{bmatrix}0\\0\\0\end{bmatrix} \tag{7}$$

The three rows in the NAM equation set corresponding to the CM nodes yield the same relation between independent voltage variables

$$V_a+V_b-2V_c=0\to V_a-V_c=-(V_b-V_c)\to V_{ac}=-V_{bc} \tag{8}$$

Since the floating mirror pair description in Equation (4) has no matrix entries for rows a, b, or c, then

$$I_a=I_b=I_c=0 \tag{9}$$

The similarity between the coefficients of rows d and e in Equation (4) imposes the constraint that the current entering (leaving) the CM at node d is equal to that entering (leaving) it at node e, and the coefficients of row f indicate that both currents are leaving (entering) the CM at node f. Equation (5) indicates KCL at node f for the currents flowing between each of nodes d and e and node f within the CM; however, the values of the currents at the terminals of the CM are unconstrained. Since the columns corresponding to nodes d, e, or f do not exist in the floating mirror description of Equation (4), there are no constraints on the terminal voltages of the CM. Thus, the NAM description in Equation (4) with $G_{mi}\to\infty$ imposes finite relationships between the nodal voltages and currents that correctly describe a floating VM–CM pair connected as shown in Figure 4.

As explained in [16], nullor elements can be represented in the NAM using infinity-variables. In this notation the variables in the NAM that are taken to an infinite limit are written as ∞_i, where ∞ indicates that the limit is taken to infinity and i refers to the active element the nullor is representing. For the nullor in Figure 2, it has been deduced in [16] that this nullor description in the NAM using infinity-variables takes the form

$$\begin{array}{c c} & \begin{array}{cc} a & b \end{array}\\ \begin{array}{c} c\\ d\end{array} & \begin{bmatrix}\infty_i & -\infty_i\\ -\infty_i & \infty_i\end{bmatrix}\end{array} \tag{10}$$

On applying this infinity-variables notation to mirror elements, the floating mirror pair description in Equation (4) becomes

$$\begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array}\\ \begin{array}{c} d\\ e\\ f\end{array} & \begin{bmatrix}\infty_i & \infty_i & -2\infty_i\\ \infty_i & \infty_i & -2\infty_i\\ -2\infty_i & -2\infty_i & 4\infty_i\end{bmatrix}\end{array} \tag{11}$$

From the set of infinity-variables describing the floating nullor pair in Equation (10), the presence of a nullator between the two nodes a and b causes the infinity-variables in the two NAM columns corresponding to nodes a and b to have equal coefficients with opposite signs. Similarly, the presence of a norator between the two nodes c and d causes the infinity-variables in the two NAM rows corresponding to nodes c and d to have equal coefficients with opposite signs. The same approach can be adopted to describe the effect of floating mirror elements on the relation between infinity-variables occupying the NAM rows and columns corresponding to the nodes at which the terminals of mirror elements are connected. The set of infinity-variables describing the floating mirror pair in Equation (11) indicates that the floating voltage mirror whose two ports are connected between each of the nodes a and b and the floating reference node c causes the infinity-variables in the two columns a and b to be equal and having the same signs while the coefficients of the infinity-variables in column c are double those in each of columns a and b and having opposite signs. Similarly, the floating CM whose two ports are connected between each of the nodes d and e and the floating reference node f causes the infinity-variables in the two rows d and e to be equal and having the same signs while the coefficients of the infinity-variables in row f are double those in each of rows d and e and having opposite signs.

2.2. *Special cases*

The NAM stamp for the floating VM–CM pair in Equation (11) includes the general descriptions of the floating mirror elements in the NAM. However, special cases arising from connecting any of the terminals of a mirror element to the ground or connecting any two terminals in a mirror element to each other will yield reduced NAM representations describing the resulting nodal voltage and/or current relationships, according to the constraints imposed by the mirror element.

Case 1: Connecting the reference terminal of the VM to ground. In this case, the NAM description for the VM does no longer contain a column corresponding to the reference node. Since the reference terminal of the VM is now connected to the ground, the infinity-variables associated with the reference terminal will occupy the column corresponding to the ground node, which is outside the NAM. This can be applied on the floating VM–CM pair described in Equation (11), and the resulting NAM representation is obtained as:

$$\begin{array}{c} \\ d \\ e \\ f \end{array}\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} \infty_i & \infty_i \\ \infty_i & \infty_i \\ -2\infty_i & -2\infty_i \end{bmatrix} \end{array}\begin{array}{c} gnd \\ -2\infty_i \\ -2\infty_i \\ 4\infty_i \end{array} \tag{12}$$

Hence, the NAM equations corresponding to rows d, e, and f will yield the relationship between the voltages at nodes a and b given by

$$V_a = -V_b \tag{13}$$

Case 2: Connecting any terminal, other than the reference terminal, in the VM to ground. In this case, the NAM description for the VM does no longer contain a column corresponding to the grounded terminal. The infinity-variables associated with the grounded terminal will occupy the column corresponding to the ground node, which is outside the NAM. This can be applied on the

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floating VM–CM pair described in Equation (11), yielding the NAM representation described by

$$\begin{array}{c} \\ d \\ e \\ f \end{array}\begin{array}{c} \begin{array}{ccc} a\ (or\ b) & c \end{array} \\ \left[\begin{array}{cc} \infty_i & -2\infty_i \\ \infty_i & -2\infty_i \\ -2\infty_i & 4\infty_i \end{array}\right] \end{array}\begin{array}{c} gnd \\ \infty_i \\ \infty_i \\ -2\infty_i \end{array} \tag{14}$$

The resulting NAM equations corresponding to rows d, e, and f imply the same voltage relationship for the remaining nodes of the VM, which is described by

$$V_{a\ (or\, b)} = 2V_c \tag{15}$$

Case 3: Connecting the reference terminal of the CM to ground. In this case, the NAM description for the CM does no longer contain a row corresponding to the reference node. Since the reference terminal of the CM is now connected to the ground, the infinity-variables associated with the reference terminal will occupy the row corresponding to the ground node, which is outside the NAM. This can be applied on the floating VM–CM pair described in Equation (11), and the resulting NAM representation described by

$$\begin{array}{cc} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} d \\ e \end{array} & \left[\begin{array}{ccc} \infty_i & \infty_i & -2\infty_i \\ \infty_i & \infty_i & -2\infty_i \end{array}\right] \\ gnd & \begin{array}{ccc} -2\infty_i & -2\infty_i & 4\infty_i \end{array} \end{array} \tag{16}$$

From the NAM description in Equation (16), the relationships between the nodal voltages and currents for the CM terminals are not changed. The infinity-variables coefficients at rows d, e, and f indicate that the current entering (leaving) the CM at node d is equal to that entering (leaving) it at node e and both currents undergo KCL at the ground node. Although the reference terminal for the CM is set to ground, the voltages at nodes d and e are still unconstrained.

Case 4: Connecting any terminal, other than the reference terminal, in the CM to ground. In this case, the NAM description for the CM does no longer contain a row corresponding to the grounded terminal. The infinity-variables associated with the grounded terminal will occupy the row corresponding to the ground node, which is outside the NAM. On applying this case on the floating VM–CM pair described by Equation (11), the resulting NAM representation is described by

$$\begin{array}{cc} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} e \\ f \end{array} & \left[\begin{array}{ccc} \infty_i & \infty_i & -2\infty_i \\ -2\infty_i & 2\infty_i & 4\infty_i \end{array}\right] \\ gnd & \begin{array}{ccc} \infty_i & \infty_i & -2\infty_i \end{array} \end{array} \tag{17}$$

The NAM equations corresponding to rows e and f imply the relationship for the remaining terminals of the CM given by

$$I_f = 2I_e \tag{18}$$

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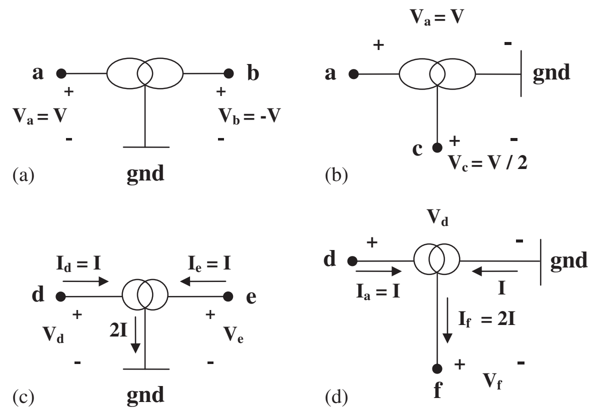

Figure 6. Special cases of the floating mirror elements.

Figure 6 shows the four special cases studied in this subsection. The special cases in which two columns (or rows) in the NAM representation of a floating mirror element are combined together as one column (or row) and their elements are added to each other, indicating that the corresponding terminals in the mirror element are connected to each other, are considered in detail in the following section.

3. REPRESENTATION OF NULLOR ELEMENTS USING MIRRORS

It has been shown in [14] that the nullator can be represented by cascading two (or an even number of) VMs as shown in Figure 7(a) and, similarly, the norator can be represented using two (or an even number of) cascaded CMs as shown in Figure 7(b). On the other hand, other elements like resistors must be combined with nullor elements in order to realize mirror elements as shown in Figure 8 [13, 14]. In this section, alternative representations of nullor elements using floating mirror elements will be derived.

For the description of the floating VM–CM pair in (11), consider the special case where either column a or b is combined with column c together as one column and their elements are added to each other. This is equivalent to connecting any of the two nodes a or b to node c. Since the coefficients of the infinity-variables in column c are double those in each of columns a and b and have opposite signs, then this combination will result in two columns with the coefficients of their infinity-variables equal and opposite in sign, as given by

$$\begin{array}{c} \\ d \\ e \\ f \end{array}\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \left[\begin{array}{ccc} \infty_i & \infty_i & -2\infty_i \\ \infty_i & \infty_i & -2\infty_i \\ -2\infty_i & -2\infty_i & 4\infty_i \end{array}\right] \end{array} \rightarrow \begin{array}{c} \\ d \\ e \\ f \end{array}\begin{array}{c} \begin{array}{cc} a & c \end{array} \\ \left[\begin{array}{cc} \infty_i & -\infty_i \\ \infty_i & -\infty_i \\ -2\infty_i & 2\infty_i \end{array}\right] \end{array} \tag{19}$$

Similarly, combining columns a and b together as one column and adding their elements to each other will yield two columns with the coefficients of their infinity-variables equal and opposite in

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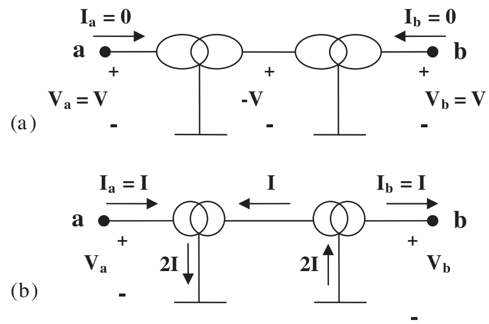

Figure 7. (a) Nullator representation using two voltage mirrors and (b) norator representation using two current mirrors.

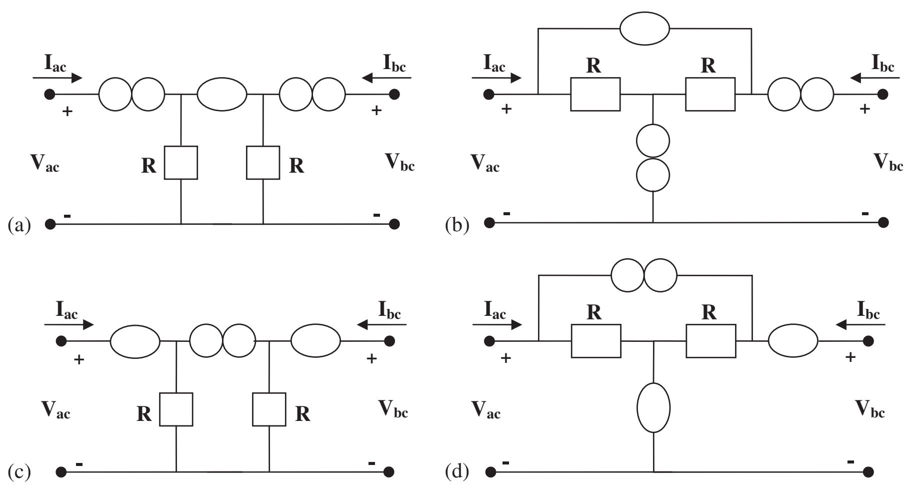

Figure 8. (a) and (b) Current mirror representations using nullor elements and two equal resistors. (c) and (d) Voltage mirror representations using nullor elements and two equal passive elements.

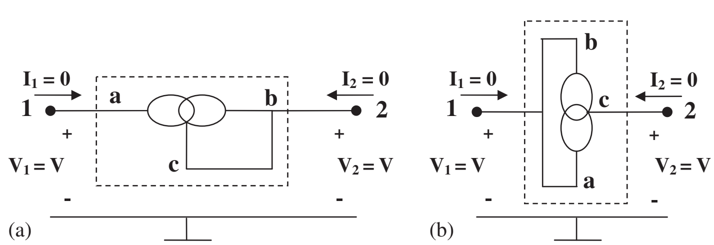

Figure 9. Nullator representations between terminals 1 and 2 using a floating voltage mirror.

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signs, as given by

$$\begin{array}{c} \\ d \\ e \\ f \end{array}\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \left[\begin{array}{ccc} \infty_i & \infty_i & -2\infty_i \\ \infty_i & \infty_i & -2\infty_i \\ -2\infty_i & -2\infty_i & 4\infty_i \end{array}\right] \end{array} \rightarrow \begin{array}{c} \\ d \\ e \\ f \end{array}\begin{array}{c} \begin{array}{cc} a & c \end{array} \\ \left[\begin{array}{cc} 2\infty_i & -2\infty_i \\ 2\infty_i & -2\infty_i \\ -4\infty_i & 4\infty_i \end{array}\right] \end{array} \tag{20}$$

The resulting sets of infinity-variables in Equations (19) and (20) describe pathological pairs including a nullator rather than a VM. Hence, the nullator description in the NAM using infinity-variables can be considered as a special case from the floating VM description, in which any two terminals in the floating VM are connected to each other causing their corresponding columns in the NAM description to be combined together as one column with their elements added to each other. Thus, a nullator can be described using a floating VM by connecting any two terminals in the floating VM to each other as one terminal. Figure 9 shows the resulting nullator descriptions, between terminals 1 and 2, using a floating VM.

The equivalence of the representations of Figure 9 to the nullator can be also demonstrated analytically as follows. For the VM configuration in Figure 9(a)

$$V_c = V_a \tag{21}$$

Then the voltage constraint in Equation (1a) will yield the following relationship between the nodal voltages:

$$V_{bc} = -V_{ac} = 0 \rightarrow V_b = V_c = V_a \tag{22}$$

Therefore,

$$V_1 = V_2 \tag{23}$$

From the VM current constraint in Equation (1b), no currents are flowing through the VM. Thus, according to KCL at node c,

$$I_c = I_a = I_b = 0 \tag{24}$$

Therefore,

$$I_1 = I_a = 0 \tag{25}$$

$$I_2 = I_b + I_c = 0 \tag{26}$$

Hence, the VM based representation in Figure 9(a) imposes finite relationships between the between the terminal voltages and currents of the VM at nodes a, b, and c which correctly describe a nullator connected between terminals 1 and 2, as deduced in (23), (25), and (26).

Now, consider the VM configuration in Figure 9(b). According to the voltage constraint in (1a), the relationship between the nodal voltages for the VM in Figure 9(b) is given by

$$V_a - V_c = -(V_b - V_c) \tag{27}$$

Since $V_a = V_b$,

$$V_c = V_a = V_b \tag{28}$$

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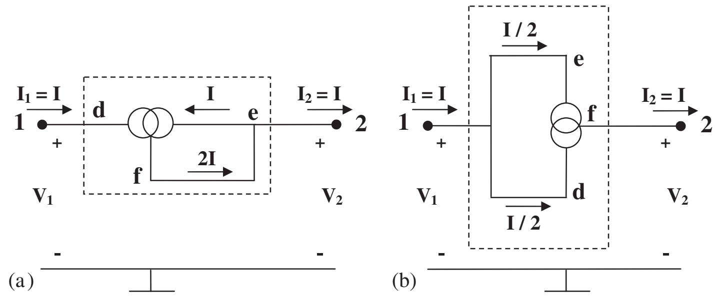

Figure 10. Norator representations between terminals 1 and 2 using a floating current mirror.

Therefore,

$$V_1 = V_2 \tag{29}$$

Applying KCL at terminals 1 and 2, therefore;

$$I_1 = I_2 = I_{ac} + I_{bc} = 0 \tag{30}$$

Hence, the VM-based representation in Figure 9(b) imposes finite relationships between the terminal voltages and currents of the VM at nodes a, b, and c, which correctly describe a nullator connected between terminals 1 and 2, as deduced in Equations (29) and (30).

Similar steps can be adopted to deduce that a norator can be represented using a single floating CM as shown in Figure 10. Thus, on connecting any two terminals in a floating mirror element to each other, the constraints imposed by the mirror element will yield nodal voltage and current relationships between the resulting two terminals that meet the constraints imposed by a nullor element. Hence, an alternative representation for nullor elements using mirror elements can be obtained, in which each nullor element is represented using only one floating mirror element.

4. SYSTEMATIC DERIVATION OF ACTIVE DEVICES

The ideal representation of active devices using pathological elements has been limited throughout the literature to using nullor and grounded mirror elements in a simple and direct way to describe ideal inverting/non-inverting single-ended voltage conveying and ideal inverting/non-inverting single-ended current conveying within the ideal representations of active building-blocks, as shown in Table I. Thus, according to the analysis in the previous two sections, only the special cases of the floating mirror elements in which the reference terminal of a mirror element is grounded (yielding a grounded mirror element) or any two terminals in a floating mirror element are connected to each other (yielding a nullor element) have been used to represent ideal analog features within active building-blocks. As a result, the domain of active devices that can be ideally represented using pathological elements was limited to the devices whose ideal descriptions possess the features described in Table I. Every one of these features involves a relationship between two signals at two single-ended terminals and hence it can be represented using only a single nullor or grounded mirror element. Thus, the pathological representations for active building-blocks possessing these analog properties, in the ideal sense, can be obtained intuitively by assuming ideal characteristics

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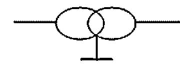
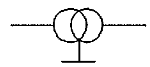
Table I. Ideal features represented using pathological elements.

Feature	Pathological element
Non-inverting voltage conveying	
Inverting voltage conveying	
Non-inverting current conveying	
Inverting current conveying	

for the active device and then mapping every single feature in the ideal device with the appropriate nullor or grounded mirror element. Table II shows examples of active building-blocks and their ideal representations using nullor-mirror elements [13], with the relationship between any two single-ended signals is ideally represented using either a nullor or grounded mirror element. However, there are many versatile active building-blocks possessing other useful signal-processing features that involve differential signals or multiple single-ended signals, like conversion between differential and single-ended voltages; differential voltage conveying; current differencing; differential current conveying; and current replication, which cannot be ideally represented using only a single nullor or grounded mirror element. It is necessary to be able to obtain ideal representations for such active building-blocks using pathological elements, so that to benefit from the advantages of using the pathological elements in the synthesis, modeling, and analysis of active circuits incorporating these devices. The key direction to obtain pathological descriptions for ideal signal-processing features involving multiple differential or single-ended signals is using the whole family of pathological elements spanning the general floating mirror elements and their associated special cases to realize pathological configurations composed of multiple pathological elements. Hence, the ideal representation of such analog signal-processing features using pathological descriptions will be more complicated than the intuitive mapping shown in Table I. This triggers the need for a formal procedure that enables the derivation of such complicated pathological configurations in a systematic way.

Since any pathological configuration can be described within a NAM using infinity-variables, then a pathological configuration that represents a certain ideal signal-processing feature within an active device can be obtained mathematically by deriving its NAM stamp. In order to ideally represent a certain transformation on applied analog signals using infinity-variables within a NAM, the infinity-variables should be distributed in the NAM stamp such that the resulting NAM equation set, after taking the limits, yields relationships correctly describing the required transformation on the applied analog signals. To be able to map a derived NAM stamp into a feasible pathological configuration, every distinctive set of infinity-variables within the derived NAM stamp should represent a pathological element or pair. A formal five-step procedure is presented to derive pathological configurations, ideally representing various analog signal-processing features possessed by active building-blocks, in a systematic way. Firstly, the properties (number of rows and/or columns and number of distinctive sets of infinity-variables) of the simplest representation using infinity-variables that can describe the required signal-processing feature are defined. Secondly, the mathematical conditions on the NAM equation set corresponding to the rows of this infinity-variables representation are specified, according to the required analog feature to be represented.

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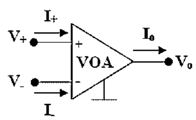
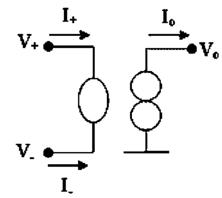
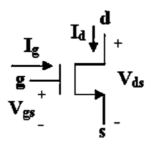
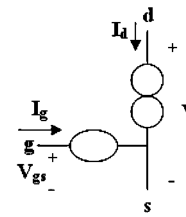
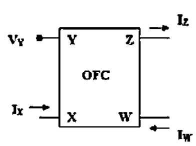
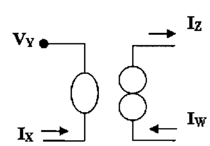
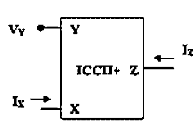
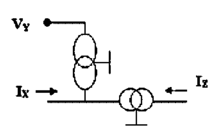
Table II. Examples for active building-blocks possessing the features in Table I and their nullor-mirror representations.

Symbol	Ideal characteristics	Pathological representation
	$I_+ = I_- = 0$ $V_o = A_v(V_+ - V_-)$ $A_v \to \infty$ V_o, I_o arbitrary	
	$I_g = 0$ $V_{ds} = A_v \cdot V_{gs}$ $A_v \to \infty$ V_{ds}, I_d arbitrary	
	$I_Y = I_X = 0$ $V_X = V_Y$ V_W arbitrary $I_Z = I_W$	
	$V_X = -V_Y$ $I_Z = I_X$ V_Z arbitrary	

VOA, voltage operational amplifier; OFC, operational floating conveyor; ICCII+, inverting second generation current conveyor positive.

Thirdly, the simplest possible combination of infinity-variables that have the properties defined in the first step and yields the mathematical constraints specified in the second step is identified. Fourthly, modifications are applied on this simple set of infinity-variables to reach a combination of NAM descriptions for pathological elements, while maintaining the relation implied by this simple set of infinity-variables. Finally, the resulting NAM stamp is mapped into a pathological configuration composed of one or more pathological elements.

In the next two subsections, the systematic procedure is explained in details in case of voltage analog features and current analog features, respectively. Pathological configurations for some common analog signal-processing features that involve differential and/or multiple single-ended signals are derived systematically in each case to illustrate the proposed systematic procedure.

4.1. Voltage analog signal-processing properties

A linear relation between voltage signals at terminals of an active device can be ideally represented in the simplest form within a NAM using infinity-variables existing on the same row and belong

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to the same set of infinity-variables. Consider a row within a set of infinity-variables having the general form described by

$$
\begin{array}{c}
\begin{array}{ccccccc} \quad 1 & \quad 2 & \quad 3 & \cdots & \cdots & \cdots & \quad n \quad \end{array} \\
\left[\begin{array}{ccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & K_1\infty_i & K_2\infty_i & K_3\infty_i & \cdots & \cdots & \cdots & K_n\infty_i & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}\right]
\end{array}
\qquad (31)
$$

where $K_1, K_2, K_3, \ldots, K_n$ are constants representing the coefficients of the infinity-variables and n is the number of columns occupied by the set of infinity-variables. On dividing the NAM equation corresponding to the above row containing the infinity-variables in Equation (31) by ∞_i and taking the limit, all finite terms in the equation will vanish and the following expression is derived:

$$\sum_{m=1}^{n} K_m V_m = 0 \qquad (32)$$

Hence, the linear relation between the voltages at the nodes corresponding to the columns occupied by the set of the ∞_i-variables is determined by the values of the coefficients $K_1, K_2, K_3, \ldots, K_n$. This mathematical abstraction shows that a linear relationship between the voltages at terminals of an active device can be ideally represented in the simplest form within a NAM using infinity-variables that belong to the same set and occupy a single row, such that the infinity-variables are located at the columns corresponding to the nodes at which these terminals are connected and the ratios between the coefficients of the infinity-variables are adjusted to yield the required relation between the voltage signals at these nodes.

However, the actual form of the required NAM stamp to be derived should be composed of NAM descriptions for pathological elements so that to yield a feasible pathological configuration constructed using one or more pathological elements. Thus, modifications should be applied on the simple row of infinity-variables, obtained in the previous steps, to yield a combination of NAM descriptions for pathological elements. The first possible modification step can be multiplying the infinity-variables at certain columns by a column scale factor or moving some infinity-variables from their columns to other columns to yield a description for a VM or a nullator. To maintain the relation between the voltage signals after this modification step, the voltages at the nodes corresponding to the columns for which the infinity-variables have undergone any changes must be adjusted accordingly. Additional sets of infinity-variables representing other VMs and/or nullators are introduced to set the voltages at these nodes to the required values. Hence, a combination of NAM descriptions for VMs and/or nullators is obtained. Note that the resulting infinity-variables NAM descriptions will be missing the other pathological elements (CMs and/or norators) needed to complete the pathological pairs in the representation; however, the derivation process is independent on the other pathological element in each pathological pair and hence it is not included in the NAM descriptions so that to limit the focus to the VMs and/or nullators that are actually realizing the required solution. There are no constraints on the other pathological element in each pair and

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it will be determined by the circuit design. The steps of the systematic process will be illustrated in details through the following example.

Consider Equation (33) that describes a conversion of the differential voltage between nodes w and x into a single-ended voltage at node y described by

$$V_y = V_w - V_x \tag{33}$$

The simplest row of infinity-variables that can describe the above relation within a NAM is given by

$$\begin{array}{c} \begin{matrix} & w & x & y & \end{matrix} \\ \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -\infty_i & \infty_i & \infty_i & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \end{array} \tag{34}$$

Let the coefficient of the $-\infty_i$ variable in column w be multiplied by a column scale factor 2 to yield a NAM description for a floating VM connected at the involved nodes (w, x, and y) with its reference terminal connected to node w, as described by

$$\begin{array}{c} \begin{matrix} & w & x & y & \end{matrix} \\ \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -\boxed{2}\infty_i & \infty_i & \infty_i & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \end{array} \tag{35}$$

This change causes the set of infinity-variables in Equation (35) to miss the required condition in Equation (33), because the contribution of the voltage at the node w in the NAM equation corresponding to the row in Equation (35) will be doubled then. For the NAM equation corresponding to the row in Equation (35) to maintain the condition in Equation (33) after taking the limit, it is necessary to restore the unity factor multiplied by the contribution of the w-voltage as in the NAM equation corresponding to the row in Equation (34). A possible solution would be to connect the reference terminal for the resulting VM described in Equation (35) to another floating node, rather than the node w, whose voltage is equal to half the voltage at node w. Hence, the contribution of the voltage at this node, which is half the voltage at node w, is doubled in the NAM equation after taking the limit, resulting in an overall unity factor multiplied by the voltage at node w in the final equation. This can be achieved using an additional set of infinity-variables (∞_j) representing another VM connected like the second special case in Section 2 shown in Figure 6(b), as described by

$$\begin{array}{c} \begin{matrix} & v & w & & & \end{matrix} \\ \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -2\infty_j & \infty_j & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \end{array} \tag{36}$$

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Thus, the voltage at node v is equal to half the voltage at node w. Then, on connecting the reference terminal for the VM described by Equation (35) to node v rather than node w, the condition imposed by Equation (33) is satisfied and the resulting NAM stamp in Equation (37) represents a pathological configuration based on two VMs ideally describing the conversion of a differential voltage between nodes w and x into a single-ended voltage at node y and vice versa.

$$\begin{array}{c} \begin{array}{cccccc} & v & w & x & y & \end{array} \\ \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -2\infty_j & \infty_j & \cdot & \cdot & \cdot \\ \cdot & -2\infty_i & \cdot & \infty_i & \infty_i & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \end{array} \tag{37}$$

An alternative scenario of modification steps can take place, in which the infinity-variables in columns x and y in Equation (34) are moved to a new column v. The resulting form in Equation (38), for the row containing the infinity-variables, describes a VM connected like the second special case in Section 2 shown in Figure 6(b) and its reference terminal connected at node v

$$\begin{array}{c} \begin{array}{ccccc} & w & x & y & \end{array} \\ \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -\infty_i & \infty_i & \infty_i & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \end{array} \rightarrow \begin{array}{c} \begin{array}{cccccc} & v & w & x & y & \end{array} \\ \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 2\infty_i & -\infty_i & 0 & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \end{array} \tag{38}$$

In order to maintain the condition in Equation (33) using the resulting infinity variables in Equation (38), the voltage at node v is given by

$$V_v = (V_x + V_y)/2 \tag{39}$$

To generate a voltage at node v equal to the average between the voltages at nodes x and y as described in Equation (39), an additional set of infinity-variables (∞_j) is introduced as described by

$$\begin{array}{c} \begin{array}{cccccc} & v & w & x & y & \end{array} \\ \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -2\infty_j & \cdot & \infty_j & \infty_j & \cdot \\ \cdot & 2\infty_i & -\infty_i & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \end{array} \tag{40}$$

The above NAM represents another floating VM whose reference terminal is connected to node v while the other two terminals are connected to nodes x and y. The resulting NAM stamp in Equation (40) is exactly the same as the previously derived Equation (37). The pathological

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configuration described by the NAM in Equations (37) and (40) can be represented using VMs as shown in Figure 11. The VM on the left results in the following relation:

$$V_v = V_w/2 \tag{41}$$

Thus, the other VM causes the single-ended voltage at node y to be

$$V_y = 2(V_w/2) - V_x = V_w - V_x \tag{42}$$

This pathological section is called the pathological differential voltage cell, and can be used to represent the conversion of a differential voltage into a single-ended voltage and vice versa. The inverting counterpart for this configuration can be easily obtained by exchanging the terminals of the differential port (w and x).

In order to maintain the condition in Equation (33) for all values of V_w, V_x, and V_y, it is necessary to avoid any modification step that will yield a cancelation for any of the infinity-variables in Equation (34). Loosing any of these infinity-variables will cause the relationship between the voltages at nodes w, x, and y to be unconstrained and hence the resulting infinity-variables are no longer describing the required ideal relation. This can be illustrated as follows. Assume that the infinity-variables within any two columns in (34) whose infinity-variables coefficients are equal in value and have opposite signs (e.g. w and x or w and y) are moved into another column in which these infinity-variables are added to each other. A description for a nullator which is connected to the ground is obtained as follows:

$$\begin{array}{c} \\ \\ \end{array}\begin{matrix} & w & x & y & \end{matrix} \qquad\qquad \begin{matrix} & w & x & y & \end{matrix}$$

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -\infty_i & \infty_i & \infty_i & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \rightarrow \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & 0 & \infty_i & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{43}$$

On dividing the NAM equation corresponding to the row of infinity-variables in Equation (43) by ∞_i and then taking the limit, the contributions of the voltages at the nodes corresponding to the two combined columns (w and x) in the resulting equation will vanish because the set of infinity-variables does no longer occupy these columns, while the remaining infinity-variable at column y will yield the following relation:

$$V_y = 0 \tag{44}$$

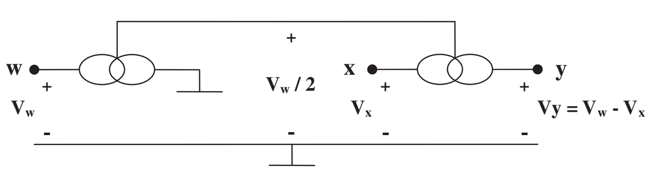

Figure 11. Pathological differential voltage cell.

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Hence, the set of infinity-variables in Equation (43) represents the relation in Equation (44) and imposes no constraints on the relation between the voltages at nodes w, x, and y. Accordingly, this modification step cannot yield a valid solution because the required set of infinity-variables should unconditionally satisfy the relation in Equation (33) for all values of V_w, V_x, and V_y; while the infinity-variable in Equation (43) satisfy the condition in Equation (33) only for the special case in which $V_w = V_x$. Therefore, it is necessary to keep the infinity-variables in Equation (34) during all the modification steps of the derivation process, so that to be able to maintain the required constraints.

The systematic approach can be adopted in a similar way to derive a pathological configuration representing the ideal differential voltage conveying, as shown in Figure 12. The two VMs have a common reference node, resulting in the following relation between the voltages at nodes w, x, y, and z:

$$(V_w + V_z)/2 = (V_x + V_y)/2 \rightarrow V_w + V_z = V_x + V_y \rightarrow V_w - V_x = V_y - V_z \tag{45}$$

This pathological section is called the pathological differential voltage conveying cell, and can be used to represent the voltage conveying between two differential ports. The inverting counterpart for this configuration can be easily obtained by either exchanging nodes w and x or exchanging nodes y and z.

4.2. *Current analog signal-processing properties*

A linear relation between current signals at terminals of an active device, without constraints on the voltages at these terminals, can be ideally represented within a NAM using sets of infinity-variables occupying the rows corresponding to the nodes at which these active device terminals are connected. Consider a signal-processing feature involving the currents at n terminals within an active device through a linear relation. The relation between these currents can be arranged as an equation with the current at one terminal existing on one hand-side and the currents at the other $n-1$ terminals existing on the other hand-side, as the general form shown in Equation (46).

$$K_n I_n = \sum_{m=1}^{n-1} K_m I_m \tag{46}$$

where K_1, K_2, $K_3, \ldots, K_n$ are constants representing the coefficients multiplied by the current signals in the analog signal-processing relation. For the general case in which the currents I_1, I_2, $I_3, \ldots, I_n$ do not include any interrelated currents, the minimum number of infinity-variables sets needed to describe the relation in Equation (22) between these n currents within a NAM is $n-1$,

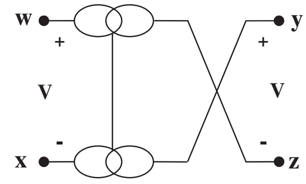

Figure 12. Pathological differential voltage conveying cell.

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as described by

$$\begin{array}{c} \\ 1 \\ 2 \\ \\ \\ n-1 \\ n \\ \\ \end{array} \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & K_1\infty_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & K_2\infty_2 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & K_{n-1}\infty_{n-1} & \cdot & \cdot \\ \cdot & \frac{K_1}{K_n}\infty_1 & \frac{K_2}{K_n}\infty_2 & \cdot & \cdot & \frac{K_{n-1}}{K_n}\infty_{n-1} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (47)$$

However, the actual form of the required NAM stamp to be derived should be composed of NAM descriptions for pathological elements so that to yield a feasible pathological configuration constructed using one or more pathological elements. Thus, modifications should be applied on the arrangement of infinity-variables, obtained in the previous step, to yield a combination of NAM descriptions for pathological elements. As in the case of voltage signals, the first possible modification step can be multiplying certain infinity-variables by a factor or moving the infinity-variables in some rows to other rows to yield a description for a CM or a norator. To maintain the relation between the current signals after this modification step, the currents due to the infinity-variables at the nodes corresponding to the rows for which the infinity-variables have undergone any changes must be adjusted accordingly. Additional sets of infinity-variables representing other CMs and/or norators are introduced to set the currents due to the infinity-variables at these nodes to the required values. Hence, a combination of NAM descriptions for CMs and/or norators is obtained. Note that the resulting infinity-variables NAM descriptions will be missing the other pathological elements (VMs and/or nullators) needed to complete the pathological pairs in the representation; however, the derivation process is independent on the other pathological element in each pathological pair and hence it is not included in the NAM descriptions so that to limit the focus to the CMs and/or norators are actually realizing the required solution. There are no constraints on the other pathological element in each pair and it will be determined by the circuit design.

As an example, consider the current differencing operation described by the following expression:

$$I_y = I_w - I_x \quad (48)$$

without constraints on the voltages at nodes w, x, and y. In order to obtain a NAM stamp using infinity-variables for a pathological configuration describing the current differencing, the infinity-variables in the NAM representation should satisfy the relation given in Equation (48). The simplest form of infinity-variables that describe Equation (48) is given by

$$\begin{array}{c} \\ w \\ x \\ y \\ \\ \end{array} \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \infty_1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \infty_2 & \cdot & \cdot \\ \cdot & \infty_1 & -\infty_2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (49)$$

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On having a close look at the NAM representation in Equation (49), it is clear that no modifications are needed to achieve a feasible pathological configuration. The ∞_1-variables represent a CM connected like the third special case in Section 2, shown in Figure 6(c), and the ∞_2-variables represent a norator. The pathological configuration described by the NAM in Equation (49) is shown in Figure 13. This pathological section is called the pathological current differencing cell. The inverting counterpart for this configuration can be easily obtained by exchanging nodes w and x.

As another example, consider the following relation describing the differential current conveying between the two differential ports formed by nodes w and x and nodes y and z

$$I_y - I_z = I_w - I_x \tag{50}$$

With no constraints on the voltages at nodes w, x, y, and z. In order to be able to describe the relation in (50) using infinity-variables within a NAM, the currents in Equation (50) can be re-arranged such that one current exists on one hand side and the other currents exist on the other hand side of the equation, as described by

$$I_y = I_w - I_x + I_z \tag{51}$$

Equation (51) can be represented in the simplest form within a NAM using three sets of infinity-variables occupying four rows as given by

$$\begin{array}{c} \\ w \\ x \\ y \\ z \\ \\ \end{array} \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \infty_1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \infty_2 & \cdot & \cdot \\ \cdot & \infty_1 & -\infty_2 & \infty_3 & \cdot \\ \cdot & \cdot & \cdot & \infty_3 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{52}$$

As in the case of current differencing, the NAM representation in Equation (52) represents a feasible pathological configuration without any modifications. The ∞_1-variables and the ∞_3-variables represent two CMs connected like the third special case in Section 2 and the ∞_2-variables represent a norator. The pathological configuration described by the NAM in Equation (52) is shown in Figure 14. This pathological section is called the pathological differential current conveying cell. The inverting counterpart for this section can be easily obtained by either exchanging nodes w and x or exchanging nodes y and z.

Now, consider the special cases of the general NAM description for current relationships in Equation (47), in which some of the currents $I_1, I_2, I_3, \ldots, I_n$ are interrelated. The interrelated

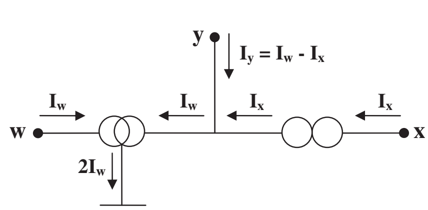

Figure 13. Pathological current differencing cell.

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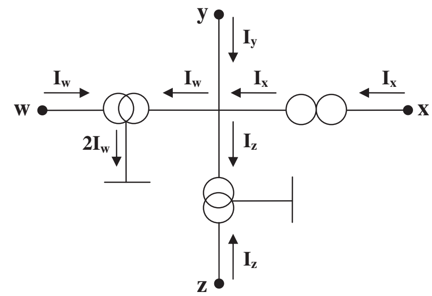

Figure 14. Pathological differential current conveying cell.

currents should be described using the same set of infinity-variables and thus a smaller number of infinity-variables sets will be needed to describe the overall current relation implied in the analog signal-processing feature. As an illustrative example focusing on the case of interrelated currents, consider the following relation describing the replication of the current entering terminal x into two currents coming out from terminals y and z

$$I_z = I_y = -I_x \tag{53}$$

without constraints on the voltages at nodes x, y, and z. In order to obtain a NAM stamp using infinity-variables for a pathological configuration describing this inverting current replication, the infinity-variables in the NAM representation should satisfy the relation in Equation (53). Since Equation (53) involves only three interrelated currents, it can be satisfied within a NAM using a single set of infinity-variables occupying three rows as described by

$$\begin{array}{c} \\ x \\ y \\ z \\ \\ \end{array} \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \infty_1 & \cdot & \cdot \\ \cdot & \cdot & -\infty_1 & \cdot & \cdot \\ \cdot & \cdot & -\infty_1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{54}$$

In order to achieve an alternative set of infinity-variables composed of one or more NAM representations for pathological elements, consider a modification step in which the coefficient of the ∞_1-variable in row x is multiplied by a row scale factor 2. This will yield a NAM description for a floating CM connected at the involved nodes (x, y, and z) with its reference terminal connected to node x, as described by

$$\begin{array}{c} \\ x \\ y \\ z \\ \\ \end{array} \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \boxed{2}\,\infty_1 & \cdot & \cdot \\ \cdot & \cdot & -\infty_1 & \cdot & \cdot \\ \cdot & \cdot & -\infty_1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{55}$$

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In the above NAM representation, the magnitude of the current at node x due to ∞_1-variables is double that at each of nodes y and z. To maintain the condition in Equation (53), the unity factor multiplied by the x current due to the ∞_1-variables should be restored. A possible solution would be to connect the reference terminal for the CM in Equation (55) to another floating node, rather than node x, for which the current due to the ∞_1-variables is double that at node x. Hence, the magnitude of each of the currents due to the ∞_1-variables at nodes x, y, and z will be equal to half the magnitude of the current due to the ∞_1-variables at this new reference node. This can be achieved using an additional set of infinity-variables (∞_2) representing another CM connected like the fourth special case in Section 2, shown in Figure 6(d), as described by

$$\begin{array}{c} \\ w \\ x \\ \\ \end{array}\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & -2\infty_2 & \cdot \\ \cdot & \infty_2 & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \tag{56}$$

In this CM description, the current leaving (entering) node w is double the current entering (leaving) node x due to the ∞_2-variables. Now, the target is to make the current entering (leaving) node x due to the ∞_2-variables equal to the current leaving (entering) each of nodes y and z due to ∞_1-variables, so that to satisfy the condition in Equation (53). This is possible by connecting the reference terminal for the CM described in Equation (55) to node w in Equation (56) rather than node x, as described by

$$\begin{array}{c} \\ w \\ x \\ y \\ z \\ \\ \end{array}\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -2\infty_2 & \cdot & 2\infty_1 & \cdot \\ \cdot & \infty_2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -\infty_1 & \cdot \\ \cdot & \cdot & \cdot & -\infty_1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{57}$$

Node w should be connected only to the reference terminals of the two CMs. Thus, according to KCL at node w, the current leaving (entering) node w due to the ∞_2-variables will be entering (leaving) node w due to the ∞_1-variables. Since this current is equal to double the current at node x due to the ∞_2-variables, the magnitudes of the currents flowing through nodes x, y, and z are equal. Thus, condition imposed by Equation (53) is satisfied and the resulting NAM stamp in Equation (57) represents a pathological configuration based on two CMs ideally describing the inverting replication for the current at terminal x into two currents at terminals y and z. An alternative scenario of modification steps would be as follows. Consider the modification on the infinity-variables in Equation (54) for which the infinity-variables in rows y and z are moved to a new row w. The resulting form for the column containing the infinity-variables, shown in Equation (58), describes a CM connected like the fourth special case in Section 2 and its reference terminal

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connected at node w

$$\begin{array}{c} \\ w \\ x \\ y \\ z \\ \\ \end{array}\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0 & \cdot & \cdot \\ \cdot & \cdot & \infty_1 & \cdot & \cdot \\ \cdot & \cdot & -\infty_1 & \cdot & \cdot \\ \cdot & \cdot & -\infty_1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \rightarrow \begin{array}{c} \\ w \\ x \\ y \\ z \\ \\ \end{array}\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -2\infty_1 & \cdot & \cdot \\ \cdot & \cdot & \infty_1 & \cdot & \cdot \\ \cdot & \cdot & 0 & \cdot & \cdot \\ \cdot & \cdot & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{58}$$

In order to maintain the condition in Equation (53) using the resulting infinity-variables in (58), the current due to the ∞_1-variables at node w must be equal to double the current due to the ∞_1-variables at each of nodes y and z, as expressed by

$$I_w = 2I_y = 2I_z \tag{59}$$

This can be achieved using an additional set of infinity-variables (∞_2), as shown in Equation (31), representing another floating CM whose reference terminal is connected to node w while the other two terminals are connected to nodes y and z.

$$\begin{array}{c} \\ w \\ x \\ y \\ z \\ \\ \end{array}\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -2\infty_1 & \cdot & 2\infty_2 & \cdot \\ \cdot & \infty_1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -\infty_2 & \cdot \\ \cdot & \cdot & \cdot & -\infty_2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{60}$$

By this way, the current entering (leaving) node x due to the ∞_1-variables is doubled at node w and then distributed equally by the ∞_2-variables into two leaving (entering) currents at nodes y and z. The resulting NAM stamp in Equation (60) is exactly the same as the previously derived one in (57). The pathological configuration described by the NAM stamps in Equations (57) and (60) can be represented using CMs as shown in Figure 15. This pathological section is called the pathological current replication cell. The ideal generation of non-inverting and inverting replicas for an input current can be also described using this section by applying the input current at terminal y or z.

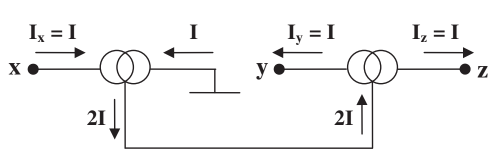

Figure 15. Pathological current replication cell.

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5. APPLICATION EXAMPLES

The pathological configurations derived so far throughout the previous examples are describing common analog properties featured by most of the recent analog building-blocks. In these systematic derivations, the modifications steps were mainly employing the NAM descriptions for mirror elements to achieve feasible pathological configurations and accordingly the resulting pathological sections are mainly based on mirror elements. Hence, the capabilities of mirrors as basic and versatile pathological elements are further demonstrated as they provide general descriptions capable of achieving ideal representations for various analog-processing features. In this section, the pathological representations of some analog building-blocks using the pathological sections derived in the previous section are presented.

In order to obtain an ideal representation for an analog building-block, a pathological element or configuration is used to describe every feature between the device terminals (assuming ideal characteristics). It is known that all the nullators (or VMs) and norators (or CMs) within the pathological representation of an active device should occur in pairs so that all the voltages and currents are uniquely and definitely determined. If every feature in the ideal description of the active device can be represented using a single pathological element, like those included in Table I, then all the nullators (or VMs) and norators (or CMs) used to represent the features between the terminals of the ideal active device will constitute complete sets of pathological pairs. However, in case of devices possessing more complicated features that are described using pathological configurations consisting of more than one pathological element, like the derived pathological sections in the previous examples, there is a possibility that the pathological sections and/or elements used to represent the ideal features between the terminals of the active device do not occur as complete sets of pathological pairs. Thus, dummy pathological elements should be added so that to complete the sets of pathological pairs and the locations of the added dummy elements are specified to maintain the assumed ideal characteristics of the active device.

5.1. Balanced output second generation current conveyor

The balanced output CCII [22] is a four-port active device that is defined by the following matrix equation:

$$\begin{bmatrix} I_Y \\ V_X \\ I_{Z+} \\ I_{Z-} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_Y \\ I_X \\ V_{Z+} \\ V_{Z-} \end{bmatrix} \qquad (61)$$

The symbolic and pathological representations for the balanced output CCII are shown in Figure 16. The pathological description for the balanced output CCII employs a nullator and a pathological current replication cell at the terminals of the active device representation. A dummy grounded nullator is added at the reference terminal of the current replication cell to form a pair with one of the CMs; meanwhile, the other CM forms a pair with the nullator connected between terminals Y and X. The location of the added dummy nullator is specified so that to maintain the device characteristics defined by Equation (61).

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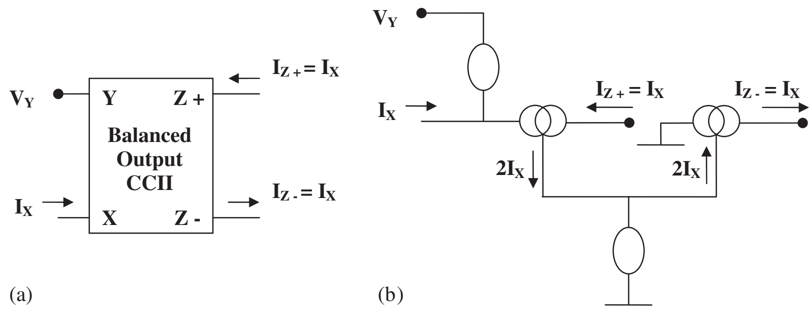

Figure 16. The balanced output CCII: (a) symbol and (b) nullor-mirror representation.

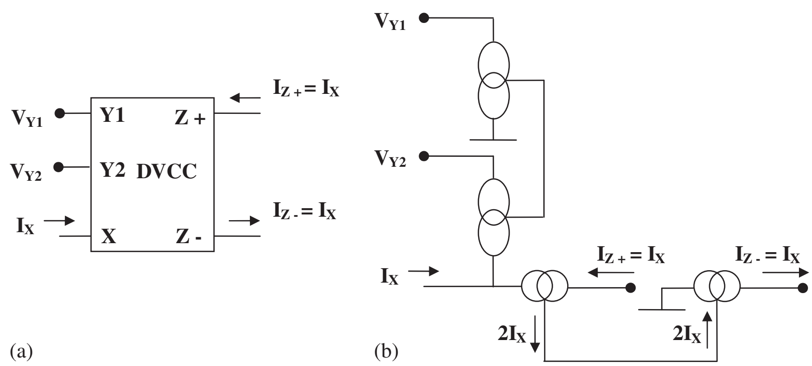

Figure 17. The differential voltage current conveyor: (a) symbol and (b) nullor-mirror representation.

5.2. *Differential voltage current conveyor*

The differential voltage current conveyor (DVCC) [23] is a five-port active device that is defined by the following matrix equation:

$$\begin{bmatrix} V_X \\ I_{Y1} \\ I_{Y2} \\ I_{Z1} \\ I_{Z2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_X \\ V_{Y1} \\ V_{Y2} \\ V_{Z1} \\ V_{Z2} \end{bmatrix} \tag{62}$$

The symbolic and pathological representations for the DVCC are shown in Figure 17. The pathological description for the DVCC employs the pathological differential voltage and current replication cells at the terminals of the active device representation. The two pathological sections used in the ideal representation of the DVCC constitute two complete pathological pairs and hence no dummy pathological elements are needed.

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5.3. Fully differential second generation current conveyor

The fully differential CCII [24] is an eight-terminal analog building-block that is defined by the following matrix equation:

$$\begin{bmatrix} V_{X+} \\ V_{X-} \\ I_{Z+} \\ I_{Z-} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{X+} \\ I_{X-} \\ V_{Y1} \\ V_{Y2} \\ V_{Y3} \\ V_{Y4} \end{bmatrix} \tag{63}$$

The symbolic and pathological representations for the fully differential CCII are shown in Figure 18. The pathological description for the fully differential CCII employs two CMs and two pathological differential voltage conveying cells at the terminals of the active device representation. A dummy norator is connected to each pathological differential voltage conveying cell to complete the pairs of pathological elements. The locations of the added dummy norators are specified so that to maintain the device characteristics defined in Equation (63).

5.4. Current differencing transconductance amplifier

The current differencing transconductance amplifier (CDTA) [25] is a five-port analog building-block shown symbolically in Figure 19(a). The CDTA is basically realized as a cascaded connection of the Modified Differential Current conveyor introduced in [26] and a transconductance amplifier [27]. The CDTA is defined by the following matrix equation:

$$\begin{bmatrix} V_{x1} \\ V_{x2} \\ I_Z \\ I_{w+} \\ I_{w-} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & G_m \\ 0 & 0 & -G_m \end{bmatrix} \begin{bmatrix} I_{x1} \\ I_{x2} \\ V_z \end{bmatrix} \tag{64}$$

For an ideal CDTA, $G_m \to \infty$. The pathological representation for the CDTA is shown in Figure 19(b). The pathological description for the CDTA employs a pathological current differencing cell, three nullators, and a norator at the terminals of the active device representation. The pathological configuration and elements used in the ideal representation of the CDTA constitute three complete pathological pairs and hence no dummy pathological elements are needed.

6. CMOS BUILDING BLOCKS REALIZATIONS

Although the main paper objective is to provide novel approach for using the pathological mirror elements in obtaining the ideal representation of linear active building-blocks, it may be suitable

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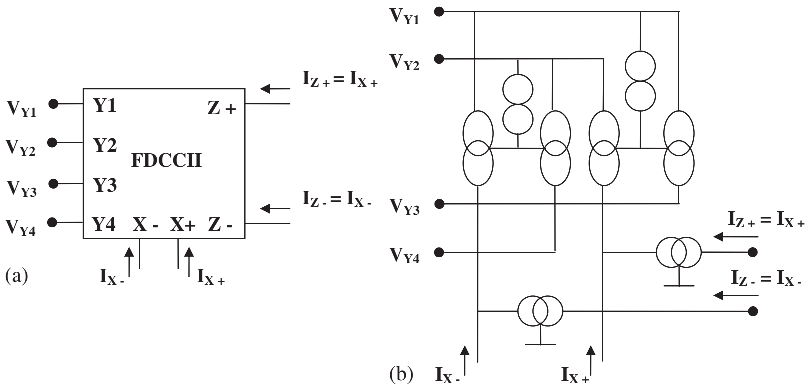

Figure 18. The fully differential CCII: (a) symbol and (b) nullor-mirror representation.

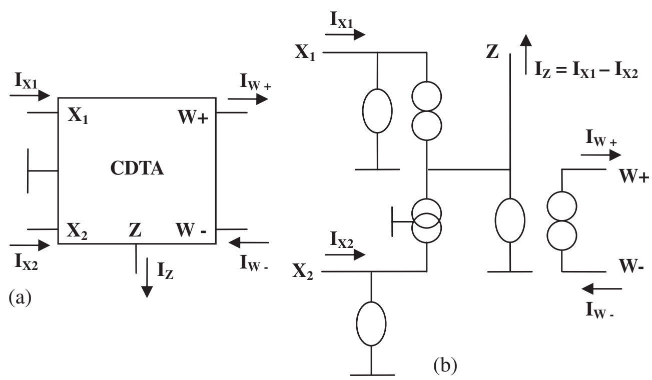

Figure 19. Current differencing transconductance amplifier: (a) symbol and (b) nullor-mirror representation.

to show a practical CMOS realization of one of the building-blocks considered in the previous section. The DVCC is taken as an example to demonstrate the practicality of CMOS circuits in providing the subtraction of two analog signals with simulation results that are very close to ideal expected ones.

Figure 20(a) represents the CMOS DVCC circuit [23], the transistor aspect ratios is given in Table III based on the 0.5 μm CMOS model from MOSIS. The supply voltages used are ± 1.5 V and $V_{B1} = -0.52$ V and $V_{B2} = 0.33$ V. The Spice simulation results are shown in Figure 20(b) representing the magnitude and phase characteristic of V_x, V_{Y1}, and V_{Y2}. As seen from the simulation results the voltage at X follows the differential voltage $(V_{Y1} - V_{Y2})$ up to 10 MHz with almost no deviation from the ideal response.

Other practical CMOS realizations for the other building blocks considered in this paper are not included to limit the paper length.

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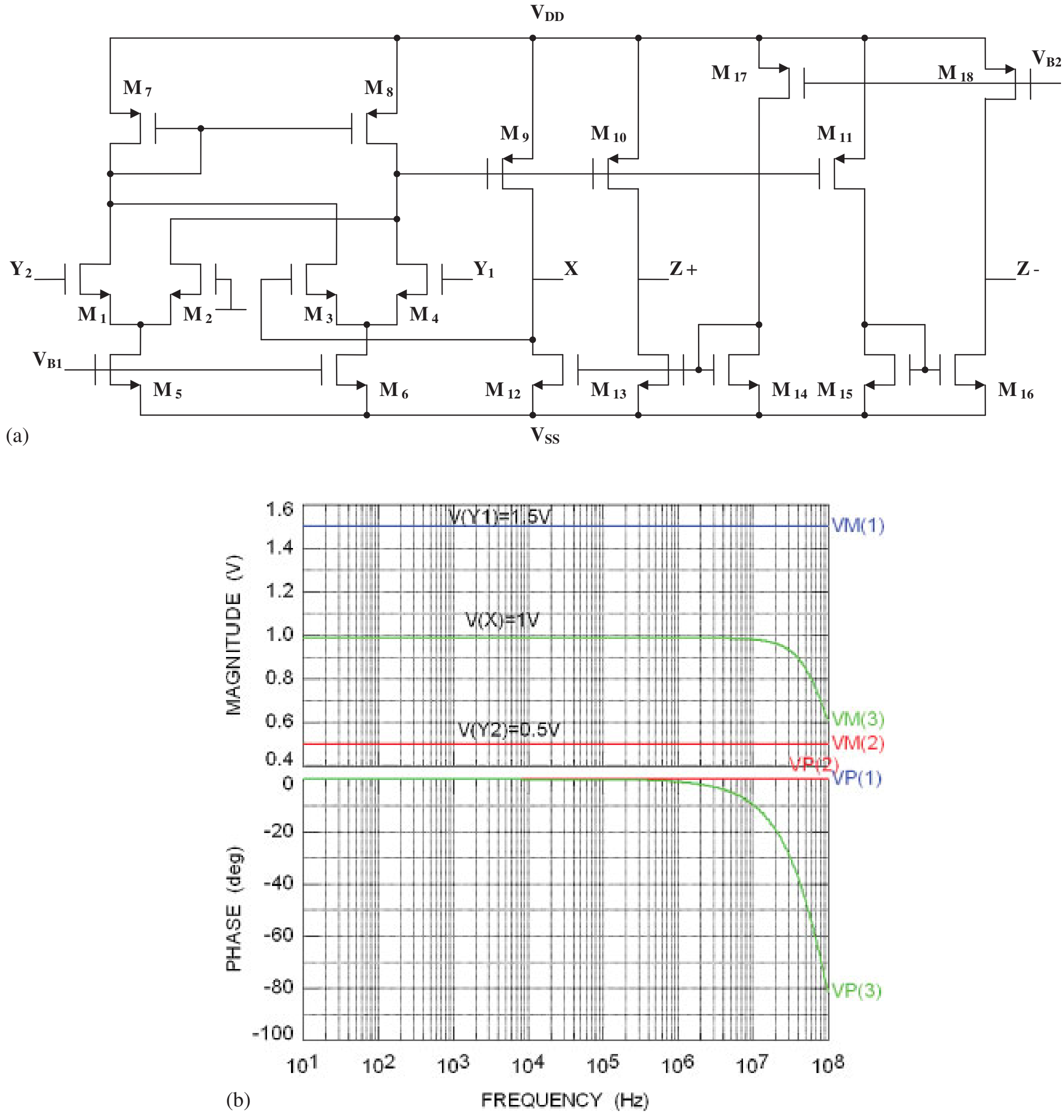

Figure 20. (a) CMOS realization of the DVCC of Figure 18 [23] and (b) simulation results of the DVCC of Figure 20(a).

7. CONCLUSIONS

The NAM descriptions for floating mirror elements using infinity-variables have been derived. The derived NAM descriptions have been used to show that the nullator and norator represent special cases from the floating VM and CM, respectively. Hence, the mirror elements can be considered as basic pathological elements with general terminal properties. A formal procedure

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Table III. Dimensions of the MOS transistors in the DVCC of Figure 20(a).

NMOS transistors	W (μm)/L (μm)
M_1, M_2, M_3, and M_4	2.5/1
M_5 and M_6	8/1
M_{12}, M_{13}, M_{14}, M_{15}, and M_{16}	20/2.5
PMOS transistors	W (μm)/L (μm)
M_7 and M_8	10/1
M_9, M_{10}, M_{11}, M_{17}, and M_{18}	40/2

has been proposed to derive pathological representations for analog signal-processing features in a systematic way. This systematic approach provides the means to derive complicated pathological configurations describing various analog features that involve differential or multiple single-ended signals in an easy and direct way. The effectiveness of this systematic procedure is basically owing to the versatility of the NAM descriptions for the mirror elements. The systematic approach has been illustrated through the derivation of pathological sections describing common analog signal-processing features; like conversion between differential and single-ended voltages, differential voltage conveying, current differencing, differential current conveying, and current replication. The derived pathological configurations are mainly based on mirror elements and hence the capabilities of the mirror elements in the ideal representation of active devices are further illustrated. The pathological descriptions for active devices possessing these common analog features have been presented as application examples and the concept of dummy pathological elements needed to complete the pathological pairs in an ideal representation has been introduced. It is emphasized that the pathological CM is different from the regular unidirectional CM [28].

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