CCII based KHN fractional order filter

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***Abstract*—This work aims to generalize the analysis of the fractional order filter to work for the low-pass, band-pass and high-pass responses. So, general expression for the maximum and minimum frequency points and the half power frequency points will be derived. In addition, the effect of the transfer function parameters on the filter poles and hence the stability is introduced. Besides, the effect of the fractional orders on the frequency response will be presented. Finally, to verify the numerical analysis and the proposed design procedure, circuit simulation will be used.**

***Index Terms*- Fractional, filters, CCII, KHN filter, fractional elements.**

I. INTRODUCTION

Recently, hundreds of research papers on the applications of the fractional calculus have been published in numerous applications. Accordingly, electronic circuits are traditionally classified as first-order, second-order or nth-order circuit where n is an integer number. Yet, these integer order differential equations represents can be considered a special case from the fractional order circuit theory [1].

Using fractional order differential equations proves to be useful in explaining many phenomena; examples include stability [2-3], diffusion [4], transmission lines [5], biological cells, tissues, diseases and medication [6], and many others. In addition, the fractional order calculus has been used with many circuits like filters [7-12], and oscillators [13]. The fractional-order element is called constant phase element or fractance [14 - 16] and its impedance is characterized by:

$$Z(s) = k_o s^{\alpha} = k_o (j\omega)^{\alpha} \tag{1}$$

where k_o is a constant and α is the fractional order. Then the magnitude and phase of Z becomes:

$$|Z| = k_o \omega^{\alpha} \tag{2.a}$$

$$\angle Z = \alpha\pi/2 \tag{2.b}$$

As $\alpha = 1,\ 0,\ -1$ the impedance Z represents an inductor; resistor; capacitor respectively. This work seeks to generalize the transfer function of the filter to the fractional domain for the three frequency responses {low-pass, band-pass and high-pass} to increase the design flexibility through the extra parameters. In addition, the proposed transfer function should work as a low-pass filter, band-pass and high-pass at the same time at different orders. So, the proposed design procedure and equations is presented in the second section. Also, this section contains subsection to discuss the critical frequency points and another one to present some case study for the filter design. After that, section III introduces circuit simulation to verify the proposed design procedure. Finally, the conclusion is presented in section IV.

II. FILTER TRANSFER FUNCTION

A fractional order Filters is characterized by its transfer function that contains rational power of s in its characteristic polynomial. The general transfer function of a fractional-order filter whose characteristic equation D(s) depends on two fractional capacitors of different orders α and β is given by:

$$T(s) = \frac{d s^{\gamma}}{s^{\beta} + a s^{\alpha} + b} \tag{3}$$

where $a, b,\ d$ are constants and α, β and γ are the fractional orders. In addition $\alpha \le 2$ and $(4 > \beta > \alpha, \gamma)$. The stability of this system depends on both α, β besides the parameters a, b. Although the fractional order γ does not impact the filter stability, but it affect the filter response. A summary of the stability analysis for some cases of the fractional orders were presented in table I, where the filter is stable for $a < 0$ at some combinations of the fractional orders (α, β) and specially for $\beta < 2$. This means that the filter can be implemented using negative impedance. On the other hand, for $\beta \ge 2$ the filter is stable only for $a > 0$. In addition, for $b = 1$ and at some case of the fractional orders (α, β) the filter transfer function of (3) does not have poles in the physical s-plane with changing value of a as shown in Table I. Moreover, when $a \ge 1$ and $\beta < 1$ the filter does not have any poles in the physical s–plane for any value of b as shown in table I. Figure 1 displays the effect of a and b on the system poles at different combinations of α, β where three different stability regions appear as follow: the stable region, the unstable region and the unphysical region for the poles. From (3), the magnitude of the characteristic equation can be obtained as follow:

$$|D(\omega, \alpha, \beta)|^2 = \omega^{2\beta} + a^2\omega^{2\alpha} + 2a\omega^{\alpha+\beta}\cos\left(\frac{(\beta-\alpha)\pi}{2}\right) + 2ab\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right) + 2b\omega^{\beta}\cos\left(\frac{\beta\pi}{2}\right) + b^2 \tag{4}$$

As a special case, for equal orders $\beta = 2\alpha$ then (4) will be the same as that presented in [8].

TABLE I. SUMMARY OF THE IMPACT OF THE TRANSFER FUNCTION PARAMETERS ON THE FILTER STABILITY

	b=1			a=1		
(α,β)	Stable range	Unstable range	No poles in the physical region	Stable range	Unstable range	No poles in the physical region
(0.4,0.8)	$-1.6 \le a \le -0.7$	$a < -1.6$	$a > -0.7$		$b \le -0.1$	$b > -0.1$
(0.4,1.2)	$-1.1 \le a \le 0.8$	$a < -1.1$	$a > 0.8$	$b \ge 1.3$	$b < 1.3$	
(0.8,1.2)	$-1.17 \le a \le 0.85$	$a < -1.17$	$a > 0.85$	$b \ge 1.62$	$b < 1.62$	
(1.2,2)	$a \ge 0.1$	$a < 0.1$		$b \ge 0.1$	$b < 0.1$	
(0.4,1.6)	$a \ge -0.69$	$a < 0.69$		$b \ge 0.1$	$b < 0.1$	
(1.2,2.8)	$a \ge 1.4$	$a < 1.4$		$0.1 \le b \le 0.6$	$b < 0.1 \& b > 0.6$	
(1.6,3.4)	$a \ge 1.94$	$a < 1.94$		$0.1 \le b \le 0.28$	$b > 0.1 \& b > 0.28$	

To complete the study of the fractional order filter, the critical frequency points like the maximum and minimum frequencies and the half power frequencies will be discussed in the following subsections [8]. After that some case studies will be introduced.

A. Critical frequency points

In filter design, there are two important frequency points that should be determined to enhance the filter design. These frequency points are the maximum/minimum ω_m and the half power frequencies(ω_h). The study of ω_m determines the ripples behavior of the pass-band of the filter. By differentiation of (4), the value of ω_m is the solution of the following equation:

$$\omega_m^{2\beta} + a^2\frac{\gamma-\alpha}{\gamma-\beta}\omega_m^{2\alpha} + \frac{2\gamma-\alpha}{\gamma-\beta}ab\cos\left(\frac{\alpha\pi}{2}\right)\omega_m^{\alpha} + \gamma b^2 + \frac{2\gamma-\beta}{\gamma-\beta}b\cos\left(\frac{\beta\pi}{2}\right)\omega_m^{\beta} + \frac{2\gamma-\alpha-\beta}{\gamma-\beta}a\cos\left(\frac{(\beta-\alpha)\pi}{2}\right)\omega_m^{\alpha+\beta} = 0 \quad (5)$$

From (5), the value of ω_m depends on the transfer function parameters $\{a, b\}$ besides the fractional orders $\{\alpha,\beta,\gamma\}$. This wide range of variables increases the design degree of freedom and hence the design flexibility. Comparing with the traditional second order system where $(\alpha,\beta,\gamma) = (1,2,2)$, it can be seen from (5) that $\omega_m = \frac{\sqrt{2}c}{\sqrt{2c-a^2}}$, which satisfies the peak frequency equation of [8]. It is very interesting to note here that (5) is a generic equation for the three well known filter responses (low-pass, band-pass and high-pass). So, the value of ω_m is calculated using (5) for the low-pass filter when $\gamma = 0$ and for the band-pass when $\gamma = \alpha$ and also for the high-pass at $\gamma = \beta$. The change of ω_m with respect to α at different values of β and α at $\gamma = 1.2$ is shown in Fig 2. The analysis of (5) generates only one solution for the cases presented in Fig 2. Also, the change of ω_m with respect to β and γ is shown in Fig 3 (a, b) respectively.

The second important frequency point is the half power frequency (ω_h) which is used to determine the filter bandwidth. Then, the half power frequency at which the power drops to half the pass-band power. Yet, in this case it is very difficult to expect the value of the transfer function at the pass-band. So for simplicity, the value of the transfer function at ω_m numerically and used as a constant in the analysis of (ω_h). Consequently, to calculate the value of (ω_h), the following relation is used.

$$\omega_h^{2\beta} + a^2\omega_h^{2\alpha} + 2ab\cos\left(\frac{\alpha\pi}{2}\right)\omega_h^{\alpha} + 2b\cos\left(\frac{\beta\pi}{2}\right)\omega_h^{\beta} + 2a\cos\left(\frac{(\beta-\alpha)\pi}{2}\right)\omega_h^{\alpha+\beta} - \frac{d^2}{\chi}\omega_h^{2\gamma} + b^2 = 0 \quad (6)$$

where χ is given as follow:

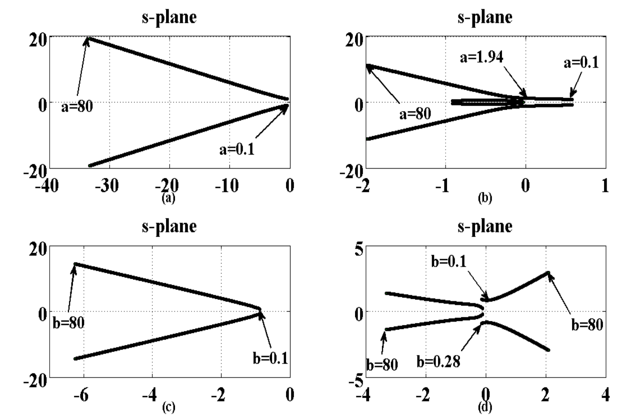


Figure 1. Movement of the poles with respect to **a** and **b**, (a) $\alpha = 0.4, \beta = 1.6, b = 1$,(b) $\alpha = 1.6, \beta = 3.4, b = 1$, (c) $\alpha = 0.4, \beta = 1.6, a = 1$, and (d) $\alpha = 1.6, \beta = 3.4, a = 1$

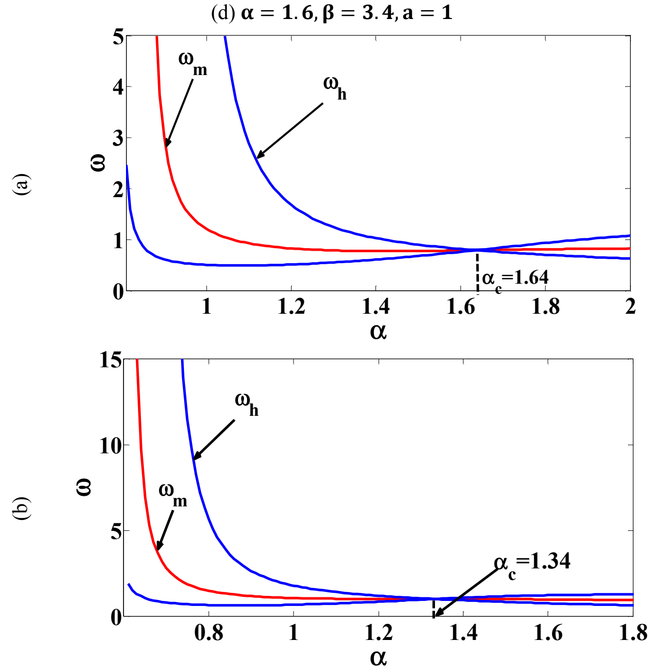


Figure 2. Impact of α on ω_m and ω_h at $a = b = d = 1$and $\gamma = 1.2$,(a) $\beta = 1.5\alpha$, and (b)$\beta = 2\alpha$.

$$\chi = \frac{1}{2}|T(j\omega_m)|^2 \quad (7)$$

So, to calculate the value of ω_h, the value of χ is calculated first from (7) then use (6) to determine the value of ω_h. Numerical analysis for (6) is depicted in Fig 2 for different values of α. In addition, the impact of β and γ on the change of ω_h is presented in Fig 3(a, b) respectively.

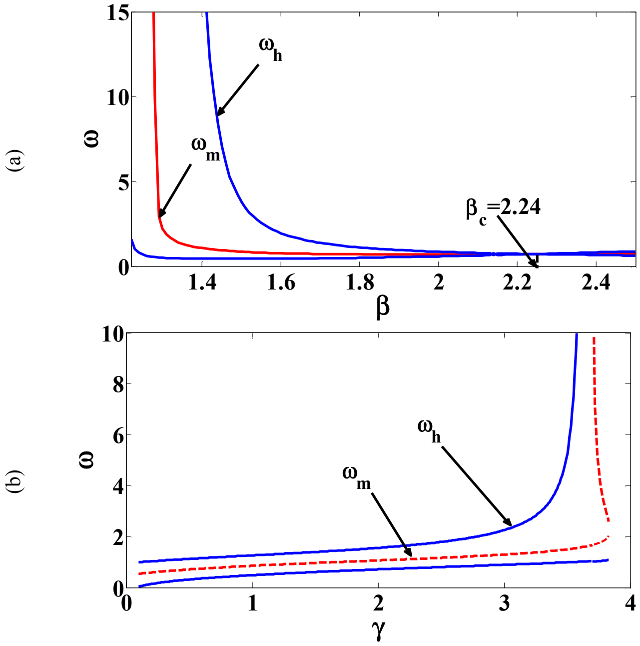


Figure 3. Impact of the fractional orders β and γon ω_m and ω_h at $a = b = d = 1$and,(a) $\alpha = 0.8\beta$ and $\gamma = 1.2$, and (b)$\beta = 3.7$ and $\alpha = 1.4$

The solution of (6) produces two values of ω_h and the difference between these two solutions starts very large at small values of α. Then this difference starts to decrease as the value of α increases as shown in Fig 2(a, b). It is important to note hare that, the curves of ω_h and the curve of ω_m intersects at a certain value for the fractional order α. At this point of the fractional order α, the filter starts to be unstable which means that the filter poles at this order lies on the imaginary axis. So this value of α is called $\alpha_c$and for the filter to be stable, the fractional order $\alpha < \alpha_c$. So, the value of $\alpha_c$can be defined as the fractional order at which $\omega_h = \omega_m$ and at which the filter begins to be unstable and it can be obtained numerically by solving (5) and (6) together. From Fig 2(a, b), the value of $\alpha_c$depends also on the value of β whereas the value of β increases the value of α_c decreases where for $\beta = (1.5\alpha, 2\alpha) \rightarrow \alpha_c = (1.65, 1.34)$. In addition, the effect of βon ω_h is similar to α as shown in Fig 3(a). Hence there is also a critical value of β and is called β_c. So, by choosing the value of β larger than the value of β_c, the filter becomes unstable. Finally, because the fractional order γ exists only at the transfer function numerator, it has no effect on the filter poles and hence the filter stability. So, the condition of $\omega_h = \omega_m$does not happen with the change of γ as shown in Fig 3(b)

B. Case study

As the value of the fractional order γ affects the number and the position of the transfer function zeros, the frequency response of (3) depends mainly on this value. So, for very small values of γ (close to zero) the filter response can be considered low-pass filter. With γ continue to increase, the zeros of (3) starts to moves in the frequency domain and hence the filter response changes from the low-pass to the band-pass when γ beomce very close to α. Then, the filter response tends to be closer to the high-pass with the increase in γ because the zeros become approximately at infinity especially at $\gamma = \beta$ as shown in Fig 4. The effect of γ on the frequency response of (3) is presented in Fig 4 where the filter response introduces the low-pass, band-pass and high-pass response when $\gamma = (0, \alpha, \beta)$ respectively. To design the fractional order filter, it is required to calculate the value of the parameters a, b, d, α, β and γ which satisfies the required filter response. This large number of the design variables increases the design degree of freedom and consequently the design flexibility. Different frequency responses at different values of the fractional orders (α, β, γ) are shown in Fig 5 at $a = b = 1$. In addition, the poles of the systems of interest are given on table II. As the value of α becomes closer to α_c($= 1.65$ in this case) the damping in the filter response increase and poles becomes near the imaginary axis as shown in Fig 5 for $(\alpha, \beta) = (1.5, 2.25)$ and its poles are given in Table II. Yet, as the value of α or β become smaller than α_c or β_c the damping of the filter decrease and also the poles move away from the imaginary axis in the direction of the stable region as depicted in Fig 5 and Table II.

III. CIRCUIT SIMULATION

To prove the reliability of the proposed design procedure, it becomes necessary to use it with the circuit design. So, the aim of this section is to apply the proposed design procedure on a practical filter like CCII based KHN filter. A finite element approximation of [14-16] is used here to model the fractional order element and it is demonstrated in the sub-figure of Fig 6(a).

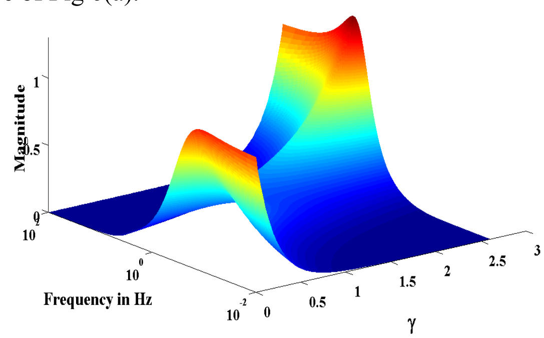


Figure 4. Magnitude change with respect **γ** at a=b=d=1 and $\boldsymbol{\alpha = 1.2}$ and $\boldsymbol{\beta = 2.6}$.

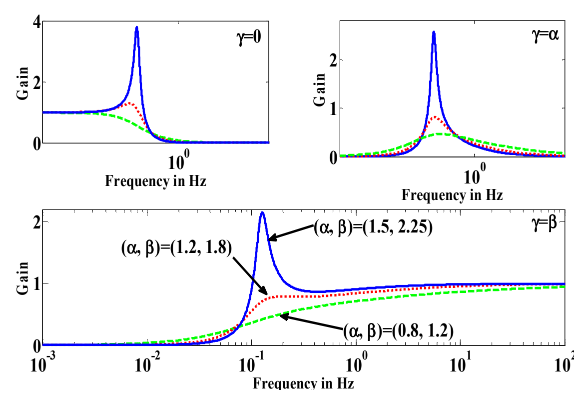


Figure 5. Magnitude change at different values of $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ for $\mathbf{a = b = d = 1}$

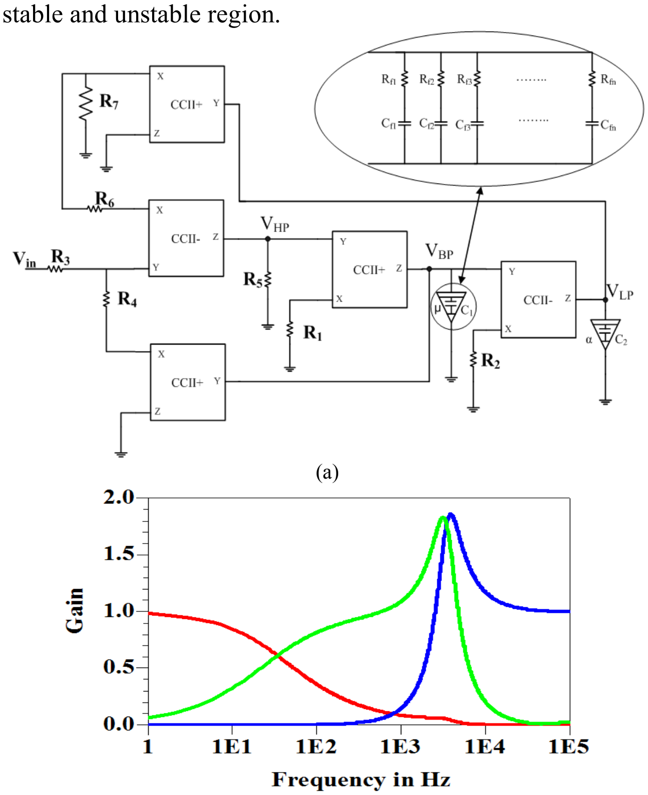
TABLE II. FILTER POLES FOR THE CASES PRESENTED IN FIG 5

(α,β)	poles
(1.5,2.25)	$-0.1102 \pm 0.7672i$
(1.2,1.8)	$-0.5 \pm 0.866i$
(0.8,1.2)	-0.5

The integer order CCII based KHN filter was first proposed in [17-18]. So, these circuits are converted to fractional order circuits by only replacing the capacitors of the integer order filter with two fractional elements of different orders (μ,α) is shown in Fig 6(a). The main advantage of this circuit is that it can give the three responses of interest simultaneously as shown in Fig 6(a). The relation between the transfer function parameters and the circuit elements is given in Table III. To design the fractional order KHN filter, the value of the parameters a, b, and d is calculated from solving (5) and (6) at the required ω_h and ω_m. Then these values are used to determine the value of the circuit components from the relations presented in Table III. As shown in Fig 6(b), the circuit simulation is close to the numerical analysis presented before.

IV. CONCLUSION

Fractional order filters can provide some additional design flexibilities compared to conventional integer order filters due to the increase in the design parameters. In addition, an approximate formula for ω_h and ω_m was derived and solved numerically. Also, the effect of the fractional orders on the stability is presented and two critical values for the fractional orders (α,β) are defined which defines the limit between the stable and unstable region.



(b)

Figure 6. (a) CCII-based fractional order KHN filter, and (b) circuit simulation for $(\boldsymbol{\alpha},\boldsymbol{\beta}) = (\mathbf{1.5},\mathbf{2.25})$.

TABLE III. RELATION BETWEEN THE TRANSFER FUNCTION PARAMETERS AND THE CIRCUIT ELEMENTS

Param.	Low-pass	Band-pass	High-pass
a	$\frac{1}{C_1R_1}\frac{R_3}{R_6R_7}\frac{R_6+R_7}{R_3+R_4}$		
b	$(R_5/R_6)/(R_1R_2C_1C_2)$		
d	$\frac{1}{C_1R_1C_2R_2}\frac{R_4}{R_6R_7}\frac{R_6+R_7}{R_3+R_4}$	$\frac{1}{C_1R_1}\frac{R_4}{R_6R_7}\frac{R_6+R_7}{R_3+R_4}$	$\frac{R_4}{R_6R_7}\frac{R_6+R_7}{R_3+R_4}$
γ	0	α	$\alpha+\mu$
β	$\mu+\alpha$		
α	α		

Finally, a circuit simulation using ADS is used to verify the numerical analysis and a good matching is found between the numerical analysis and the circuit simulation.

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