Active Compensation of RC Oscillators

Aktive Kompensation von RC-Oszillatoren

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Abstract:

Due to the frequency dependent nature of the Op-Amp gain, the actual performance of the RC oscillator differs from the ideal performance. This can be overcome, somehow by the use of compensated amplifiers. In this paper, a general analysis of active compensation is introduced and many compensated amplifiers are used to approximate the performance to the ideal level. Experimental results show excellent agreement with theory.

Übersicht:

Wegen der Frequenzabhängigkeit der Verstärkung von Operationsverstärkern weichen die tatsächlichen Eigenschaften von RC-Oszillatoren vom Idealwert ab. Solche Abweichungen können durch die Verwendung von kompensierten Verstärkern verringert werden. In diesem Beitrag wird die aktive Kompensation allgemein analysiert. Zahlreiche kompensierte Verstärkerschaltungen zur Annäherung der Eigenschaften an den Idealzustand werden gezeigt. Experimentelle Ergebnisse stimmen ausgezeichnet mit der Theorie überein.

Für die Dokumentation:
RC-Oszillator / Verstärker / Kompensation

1. Introduction

The actual performance of any RC oscillator differs in many ways from the ideal. At low frequencies, there is no noticeable difference; however, as the frequency in creases, the deviation from the ideal case also increases. This is due to the frequency dependent nature of the Op-Amp gain [1]. These differences may be especially noticed in the condition of oscillation, the frequency of oscillation, and the amplitude of the output signal. To minimize the undesired deviations in these three parameters, compensated amplifiers may be used. Under some conditions, it is possible to extend the useful operating frequency range and to approximate the actual performance to the ideal. The following study shows how this can be done.

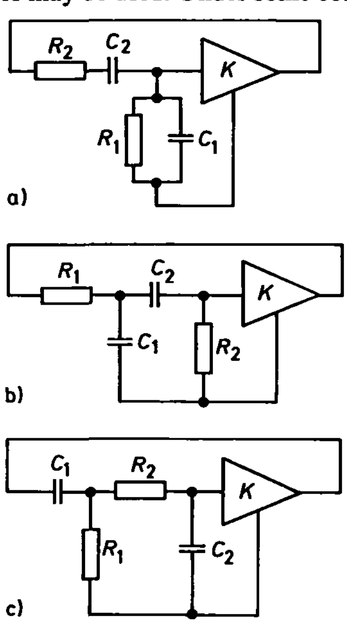

Fig. 1: Three types of oscillators
a) The Wien-bridge oscillator
b) The second RC-oscillator
c) The third RC-oscillator

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2. The ideal 2nd-order RC oscillator

Three different RC oscillators are shown in **Fig. 1.** The very popular KRC-Oscillator, known as The Wien-Bridge Oscillator, is that one shown in Fig. 1a. The circuit consists of an amplifier $K(s)$ and a bandpass feedback provided through the RC network. The two other circuits are generated from the same 2nd-order RC network of Fig. 1a [2]. They are related by RC-CR transformation.

The transfer function of the RC network $T_{RC}(s)$ can be written for the three networks in the form

$$1/T_{RC}=\frac{V_0}{V_i}=K_0+\mathrm{j}\,\frac{\omega^2-\omega_0^2}{\omega\omega_0(n/\sqrt{AB})} \tag{1}$$

where

$$K_0=(1+A+B)/n, \tag{2}$$

$$\omega_0=1/(RC\sqrt{AB}), \tag{3}$$

$$R_2/R_1=A,\ R_1=R, \tag{4}$$

$$C_1/C_2=B,\ C_2=C, \tag{5}$$

and

$$n=\begin{cases}1 & \text{for the circuit in Fig. 1a,} \quad (6)\\ A & \text{for the circuit in Fig. 1b,} \quad (7)\\ B & \text{for the circuit in Fig. 1c.} \quad (8)\end{cases}$$

When the circuit oscillates, the unity loop-gain condition must be satisfied, i.e.

$$LG(s)=K(s)\cdot T_{RC}(s)=1$$

or

$$K(s)=1/T_{RC}(s). \tag{9}$$

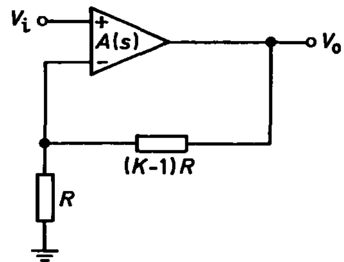

Fig. 2: The uncompensated amplifier

With ideal Op-Amps, $K(s)=K_0$, and equations (2) and (3) give the condition and the frequency of oscillation, respectively. At this frequency, the loop gain has a zero phase and unity magnitude which are the required criteria for oscillation.

3. The single Op-Amp realization of $K(s)$

The conventional single Op-Amp realization of $K(s)$ is shown in **Fig. 2.** With the finite gain-bandwidth product of the Op-Amp, *GB*, the transfer function of the amplifier becomes

$$K(s)=K_1\frac{1}{1+K_1 s\tau} \tag{10}$$

where $\tau=1/GB$ is the time constant of the Op-Amp and typically equal to $(1/2\pi)\cdot 10^{-6}$ s.

With equal resistors and capacitors in the passive feedback ($R_1=R_2$, $C_1=C_2$) the three circuits of Fig. 1 yield the same transfer function of

$$T_{RC}(s)=\frac{V_i}{V_0}=\frac{s\omega_0}{s^2+3\omega_0 s+\omega_0^2}. \tag{11}$$

From (9), (10), and (11) the characteristic equation becomes

$$K_1\tau s^3+(1+3K_1\omega_0\tau)s^2+(3-K_1)\omega_0 s + K_1\omega_0^2\tau s+\omega_0^2=0. \tag{12}$$

With $K_1=3$, the above equation can only be satisfied with the ideal Op-Amp ($\tau=0$) or in DC operation. In the actual case, K_1 must be set to a value greater than 3 by which the above equation can be satisfied. The gain deviation can be defined as

$$\frac{\Delta K_1}{K_0}=\frac{K_1-K_0}{K_0} \tag{13}$$

and it represents an error term depending on the frequency. Hence, the critical value of the gain must be set higher than for the ideal case to sustain oscillations. Consequently, the frequency of oscillation will not be ω_0. It will decrease to some value ω_a smaller than ω_0. The frequency deviation can be defined as

$$\frac{\Delta\omega_0}{\omega_0}=\frac{\omega_a-\omega_0}{\omega_0}. \tag{14}$$

The quantities $\Delta K_1/K_0$ and $\Delta\omega_0/\omega_0$ describe the performance of any realization of such type of oscillators. In fact, the oscillator is a special filter with zero input signal and infinite Q. For an active filter, we usually calculate $\Delta Q_1/Q_0$ and $\Delta\omega_0/\omega_0$ [1]. The two figures $\Delta K_1/K_0$ and $\Delta Q_1/Q_0$ are identical in the analysis. All studies prepared to reduce $\Delta Q_1/Q_0$ can be modified for this new figure in RC oscillators.

The amplitude error $\gamma(\omega)$ is another figure of merit. It is defined as

$$\gamma(\omega)=\frac{|K(s)|-|K(0)|}{|K(0)|}=|\varepsilon(j\omega)|-1 \tag{15}$$

where $\varepsilon(s)$ and $K(s)=K_1\cdot\varepsilon(s)$ are error terms depending on the frequency. Practically, $\gamma(\omega)$ is the per-unit loss in the output signal amplitude of the amplifier $K(s)$ due to the frequency dependence of the Op-Amp gain. It is itself the per-unit loss in the amplitude of the oscillation.

The most important figure in the design is $\Delta K_1/K_0$. In order to explain this fact, return to (12). The 3rd-order polynomial can be written as

$$as^3+bs^2+cs+d=0 \tag{16}$$

where a, b, c, and d are constants. By putting $s=j\omega_a$, the following complex equation is obtained:

$$\omega_a^2=d/b=c/a. \tag{17}$$

Substituting the constants as in (12), ω_a may be written in two forms. The first one is

$$\omega_a=\sqrt{d/b}=\omega_0(1+3K_1\omega_0\tau)^{-1/2} \tag{18}$$

and the frequency deviation becomes

$$\Delta\omega_0/\omega_0=-1.5K_1\omega_0\tau. \tag{19}$$

The second form of (17) is

$$\omega_a=\sqrt{c/a}=\omega_0\left(1+\frac{3-K_1}{K_1\omega_0\tau}\right)^{1/2}. \tag{20}$$

Now, if we neglect the gain deviation and set $K_1=3$, we can notice the error made by this invalid approximation. However, from (17), the condition of oscillation becomes

$$K_1\tau=(1+3K_1\omega_0\tau)\cdot(K_1\tau+(3-K_1)/\omega_0). \tag{21}$$

Let $K_1=3$, $\Delta K_1\ll 3$, and solve for ΔK_1

$$\Delta K_1=3K_1^2\omega_0^2\tau^2. \tag{22}$$

Now, using (22) and (20), an equation similar to (18) may be obtained. This can be written as

$$\omega_a=\omega_0(1-3K_1\omega_0\tau)^{1/2}. \tag{23}$$

With $\omega_0\tau\ll 1$, it is clear that (18) and (23) are identical. The problem of the invalid approximation of ΔK_1 obtained by setting $K_1=3$ in (20), which seemed to be simple in the previous discussion, is very effective and critical. (In some cases, it is not easy to deal with this problem and a lot of attention and effort may be required.)

From the general case, explained later, the single Op-Amp realization can be obtained with $b_1=b_2=b_3=b_4=0$ in (31). The gain and frequency deviations and the magnitude error can be obtained from the general case as

$$K_1=(1+A+B)/n, \tag{24}$$

$$\Delta K_1/K_0=K_1^2\omega^2\tau^2, \tag{25}$$

$$\Delta\omega_0/\omega_0=-K_1^2(n/2\sqrt{AB})\,\omega_0\tau, \tag{26}$$

$$\gamma(\omega)=-(K_1\omega\tau)^2/2. \tag{27}$$

Table 1: A summary of the different realizations of the circuits of Figs. 1a, b and c.
Note: All the results of Fig. 1c are the same as those of Fig. 1b with the exchange *A* and *B*.

	Amplifier	Passive Network	A	B	K_1	K_2	Additional Conditions	$\frac{\Delta K_1}{K_0}$	$\frac{\Delta\omega_0}{\omega_0}$	$\gamma(\omega)$
1-	Uncompensated	Figs. 1a, b, c	1.0	1.0	3.0	—	—	$9.0\,\omega_0^2\tau^2$	$-4.5\,\omega_0\tau$	$-4.5\,\omega^2\tau^2$
	Fig. 2	Fig. 1a	0.5	0.5	2.0	—	—	$4.0\,\omega_0^2\tau^2$	$-4.0\,\omega_0\tau$	$-2.0\,\omega^2\tau^2$
		Fig. 1b	5.882	2.4	1.578	—	—	$2.49\,\omega_0^2\tau^2$	$-1.95\,\omega_0\tau$	$-1.245\,\omega^2\tau^2$
2-	Reddy design	Figs. 1a, b, c	1.0	1.0	3.0	3.0	—	$-4.5\,\omega_0^2\tau^2$	$-4.5\,\omega_0\tau$	$4.5\,\omega^2\tau^2$
	Budak design	Figs. 1a, b, c	1.0	1.0	3.0	$\sqrt{3}$	—	$50.38\,\omega_0^4\tau^4$	$-2.6\,\omega_0\tau$	$1.5\,\omega^2\tau^2$
	Fig. 6	Fig. 1b	12.0	5.0	1.5	$\sqrt{1.5}$	—	$66.82\,\omega_0^4\tau^4$	$-1.42\,\omega_0\tau$	$0.75\,\omega^2\tau^2$
3-	Soliman (i)	Figs. 1a, b, c	1.0	1.0	3.0	3.0	—	$81.0\,\omega_0^4\tau^4$	$-4.5\,\omega_0\tau$	$4.5\,\omega^2\tau^2$
	Fig. 7	Fig. 1a	0.5	0.5	2.0	2.0	—	$16.0\,\omega_0^4\tau^4$	$-4.0\,\omega_0\tau$	$2.0\,\omega^2\tau^2$
		Fig. 1b	5.882	2.4	1.578	1.578	—	$6.2\,\omega_0^4\tau^4$	$-1.95\,\omega_0\tau$	$1.245\,\omega^2\tau^2$
4-	Soliman-Ismail	Figs. 1a, b, c	1.0	1.0	3.0	1.5	—	$20.25\,\omega_0^4\tau^4$	$-2.25\,\omega_0\tau$	$(9/8)\,\omega^2\tau^2$
	Fig. 8	Fig. 1a	0.5	0.5	2.0	1.0	—	$4.0\,\omega_0^4\tau^4$	$-2.0\,\omega_0\tau$	$0.5\,\omega^2\tau^2$
		Fig. 1b	5.75	4.75	2.0	1.0	—	$4.0\,\omega_0^4\tau^4$	$-1.1\,\omega_0\tau$	$0.5\,\omega^2\tau^2$
5-	Soliman (ii)	Figs. 1a, b, c	1.0	1.0	3.0	—	$a=3/4$	$20.25\,\omega_0^4\tau^4$	$-2.25\,\omega_0\tau$	$(9/8)\,\omega^2\tau^2$
	Fig. 9	Fig. 1a	0.5	0.5	2.0	—	$a=0.0$	$4.0\,\omega_0^4\tau^4$	$-2.0\,\omega_0\tau$	$0.5\,\omega^2\tau^2$
		Fig. 1b	5.75	4.75	2.0	—	$a=0.0$	$4.0\,\omega_0^4\tau^4$	$-1.1\,\omega_0\tau$	$0.5\,\omega^2\tau^2$
6-	Soliman (iii)	Figs. 1a, b, c	1.0	1.0	3.0	8/3	$K_3=1.0$	$64.0\,\omega_0^6\tau^6$	$-2.0\,\omega_0\tau$	$(8/9)\,\omega^2\tau^2$
	Fig. 10	Fig. 1b	8.0	15.0	3.0	8/3	$K_3=1.0$	$64.0\,\omega_0^6\tau^6$	$-1.46\,\omega_0\tau$	$(8/9)\,\omega^2\tau^2$
7-	Natarajan-Bhattacharyya	Figs. 1a, b, c	1.0	1.0	3.0	5/3	$a_1=0.6$, $a_2=1.0$	$64.0\,\omega_0^6\tau^6$	$-2.0\,\omega_0\tau$	$(8/9)\,\omega^2\tau^2$
	Fig. 11	Fig. 1b	8.0	15.0	3.0	5/3	$a_3=0.0$	$64.0\,\omega_0^6\tau^6$	$-1.46\,\omega_0\tau$	$(8/9)\,\omega^2\tau^2$

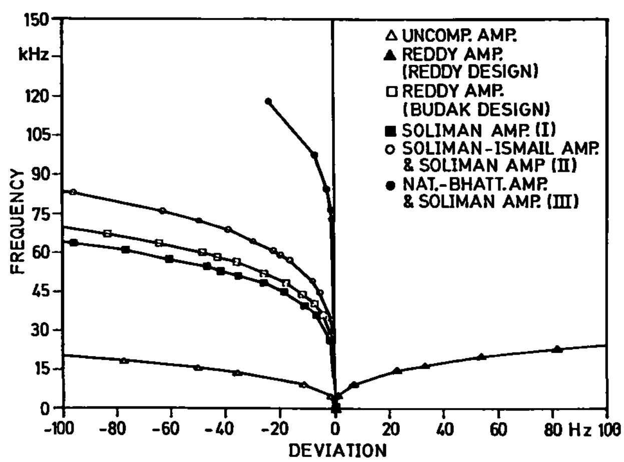

Fig. 3: Upper half-plane pole loci (Equal R, C design)

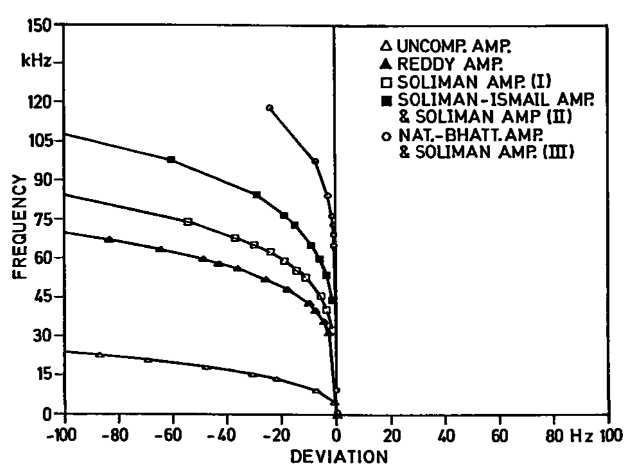

Fig. 4: Upper half-plane pole loci (The optimum design of the circuit of Fig. 1a)

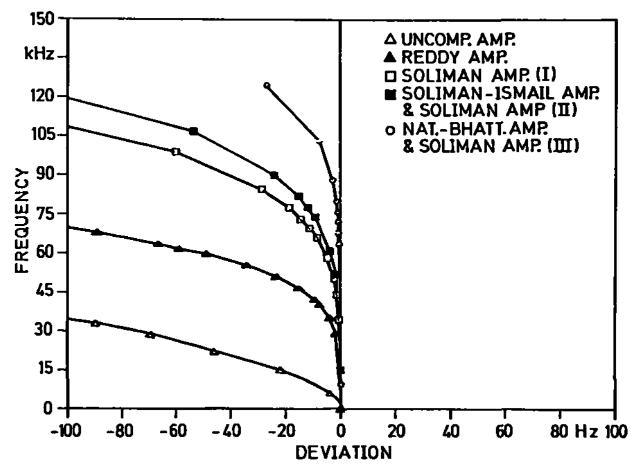

Fig. 5: Upper half-plane pole loci (The optimum design of the circuits of Figs. 1b and c)

With this realization of $K(s)$, equal RC design of the circuits of Fig. 1 results in the same characteristic equation of

$$3\tau s^3+(1+9\,\omega_0\tau)\,s^2+3\,s\omega_0^2\tau+\omega_0^2=0. \qquad (28)$$

With non-unity A and B in (26), we can in most cases obtain the optimum design by choosing the ratios of the resistors and the capacitors in Figs. 1a, b and c. For the different realizations, the values of A and B are listed in **Table 1.** For the present case, the optimum design of Fig. 1a results in

$$2\tau s^3+(1+8\,\omega_0\tau)\,s^2+2\,s\omega_0^2\tau+\omega_0^2=0 \qquad (29)$$

and the optimum design of Figs. 1b and c results in

$$1.578\,\tau s^3+(1+3.898\,\omega_0\tau)\,s^2+5.78\,s\omega_0^2\tau+\omega_0^2=0. \qquad (30)$$

For the different realizations **Fig. 3** shows the upper half plane loci of the equal R, C design. **Fig. 4** shows that of the optimum design of the circuit shown in Fig. 1a. Finally, **Fig. 5** shows that of the optimum design of the circuits shown in Figs. 1b and c which are related by RC-CR transformation. All results are listed in Table 1.

4. General analysis of the compensated RC oscillator

The general form of any one-Op-Amp, two-Op-Amps, or three-Op-Amps realization of $K(s)$ can be written – with matched Op-Amps – in the form

$$K(s)=K_1\,\frac{1+b_3 s\tau+b_4 s^2\tau^2}{1+b_0 s\tau+b_1 s^2\tau^2+b_2 s^3\tau^3} \quad (31)$$

where b_0, b_1, b_2, b_3 and b_4 are constants by which the amplifier can be characterized. With $s=\mathrm{j}\omega$, (31) becomes

$$K(s)=\frac{V_0}{V_i}=K_1\left(\frac{X}{Z}+\mathrm{j}\,\frac{Y}{Z}\right) \quad (32)$$

where

$$X=1+(b_0 b_3-b_1-b_4)\,\omega^2\tau^2+(b_1 b_4-b_2 b_3)\,\omega^4\tau^4, \quad (33)$$

$$Y=[(b_3-b_0)+(b_0 b_4+b_2-b_1 b_3)\omega^2\tau^2-b_2 b_4\omega^4\tau^4]\,\omega\tau, \quad (34)$$

$$Z=1+(b_0^2-2b_1)\,\omega^2\tau^2+(b_1^2-2b_0 b_2)\,\omega^4\tau^4+b_2^2\omega^6\tau^6. \quad (35)$$

When the amplifier is used in any oscillator circuit of Fig. 1, the loop-gain condition of (9) must be satisfied. Thus from the real parts of (1) and (32) we can deduce that

$$K_0=(1+A+B)/n \quad (36)$$

$$\Delta K_1/K_0=f_1\omega^2\tau^2+(f_2-f_1 f_4)\,\omega^4\tau^4+(f_3-f_1 f_5-f_2 f_4+f_1 f_4{}^2)\cdot\omega^6\tau^6+\text{higher-order terms} \quad (37)$$

where

$$f_1=b_0^2-b_1+b_4-b_0 b_3, \quad (38)$$

$$f_2=b_1^2+b_2 b_3-b_1 b_4-2\,b_0 b_2, \quad (39)$$

$$f_3=b_2^2, \quad (40)$$

$$f_4=b_0 b_3-b_1-b_4, \quad (41)$$

$$f_5=b_1 b_4-b_2 b_3. \quad (42)$$

The equations (36) and (37) describe the condition of oscillation and the gain deviation, respectively. With $\Delta K_1 \ll K_0$, the imaginary parts of (1) and (32) can be solved together for ω_a which is

$$\omega_a=\omega_0\,(1+(b_0-b_3)\cdot(K_1 n/\sqrt{AB})\,\omega_0\tau)^{-1/2} \quad (43)$$

and finally, the frequency deviation becomes

$$\frac{\Delta\omega_0}{\omega_0}=-\frac{(b_0-b_3)\,K_1 n}{2\sqrt{AB}}\,\omega_0\tau+\text{higher-order terms.} \quad (44)$$

From (31), the magnitude error defined by (15) can be written as

$$\gamma(\omega)=\frac{b_3^2-b_0^2+2\,(b_1-b_4)}{2}\,\omega^2\tau^2. \quad (45)$$

The minimum gain deviation can be obtained from (37) itself. For the one Op-Amp realization, with $b_1=b_2=b_3=b_4=0$, we can write

$$\frac{\Delta K_1}{K_0}=f_1\omega^2\tau^2=b_0^2\omega^2\tau^2. \quad (46)$$

For the two Op-Amps realizations, with $b_2=b_4=0$, we can reduce the gain deviation to a fourth-order term by setting $f_1=0$. Thus, we obtain

$$\frac{\Delta K_1}{K_0}=f_2\omega^4\tau^4=b_1^2\omega^4\tau^4. \quad (47)$$

Finally, for the three Op-Amps realizations, the gain deviation can be reduced to a sixth-order term by setting $f_1=0$ and $f_2=0$. This means that

$$\frac{\Delta K_1}{K_0}=f_3\,\omega^6\tau^6=b_2^2\omega^6\tau^6. \quad (48)$$

In order to delete the first-order term of $\Delta\omega_0/\omega_0$, it is required that $b_0=b_3$. For the one Op-Amp realization, this is impossible since $b_0 \neq 0$. For two Op-Amps realizations, with $f_1=0$ (the condition of minimum gain deviation), $\Delta\omega_0/\omega_0$ must be a first-order term since $b_1 \neq 0$. Finally, for the three-Op-Amps realizations, with $f_1=0$ and $f_2=0$, $\Delta\omega_0/\omega_0$ must be again a first-order term since $b_2 \neq 0$; see (38) and (39).

In order to reduce $\Delta\omega_0/\omega_0$, it is possible to use unequal resistors and capacitors in the passive feedback. The optimum selection of A and B can reduce both $\Delta\omega_0/\omega_0$ and $\Delta K_1/K_0$. The following examples show this fact.

5. The active compensated RC oscillators

5.1 The first realization

In order to extend the useful range of RC oscillators, Reddy recommended the use of the variable phase-shift amplifier shown in **Fig. 6** [3]. The transfer function of the circuit is

$$K(s)=K_1\,\frac{1+\dfrac{K_2}{(K_2-1)}\,s\tau}{1+\dfrac{K_1}{(K_2-1)}\,s\tau+\dfrac{K_1K_2}{(K_2-1)}\,s^2\tau^2}. \quad (49)$$

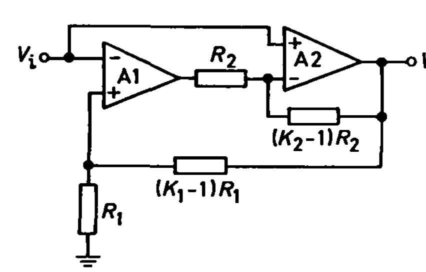

Fig. 6: Reddy's amplifier

In order to obtain a minimum phase-error in $K(s)$, Reddy uses $K_2=3$ in which case Equal R, C design results in $K_1=K_2=3$. Taking the effect of the non-ideal Op-Amp on the amplifier only, he concluded that the actual frequency of oscillation is given as in (7) [3]. Accordingly, the frequency deviation is

$$\Delta\omega_0/\omega_0 \cong -(81/8)\,\omega_0^3\tau^3. \qquad (50)$$

The characteristic equation of the overall circuit is

$$4.5\,\tau^2 s^4+(1+9\,\omega_0\tau)\,1.5\,\tau s^3+(1+4.5\,\omega_0^2\tau^2)\,s^2+1.5\,\omega_0^2 s\tau+\omega_0^2=0. \qquad (51)$$

To calculate the correct value of the frequency deviation, let us consider the fourth-order polynomial

$$as^4+bs^3+cs^2+ds+e=0 \qquad (52)$$

from which the actual frequency can be obtained as $\omega_a=\sqrt{d/b}$. Using (51) and (52), the frequency deviation becomes

$$\Delta\omega_0/\omega_0=-4.5\,\omega_0\tau. \qquad (53)$$

Notice that the frequency deviation is described by a first-order term with respect to $\omega_0\tau$. The reason for Reddy's result is the invalid approximation of $\Delta K_1/K_0$ which can be obtained from (37) to be

$$\Delta K_1/K_0=-4.5\,\omega_0^2\tau^2. \qquad (54)$$

Reddy's design was first discussed by Budak and Nay [4]. They noticed that as the frequency increases, the poles of the oscillator will be forced into the right half-plane. From the characteristic equation, with $K_1=3$, they obtained the correct value of K_2 by which the criteria of oscillation can be satisfied.

Actually, their result was the starting point of our study. Their design of K_2 is the condition of oscillation with minimum gain deviation. To generalize their work for the circuits shown in Figs. 1 a, b and c we can obtain from the general case introduced above that

$$K_1=(1+A+B)/n, \quad K_2=\sqrt{K_1}, \qquad (55)$$

$$\Delta K_1/K_0=(K_1^3/(\sqrt{K_1}-1)^2)\,\omega_0^4\tau^4, \qquad (56)$$

$$\Delta\omega_0/\omega_0=-K_1\sqrt{K_1}\,(n/2\sqrt{AB})\,\omega_0\tau, \qquad (57)$$

$$\gamma(\omega)=((K_1^2+K_1-2K_1\sqrt{K_1})/2\,(\sqrt{K_1}-1)^2)\,\omega^2\tau^2. \qquad (58)$$

Thus, Budak-Nay's design with equal R, C results in

$$3\sqrt{3}\,\tau^2 s^4+(1+3\sqrt{3}\,\omega_0\tau)\,3\,\tau s^3$$
$$+[(\sqrt{3}-1)+3\sqrt{3}(\sqrt{3}-1)\,\omega_0\tau+3\sqrt{3}\,\omega_0^2\tau^2]\,s^2$$
$$+3\,\omega_0^2\tau s+(\sqrt{3}-1)\,\omega_0^2=0 \qquad (59)$$

which is itself the optimum design of the circuit in Fig. 1 a. The optimum design of the circuits of Figs. 1 b and c results in

$$8.174\,\tau^2 s^4+(1+2.8466\,\omega_0\tau)\,6.674\,\tau s^3$$
$$+(1+2.8466\,\omega_0\tau+8.174\,\omega_0^2\tau^2)\,s^2$$
$$+6.67\,\omega_0^2\tau s+\omega_0^2=0. \qquad (60)$$

5.2 The second realization

The compensated amplifier designed by Soliman [5] can be also used to realize $K(s)$. From the circuit shown in **Fig. 7**, the transfer function is

$$K(s)=K_1\,\frac{1}{1+K_2 s\tau+K_1K_2 s^2\tau^2}. \qquad (61)$$

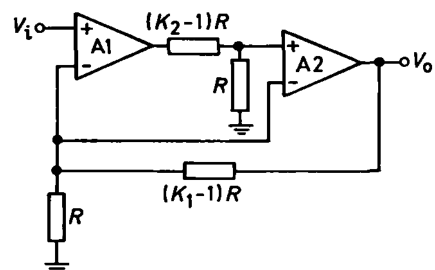

Fig. 7: Soliman's amplifier (i)

From the general case introduced earlier, we can deduce that

$$K_1=(1+A+B)/n, \quad K_1=K_2, \qquad (62)$$

$$\Delta K_1/K_0=K_1^4\omega_0^4\tau^4, \qquad (63)$$

$$\Delta\omega_0/\omega_0=-K_1^2\,(n/2\sqrt{AB})\,\omega_0\tau, \qquad (64)$$

$$\gamma(\omega)=(K_1^2/2)\,\omega^2\tau^2. \qquad (65)$$

For the circuits of Figs. 1 a, b and c, equal R, C design results in

$$9\,\tau^2 s^4+(1+9\,\omega_0\tau)\,3\,\tau s^3+(1+9\,\omega_0\tau+9\,\omega_0^2\tau^2)\,s^2$$
$$+3\,\omega_0^2\tau s+\omega_0^2=0. \qquad (66)$$

For the circuit of Figs. 1 a, the optimum design results in

$$4\,\tau^2 s^4+(1+8\,\omega_0\tau)\,2\,\tau s^3+(1+8\,\omega_0\tau+4\,\omega_0^2\tau^2)\,s^2$$
$$+2\,\omega_0^2\tau s+\omega_0^2=0. \qquad (67)$$

For the circuits of Figs. 1 b and c, the optimum design results in

$$2.49\,\tau^2 s^4+(1+3.898\,\omega_0\tau)\,1.578\,\tau s^3$$
$$+(1+3.898\,\omega_0\tau+2.49\,\omega_0^2\tau^2)\,s^2$$
$$+1.578\,\omega_0^2\tau s+\omega_0^2=0. \qquad (68)$$

5.3 The third realization

Fig. 8 shows the compensated amplifier designed by Soliman and Ismail [6]. The transfer function of the circuit (Fig. 8 a) is

$$K(s)=K_1\,\frac{1+K_2 s\tau}{1+K_1 s\tau+K_1K_2 s^2\tau^2}. \qquad (69)$$

From the general case introduced earlier, we can deduce that

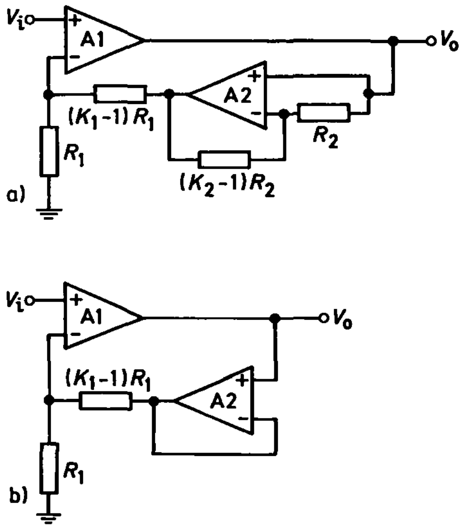

Fig. 8: Soliman-Ismail's amplifier
a) general design
b) optimal design with $K_2=1$

$$K_1=(1+A+B)/n,\quad K_2=0.5\,K_1, \tag{70}$$

$$\Delta K_1/K_0=0.25\,K_1^4\omega_0^4\tau^4, \tag{71}$$

$$\Delta\omega_0/\omega_0=-0.25\,K_1^2(n/\sqrt{AB})\,\omega_0\tau, \tag{72}$$

$$\gamma(\omega)=0.125\,K_1^2\omega_0^2\tau^2. \tag{73}$$

The optimum design of this realization causes $K_2=1$. Consequently, the amplifier circuit must be modified as in Fig. 8b.

For the circuits of Figs. 1a, b and c, equal R, C design results in

$$4.5\,\tau^2s^4+(1+4.5\,\omega_0\tau)\,3\,\tau s^3 +(1+4.5\,\omega_0\tau+4.5\,\omega_0^2\tau^2)\,s^2 +3\,\omega_0^2\tau s+\omega_0^2=0. \tag{74}$$

For the circuit of Fig. 1a, the optimum design results in

$$2\tau^2s^4+(1+4\,\omega_0\tau)\,2\tau s^3+(1+4\,\omega_0\tau+2\,\omega_0^2\tau^2)\,s^2 +2\,\omega_0^2\tau s+\omega_0^2=0. \tag{75}$$

For the circuits of Figs. 1b and c, the optimum design results in

$$2\tau^2s^4+(1+2.2\,\omega_0\tau)\,2\tau s^3+(1+2.2\,\omega_0\tau+2\omega_0^2\tau^2)\,s^2 +2\,\omega_0^2\tau s+\omega_0^2=0, \tag{76}$$

5.4 *The fourth realization*

Fig. 9 shows another compensated amplifier designed by Soliman [7]. The transfer function of the circuit (Fig. 9a) is

$$K(s)=K_1\,\frac{1+(1+a-a/K_1)\,s\tau}{1+K_1s\tau+(K_1+a(K_1-1))\,s^2\tau^2}. \tag{77}$$

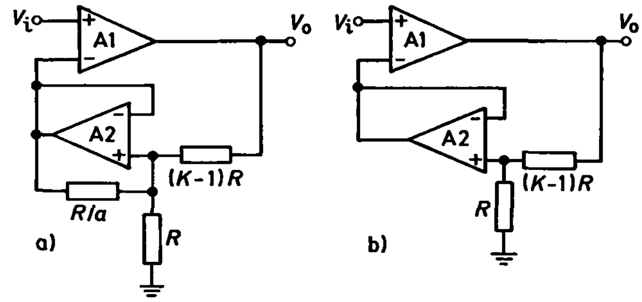

Fig. 9: Soliman's amplifier (ii)
a) general design
b) optimal design with $a=0$

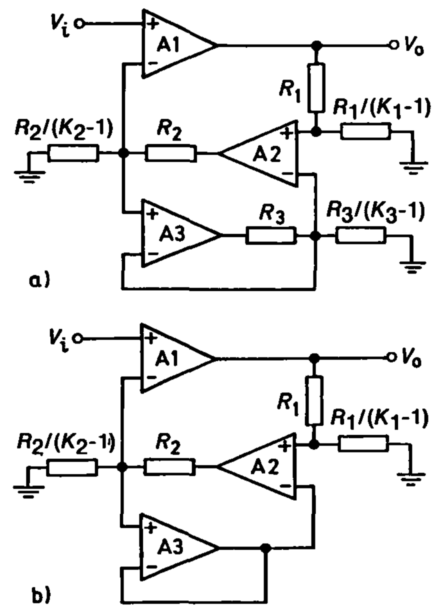

Fig. 10: Soliman's amplifier (iii)
a) general design
b) optimal design with $K_3=1$

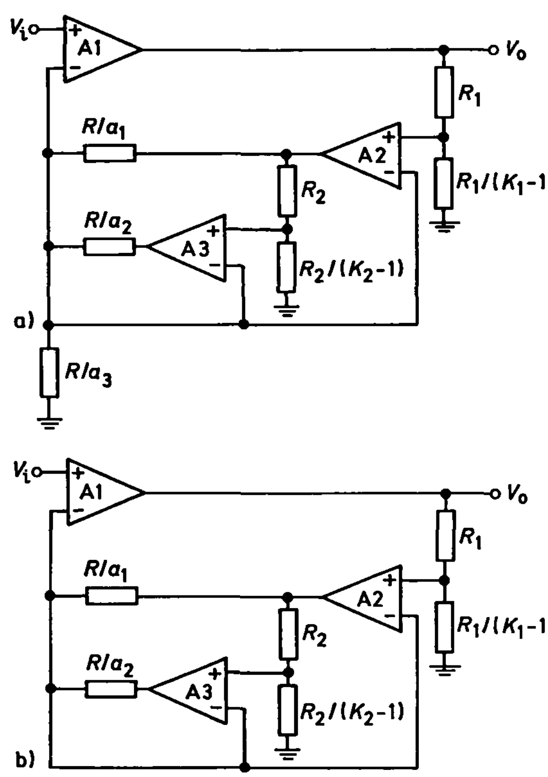

Fig. 11: Natarajan-Bhattacharyya's amplifier
a) general design
b) optimal design with $a_3=0$

From the general case introduced earlier, we can deduce that

$$a=(K_1^2-2K_1)/2(K_1-1) \qquad (78)$$

and the other results (71) to (76) of the previous realization. The optimum design of this realization causes $a=0$. Consequently, the amplifier circuit becomes as shown in Fig. 9b.

5.5 The fifth realization

The three-Op-Amps amplifier designed by Soliman is the one shown in **Fig. 10** [8]. The transfer function of the circuit (Fig. 10a) is

$$K(s)=K_1\frac{1+K_2s\tau+K_2K_3s^2\tau^2}{1+(K_1+K_3)s\tau+K_1K_2s^2\tau^2+K_1K_2K_3s^3\tau^3}. \qquad (79)$$

From the general case introduced earlier, we can deduce that

$$K_1=(1+A+B)/n, \quad K_2=(8/9)K_1, \qquad (80)$$

$$K_3=(1/3)K_1, \qquad (81)$$

$$\Delta K_1/K_0=(2/3)^6K_1^6\omega_0^6\tau^6, \qquad (82)$$

$$\Delta\omega_0/\omega_0=-(2/9)K_1^2(n/\sqrt{AB})\,\omega_0\tau, \qquad (83)$$

$$\gamma(\omega)=(8/81)K_1^2\omega^2\tau^2. \qquad (84)$$

With $K_1=3$, $K_3=1$, and the amplifier circuit becomes as shown in Fig. 10b. For the circuits of Figs. 1a, b and c, equal R, C design results in

$$8\tau^3s^5+(1+3\omega_0\tau)\,8\tau^2s^4+(1+4\omega_0\tau+2\omega_0^2\tau^2)\,4\tau s^3$$
$$+(1+4\omega_0\tau+8\omega_0^2\tau^2)s^2+4\omega_0^2\tau s+\omega_0^2=0. \qquad (85)$$

For the circuit of Fig. 1a, the optimum design is equal R, C design itself. For the circuits of Figs. 1b and c, the optimum design results in

$$8\tau^3s^5+(1+2.191\,\omega_0\tau)\,8\tau^2s^4$$
$$+(1+2.921\,\omega_0\tau+2\omega_0^2\tau^2)\,4\tau s^3$$
$$+(1+2.921\,\omega_0\tau+8\omega_0^2\tau^2)s^2+4\omega_0^2\tau s+\omega_0^2=0. \qquad (86)$$

5.6 The sixth realization

Finally, **Fig. 11** shows the three Op-Amps amplifier designed by Natarajan and Bhattacharyya [9]. The transfer function of the circuit (Fig. 11a) is

$$K(s)=K_1\frac{1+K_2(1+a_1/a_2)s\tau+(K_2/a_2)(a_1+a_2+a_3)s^2\tau^2}{1+(K_1+K_2a_1/a_2)s\tau+K_1K_2(1+a_1/a_2)s^2\tau^2+(K_1K_2/a_2)(a_1+a_2+a_3)s^3\tau^3}. \qquad (87)$$

From the general case introduced earlier, we can deduce that

$$K_2=(5/9)K_1, \qquad (88)$$

$$a_1=0.6a_2, \quad a_2\neq 0, \qquad (89)$$

$$a_3=(K_1/3-1)\,1.6a_2 \qquad (90)$$

and the other results (82) to (86) of the previous realization. With $K_1=3$, $a_3=0$, and the amplifier circuit becomes as shown in Fig. 11b.

6. Experimental results

The performance of the considered oscillators was tested using two approaches; open-loop and closed-loop measurements as in article [4]. The experimental results were too many to be listed in this limited paper. Budak-Nay's results were some of the different results obtained through our work.

In order to limit the length of the paper, only two samples of the results are included for the fourth realization: Equal R, C design of Fig. 1b as an open-loop test and the optimum design of Fig. 1a as a closed-loop test.

The overall loop-gain of the present realization is

$$LG(s)=K(s)\cdot T_{RC}(s)=\frac{s\tau+\dfrac{K_1}{K_1(1+a)-a}}{s^2\tau^2+\dfrac{K_1}{K_1(1+a)-a}s\tau+\dfrac{1}{K_1(1+a)-a}}\cdot\frac{(n/\sqrt{AB})\,\omega_0s}{s^2+K_0(n/\sqrt{AB})\,\omega_0s+\omega_0^2}. \qquad (91)$$

Equal R, C design of Fig. 1b results in

$$\theta_{LG}=\tan^{-1}1.5\,\omega\tau-\tan^{-1}\frac{3\,\omega\tau}{1-4.5\,\omega^2\tau^2}-\tan^{-1}\frac{3\,\omega_0\omega}{\omega_0^2-\omega^2}+\frac{\pi}{2} \qquad (92)$$

$$|LG|^2=\frac{\omega^2\tau^2+4/9}{(81/4-\omega^2\tau^2)^2+(2\,\omega\tau/3)^2}\cdot\frac{\omega^2\omega_0^2}{(\omega_0^2-\omega^2)^2+9\,\omega^2\omega_0^2}. \qquad (93)$$

Fig. 12a shows the test circuit; in **Fig. 12b**, the theoretical loop-gain phase and magnitude characteristics given by the above two equations are plotted as a function of f_a/f_0 using $\omega\tau$ as a parameter. With $\tau=(1/2\pi)\times 10^{-6}$ s, constant $\omega\tau$ curves are plotted for both θ_{LG} and $|LG|$. A vertical line drawn at the frequency resulting in $\theta_{LG}=0$ would intersect the corresponding $|LG|$ curve at a point very near to the unity. For each ideal frequency f_0, the actual frequency f_a was measured together with the corresponding $|LG|$. The results are also plotted in Fig. 12b as X's. These points can be easily compared with those obtained by theory. As can be seen, the agreement is quite close.

The closed-loop test was carried out for the optimum design of Fig. 1a. With $a=0$, $A=B=0.5$, the characteristic equation becomes

$$K_1\tau^2s^4+(1+4\,\omega_0\tau)\,K_1\tau s^3$$
$$+(1+2K_1\omega_0\tau+K_1\omega_0^2\tau^2)s^2$$
$$+[2(2-K_1)\omega_0+K_1\omega_0^2\tau]s+\omega_0^2=0. \qquad (94)$$

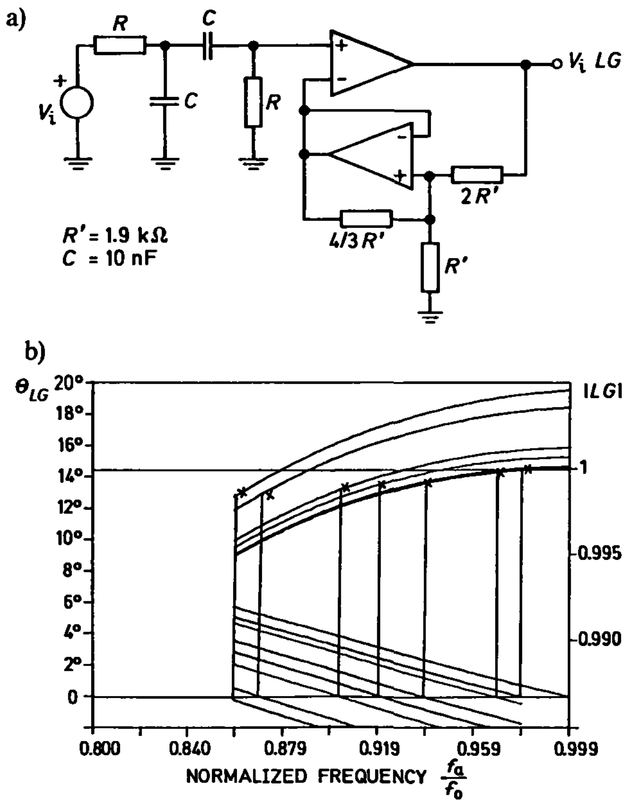

Fig. 12: Open-loop test of the fourth realization
a) Test circuit
b) Experimental and theoretical loop-gain response

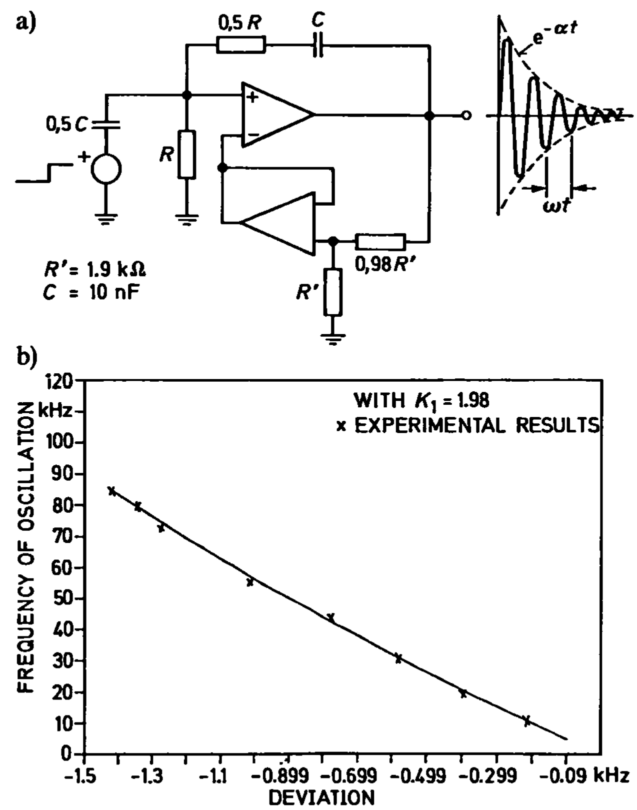

Fig. 13: Closed-loop test of the fourth realization
a) Test circuit
b) Experimental and theoretical pole positions as a function of f_0

The circuit will not oscillate with $K_1 = 1.98$. It will be kept on the verge of oscillation. So, with a square wave as in **Fig. 13a**, the real and the imaginary parts of the poles of (94) can be measured. These are plotted in **Fig. 13b** together with those determined from (94). Again, the agreement is very close.

7. Conclusions

From Table 1, we can note that the best design can be obtained by the fourth and fifth realization. In these two realizations, the minimum number of resistors can be used with minimum amplitude distortion in practice.

For the sixth realization, there are redundant factors for the design. Thus, a number of additional assumptions were required. These were done in such a way that the transfer functions of this realization and the preceding one are identical.

References:

[1] Budak, A.: Passive and Active Network Analysis and Synthesis. Boston, MA: Houghton Mifflin, 1974.
[2] Bhattacharyya, B. B; Sundaramurthy, M.; Swamy, M. N. S.: Systematic generation of canonic sinusoidal RC-active oscillators. IEE Proc. Vol. 28 (June 1981), No. 3, pp. 114—126.
[3] Reddy, M. A.: Operational-amplifier circuits with variable phase-shift and their application to high-Q active RC filters and RC-oscillators. IEEE Trans. Circuits and Syst., Vol. CAS-23 (June 1976), pp. 384—389.
[4] Budak, A.; Nay, K.: Operational-amplifier circuits for Wien-bridge oscillators. IEEE Trans. Circuits and Syst., Vol. CAS-28 (Sept. 1981), No. 9, pp. 930—934.
[5] Soliman, A. M.: Economical wide-band voltage controlled voltage source. Int. J. Electronics, Vol. 49 (1981), pp. 77—81.
[6] Soliman, A. M.; Ismail, M.: Active compensation of Op-Amps. IEEE Trans. Circuits and Syst., Vol. CAS-26 (Febr. 1979), pp. 112—117.
[7] Soliman, A. M.: Classification and generation of active compensated non-iverting VCVS building blocks. Int. J. of Circuit Theory and Applications 8 (1980), pp. 395—405.
[8] Soliman, A. M.: Design of high frequency amplifiers. IEEE Circuits and Syst. Magazine, Vol. 5 (June 1983), pp. 9—11.
[9] Natarajan, S.; Bhatachnaryya, B. B.: Design of actively compensated finite gain amplifiers for high frequency applications. IEEE Trans. Circuits and Syst., Vol. CAS-27 (Dec. 1980), pp. 1133—1139.

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(Received on December 8, 1987,
revised version on June 16, 1988)