INT. J. ELECTRONICS, 1978, VOL. 44, NO. 3, 317–323

Realizations of ideal FDNC and FDNR elements using new types of mutators

AHMED M. SOLIMAN†

New types of mutators for realizing a one-port ideal FDNC (D) and FDNR (N) are defined. The mutators introduced are D–R, D–C, N–R and N–C mutators. Two general types are defined for each mutator. The first type belongs to the generalized positive impedance converter (GPIC) and the second type belongs to the generalized positive impedance inverter (GPII). Most of the active RC realizations available for simulating an ideal FDNC or FDNR are included as special cases.

1. Introduction

One of the most practical methods for designing active RC filters is to apply Bruton's (1969) transformation to the classical passive filters. Generally the realizations obtained using this method include a grounded FDNC and FDNR elements (Antoniou 1971, Hasler and Neirynck 1976, Panzer 1976).

The purpose of this paper is to provide a general approach for realizing grounded FDNC and FDNR elements using new types of mutators. Some of the mutators defined are realizable from other well-known types (Chua 1968) using the RC : CR transformation. The intention here is to direct the attention of the network designer to a generalized approach which includes most of the available realizations for FDNC or FDNR as special cases, rather than to give specific circuit realizations.

It is to be noted that although the mutators were originally introduced by Chua (1968) for transforming one type of non-linear network element into another type, they proved to be a very useful active RC two-port networks for simulating a linear inductor using L–C or L–R mutators (Soliman 1972, 1973).

2. Definitions

2.1. *The FDNC element* (D)

The frequency-dependent negative conductance D, Fig. 1 (a) (Bruton 1969), is a one-port network element having $Y(s)=Ds^2$, where D is a positive constant.

2.2. *The FDNR element* (N)

The frequency-dependent negative resistance N, Fig. 1 (b) (Antoniou 1971), is a one-port network element having $Z(s)=Ns^2$, where N is a positive constant.

Received 14 March 1977.
† Electronics and Communications Engineering Department, Cairo University, Giza, Egypt.

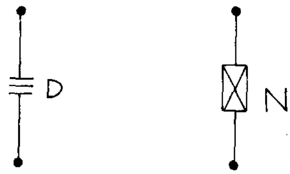

Figure 1. (*a*) Ideal FDNC element. (*b*) Ideal FDNR element.

3. The D–R mutator

3.1. *Definition*

The D–R mutator, Fig. 2 (*a*), is an active two-port linear network which when terminated at port 2 in a resistor R will present to port 1 a D element. Two types of mutators are defined next.

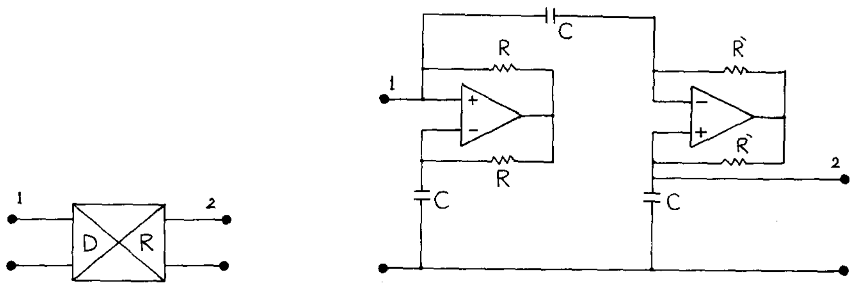

Figure 2. (*a*) Symbolic representation of the D–R mutator. (*b*) Realization of a type 2 a D–R mutator.

3.2. *Type* 1 *D–R mutator*

This two-port is defined by the following transmission matrix :

$$T1_{DR}(s)=\begin{bmatrix} \pm a_1 s^n & 0 \\ 0 & \pm a_2 s^m \end{bmatrix} \tag{1}$$

where a_1 and a_2 are positive constants, n and m are arbitrary integers such that $m=n+2$. The mutator in this case belongs to the generalized positive impedance converter (GPIC).

3.3. *Type* 2 *D–R mutator*

The mutator in this case is of the generalized positive impedance inverter (GPII) type and its transmission matrix is given by

$$T2_{DR}(s)=\begin{bmatrix} 0 & \pm b_1 s^k \\ \pm b_2 s^l & 0 \end{bmatrix} \tag{2}$$

where b_1 and b_2 are positive constants, k and l are arbitrary integers such that $l=k+2$.

It is clear that a family of different *D–R* mutators exist for each type.

3.4. *Realizations of D–R mutators*

Although new active *RC* realizations for these two-port active networks may be found, the numerous circuits available in the literature for realizing *L–C* mutators (which include gyrators as special class) (Chua 1968, Mitra 1969, Antoniou 1970 and Soliman 1972) can be utilized to realize *D–R* mutators. The following lemma derived directly from Mitra's theorem (1967) demonstrates that *D–R* mutators are realizable from active *RC*, *L–C* mutators using the *RC* : *CR* transformation.

Lemma

If

$$[T(s)]=\begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix}$$

represents the transmission matrix of an active *RC* two-port network. The corresponding transmission matrix $[T'(s)]$ of the transformed active *RC* two-port network obtained by applying the *RC* : *CR* transformation on the original network is given by

$$[T'(s)]=\begin{bmatrix} A\left(\frac{1}{s}\right) & \frac{1}{s}B\left(\frac{1}{s}\right) \\ sC\left(\frac{1}{s}\right) & D\left(\frac{1}{s}\right) \end{bmatrix} \tag{3}$$

Table 1 includes the transmission matrix for six special types of the *L–C* mutators and the corresponding *D–R* mutators. (Note that Chua's (1968) type 1 *L–C* mutator, gyrator is identified here as type 2*a* *L–C* and Chua's type 2 *L–C* mutator is identified as type 1*a* *L–C* mutator in order to have a consistency for classifying converter type as type 1 and inverter type as type 2.)

Most of the realizations available in the literature (Antoniou 1971, Bruton 1974 and Wait *et al.* 1975) for realizing an ideal *D* element are based on using Antoniou's GPIC (1970). The realization in this case is a special class from type 1*c* *D–R* mutator with $a_1=1$. Many new realizations for *D–R* mutators can be obtained from available realizations of the *L–C* mutators.

The use of type 2*a* *D–R* mutator for simulating a *D* element will lead to many new realizations for the ideal *D* element utilizing the numerous gyrator circuits available in the literature. It should be noted that in realizing a *D–R* mutator of the type 2*a* by applying the *RC* : *CR* transformation to a specific gyrator network, the resistors which appear as ratios in the gyrator analysis need not be transformed to capacitors in order to minimize the number of capacitors used.

Mutator type		Transmission matrix of L–C mutator	Transmission matrix of D–R mutator
		Related by the $RC:CR$ transformation	
GPIC (Type 1)	1 (a)	$\begin{bmatrix} a_1 s & 0 \\ 0 & a_2/s \end{bmatrix}$ Type 2, L–C (Chua 1968)	$\begin{bmatrix} a_1/s & 0 \\ 0 & a_2 s \end{bmatrix}$
	1 (b)	$\begin{bmatrix} a_1 s^2 & 0 \\ 0 & a_2 \end{bmatrix}$ Type 2(b), L–C (Soliman 1972)	$\begin{bmatrix} a_1/s^2 & 0 \\ 0 & a_2 \end{bmatrix}$
	1 (c)	$\begin{bmatrix} a_1 & 0 \\ 0 & a_2/s^2 \end{bmatrix}$ Type 2 (c), L–C (Soliman 1972)	$\begin{bmatrix} a_1 & 0 \\ 0 & a_2 s^2 \end{bmatrix}$
GPII (Type 2)	2 (a)	$\begin{bmatrix} 0 & b_1 \\ b_2 & 0 \end{bmatrix}$ Type 1, L–C (Chua 1968) (also Gyrator)	$\begin{bmatrix} 0 & b_1/s \\ b_2 s & 0 \end{bmatrix}$
	2 (b)	$\begin{bmatrix} 0 & b_1 s \\ b_2 s & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & b_1/s^2 \\ b_2 & 0 \end{bmatrix}$
	2 (c)	$\begin{bmatrix} 0 & b_1/s \\ b_2/s & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & b_1 \\ b_2 s^2 & 0 \end{bmatrix}$

Table 1.

As an example Fig. 2 (b) demonstrates the conversion of Deboo's (1967) gyrator to realize a type $2a$ D–R mutator. The realization obtained in this case is not canonical and the three capacitors must be equal. Note also that a type 1 D–R mutator is realizable from a type 1 L–C mutator by interchanging its two ports, that is by terminating port 1 of the type 1 L–C mutator in a resistor will present to port 2 a D element.

4. The D–C mutator

4.1. *Definition*

The D–C mutator is an active two-port linear network which when terminated at port 2 in a capacitor will present to port 1 a D element.

Two types are defined next.

4.2. *Type* 1 *D–C mutator*

The transmission matrix in this case is given by

$$T1_{DC}(s)=\begin{bmatrix} \pm a_1 s^n & 0 \\ 0 & \pm a_2 s^m \end{bmatrix} \tag{4}$$

where $m=n+1$.

4.3. *Type* 2 *D–C mutator*

In this case

$$T2_{DC}(s)=\begin{bmatrix} 0 & \pm b_1 s^k \\ \pm b_2 s^l & 0 \end{bmatrix} \tag{5}$$

with $l=k+3$.

Mutator type		Transmission matrix of *L–R* mutator	Transmission matrix of *D–C* mutator
		Related by the *RC* : *CR* transformation	
GPIC (Type 1)	1 (*a*)	$\begin{bmatrix} a_1 s & 0 \\ 0 & a_2 \end{bmatrix}$ Type 1, *L–R* (Chua 1968)	$\begin{bmatrix} a_1/s & 0 \\ 0 & a_2 \end{bmatrix}$
	1 (*b*)	$\begin{bmatrix} a_1 & 0 \\ 0 & a_2/s \end{bmatrix}$ (Soliman 1973)	$\begin{bmatrix} a_1 & 0 \\ 0 & a_2 s \end{bmatrix}$
	1 (*c*)	$\begin{bmatrix} a_1 s^2 & 0 \\ 0 & a_2 s \end{bmatrix}$	$\begin{bmatrix} a_1/s^2 & 0 \\ 0 & a_2/s \end{bmatrix}$
GPII (Type 2)	2 (*a*)	$\begin{bmatrix} 0 & b_1 s \\ b_2 & 0 \end{bmatrix}$ Type 2, *L–R* (Chua 1968)	$\begin{bmatrix} 0 & b_1/s^2 \\ b_2 s & 0 \end{bmatrix}$
	2 (*b*)	$\begin{bmatrix} 0 & b_1 \\ b_2/s & 0 \end{bmatrix}$ (Soliman 1973)	$\begin{bmatrix} 0 & b_1/s \\ b_2 s^2 & 0 \end{bmatrix}$
	2 (*c*)	$\begin{bmatrix} 0 & b_1/s \\ b_2/s^2 & 0 \end{bmatrix}$ (Soliman 1976)	$\begin{bmatrix} 0 & b_1 \\ b_2 s^3 & 0 \end{bmatrix}$

Table 2.

4.4. *Realizations of D–C mutators*

From eqn. (4) it is clear that a type 1 *C–R* mutator is equivalent to a type 1 *D–C* mutator.

From eqns. (3), (4) and (5) it follows that *D–C* mutators of either type are realizable from the active *RC*, *L–R* mutator of the corresponding type using the *RC : CR* transformation. Table 2 represents the transmission matrix for three types of *L–R* mutators from each family and the corresponding *D–C* mutators. Many more higher-order mutators could be defined.

5. The N–R and the N–C mutators

5.1. *Definition*

The *N–R* (*N–C*) mutator is an active two-port linear network which when terminated at port 2 in a resistor (a capacitor) will present to port 1 an *N* element.

5.2. *Realizations*

Two types exist for each of the *N–R* and *N–C* mutators. For brevity, only the generalized definition is given in Table 3. It is clear that a type 1 *N–R* mutator is the same as a type 1 *L–C* mutator (that is, terminating port 2 of any type 1 *L–C* mutator in a resistor will present to port 1 an *N* element). Note also that a type 2 *L–R* mutator can be used as a type 2 *N–C* mutator.

It can be easily seen that a type 1 *N–C* mutator is realizable by cascading a type 1 *L–C* mutator and a type 1 *L–R* mutator.

Transmission matrix / Mutator type	GPIC (Type 1)	GPII (Type 2)
	$\begin{bmatrix} \pm a_1 s^n & 0 \\ 0 & \pm a_2 s^m \end{bmatrix}$	$\begin{bmatrix} 0 & \pm b_1 s^k \\ \pm b_2 s^l & 0 \end{bmatrix}$
	$n-m$	$k-l$
L–C	2	0
L–R	1	1
C–R	−1	−1
D–R	−2	−2
D–C	−1 (Same as Type 1, *C–R*)	−3
N–R	2 (Same as Type 1, *L–C*)	2
N–C	3	1 (Same as Type 2, *L–R*)
N–D	4	0 (Same as Type 2, *L–C*)
P–R	−4	−4

Table 3.

6. More types of mutators

Other types of mutators could be defined. For example, an N–D mutator is defined in Table 3. Note that any type 2 L–C mutator (including gyrators) can be used as a type 2 N–D mutator. A type 1 N–D mutator however is realizable by cascading two type 1 L–C mutators.

The frequency-dependent positive conductance FDPC (P) which is a one-port network element having $Y(s) = Ps^4$ may be realized using the P–R mutator whose general definition is also included very briefly in Table 3. Notice that any mutator from the type 1 N–D mutator family realizes a type 1 P–R mutator by interchanging its two ports or applying the $RC:CR$ transformation to it. Furthermore, a type 2 P–R mutator is realizable from a type 2 N–R mutator using the $RC:CR$ transformation.

7 Conclusions

New types of mutators for realizing an ideal FDNC, FDNR and FDPC are defined. Many more types of mutators could be defined. It is hoped here to direct the attention of the network designer to the utilization of the numerous circuit realizations available for the GPIC and the GPII in the simulation of ideal FDNC, FDNR or FDPC. Of course, in choosing a specific realization the practical limitations of the network including the effect of the non-ideal active elements and the stability of the active two-port must be examined. Also the number of capacitors to be used is an important factor to be considered.

References

Antoniou, A., 1970, *I.E.E.E. Trans. Circuit Theory*, **17,** 212 ; 1971, *Ibid.*, **18,** 297.

Bruton, L. T., 1969, *I.E.E.E. Trans. Circuit Theory*, **16,** 406.

Bruton, L. T., and Haase, A. B., 1974, *Int. J. Circuit Theory Applic.*, **2,** 187.

Chua, L., 1968, *Proc. I.E.E.E.*, **56,** 1325.

Deboo, G. L., 1967, *I.E.E.E. Trans. Circuit Theory*, **14,** 101.

Hasler, M., and Neirynck, J., 1976, *European Conference on Circuit Theory and Design.*

Mitra, S. K., 1967, *Proc. I.E.E.E.*, **55,** 2021 ; 1969, *Analysis and Synthesis of Linear Active Networks* (New York : Wiley).

Panzer, K., 1976, *I.E.E.E. Int. Symposium on Circuits and Systems*, p. 38.

Soliman, A. M., 1972, *I.E.E.E. Trans. Circuit Theory*, **19,** 371 ; 1973, *Int. J. Electron.*, **34,** 753 ; 1976, *Ibid.*, **41,** 59.

Wait, J. V., Huelsman, L. P., and Korn, G. A., 1975, *Introduction to Operational Amplifier Theory and Applications* (New York : McGraw-Hill), p. 317.