Use of Mirror Elements in the Active Device Synthesis by Admittance Matrix Expansion

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Abstract—**This paper proposes a modification for the symbolic synthesis method of analog circuits using admittance matrix expansion. The modification involves a generalization of the synthesis approach to employ mirror elements (voltage mirrors and current mirrors) in the admittance matrix expansion and ideal description of active elements, rather than using only nullor elements (nullators and norators). Accordingly, more alternative ideal representations, based on nullor–mirror elements, can be realized and a wide range of active elements can be used in the circuit synthesis. Systematic synthesis of the CCII-based generalized impedance converters (GICs) is presented as an application example to illustrate the potential of this generalized approach. Multiple equivalent nullor–mirror realizations for the GIC could be extracted easily, by virtue of using mirror elements in the admittance matrix expansion. Consequently, numerous circuit realizations, spanning various combinations of CCII types, have been generated in a simple and direct way.**

Index Terms—**Active-*RC* circuits, active circuit synthesis, admittance matrix, nullor, pathological element.**

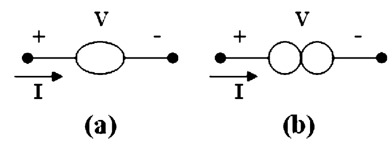

Fig. 1. Nullor elements. (a) Nullator. (b) Norator.

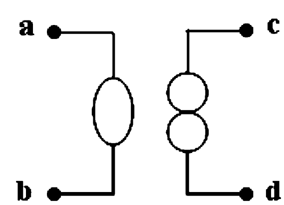

Fig. 2. Two-port nullor with a nullator connected between nodes a and b and a norator connected between nodes c and d.

I. Introduction

In THE PROCESS of designing active circuits, it is useful to follow systematic methodologies to obtain novel circuits. This is beneficial in developing analog tools for circuit design automation. In the literature of systematic synthesis for active circuits, some methods adopted admittance matrix expansion with the motivation to synthesize active circuits from a mathematical point of view without any detailed prior knowledge of the circuit form. There was an early work in this area employing admittance matrix expansion by node induction to realize a network consisting of amplifiers and resistors only [1].

Recently, a symbolic framework for systematic synthesis of linear active circuits based on admittance matrix expansion was presented in [2]–[5], in which a $p \times p$ port admittance matrix describing a certain circuit function is expanded to a port-equivalent $n \times n$ nodal admittance matrix (NAM) describing the synthesized circuit, such that $n > p$. The matrix expansion process begins by introducing blank rows and columns, representing new internal nodes, in the admittance matrix. Pivotal expansion [2], [4], which is the reverse operation of the Gaussian elimination, is applied thereafter to all matrix elements including products and quotients of admittance terms; until every element in the admittance matrix becomes a single admittance term. Then, nullators and norators (shown in Fig. 1) are used to move the resulting admittance matrix elements to their final locations, properly describing either floating or grounded passive elements. Thus, the final NAM is obtained including finite elements representing passive circuit components and unbounded elements, so called infinity-variables, representing nullators and norators.

In this framework, nullators and norators ideally describe active elements in the circuit [6], [7]. The nullator and norator are pathological elements that possess ideal characteristics and are specified according to the constraints they impose on their terminal voltages and currents. For the nullator, $V = I = 0$, while the norator imposes no constraints on its voltage and current. A nullator–norator pair constitutes a universal active two-port network element [8] called the nullor (shown in Fig. 2), and, hence, nullator and norator are also called nullor elements. The attractive feature of the two nullor elements is their ability to model active circuits independently of the particular realization of the active devices with the possibility of generating a number of ideally equivalent circuits from which the best practical ones can thereafter be selected [9], [10].

Despite the ability of nullor elements to describe many active building blocks, they fail to represent devices like the positive-type second-generation current conveyor (CCII+) proposed in [11]. Other passive elements like resistors are combined with nullators and norators in order to obtain the nullor representation of the CCII+ [12]. In order to avoid the use of passive elements in the nullor representation of any building block, additional pathological elements called mirror elements (shown in Fig. 3) were introduced in [13] to describe the voltage and current reversing actions. The voltage mirror (VM), shown in

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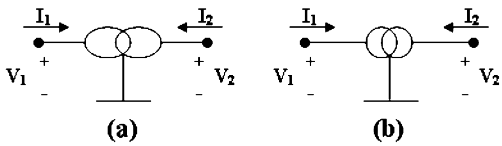


Fig. 3. Mirror elements. (a) Voltage mirror. (b) Current mirror.

Fig. 3(a), is a lossless two-port network element used to represent an ideal voltage reversing action, and it is described by

$$V_1 = -V_2 \tag{1a}$$
$$I_1 = I_2 = 0. \tag{1b}$$

The current mirror (CM), shown in Fig. 3(b), is a two-port network element used to represent an ideal current reversing action and it is described by

$$V_1 \text{ and } V_2 \text{ are arbitrary}$$
$$I_1 = I_2 \text{and they are also arbitrary.} \tag{2}$$

Although the current mirror element shown in Fig. 3(b) has the same symbol as the regular current mirror, it is a bi-directional element and has a theoretical existence. It is worth noting that each of the voltage mirror and the current mirror symbols shown in Fig. 3 has a reference node, which is set to ground, and, although these elements are two-port network elements, they are used as two terminal elements with the reference node unused [14].

In this paper, the conventional systematic synthesis framework using admittance matrix expansion, presented in [2]–[5], is extended to use all types of pathological elements (mirror elements and nullor elements), rather than using nullors only, for admittance matrix expansion and ideal description of active elements. This modification facilitates the extraction of more alternative ideal representations based on pathological elements, and hence, more circuit realizations for a certain function can be achieved. Moreover, a wide range of active elements can be used in the circuit synthesis.

The NAM stamp for the VM–CM pair is deduced in Section II, followed by the representation of mirror elements in the NAM using infinity-variables notation and bracket notation [2], [4], and the transformations guiding the admittance matrix expansion using mirror elements. In Section III, the systematic synthesis of the CCII-based GIC is presented as an application to demonstrate the potential of this generalized approach and its flexibility over the conventional approach in providing multiple alternative realizations using various building blocks. Conclusions are drawn in Section IV.

II. Mirror Elements in the NAM

A. NAM Stamp for Mirror Elements

It is known that, for physically realizable circuits, all of the voltages and currents are always uniquely and definitely determined. This in turn implies that, in the ideal representation of a physically realizable circuit, nullators (or voltage mirrors) and norators (or current mirrors) must occur in a pair [2], [15]. Since the representation of active devices using pairs of pathological elements necessarily imposes ideal description, then the gain of any dependent source used to represent the relation between the signals at two ports within a pair of pathological elements (nullor elements, mirror elements, or a combination of them) is always taken to infinity [2], [4]. Hence, the two-port nullor can be derived as a limit of any one of the four types of dependent sources (VCVS, CCCS, VCCS, and CCVS) when its gain tends to infinity [2].

Consider the general four-port VM-CM pair in Fig. 4, consisting of a voltage mirror between nodes a and b and a current mirror between nodes c and d and defined by

$$V_a = -V_b \tag{3a}$$
$$I_a = I_b = 0 \tag{3b}$$
$$V_c \text{ and } V_d \text{ are arbitrary}$$
$$I_c = I_d, \quad \text{and they are also arbitrary.} \tag{3c}$$

This VM–CM pair can be represented using four dependent sources when their gains tend to infinity, with every dependent source describing the relation between the signal at one port in one of the mirror elements and the signal at one port in the other mirror element. Since the considered synthesis framework uses the admittance matrices and the VCCS is the only dependent source that possesses an admittance matrix [2], the mirror pair in Fig. 4 should be described in terms of voltage-controlled current sources (VCCSs) for which the transconductance gains tend to infinity. Hence, the admittance matrix stamp for the representation of a four-port VM-CM pair with voltage mirror connected between nodes a and b and current mirror connected between nodes c and d, as in Fig. 4, can be considered as that for the VCCSs-based ideal model in Fig. 5. This VCCSs-based ideal model can be entered into the NAM in the following form:

$$\begin{array}{c} \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cc} a & \quad b \end{array} \\ \begin{bmatrix} G_{mi} & G_{mi} \\ G_{mi} & G_{mi} \end{bmatrix} \end{array} \tag{4}$$

where G_{mi} is the transconductance gain of every VCCS and is taken to a limit of infinity. Then, rows c and d of the NAM equation set will have the form

$$\begin{bmatrix} I_c \\ I_d \end{bmatrix} = \begin{bmatrix} G_{mi} & G_{mi} \\ G_{mi} & G_{mi} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} + \begin{bmatrix} \text{finite terms} \\ \text{finite terms} \end{bmatrix} \tag{5}$$

where $G_{mi} \to \infty$. Dividing both sides by G_{mi} and applying the limit

$$\mathrm{Lim}_{G_{mi}\to\infty} \begin{bmatrix} \frac{I_c}{G_{mi}} \\ \frac{I_d}{G_{mi}} \end{bmatrix} = \mathrm{Lim}_{G_{mi}\to\infty} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} + \mathrm{Lim}_{G_{mi}\to\infty} \begin{bmatrix} \frac{\text{finite terms}}{G_{mi}} \\ \frac{\text{finite terms}}{G_{mi}} \end{bmatrix}. \tag{6}$$

Thus, dependent current terms on the right-hand side and finite terms on the left-hand side will vanish to yield

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{7}$$

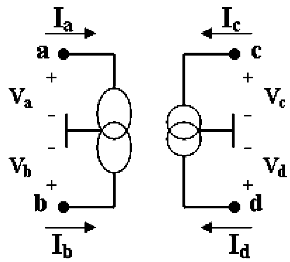


Fig. 4. Four-port mirror pair with a voltage mirror connected between nodes a and b and a current mirror connected between nodes c and d.

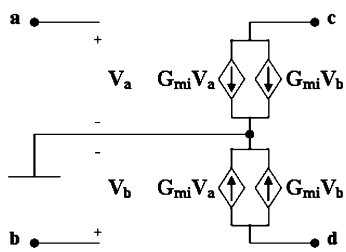


Fig. 5. Voltage controlled current sources based model for a mirror pair with a voltage mirror connected between nodes a and $b$and a current mirror connected between nodes c and d.

Both rows in the NAM equation set corresponding to the current mirror nodes yield the same relation between independent voltage variables

$$V_a + V_b = 0 \rightarrow V_a = -V_b. \quad (8)$$

Since no matrix entries for rows a and b, then

$$I_a = I_b = 0. \quad (9)$$

The similarity between the coefficients of rows c and d imposes the constraint that the current entering the current mirror at node c is equal to that entering it at node d; however, the values of the terminal voltages and currents for the current mirror are unconstrained.

Thus, the mirror pair description in (4) with $G_{\mathrm{mi}} \rightarrow \infty$ imposes finite relationships between the nodal voltages and currents for nodes a and b which correctly describe the voltage mirror and implies a finite relationship between the current entering the mirror pair at node c and the current entering the mirror pair at node d, without constrains on the nodal voltages and currents for nodes c and d, which correctly describes the current mirror.

B. Notations

As explained in [2]–[5], nullor elements can be represented in the NAM using infinity-variables. In this notation the variables in the NAM that are taken to an infinite limit are written as ∞_i, where ∞ indicates that the limit is taken to infinity and i refers to the active element the nullor is representing.

Using the infinity-variable as a placeholder for a variable that can tend to infinity at some point is just a notational convenience [2]. Replacement of variables that tend to infinity in an admittance matrix by infinity-variables implies that these variables can no longer be given numerical values and have to be treated as symbolic variables that may be manipulated by hand or by using symbolic computation [2]. Since infinity-variables are shorthand for finite variables with infinite limits, and algebraic transformations may be applied before taking the limit, it follows that infinity-variables must conform to the rules of algebra for regular variables, including Gaussian elimination and pivotal expansion of admittance matrices [2].

Thus, the infinity-variables share some properties with the normal infinity but not all [16]. Properties which are not shared [16] are

$$\alpha\infty_i - \beta\infty_i = \text{finite}, \qquad \text{if and only if} \quad \alpha = \beta. \quad (10)$$

$$\infty_i / \infty_i = 1 \quad (11)$$

$$\infty_i - \infty_i + G = G, \qquad \text{where } G \text{ is finite} \quad (12)$$

$$\infty_i - \infty_i + \infty_j = \infty_j. \quad (13)$$

A property which is shared [16] is

$$\infty_i \times 0 = \text{Intermediate}. \quad (14)$$

For the nullor in Fig. 2, the description in the NAM using infinity-variables takes the following form, which was introduced in [2]:

$$\begin{array}{c} \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} \infty_i & -\infty_i \\ -\infty_i & \infty_i \end{bmatrix} \end{array}. \quad (15)$$

On applying the infinity-variables notation to the mirror elements, the mirror pair description in (4) becomes

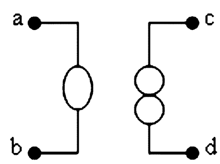
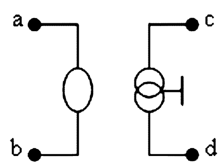
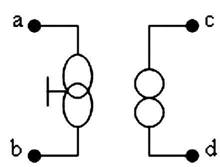
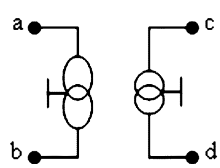
$$\begin{array}{c} \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} \infty_i & \infty_i \\ \infty_i & \infty_i \end{bmatrix} \end{array}. \quad (16)$$

The NAM stamps for the four possible pairs of pathological elements can be shown using the general set of infinity-variables in (17), which was introduced in [2], with a row-scaling factor α and a column-scaling factor β

$$\begin{array}{c} \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} \infty_i & \beta\infty_i \\ \alpha\infty_i & \alpha\beta\infty_i \end{bmatrix} \end{array}. \quad (17)$$

According to the voltage and current constraints imposed by nullor–mirror elements, the NAM representations for the four possible pairs of pathological elements correspond to the NAM stamps described by the above set of infinity-variables when α and β are ± 1. The value of α determines whether the pathological pair includes a norator or a current mirror, such that $\alpha = -1$ indicates a norator while $\alpha = 1$ indicates a current mirror. Similarly, the value of β determines whether the pair includes a nullator or a voltage mirror, such that $\beta = -1$ indicates a nullator while $\beta = 1$ indicates a voltage mirror. Table I shows the symbolic representations for the four possible pairs of pathological elements that can be described by the general set of infinity-variables in (17), specified according to the values of α and β. Note that using the infinity-variable as a placeholder for a variable

TABLE I
NULLOR–MIRROR PAIRS DESCRIBED BY (17) ACCORDING TO THE VALUES OF α AND β

α	β	Symbolic description
-1	-1	
1	-1	
-1	1	
1	1	

that can tend to infinity at some point is just a notational convenience [2], [4].

An alternative notation for the nullor elements was also used in [2]–[5], which is called the bracket notation. In this notation, nodes linked by nullators are indicated by brackets linking the corresponding columns in the NAM. Similarly, nodes linked by norators are indicated by brackets linking the corresponding rows in the NAM. Using this notation, the nullor description in (15) takes the form

$$\begin{array}{c} \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}} \end{array}\Big] \tag{18}$$

The bracket notation can be applied to mirror elements in a similar way such that, nodes linked by voltage mirrors are indicated by brackets linking the corresponding columns in the NAM and nodes linked by current mirrors are indicated by brackets linking the corresponding rows in the NAM. However, a different shape is used for brackets that indicate mirror elements in order to be able to distinguish between mirror and nullor elements when both occur in a realization. Using bracket notation, the mirror pair description in (16) takes the form

$$\begin{array}{c} \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}} \end{array}\Big\} \tag{19}$$

Bracket notation is not a mathematical one and thus it is not suitable for analysis. On the other hand it is more general than the $\pm\infty_i$ notation, because in the case where there is more than one pair of pathological elements, the bracket notation allows pathological elements to be paired in different ways resulting in various equivalent realizations [4].

C. Equivalence Transformations for Admittance Matrices Containing Descriptions of Mirror Elements

Based on the arbitrary element theorem and its corollary, the element shift theorem, for admittance matrices with unbounded elements [2], the NAM equivalence transformations associated with nullor elements have been deduced in [2] and [4]. Similarly, the equivalence transformations associated with mirror elements descriptions in the NAM will be deduced in this subsection using the arbitrary element and element shift theorems.

For the general set of infinity-variables in (17), the arbitrary element theorem indicates the equivalence described in

$$\begin{array}{c} \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} \infty_i & \beta\infty_i \\ \alpha\infty_i & \alpha\beta\infty_i \end{bmatrix} \end{array} \equiv \begin{array}{c} \\ c \\ d \\ f \end{array}\begin{array}{c} \begin{array}{ccc} a & b & e \end{array} \\ \begin{bmatrix} \infty_i & \beta\infty_i & n_e \\ \alpha\infty_i & \alpha\beta\infty_i & \alpha n_e \\ m_f & \beta m_f & \ddots \end{bmatrix} \end{array}. \tag{20}$$

Variables n_e and m_f are arbitrary finite expressions and f and e represent any row or column including the rows and columns occupied by the set of infinity-variables. The element shift theorem is obtained by applying the arbitrary element theorem in the special case where there already exist matrix elements q_{ce} and p_{fa} while $n_e = -p_{fa}$ and $m_f = -q_{ce}$, as described by

$$\begin{array}{c} \\ c \\ d \\ f \end{array}\begin{array}{c} \begin{array}{ccc} a & b & e \end{array} \\ \begin{bmatrix} \infty_i & \beta\infty_i & q_{ce} \\ \alpha\infty_i & \alpha\beta\infty_i & 0 \\ p_{fa} & 0 & \ddots \end{bmatrix} \end{array} \equiv \begin{array}{c} \\ c \\ d \\ f \end{array}\begin{array}{c} \begin{array}{ccc} a & b & e \end{array} \\ \begin{bmatrix} \infty_i & \beta\infty_i & 0 \\ \alpha\infty_i & \alpha\beta\infty_i & -\alpha q_{ce} \\ 0 & -\beta p_{fa} & \ddots \end{bmatrix} \end{array}. \tag{21}$$

The description of a VM–CM pair corresponds to setting $\alpha = \beta = 1$ in (17), in which case the arbitrary element theorem in (20) takes the form

$$\begin{array}{c} \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} \infty_i & \infty_i \\ \infty_i & \infty_i \end{bmatrix} \end{array} \equiv \begin{array}{c} \\ c \\ d \\ f \end{array}\begin{array}{c} \begin{array}{ccc} a & b & e \end{array} \\ \begin{bmatrix} \infty_i & \infty_i & n_e \\ \infty_i & \infty_i & n_e \\ m_f & m_f & \ddots \end{bmatrix} \end{array}. \tag{22}$$

Under the same conditions, the element shift theorem in (21) takes the following form:

$$\begin{array}{c} \\ c \\ d \\ f \end{array}\begin{array}{c} \begin{array}{ccc} a & b & e \end{array} \\ \begin{bmatrix} \infty_i & \infty_i & q_{ce} \\ \infty_i & \infty_i & 0 \\ p_{fa} & 0 & \ddots \end{bmatrix} \end{array} \equiv \begin{array}{c} \\ c \\ d \\ f \end{array}\begin{array}{c} \begin{array}{ccc} a & b & e \end{array} \\ \begin{bmatrix} \infty_i & \infty_i & 0 \\ \infty_i & \infty_i & -q_{ce} \\ 0 & -p_{fa} & \ddots \end{bmatrix} \end{array}. \tag{23}$$

From the arbitrary element and element shift theorems in case of VM–CM pair described in (22) and (23), the equivalence transformations associated with mirror elements descriptions in a NAM are deduced as follows. Where a set of infinity-variables in a NAM describing a pair of pathological elements including a voltage mirror, then: 1) arbitrary finite elements can be added anywhere in the rows containing this set of infinity-variables, provided that the set of added elements has a row scale factor equal to that of the infinity-variables set and 2) matrix elements can be moved arbitrarily between the two columns corresponding to the nodes linked by the voltage mirror, but the signs of the moved elements must be reversed. Similarly, where a set of infinity-variables in a NAM describing a pair of pathological elements including a current mirror, then: 1) arbitrary finite elements can be added anywhere in the columns containing this set of infinity-variables, provided that the set of added elements has a column scale factor equal to that of the infinity-variables set and 2) matrix elements can be moved arbitrarily between the two rows corresponding to the nodes linked by the current mirror, but the signs of the moved elements must be reversed.

These equivalence transformations associated with the mirror elements represent the key contribution of the usage of voltage mirrors and current mirrors in the admittance matrix expansion. The interesting feature of reversing the signs of the matrix elements when moving them between rows or columns, corresponding to nodes linked by mirror elements, facilitates more choice in realizing alternative ideal descriptions for the circuits to be synthesized. Consider the case where there is a negative element in the admittance matrix. If nullor elements only are to be used for the matrix expansion, then the convenient direction of the transformations will be to use this negative matrix element in describing a floating component. Hence, in the general case, one or two nullor elements are used to move the matrix element to its final position and other nullor elements are used to retrieve the missing elements, which are needed to completely represent a floating element, from the rows and/or columns corresponding to other nodes (e.g., ground node). If mirror elements are to be used in the matrix expansion, then their associated sign-reversing feature enables changing this negative element into a positive one, and it can be moved simply to the diagonal using lower number of pathological elements. Accordingly, the option of having this element describing a grounded element becomes possible in a flexible and easy way. The potential of this approach will be illustrated by examples in the following section, through the systematic synthesis of CCII-based generalized impedance converters.

III. CCII-Based Generalized Impedance Converters Synthesis Example

The generalized impedance converter is an active two-port network in which $Z_{\rm in} = f(s)Z_L$, where $Z_{\rm in}$ and Z_L are the input and output load impedances, respectively, and $f(s)$ is a general function referred to as the impedance conversion function [17], [18]. The second-generation current conveyors (CCIIs) with all their types (CCII+, CCII-, ICCII+, and ICCII-) are versatile building blocks and can be ideally represented using nullor–mirror elements. The symbols and the nullor–mirror representations for the four types of the CCII are shown in Fig. 6 [13], [15]. Here, the systematic synthesis of the CCII-based GICs is taken as an example on using the generalized synthesis approach. This is to illustrate the flexibility added to the synthesis method by providing the means of using the mirror elements in addition to nullor elements in the admittance matrix expansion and the ideal representation of active elements in the synthesized circuits.

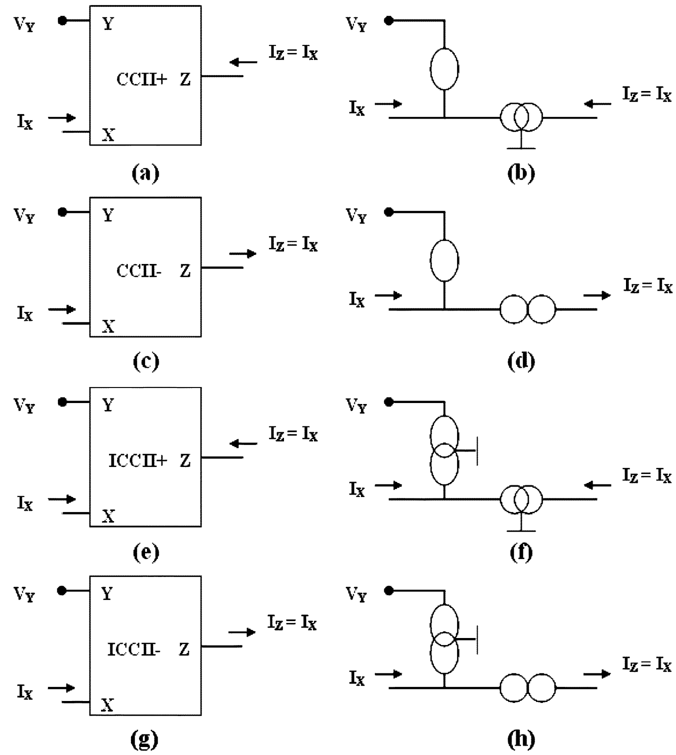


Fig. 6. (a) CCII+ symbol. (b) CCII+ nullor–mirror representation. (c) CCII- symbol. (d) CCII- nullor–mirror representation. (e) ICCII+ symbol. (f) ICCII+ nullor–mirror representation. (g) ICCII- symbol. (h) ICCII- nullor–mirror representation.

A. Synthesis

It is known that the GIC is described using the following relationship between its terminal voltages and currents:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{Y_1 Y_3}{Y_2 Y_4} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}. \tag{24}$$

Thus, the GIC cannot be described in admittance matrix using only unbounded elements [2], [4]. On the other hand, the synthesis framework uses the admittance matrix for the synthesis operation. In the process of achieving a conversion of transmission matrices for useful circuit functions to admittance matrix form, five expanded admittance matrix stamps using infinity-variables and a single expanded stamp without infinity-variables for the impedance converter were deduced in [2]. It can be shown that, using the impedance converter expansions Type i or Type ii in [2] and applying row and column operations, optimized ideal realizations with only one pair of nullor–mirror elements in addition to the intrinsic nullor (represented by $\pm\infty_1$) can be achieved [4]. However, in such realizations, there is no common node between any two pathological elements that can constitute a pair. In order to reach a CCII-based realization, each pair of pathological elements in the symbolic ideal description must possess a common node. This imposes a constraint that at least two pairs of nullor–mirror elements in addition to the intrinsic nullor are needed to realize a CCII-based

GIC using the impedance converter expanded admittance matrix stamps in [2]. Also, it is known that grounded passive elements are more amenable to monolithic integration than floating ones. Hence, the most optimized CCII-based GICs that can be generated using the impedance converter admittance matrix expansions in [2] are those built using three current conveyors and four grounded passive elements.

The following expansion of the impedance converter, which is expansion Type i in [2], will be used:

$$\begin{bmatrix} 0 & 0 & -N_I & 0 \\ 0 & \infty_1 & 0 & -\infty_1 \\ 0 & -\infty_1 & D_I & \infty_1 \\ -N_v & 0 & 0 & D_v \end{bmatrix} \tag{25}$$

where the forward voltage gain $A_V = N_V/D_V$ and the reverse current gain $A_I = N_I/D_I$. The GIC is described by $A_V = 1$ and $A_I = Y_1Y_3/Y_2Y_4$. Gaussian elimination of row 4 and column 4 in the above NAM stamp will yield the following reduced stamp:

$$\begin{bmatrix} 0 & 0 & -N_I \\ -\infty_1 & \infty_1 & 0 \\ \infty_1 & -\infty_1 & D_I \end{bmatrix}. \tag{26}$$

In order to achieve a final expanded NAM with all finite matrix elements describing grounded components and using minimum number of pathological elements, let $N_I = Y_1Y_3/Y_2$ and $D_I = Y_4$. The stamp in (26) will become

$$\begin{bmatrix} 0 & 0 & \frac{-Y_1Y_3}{Y_2} \\ -\infty_1 & \infty_1 & 0 \\ \infty_1 & -\infty_1 & Y_4 \end{bmatrix}. \tag{27}$$

Applying the pivotal expansion [2], [4] to the term Y_1Y_3/Y_2, the expanded stamp may take any of the following two forms:

$$\begin{bmatrix} 0 & 0 & 0 & -Y_1 \\ -\infty_1 & \infty_1 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 \\ 0 & 0 & -Y_3 & Y_2 \end{bmatrix} \tag{28}$$

or

$$\begin{bmatrix} 0 & 0 & 0 & Y_1 \\ -\infty_1 & \infty_1 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 \\ 0 & 0 & Y_3 & Y_2 \end{bmatrix}. \tag{29}$$

For the expansion in (28), to get the off-diagonal elements on to the diagonal, blank rows and columns 5 and 6 are introduced, and then pathological elements are added to use their associated equivalence transformations in the matrix expansion. In order to convert the element $-Y_1$ in 1, 4 to an on-diagonal positive element, a nullator is added between nodes 4 and 5 to move $-Y_1$ to the position 1, 5 and then a current mirror is added between nodes 1 and 4 to move $-Y_1$ to the on-diagonal position 5, 5 and change its sign. Thus, Y_1 becomes a positive element existing on the diagonal, as shown in (30). The added nullor–mirror elements are represented by bracket notation.

$$\begin{bmatrix} 0 & 0 & 0 & -Y_1 \\ -\infty_1 & \infty_1 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 \\ 0 & 0 & -Y_3 & Y_2 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 0 & 0 & 0 & 0 & -Y_1 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 \\ 0 & 0 & -Y_3 & Y_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 \\ 0 & 0 & -Y_3 & Y_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{30}$$

Similarly, $-Y_3$ at 4, 3 can be changed into Y_3 at 6, 6. The resulting expanded matrix is shown in

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 \\ 0 & 0 & -Y_3 & Y_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2 & 0 & -Y_3 \\ 0 & 0 & 0 & 0 & Y_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_3 \end{bmatrix} \tag{31}$$

The nullor–mirror equivalent circuit for the GIC described by the NAM in (31) is shown in Fig. 7. Since nullor–mirror elements are connected in a continuous loop, pathological elements can be paired as current conveyors in two alternative ways; resulting in the two equivalent CCII-based GICs in Fig. 8(a) and (b). The GIC in Fig. 8(a) has its input fed to a Y–Z-terminal, while the GIC in Fig. 8(b) is characterized by having an X-input. Applying all possible combinations of the added nullor–mirror elements to realize a GIC with three active devices and four grounded passive elements using the expansions in (28) and (29) will yield the eight equivalent

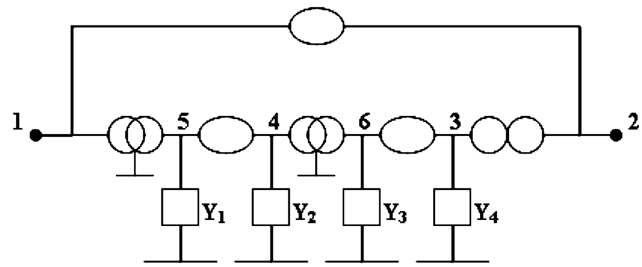


Fig. 7. Nullor–mirror representation for the GIC.

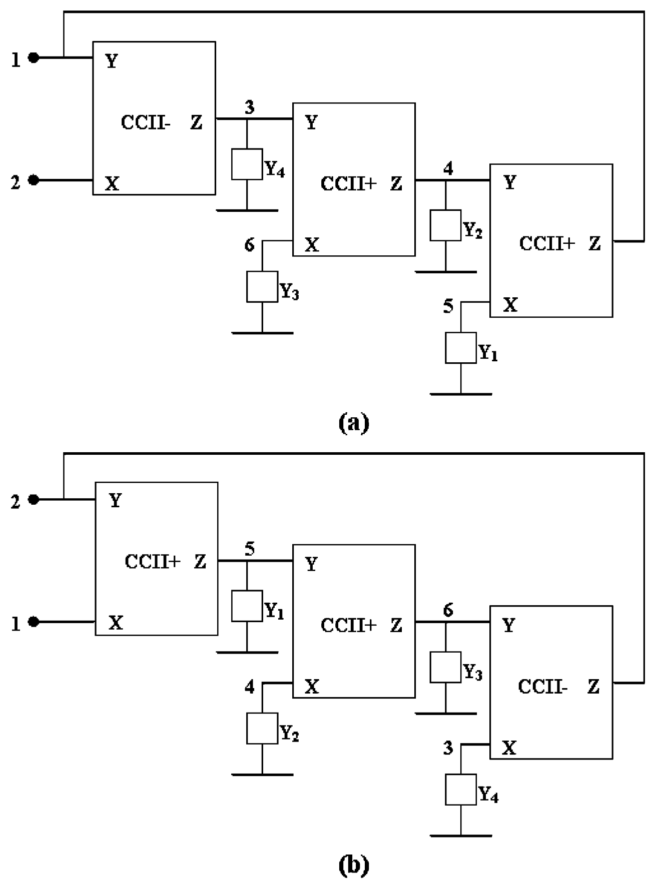


Fig. 8. CCII-based GIC. (a) Y–Z-input. (b) X-input.

nullor–mirror circuits shown in Fig. 9. Consequently, 16 different circuit realizations (eight with Y-Z-input and eight with X-input) for the CCII-based GIC can be achieved.

B. Discussion

Mirror elements can be represented using nullor elements and resistors as shown in Figs. 10 and 11, [13], [14]. In order to appreciate the flexibility added to the systematic synthesis approach after including mirror elements in the admittance matrix expansion and ideal description of active elements, consider the expansion of the NAM in (28) to the final form in (31). The current mirror between nodes 4 and 6 is used to get $-Y_3$ from row 4 to row 6 and change it to a positive element. If this task were to be done using nullor elements, then the following complex scenario would take place. Blank rows and columns 7 and 8 are introduced to be used in the NAM operations and transformations targeting the movement of $-Y_3$ from row 4 to row 6 and changing its sign, while blank rows and columns 5 and 6 are kept because they are used in the other expansion steps from (28)–(31). Row 4 is multiplied by -1 and added to rows 7 and 8 to yield

$$\begin{bmatrix} 0 & 0 & 0 & -Y_1 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -Y_3 & Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -Y_1 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -Y_3 & Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & -Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & -Y_2 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (32)$$

A norator is connected between nodes 6 and 7 to move the Y_3 from row 7 to row 6 to yield

$$\begin{bmatrix} 0 & 0 & 0 & -Y_1 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -Y_3 & Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & -Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & -Y_2 & 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 & 0 & -Y_1 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -Y_3 & Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & -Y_2 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (33)$$

Then, another norator is added between nodes 4 and 8, and accordingly Y_3 in row 8 can be moved to row 4. The position 4, 3 will then have $Y_3 - Y_3 = 0$:

$$\begin{bmatrix} 0 & 0 & 0 & -Y_1 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -Y_3 & Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & -Y_2 & 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 & 0 & -Y_1 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Y_2 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (34)$$

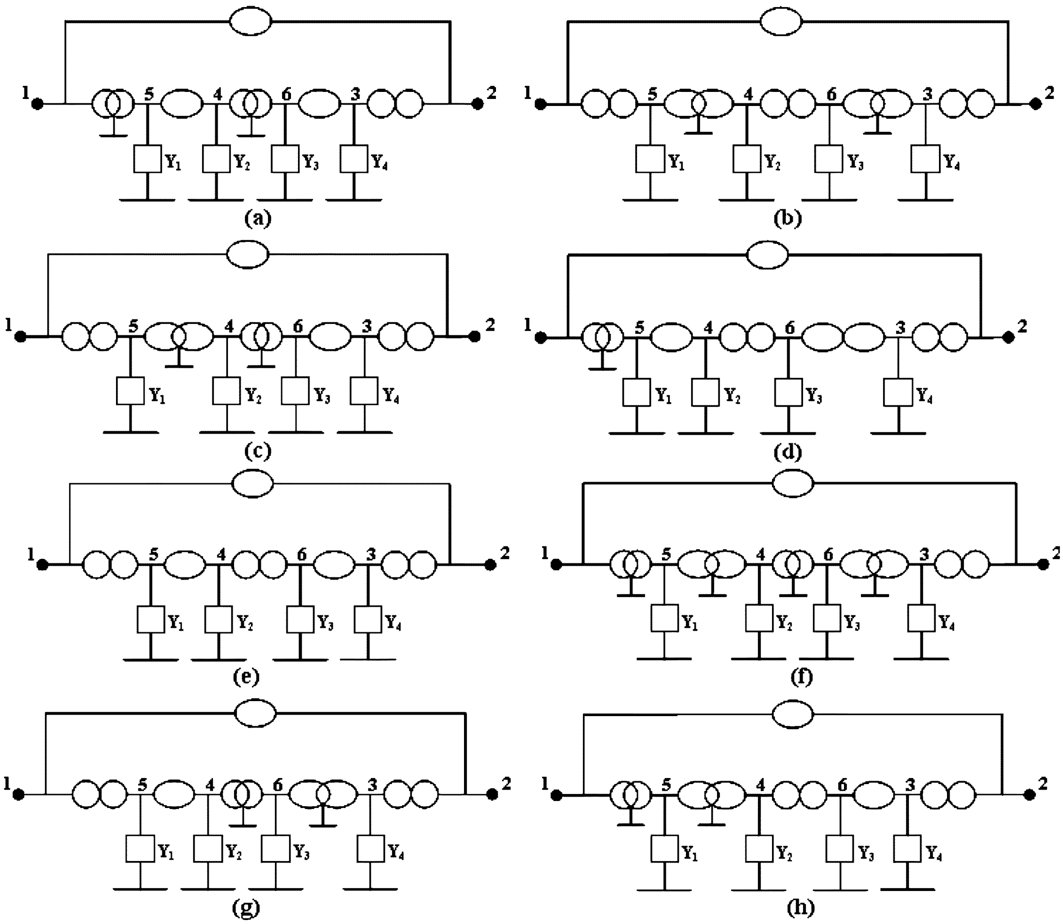


Fig. 9. Eight equivalent nullor–mirror descriptions for a GIC with three active devices and four grounded passive elements.

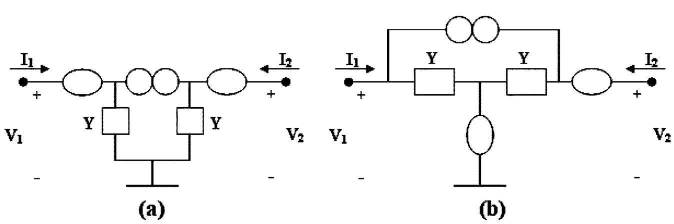


Fig. 10. (a) Voltage mirror representation using nullor elements and two equal passive elements. (b) Equivalent voltage mirror representation using nullor elements and two equal passive elements.

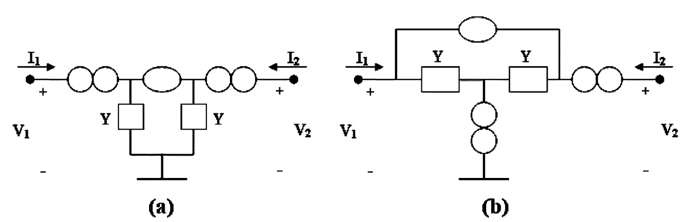


Fig. 11. (a) Current mirror representation using nullor elements and two equal passive elements. (b) Equivalent current mirror representation using nullor elements and two equal passive elements.

Connection of a nullator between nodes 4 and 8 and a nullator between nodes 7 and 8 allows the $-Y_2$ terms in column 4 to be moved to columns 7 and 8 and become diagonal elements. The resulting expansion takes the form in The dashed frame in Fig. 12 surrounds the equivalent nullor circuit used to get $-Y_3$ from row 4 to row 6 and change it to a positive element. This nullor circuit is the nullor equivalent for a current mirror connected between nodes 4 and 6. There is a nullator connected between nodes 4 and 8, which is not included in the current mirror representation using nullor elements in Fig. 11(a). However, the nullor circuit in Fig. 12 indicates that the current flowing through the grounded $-Y_2$ connected at node 8 is the same current flowing through the grounded Y_2 connected at node 4, and hence, this current causes V_4 to be equal to V_8. Thus, connecting node 4 and node 8 by a nullator will not change the operation of the nullor circuit surrounded by the dashed frame in Fig. 12 as a current mirror.

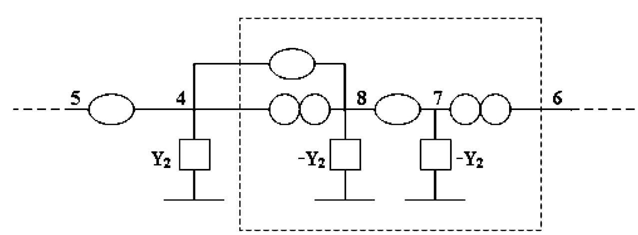


Fig. 12. Equivalent nullor circuit for the current mirror connected between nodes 4 and 6 in the GIC described in (31).

The above discussion describes the complexity experienced in the expansion process to realize circuits similar to those in

Fig. 8(a) and (b) without using mirror elements in the ideal description. All of the steps from (32) to (35) have been taken to perform a simple transformation step that can be done using a single mirror element.

$$\begin{bmatrix} 0 & 0 & 0 & -Y_1 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Y_2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 0 & 0 & 0 & -Y_1 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -Y_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -Y_2 & 0 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 0 & 0 & 0 & -Y_1 & 0 & 0 & 0 & 0 \\ -\infty_1 & \infty_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \infty_1 & -\infty_1 & Y_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -Y_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Y_2 \end{bmatrix} \quad (35)$$

Thus, the potential of using mirror elements in the admittance matrix expansion are illustrated accordingly and it is clear now how the ability to use mirror elements in the ideal representation facilitates the synthesis process and enables extraction of all possible alternative ideal realizations with specific features that perform a certain task. Besides, more circuit realizations can be achieved from the ideal descriptions containing mirror elements by replacing the mirror elements by their nullor equivalent descriptions in Figs. 10 and 11. Due to limited space allowed for this paper, a second application example for the generalized systematic synthesis framework has been presented in [19].

IV. Conclusion

The systematic synthesis method based on admittance matrix expansion using nullor elements has been extended to accommodate mirror elements. This results in a generalized framework encompassing all pathological elements for ideal description of active elements. Accordingly, more alternative realizations are possible and a wide range of active devices can be used in the synthesis. The advantages of this generalized synthesis approach have been demonstrated through an application example, which is systematic synthesis of CCII-based GIC. Multiple nullor-mirror descriptions could be easily generated. Additionally, active blocks that are easily described using mirror elements like CCII+, ICCII-, and ICCII+ were used in the synthesized circuits. Hence, various equivalent realizations for the GIC have been achieved.

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