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ADJOINT NETWORK THEOREM AND FLOATING ELEMENTS IN THE NAM

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Although the adjoint network theorem preserves all the circuit properties it does not, however, guarantee that the floating property of an element is maintained. In other words, the adjoint of a floating element may not be floating and vice-versa a nonfloating element may have an adjoint floating element as will be explained in this paper.

An important and new property of the Nodal Admittance Matrix (NAM) is that it can identify any element as a floating or nonfloating. The four floating basic building blocks including the nullor are tabulated. It is shown that the nullor and the Voltage Mirror (VM)–Current Mirror (CM) pair are self adjoint. The other two floating elements namely Nullator–CM pair and the VM–Norator pair are adjoint to each other.

The NAM of the Op Amp family and Current Conveyor (CCII) family are also given. Two examples are given demonstrating the generation of two families of CCII filters from two known two-CCII filter circuits with demonstration of the floatation property in each of the two filters.

Although the paper has a tutorial nature it also includes new important results.

Keywords: Adjoint theorem; nodal admittance matrix; floating elements.

1. Introduction

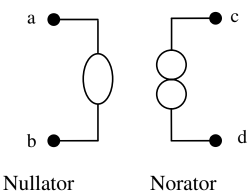
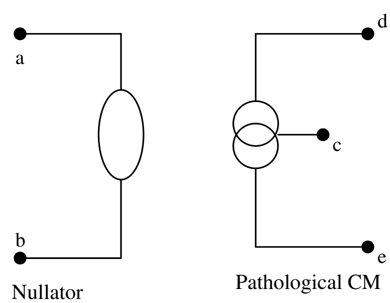
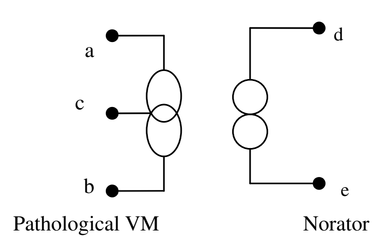
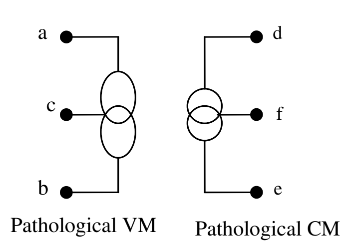
The reciprocity theorem introduced in Ref. 1 defines a circuit as reciprocal if the same transfer function is obtained when the input and output are interchanged. Circuits containing active elements generally do not satisfy the reciprocity theorem. The scope of the reciprocity theorem was extended in Ref. 2 by defining the concept of inter-reciprocity. An inter-reciprocal circuit is known as the adjoint of the original circuit. The adjoint circuit can be found by following the rules given in Ref. 1 and summarized in Refs. 3–6. As stated in Ref. 5 since the circuit and its adjoint are inter-reciprocal, they are exactly equivalent in terms of signal transfer, sensitivity, power dissipation, etc. The properties of the adjoint circuit can therefore be inferred from the properties of the original circuit without requiring any further analysis.

In this paper, it is shown that all the properties are preserved to the adjoint circuit except one important property namely the floating status. It will be shown that the adjoint of a floating element may not be floating and vice-versa a nonfloating element may have an adjoint floating element as will be explained by examples.

2. The Nodal Admittance Matrix

The Nodal Admittance Matrix (NAM) stamp for representation of a nullor[7] with nullator connected between nodes a and b and norator connected between nodes c and d as shown in Table 1, can be considered as that of a Voltage Controlled Current

Table 1. NAM description of the four basic floating building blocks.

	NAM	Pathological element representation
Nullor	$\begin{matrix} & a & b \\ \begin{matrix} c \\ d \end{matrix} & \multicolumn{2}{l}{\begin{bmatrix} \infty_i & -\infty_i \\ -\infty_i & \infty_i \end{bmatrix}} \end{matrix}$	
Nullator CM pair	$\begin{matrix} & a & b \\ \begin{matrix} d \\ e \\ c \end{matrix} & \multicolumn{2}{l}{\begin{bmatrix} \infty_i & -\infty_i \\ \infty_i & -\infty_i \\ -2\infty_i & 2\infty_i \end{bmatrix}} \end{matrix}$	
VM–Norator pair	$\begin{matrix} & a & b & c \\ \begin{matrix} d \\ e \end{matrix} & \multicolumn{3}{l}{\begin{bmatrix} \infty_i & \infty_i & -2\infty_i \\ -\infty_i & -\infty_i & 2\infty_i \end{bmatrix}} \end{matrix}$	
VM–CM pair	$\begin{matrix} & a & b & c \\ \begin{matrix} d \\ e \\ f \end{matrix} & \multicolumn{3}{l}{\begin{bmatrix} \infty_i & \infty_i & -2\infty_i \\ \infty_i & \infty_i & -2\infty_i \\ -2\infty_i & -2\infty_i & 4\infty_i \end{bmatrix}} \end{matrix}$	

Source with trans-conductance G_{mi} as described in Refs. 8 and 9 and given by

$$\begin{array}{c} \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} G_{mi} & -G_{mi} \\ -G_{mi} & G_{mi} \end{bmatrix} \end{array}. \tag{1}$$

G_{mi} is taken to a limit of infinity. The infinite limits of elements in the NAM may be used with the understanding that the limit applies to the NAM equation rather than the NAM elements in isolation.[8]

As stated in Ref. 8, the symmetry of the coefficients in (1) imposes the constraint that the current entering the norator is equal to that leaving it and (1) imposes KCL at nodes c and d; however, the norator voltage and current are otherwise unconstrained. This statement can be generalized as an important property of the NAM and stated in the following form:

Property 1. *A necessary condition that an active element is floating is that the summation of each column in the NAM be zero.*

On the other hand, if the summation of any of the columns is not zero implies that the element is not floating.

As seen from (1), *the nullor is a floating element and is also known by the name Four Terminal Floating Nullor.*

Table 1 *includes the four basic floating building blocks using the four pathological elements namely nullator, norator, pathological Voltage Mirror* (*VM*), *and pathological Current Mirror* (*CM*).[10–14]

It is seen that each of the four building blocks satisfies the condition stated above and it is a floating active element.

Property 2. *If the NAM of an active element is symmetrical then the active element is self adjoint.*

On the other hand, if the NAM of the active element is not symmetrical then the adjoint element NAM is the transpose of the original one. This property was reported in Ref. 4 *with respect to admittance matrices of nonreciprocal active elements.*

It is well known that the nullor is self adjoint.[5] *The interchange of nullators and norators and vice-versa was given in Ref.* 15 *in explaining the Voltage Controlled Voltage Source* (*VCVS*) *to Current Controlled Current Source* (*CCCS*) *transformations and then in Ref.* 16 *to obtain an alternative oscillator circuit from a given one with the same characteristic equation. In Ref.* 17, *the interchange of nullators and norators and vice-versa was generalized in the transformation of voltage mode to current mode circuits. In a similar way the generalization to pathological mirror elements in applying the adjoint transformation was introduced in Ref.* 10, *where it is stated that*: *the adjoint of any building block can be obtained by the interchange of VM and CM and vice-versa.*

From Table 1, it is also seen that the NAM of the VM–CM pair is symmetrical implying that the VM–CM pair is self adjoint a new property for the pathological mirror elements.

From Table 1, it is also seen that the NAM of the Nullator–CM pair is the transpose of the NAM of the VM–Norator pair, therefore they are adjoint to each other.

It is worth noting that the well-known statement that; in any circuit the nullator and norator must always exist in pairs should be revised after the invention of the pathological mirror elements in 1999.[10] It should be stated that the nullator or VM and the norator or CM must always exist in pairs.[10–14] Of course, second and third rows of Table 1 are good examples for the above revised statement.

3. The NAM and the Op Amp Family

It is well-known that the Voltage Operational Amplifier (VOA) is represented by a nullator and a grounded norator[5,15,18–22] as given in Table 2 together with its NAM representation. Based on Property 1, the VOA is of course a nonfloating element.

The adjoint of the two input single output VOA is a single input two output Current Operational Amplifier (COA) as stated clearly in Ref. 23.

The second row in Table 2 includes the NAM of the COA which is the transpose of the VOA–NAM. Based on Property 1, it can be seen that the COA is a floating active element although its adjoint namely the VOA is not floating. In general, it is stated that although the adjoint transformation maintains most of the properties of the transformed circuits it may not maintain the floating status.

Of course, the COA has frequency limitations exactly like the VOA and methods of active compensation of the VOA given in Ref. 24, applies directly to the COA using the adjoint network theorem as explained in Ref. 25.

The single input single output VOA has one row one column NAM as given in third row of Table 2. This is of course a self adjoint element as clearly given in Refs. 26 and 27 and then was used in amplifiers and filter realizations in Refs. 28 and 29.

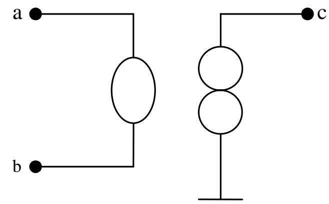
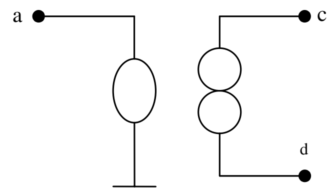
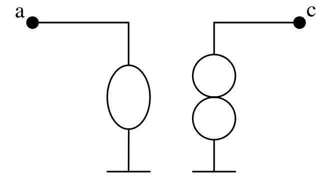
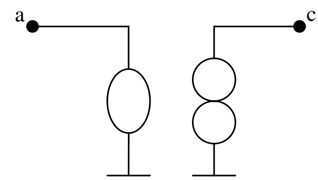
The single input single output COA also has one row and one column NAM as given in fourth row of Table 2. This is of course a self adjoint element also. It is also the adjoint of the single input single output VOA.

From the above, it is clear that the single input single output VOA is equivalent to the single input single output COA, this was observed in Ref. 26.

Although the application of the adjoint network theorem to VCVS using VOA was given in Refs. 5, 17, 23, and 25 and since this is a partially tutorial paper it will be further be demonstrated here as examples with a focus on the floating nature of the COA.

Figure 1(a) represents the well-known noninverting VCVS of gain equal to $(K + 1)$. The VOA is represented by the nullator and grounded norator as shown in Fig. 1(b). The adjoint network theorem is applied to Fig. 1(b) and the nullator and norator are exchanges as well as input and output resulting in Fig. 1(c). Replacing the grounded nullator and norator by the COA the CCCS of gain $(K + 1)$ is

Table 2. NAM description of the Op Amp family (VOA and COA).

	NAM	Pathological element representation
VOA	$\begin{matrix} & a & b \\ c & [\infty_i & -\infty_i] \end{matrix}$	
COA single input two outputs	$\begin{matrix} & a \\ c & \infty_i \\ d & -\infty_i \end{matrix}$	
Single input VOA	$\begin{matrix} & a \\ c & [\infty_i] \end{matrix}$	
COA single input single output	$\begin{matrix} & a \\ c & [\infty_i] \end{matrix}$	

obtained as shown in Fig. 1(d). It is interesting to observe that the current leaving the circuit to ground equals to zero, this is of course due to the floating nature of the COA.

Figure 2(a) represents the well-known inverting VCVS of gain equal to $-K$. The single input VOA is represented by the grounded nullator and grounded norator as shown in Fig. 2(b). The adjoint network theorem is applied to Fig. 2(b) and the nullator and norator are exchanges as well as input and output resulting in Fig. 2(c). Replacing the grounded nullator and grounded norator by the single input VOA the CCCS of gain $-K$ is obtained as shown in Fig. 2(d). Replacing the grounded nullator and grounded norator by the single input single output COA another equivalent circuit to Fig. 2(d) of the CCCS of gain $-K$ is obtained as shown in Fig. 2(e). It is interesting to observe that the current leaving the circuit to ground equals to zero. This is of course due to the floating nature of the COA. This floating nature does not exist in VOA equivalent circuit of Fig. 2(d).

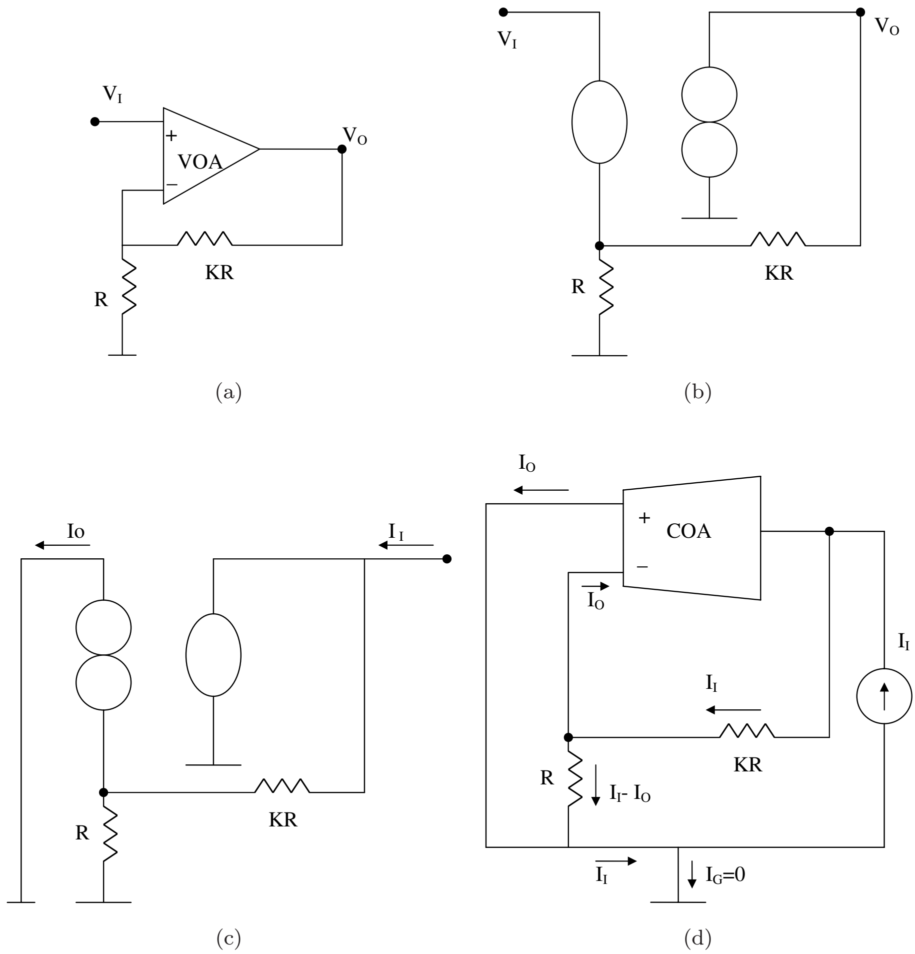


Fig. 1. (a) Noninverting amplifier using VOA, (b) equivalent circuit to (a), (c) adjoint circuit to (b), and (d) equivalent circuit to (c) using COA.

4. The NAM and the Current Conveyor Family

The first two members of the second generation current conveyor (CCII) family namely CCII+ and CCII–, were introduced in 1970.[30] At this time, the technology was not ready for this great invention by Sedra and Smith. It attracts the attention of many authors and no one knows that the CCII+ and CCII– are a part from a family till 1999 when Ref. 10 was published showing that the other missing two members are ICCII+ and ICCII–.

Tables 3(a) and 3(b) include the NAM representation of the CCII family, which consists of the three CCII members and the three ICCII members.

Indeed the Inverting CCII members are necessary to complete the family of the CCII, as the CCII+ is the adjoint of the ICCII– as demonstrated in Table 4.

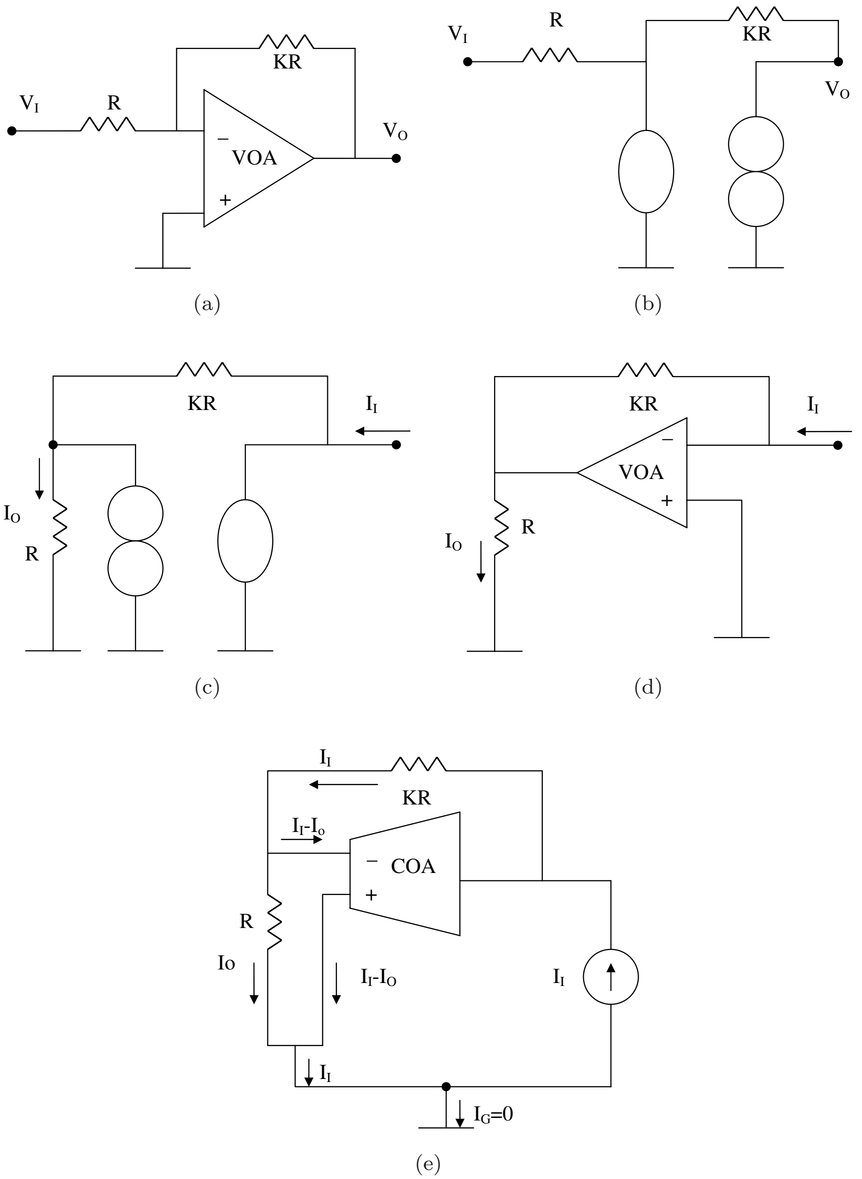
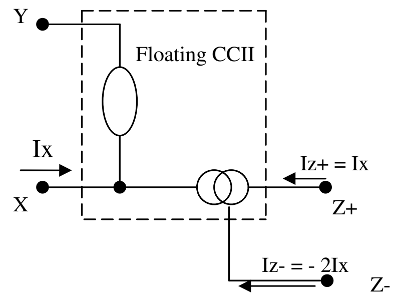
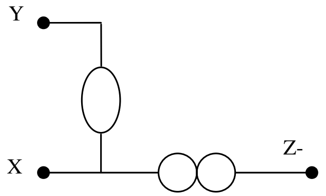
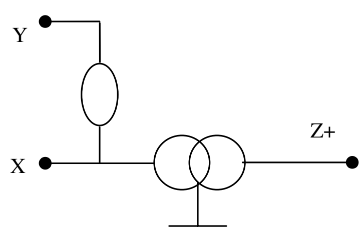
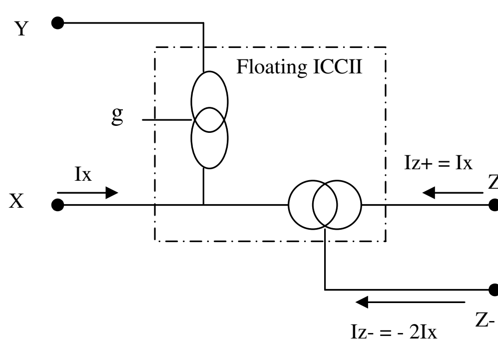


Fig. 2. (a) Inverting amplifier using VOA, (b) equivalent circuit to (b), (c) adjoint circuit to (b), (d) equivalent circuit to (c) using VOA, and (e) alternative equivalent circuit to (c) using COA.

The floating CCII is a four port building block as shown in Table 3(a) and is defined by the following matrix equation[31]:

$$\begin{bmatrix} V_X \\ I_Y \\ I_{Z+} \\ I_{Z-} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_X \\ V_Y \\ V_{Z+} \\ V_{Z-} \end{bmatrix}. \tag{2}$$

Table 3. NAM description of the (a) CCII members and (b) ICCII members.

<table>
<tr><th></th><th>NAM</th><th>Pathological element representation</th></tr>
<tr><td colspan="3">(a)</td></tr>
<tr><td>Floating CCII</td><td>$\begin{array}{c} \\ X \\ Z+ \\ Z- \end{array}\begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} \infty_i & -\infty_i \\ \infty_i & -\infty_i \\ -2\infty_i & 2\infty_i \end{bmatrix} \end{array}$</td><td>
</td></tr>
<tr><td>CCII–</td><td>$\begin{array}{c} \\ X \\ Z- \end{array}\begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} \infty_i & -\infty_i \\ -\infty_i & \infty_i \end{bmatrix} \end{array}$</td><td>
</td></tr>
<tr><td>CCII+</td><td>$\begin{array}{c} \\ X \\ Z+ \end{array}\begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} \infty_i & -\infty_i \\ \infty_i & -\infty_i \end{bmatrix} \end{array}$</td><td>
</td></tr>
<tr><td colspan="3">(b)</td></tr>
<tr><td>Floating ICCII</td><td>$\begin{array}{c} \\ X \\ Z+ \\ Z- \end{array}\begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} \infty_i & \infty_i \\ \infty_i & \infty_i \\ -2\infty_i & -2\infty_i \end{bmatrix} \end{array}$</td><td>
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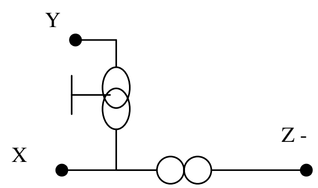
<tr><td>ICCII–</td><td>$\begin{array}{c} \\ X \\ Z- \end{array}\begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} \infty_i & \infty_i \\ -\infty_i & -\infty_i \end{bmatrix} \end{array}$</td><td>
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</table>

Table 3. (*Continued*)

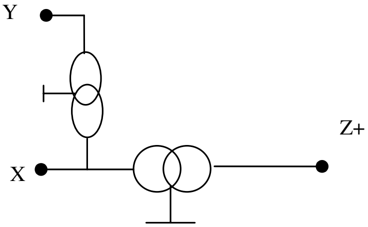
NAM		Pathological element representation
ICCII+	$\begin{matrix} & X & Y \\ X & \infty_i & \infty_i \\ Z+ & \infty_i & \infty_i \end{matrix}$	

Table 4. Active elements and the adjoint element.

Active element	Adjoint element
Nullor	Nullor
VOA (2 inputs, 1 output)	COA (1 input, 2 outputs)
VOA (1 input, 1 output)	VOA (1 input, 1 output)
	OR: COA (1 input, 1 output)
Nullator CM pair	VM–norator pair
VM–norator pair	Nullator CM pair
VM–CM pair	VM–CM pair
CCII–	CCII–
CCII+	ICCII–
ICCII–	CCII+
ICCII+	ICCII+
OMA (nullator–two terminal CM)	Norator–two terminal VM

Table 5. Floating status of active elements.

Active element	Floating status
VOA (2 inputs, 1 output)	No
COA (1 input, 2 outputs)	Yes
Nullor	Yes
Nullator CM pair	Yes
VM–norator pair	Yes
VM–CM pair	Yes
Floating CCII	Yes
CCII–	Yes
CCII+	No
Floating ICCII	Yes
ICCII–	Yes
ICCII+	No
OMA	No
Two terminal VM–norator pair	Yes

It is seen that this four port-active building block includes the CCII+ as special case with the $Z-$ port grounded. If the two Z output terminals are connected together it realizes the CCII– as special case.

The floating ICCII is a four port building block as shown in Table 3(b) and is defined by the following matrix equation[31]:

$$\begin{bmatrix} V_X \\ I_Y \\ I_{Z+} \\ I_{Z-} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_X \\ V_Y \\ V_{Z+} \\ V_{Z-} \end{bmatrix} . \tag{3}$$

It is seen that this four port-active building block realizes the ICCII+ as special case with the $Z-$ port grounded. If the two Z output terminals are connected together it realizes the ICCII– also as special case.

From the NAM representation and using Property 1, it is seen that all the CCII family members are floating except the CCII+ and ICCII+ as given in Table 5.

5. The NAM of the Operational Mirror Amplifier

In this section, an important active element which does not belong to the Op Amp and CCII families will be discussed as an example of a nonfloating element whose adjoint is floating.

The operational mirror amplifier (OMA) was introduced in Ref. 32 and is shown symbolically in Fig. 3(a) and it has very high trans-conductance gain, ideally infinity. It is represented by the following equations

$$V_a = V_b \,, \quad I_c = I_d \,. \tag{4}$$

Although the OMA was introduced in 1981,[32] the pathological representation of the OMA was given almost after 20 years in Ref. 12 as shown in Fig. 3(b).

The NAM stamp of the OMA is given by

$$\begin{array}{c} \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cc} a & \quad b \end{array} \\ \begin{bmatrix} \infty_i & -\infty_i \\ \infty_i & -\infty_i \end{bmatrix} \end{array} . \tag{5}$$

From the above NAM representation and based on Property 1, the OMA is a nonfloating building block.

Applying the rules of the adjoint transformation, the two terminal CM is changed to two terminal VM and the nullor to a norator as shown in Fig. 3(c). The NAM stamp for Fig. 3(c) is the transpose of the NAM in (5) and is given by

$$\begin{array}{c} \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cc} a & \quad b \end{array} \\ \begin{bmatrix} \infty_i & \infty_i \\ -\infty_i & \infty_i \end{bmatrix} \end{array} . \tag{6}$$

Based on Property 1 this is a floating element although its adjoint is a nonfloating element.

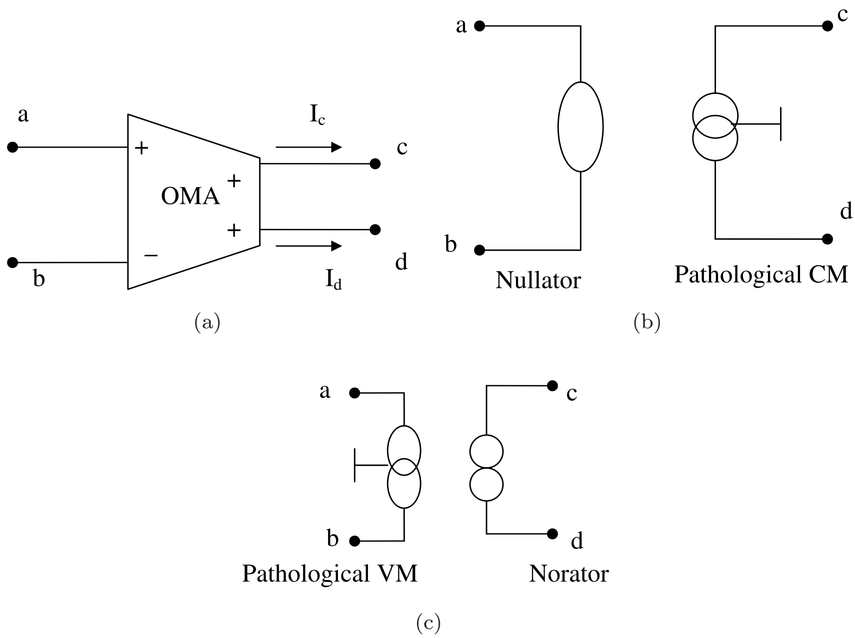


Fig. 3. (a) Symbol of the OMA, (b) the OMA as a nonfloating element, (c) adjoint of the OMA is a floating element.

6. Examples

In this section, two examples are given demonstrating the generation of two families of CCII filters from two known two-CCII filter circuits with demonstration of the floatation property in each of the two filters.

The first example is given demonstrating the generation of a family of CCII low-pass filters from a known two-CCII low-pass filter circuit. Another example is given demonstrating the generation of a family of CCII band-pass filters from a known two-CCII band-pass filter circuit.

6.1. *Generation of a family of CCII low-pass filters*

Figure 4(a) represents a two CCII+ minimum passive component grounded capacitor low-pass filter.[33] The generation of this noninverting low-pass filter from a passive RLC filter was demonstrated in Refs. 34–35.

The voltage transfer function for this circuit is given by

$$\frac{V_{\mathrm{LP}}}{V_I} = \frac{1}{s^2 C_1 C_2 R_1 R_2 + s(C_1 + C_2)R_2 + 1}\,. \tag{7}$$

A new property for this circuit which was not reported before is that this circuit has a floating nature. In other words, the current flowing in the ground terminal I_G is zero.

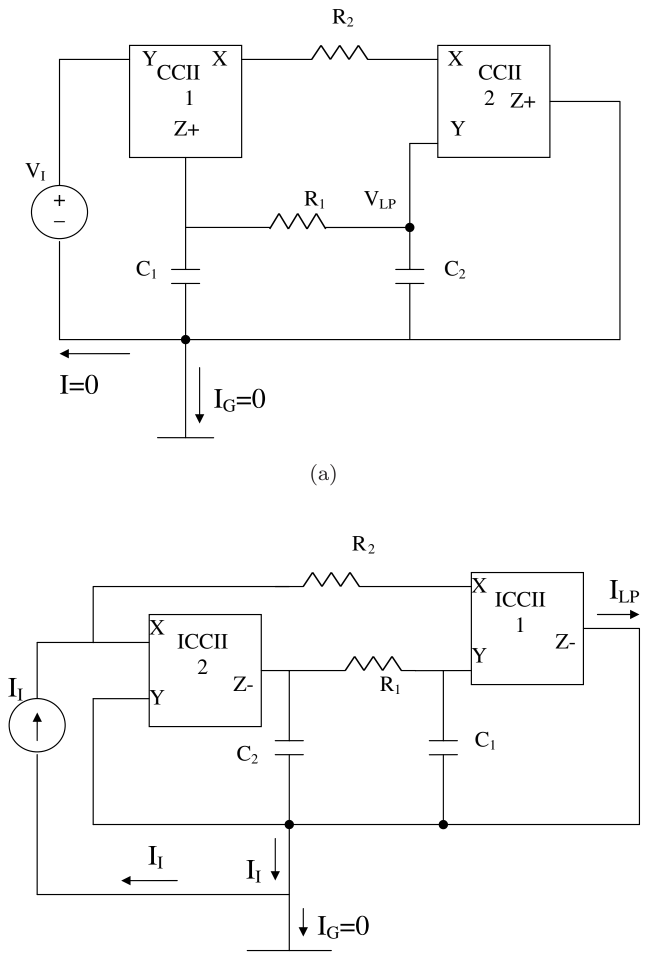


Fig. 4. (a) Voltage mode low-pass filter using two CCII+[33–35] and (b) current mode low-pass filter using two ICCII–.

By applying the adjoint network theorem to this Voltage-Mode (VM) circuit result in the Current-Mode (CM) low-pass filter shown in Fig. 4(b) given in Ref. 36.

It should be noted that in applying the adjoint network theorem to the circuit in Fig. 4(a) the output voltage node is considered to be the X terminal of the second CCII+ which is equal to V_{LP} at the Y terminal of the same CCII+.

The current transfer function for this circuit is given by

$$\frac{I_{\mathrm{LP}}}{I_I} = \frac{1}{s^2 C_1 C_2 R_1 R_2 + s(C_1 + C_2) R_2 + 1} \,. \tag{8}$$

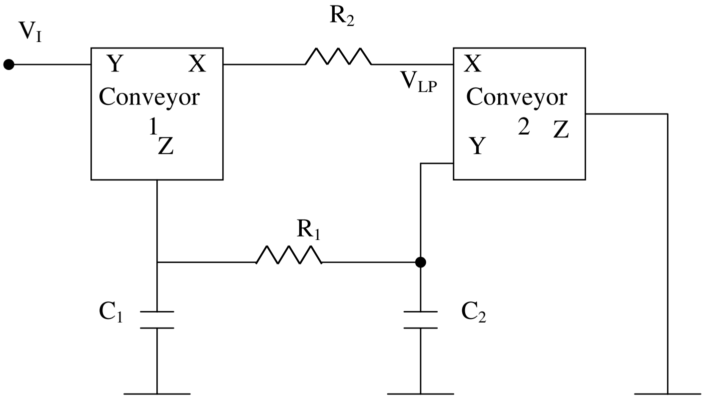


Fig. 5. Generalized voltage mode low-pass filter using two conveyors.

Table 6. Properties of the VM low-pass filter family belonging to Fig. 5

Circuit	Conveyor 1	Conveyor 2	LP polarity	Reference	Floating	Adjoint of
VM-1	CCII+	CCII+	Noninverting	33–35	Yes	CM-1
VM-2	CCII+	CCII–	Noninverting	34, 35	No	CM-5
VM-3	ICCII–	ICCII–	Both	36	Yes	CM-3
VM-4	ICCII–	ICCII+	Both	36	No	CM-7
VM-5	ICCII+	CCII+	Inverting	New	Yes	CM-2
VM-6	ICCII+	CCII–	Inverting	New	No	CM-6
VM-7	CCII–	ICCII–	Both	New	Yes	CM-4
VM-8	CCII–	ICCII+	Both	New	No	CM-8

A new property for this circuit which was not reported before is that this circuit has a floating nature. In other words, the current flowing in the ground terminal I_G is zero. The two circuits of Figs. 4(a) and 4(b) are both noninverting and both are having a floating nature.

Next it will be demonstrated that the floating nature of a VM circuit does not have to be maintained in its CM adjoint.

Figure 5 represents the generalized circuit obtained from Fig. 4(a). Conveyors 1 and 2 can be a CCII or an ICCII with the proper Z polarity as given in Table 6. It is seen that eight VM circuits are generated from Fig. 5, four of them are new.

Each of the eight VM low-pass filter circuits has the same equation which is given by (7) except for the proper sign inverting or noninverting as given in Table 6.

Of course, the four circuits with the second conveyor as ICCII will have two low-pass outputs of opposite polarity at the X and Y terminals of the ICCII which is an advantage in having both low-pass polarities available as shown in Table 6.

It is observed that four circuits are having a floating property and the other four have a nonfloating property.

To demonstrate that the adjoint theorem does not necessarily preserve the floating property, the CM low-pass filter family is generated next. Figure 6 is the

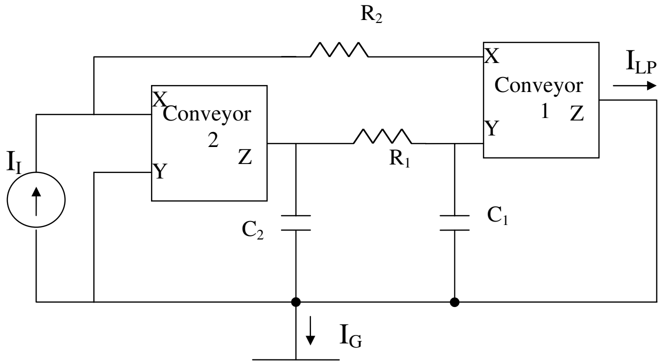


Fig. 6. Generalized current mode low-pass filter using two conveyors.

Table 7. Current mode low-pass filter family belong to Fig. 6.

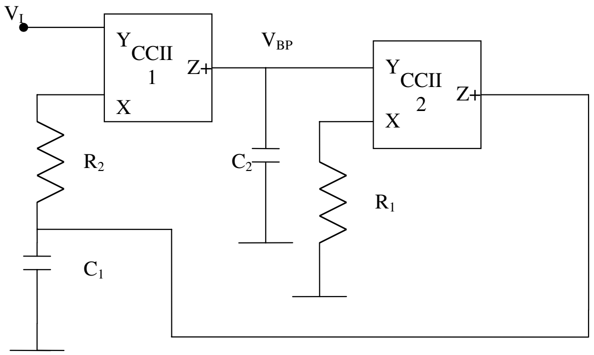
Circuit	Conveyor 2	Conveyor 1	LP polarity	Reference	Floating	Adjoint of
CM-1	ICCII–	ICCII–	Noninverting	36	Yes	VM-1
CM-2	ICCII–	ICCII+	Inverting	New	No	VM-5
CM-3	CCII+	CCII+	Inverting	36	No	VM-3
CM-4	CCII+	CCII–	Noninverting	New	No	VM-7
CM-5	CCII–	ICCII–	Noninverting	New	Yes	VM-2
CM-6	CCII–	ICCII+	Inverting	New	No	VM-6
CM-7	ICCII+	CCII+	Inverting	New	No	VM-4
CM-8	ICCII+	CCII–	Noninverting	New	No	VM-8

generalized CM low-pass filter obtained from Fig. 4(b). Conveyors 1 and 2 can be a CCII or an ICCII with the proper Z polarity as given in Table 7. It is seen that eight CM circuits are generated from Fig. 6, six of them are new. Each of the eight circuits has the same equation which is given by (8) except for the proper sign inverting or noninverting as given in Table 7.

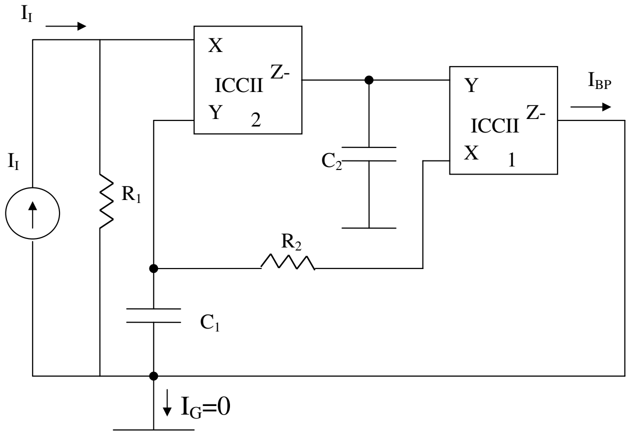
It is observed that only two CM-circuits are having a floating property and the other six have a nonfloating property.

From Tables 6 and 7, it is concluded that among the eight VM circuits and the eight adjoint CM circuits, only one VM-1 circuit as well as its adjoint namely CM-1 are both floating. Three VM circuits namely VM-4, VM-6, and VM-8 have the same nonfloatation property as their adjoint CM circuits namely CM-7, CM-6, and CM-8, respectively. The other four VM circuits namely VM-2, VM-3, VM-5, and VM-7 have different floatation property from their adjoint circuits namely CM-5, CM-3, CM-2, and CM-4, respectively. The above example demonstrates clearly one of the most important new results of this paper namely the adjoint transformation may not preserve the floatation property.

A second example of a band-pass filter is considered next.



(a)



(b)

Fig. 7. (a) Voltage mode band-pass filter using two CCII+[33,35] and (b) current mode floating band-pass filter using two ICCII–.[37]

6.2. *Generation of a family of CCII band-pass filters*

Figure 7(a) represents a two CCII+ minimum passive component grounded capacitor band-pass filter.[33]

The voltage transfer function for this circuit is given by

$$\frac{V_{\mathrm{BP}}}{V_I} = \frac{sC_1R_1}{s^2C_1C_2R_1R_2 + sC_2R_1 + 1}\,. \tag{9}$$

By applying the adjoint network theorem to this Voltage-Mode (VM) circuit result in the Current-Mode (CM) band-pass filter shown in Fig. 7(b) generated from a passive parallel RLC circuit in Ref. 37. It should be noted that in applying the adjoint network theorem to the circuit in Fig. 7(a) the output voltage node is considered to be the X terminal of the second CCII+ which is equal to V_{BP} at the Y terminal of the same CCII+.

The current transfer function for this circuit is given by

$$\frac{I_{\rm BP}}{I_I} = \frac{sC_1R_1}{s^2C_1C_2R_1R_2 + sC_2R_1 + 1}\,. \tag{10}$$

A new property for this circuit which was not reported before in Ref. 37 is that this circuit has a floating nature. In other words, the current flowing in the ground terminal I_G is zero. It is noted that the two circuits of Figs. 7(a) and 7(b) although having the same noninverting transfer function are having a different floating nature.

It is worth noting a similar topology band-pass filter using two-CCII was also obtained earlier from a passive RLC filter as demonstrated in Ref. 38.

It is observed that only two VM band-pass filter circuits are having a floating property and the other six have a nonfloating property (Fig. 8; Table 8).

To demonstrate that the adjoint theorem does not necessarily preserve the floating property, the CM band-pass filter family is generated next. Figure 9 is the generalized CM band-pass filter obtained from Fig. 7(b). Conveyors 1 and 2 can be a CCII or an ICCII with the proper Z polarity as given in Table 9.

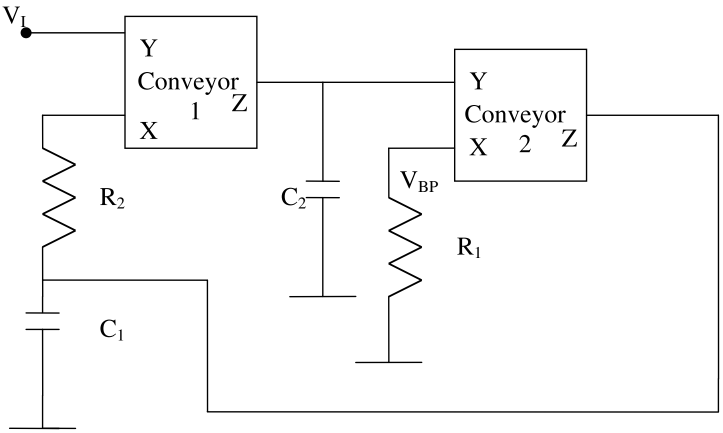


Fig. 8. Generalized voltage mode band-pass filter using two conveyors.

Table 8. Voltage mode band-pass filter family belong to Fig. 8.

Circuit	Conveyor 1	Conveyor 2	BP polarity	Reference	Floating	Adjoint of
VM-1	CCII+	CCII+	Noninverting	33, 35	No	CM-1
VM-2	CCII–	CCII–	Inverting	35	Yes	CM-2
VM-3	ICCII–	ICCII+	Both	37	No	CM-3
VM-4	ICCII+	CCII+	Inverting	New	No	CM-4
VM-5	ICCII–	CCII–	Noninverting	New	Yes	CM-5
VM-6	CCII–	ICCII+	Both	New	No	CM-6
VM-7	CCII+	ICCII–	Both	New	No	CM-7
VM-8	ICCII+	ICCII–	Both	New	No	CM-8

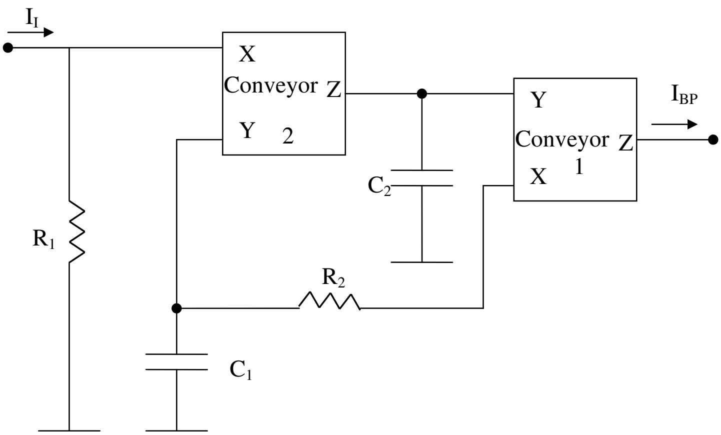


Fig. 9. Generalized current mode band-pass filter using two conveyors.

Table 9. Current mode low-pass filter family belong to Fig. 9.

Circuit	Conveyor 2	Conveyor 1	BP polarity	Reference	Floating	Adjoint of
CM-1	ICCII–	ICCII–	Noninverting	37	Yes	VM-1
CM-2	CCII–	CCII–	Inverting	38	Yes	VM-2
CM-3	ICCII+	CCII+	Inverting	37	No	VM-3
CM-4	ICCII–	ICCII+	Inverting	New	No	VM-4
CM-5	CCII–	CCII+	Noninverting	38	No	VM-5
CM-6	ICCII+	CCII–	Noninverting	New	No	VM-6
CM-7	CCII+	ICCII–	Inverting	New	No	VM-7
CM-8	CCII+	ICCII+	Noninverting	New	No	VM-8

It is seen that eight CM circuits are generated from Fig. 9, four of them are new. Each of the eight circuits has the same equation which is given by (10) except for the proper sign inverting or noninverting as given in Table 9.

It is observed that two CM-circuits are having a floating property and the other six have a nonfloating property.

From Tables 8 and 9, it is concluded that among the eight VM circuits and the eight adjoint CM circuits, only one circuit and its adjoint shares the flotation property namely VM-2 and its adjoint CM-2. Five VM circuits and their adjoint shares the same nonfloatation property namely VM-3, VM-4, VM-6, VM-7, VM-8 and their adjoint CM-3, CM-4, CM-6, CM-7, CM-8, respectively. The other two VM circuits namely VM-1, VM-5 have different floatation property from their adjoint circuits namely CM-1, CM-5, respectively.

7. Conclusions

The inter-reciprocity theorem[2] also known as the adjoint network theorem[3,5,6] and the transposition theorem[4] is demonstrated by simple examples on the VOA and COA.

Two important properties of the NAM that can be used in the identification of floating active elements and self adjoint elements are reported. Table 1 summarizing the four basic floating elements is given.

It is noted that the nullor and the VM–CM pair are both self adjoint. It is also stated that the nullator–CM pair is the adjoint of the VM–norator pair and they have NAM stamps that are transpose of each other.

The NAM description of the Op Amp family (VOA and COA) is given in Table 2.

The NAM description of the CCII family (CCII and ICCII members) is given in Tables 3(a) and 3(b). It is pointed out that the CCII– and ICCII– are floating elements whereas the CCII+ and ICCII+ are nonfloating elements.

Examples of the grounded-capacitor minimum passive component low-pass and band-pass filters are given to show that the ICCII members are necessary to obtain all possible equivalent realizations and to demonstrate that a given VM filter and its adjoint CM filter may have different floating nature.

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