Eqn. 14 gives the optimal Δ for given values of ω_1 and L. When $\omega_1 L$ is small, $k = 1$ is preferred for practical reasons. (For $\pi < \omega_1 L < 2\pi$, the above equation is to be modified to $\omega_1(L + 2\Delta - 1) = (2k + 1)\pi$ so that the overall value of the fourth term of eqn. 13 is positive.)

Theoretical and simulated results: Figs. 2*a* and 3*a* show the plots of normalising magnitude of $G(\omega)$ in dB for $\Delta = 1$ and the corresponding optimal values of Δ. The frequency of the sinusoid, the number of weights of the a.l.e. and σ_1^2/σ_0^2 (s.n.r.) that are used in the examples are shown in the Figures. The results clearly show that the a.l.e. with optimal Δ yields much better estimate of the sinusoid frequency than with $\Delta = 1$.

The a.l.e. is simulated using eqn. 1 with input consisting of samples of sinusoidal signal and white Gaussian noise. The step-size parameter in eqn. 1 is replaced by $\mu = \alpha/2\hat{r}(j)$, where

$$\hat{r}(j) = (1 - \gamma)\hat{r}(j - 1) + \gamma \sum_{k=0}^{L-1} x(j - \Delta - k)^2$$

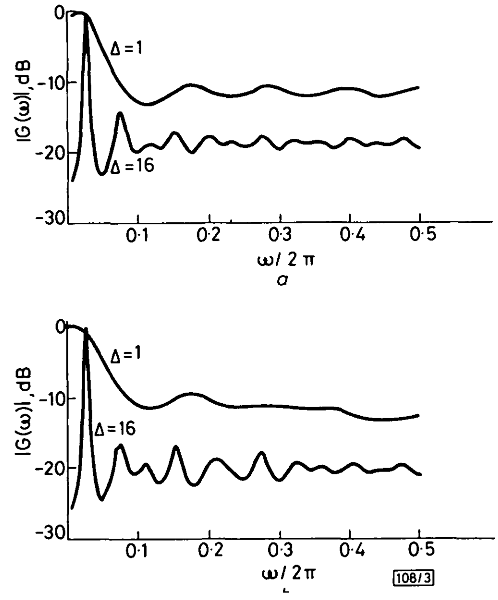

Fig. 3 *Plots of normalised magnitude of* $G(\omega)$
a Theoretical
b Simulated
$\omega_1 = \pi/20$; $L = 9$; s.n.r. = 0 dB

α and γ are real constants. The input data are normalised to unit variance. Choosing $\alpha = 0{\cdot}01$, $\gamma = 0{\cdot}01$ and $\Delta = 1$, and the initial value of the weight as zero, eqn. 1 is simulated for each a_k. The weight values obtained after 2000 iterations are used to compute $H(\omega)$ and $G(\omega)$. This is repeated for different frequencies of the sinusoid and the respective optimal values of Δ, computed from eqn. 14. The corresponding plots of normalised magnitude of $G(\omega)$ are shown in Figs. 2*b* and 3*b*.

The plots of Figs. 2 and 3 show that the simulated results agree very closely with the predicted results. In fact, the simulated results are slightly better than the theoretical and the reason may be that the spectrum of the white noise used in the simulation is not exactly flat.

For practical applications, a method to estimate the optimal value of Δ is to be derived. A recursive technique for estimating the optimal Δ and its implementation using ladder forms are currently being worked out, and the results will be presented in future publication.

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NOVEL PHASE LEAD INVERTING INTEGRATOR AND ITS APPLICATION IN TWO-INTEGRATOR LOOP FILTERS

Indexing terms: Active networks, Filters

A new variable phase inverting integrator is given. The proposed integrator is suitable for phase correction in the two integrator loop filters.

New circuit: The proposed variable phase inverting integrator is shown in Fig. 1. Straightforward analysis of the circuit yields the following transfer function:

$$\frac{V_o}{V_i} = -\frac{K+1}{sCR_1} \times \frac{1 + \left[\dfrac{K(a+1)+1}{K+1}\right]\dfrac{1}{A_2}}{1 + \left\{(K+1)\dfrac{1}{A_1} + [K(a+1)+1]\dfrac{1}{A_1 A_2}\right\}\left\{1 + \dfrac{1}{sCR_1}\right\}} \qquad (1)$$

Assume that matched o.a.s are used which are internally compensated to have a single pole open-loop response with a unity gain-bandwidth ω_t, thus:[1]

$$A(s) \simeq \omega_t/s \qquad (2)$$

From eqn. 2 in eqn. 1, therefore:

$$V_o/V_i = -(\omega_0/s)\varepsilon(s) \qquad (3)$$

where

$$\omega_0 = \frac{K+1}{CR_1} \qquad (4)$$

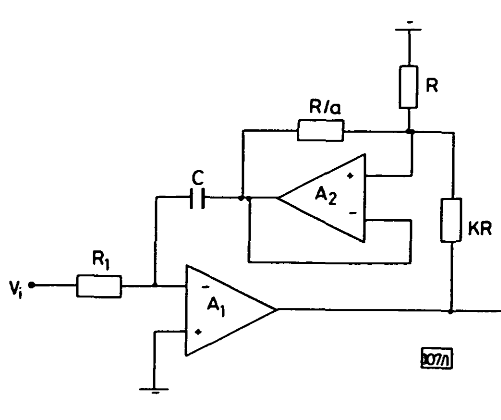

Fig. 1 *Proposed phase lead inverting integrator*

$$\varepsilon(s) = \frac{1 + \left\{\frac{K(a+1)+1}{K+1}\right\}\left(\frac{s}{\omega_t}\right)}{\left(1 + \frac{\omega_0}{\omega_t}\right) + \left[(K+1) + \frac{K(a+1)+1}{K+1}\frac{\omega_0}{\omega_t}\right] \times \left(\frac{s}{\omega_t}\right) + [K(a+1)+1]\left(\frac{s}{\omega_t}\right)^2} \tag{5}$$

For $\omega_0 \ll \omega_t$, $\omega_0 \ll [(K+1)^2/K(a+1)+1]\omega_t$, the above equation reduces to

$$\varepsilon(s) \simeq \frac{1 + \left[\frac{K(a+1)+1}{K+1}\right]\left(\frac{s}{\omega_t}\right)}{1 + (K+1)\left(\frac{s}{\omega_t}\right) + [K(a+1)+1]\left(\frac{s}{\omega_t}\right)^2} \tag{6}$$

For frequencies such that

$$\omega \ll \left[\frac{K+1}{K(a+1)+1}\right]\omega_t; \qquad \omega \ll \left[\frac{1}{K+1}\right]\omega_t \tag{7}$$

the excess phase of the integrator is given by

$$\phi \simeq K\frac{\omega}{\omega_t}\left[\frac{a}{K+1} - 1\right] \tag{8}$$

Thus it is seen that the resistor R/a controls the phase of the integrator which can be made leading. The resistor KR controls the stability of the integrator.

Stability analysis: Taking the second o.a. pole into account and assuming that it occurs at a frequency $\omega_2(\omega_2 > \omega_t)$, the open-loop gain A can be expressed as:[2]

$$A(s) \simeq \omega_t/[s(1 + s/\omega_2)] \tag{9}$$

Substituting from eqn. 9 into eqn. 1, and after some approximation, it follows that a necessary condition for the integrator stability is that

$$\omega_2 > \left(\frac{2}{K+1} - \frac{K+1}{2[K(a+1)+1]}\right)\omega_t \tag{10}$$

Taking $K = 1$ will ensure the integrator stability. In this case, eqn. 8 reduces to

$$\phi \simeq \frac{\omega}{\omega_t}\left[\frac{a}{2} - 1\right] \tag{11}$$

Application of this novel phase lead inverting integrator in the Tow-Thomas (TT) biquad[3,4] and the Kerwin-Huelsman-Newcomb[5] (KHN) biquand is discussed next.

Application in two-integrator loop filters: The TT and KHN biquad filters are straightforward implementations of the two integrator loop principle. Although both circuits exhibit very low sensitivities with respect to the passive components, they suffer from a rather drastic Q-factor enhancement effect due to the o.a.s' limited gain-bandwidth. The Q-factor enhancement is due to the excess phase lag around the loop, and hence may be compensated for by the addition of an equal amount of phase lead. Although several methods[6–12] for phase correction in the TT biquad have been reported, only very few active compensation methods for phase correction in the KHN biquad have been introduced.[13,14] The proposed method is suitable for both the TT and the KHN biquad filters. Phase correction in the TT or the KHN biquads can be achieved if the proposed phase lead inverting integrator is used in place of one of the two biquad integrators as shown in Figs. 2*a*, *b*, respectively. To provide the amount of phase lead necessary for phase correction around the loop, a should be taken equal to 8 (assuming matched o.a.s are used for the biquad). This will yield an extra phase lead at resonance equal to $(3\omega_0/\omega_t)$ which is the amount necessary for phase correction at ω_0.

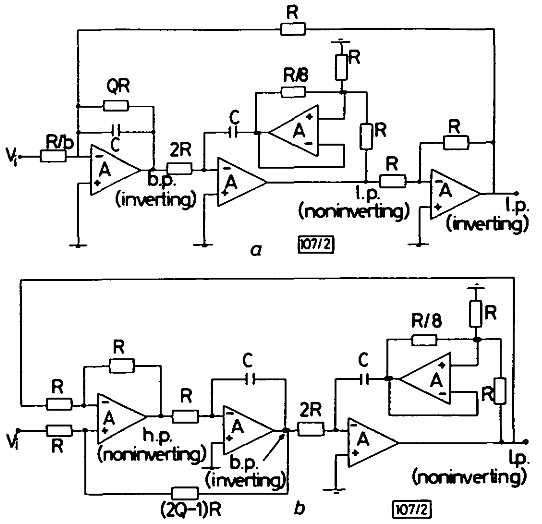

Fig. 2

a Improved Tow-Thomas biquad suitable for high Q and high frequency. Gain magnitude at ω_0 at each of the 3 outputs = bQ; $\omega_0 = 1/CR$
b Improved KHN biquad suitable for high Q and high frequency. Gain magnitude at ω_0 at each of the 3 outputs = $2Q - 1$; $\omega_0 = 1/CR$

It is worth noting that the active phase lead inverting integrator reported in Reference 15 may also be used for phase correction in the TT or the KHN biquad filters. In this case however, for stable operation it is necessary to use o.a.s having $\omega_2 > 1{\cdot}875\omega_t$.

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