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SHORT COMMUNICATIONS

TWO ACTIVE RC CONFIGURATIONS FOR REALIZING NONMINIMUM PHASE TRANSFER FUNCTIONS

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SUMMARY

Two different realizations are given for synthesizing a second order nonminimum phase transfer function. The first circuit uses a single operational amplifier and a passive RC bandpass building block and has the advantage of being always stable. The second configuration uses two operational amplifiers to activate a symmetrical twin T, and has the advantage of providing a unity gain factor. Sensitivities to all circuit components and the frequency limitation equations for both networks are given.

INTRODUCTION

Many active RC configurations for realizing all-pass or notch characteristics are available.[1–15] Most of them are limited to poles on the negative real axis, that is $Q_p \leqslant 0{\cdot}5$. The realization of all-pass characteristics having complex poles and zeros and using a single operational amplifier (OA) was given first by Holt and Gray.[16] Their synthesis procedure, however, is based on the factorization of the denominator of the transfer function as the difference of two polynomials, hence the pole sensitivity with respect to the gain of the OA is high.[17] Recently Moschytz[18] gave a general configuration based on the Sallen-Key circuit for realizing low Q all-pass characteristics having complex poles and using a single OA. The Moschytz realization, however, requires four capacitors.

In this paper two configurations are given for realizing a second order nonminimum phase transfer function of the form

$$G(s) = H\frac{s^2-(\omega_0/Q_0)s+\omega_0^2}{s^2+(\omega_0/Q_p)s+\omega_0^2} \tag{1}$$

having a gain factor $H = G(0) = G(\infty) \leqslant 1$.

Design equations for realizing all-pass transfer characteristics are given as a special case.

A SINGLE OA NONMINIMUM PHASE NETWORK

The general configuration

Figure 1(a) represents the general configuration. Analysis of the circuit taking the effect of the OA gain into consideration leads to the following open circuit voltage transfer function

$$G(s) \equiv \frac{V_o}{V_{in}} = \frac{a-T(s)}{1+(1/A)-T(s)} \tag{2}$$

where $T(s)$ is the open circuit voltage transfer function of the network N.

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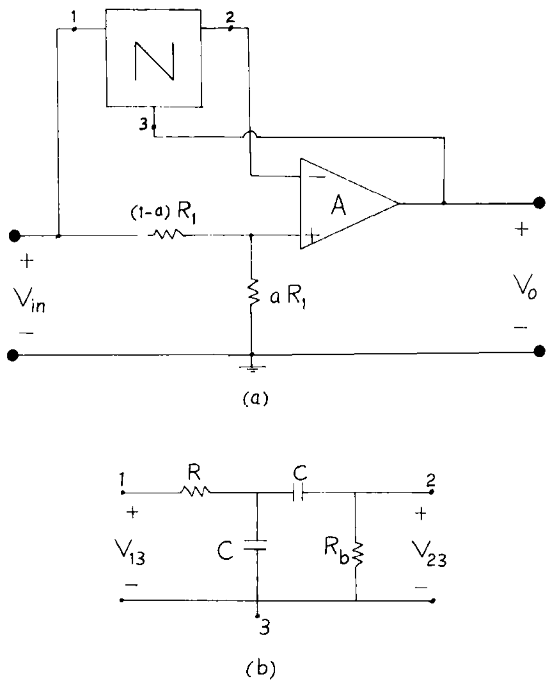

Figure 1. A single OA nonminimum phase network, (a) general configuration, (b) network N

Using the passive RC bandpass network shown in Figure 1(b) for N* results in

$$T(s) = \frac{V_{23}}{V_{13}} = \frac{\left(\frac{1}{CR}\right)s}{s^2 + \left(\frac{b+2}{bCR}\right)s + \frac{1}{C^2R^2b}} \tag{3}$$

where

$$b = \frac{R_b}{R} \tag{4}$$

Substituting (3) in (2) gives

$$G(s) = \frac{a}{1+(1/A)} \cdot \frac{s^2 - \frac{1}{CR\sqrt{b}}\left(\frac{\sqrt{b}}{a} - \frac{b+2}{\sqrt{b}}\right)s + \left(\frac{1}{CR\sqrt{b}}\right)^2}{s^2 + \frac{1}{CR\sqrt{b}}\left(\frac{b+2}{\sqrt{b}} - \frac{\sqrt{b}}{1+(1/A)}\right)s + \left(\frac{1}{CR\sqrt{b}}\right)^2} \tag{5}$$

As A approaches infinity the above equation becomes

$$G(s) = a\,\frac{s^2 - \frac{1}{CR\sqrt{b}}\left(\frac{\sqrt{b}}{a} - \frac{b+2}{\sqrt{b}}\right)s + \left(\frac{1}{CR\sqrt{b}}\right)^2}{s^2 + \left(\frac{2}{CRb}\right)s + \left(\frac{1}{CR\sqrt{b}}\right)^2} \tag{6}$$

* It has been brought to the author's attention that this circuit may also be considered as a special case of the very general structure described already by Mitra[19] or by Rupprecht,[20] also by Deliyannis.[21] However, the analysis presented here is more detailed and exact and the derived formulas are of real use for the practical design of the network. Also the general structure given here in Figure 1(a) can employ other passive RC bandpass circuits different from that in Figure 1(b).

Comparing with (1) it is seen that the circuit realizes a nonminimum phase transfer function having

$$\omega_0 = \frac{1}{CR\sqrt{b}} = \frac{1}{C\sqrt{(RR_b)}} \tag{7}$$

$$Q_0 = \frac{a\sqrt{b}}{b(1-a)-2a} \tag{8}$$

$$Q_p = \frac{\sqrt{b}}{2} = \frac{1}{2}\sqrt{\frac{R_b}{R}} \tag{9}$$

R_b controls Q_p; a controls Q_0 which determines the type of the filter, without affecting ω_0 or Q_p; ω_0 is tuned by varying the two ganged capacitors without affecting Q_0 or Q_p of the filter. The design formulas are

$$R_b = 4Q_p^2 R \tag{10}$$

$$C = \frac{1}{2\omega_0 Q_p R} \tag{11}$$

$$a = \frac{1}{1+\frac{1}{2Q_p}\left(\frac{1}{Q_0}+\frac{1}{Q_p}\right)} \tag{12}$$

For a notch filter, the last equation reduces to

$$a = 1/[1+(1/2Q_p^2)] \tag{13}$$

and for an all-pass transfer characteristic

$$a = 1/[1+(1/Q_p^2)] \tag{14}$$

Sensitivity analysis

The ω_0 and the Q_p sensitivities to the passive element variations are

$$S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = S_R^{\omega_0} = S_{R_b}^{\omega_0} = -\tfrac{1}{2} \tag{15}$$

$$\left.\begin{aligned} S_{C_1}^{Q_p} &= S_{C_2}^{Q_p} = 0 \qquad \text{for } C_1 = C_2 \\ S_{R_b}^{Q_p} &= -S_R^{Q_p} = \tfrac{1}{2} \end{aligned}\right\} \tag{16}$$

The sensitivities to the OA gain are derived next. From (5) and assuming the OA gain to be real and equal to A_0, the actual values of H, ω_0, Q_0 and Q_p are

$$H_a = \frac{H}{1+(1/A_0)} \tag{17}$$

$$\omega_{0a} = \omega_0 \tag{18}$$

$$Q_{0a} = Q_0 \tag{19}$$

$$Q_{p_a} \simeq \frac{Q_p}{1+(2Q_p^2/A_0)} \qquad \text{for } A_0 \gg 1 \tag{20}$$

It is seen that A_0 has no effect on ω_0 and Q_0, thus

$$S_{A_0}^{\omega_0} = 0, \qquad S_{A_0}^{Q_0} = 0 \tag{21}$$

That is, the circuit is absolutely ω_0 and Q_0 invariant to the OA gain.

$$S_{A_0}^{Q_{p_a}} \simeq \frac{2Q_p^2}{A_0} \qquad \text{for } A_0 \gg 2Q_p^2 \tag{22}$$

Effect of the rolloff of the OA gain

Here the frequency limitation equations of the network are derived, based on the one-pole rolloff model of the OA, which is characterized by

$$A = \frac{A_0\omega_1}{s+\omega_1} \simeq \frac{GB}{s} \tag{23}$$

where A_0 is the open loop dc gain of the OA, ω_1 is the open loop 3-dB bandwidth, and $GB = A_0\omega_1$ is the gain-bandwidth product. When (23) is substituted in (5), the denominator of $G(s)$ becomes

$$D(s) = \left(s^2 + \frac{\omega_0}{Q_p}s + \omega_0^2\right) + \frac{s}{GB}\left(s^2 + \frac{\omega_0}{Q_p}(1+2Q_p^2)s + \omega_0^2\right) \tag{24}$$

Following the analysis of Budak–Petrela[22] the actual values of ω_0 and Q_p are given by

$$\omega_{0a} = \omega_0\left(1 - \frac{\omega_0 Q_p}{GB}\right) \tag{25}$$

$$Q_{p_a} = Q_p\left(1 + \frac{\omega_0 Q_p}{GB}\right) \tag{26}$$

As an example, using the μA 741 OA having $GB = 1$ MHz and for $f_0 = 1$ kHz and $Q_p = 10$, the deviation in $f_0 = -1$ per cent and the deviation in $Q_p = 1$ per cent. On the other hand if f_0 and Q_p are not to change by more than 10 per cent from their nominal values, and for $Q_p = 5$, the frequency limitation of this filter is $f_0 \leqslant 20$ kHz.

From (24) and using Routh's criterion, it is seen that the circuit is always stable, and the rolloff of the OA gain will not lead to any instability problems.

A DOUBLE OA NONMINIMUM PHASE NETWORK

The general configuration

The circuit shown in Figure 2 consists of a symmetrical twin T network excited by the input and two voltage controlled voltage sources (VCVS). The first is an inverting VCVS controlled by the input voltage

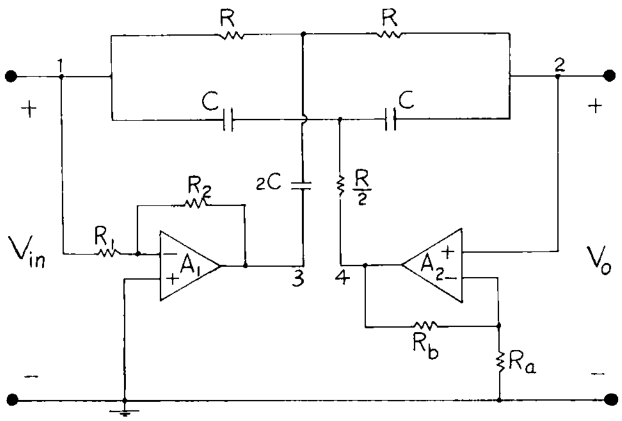

Figure 2. A double OA nonminimum phase network

and activates the twin T at terminal 3 to control the zeros of the transfer function. The second is a non-inverting VCVS controlled by the output voltage and provides a feedback to the network to control the position of the complex poles. Analysis leads to

$$G(s) \equiv \frac{V_o}{V_{in}} = \frac{s^2 - \left(\frac{2K_{1a}}{CR}\right)s + \left(\frac{1}{CR}\right)^2}{s^2 + \frac{2(2-K_{2a})}{CR}s + \left(\frac{1}{CR}\right)^2} \tag{27}$$

where K_{1a} and K_{2a} are the actual values of the magnitudes of the two VCVS's taking the finite dc gain of the OAs into effect and are given by

$$K_{1a} = \frac{K_1}{1+(1/A_1)(K_1+1)}, \qquad K_1 = \frac{R_2}{R_1} \tag{28}$$

$$K_{2a} = \frac{1}{(1/K_2)+(1/A_2)}, \qquad K_2 = \frac{R_b}{R_a}+1 \tag{29}$$

As A_i approaches infinity, $K_{ia} = K_i$ ($i = 1, 2$), and from (27) it is seen the network realizes a nonminimum phase transfer characteristic having

$$H = 1 \tag{30}$$

$$\omega_0 = \frac{1}{CR} \tag{31}$$

$$Q_0 = \frac{1}{2K_1} \tag{32}$$

$$Q_p = \frac{1}{2(2-K_2)} \tag{33}$$

An interesting feature of this circuit is that each of ω_0, Q_0 and Q_p can be controlled independently by varying C (or R), K_1 and K_2 respectively without affecting any of the other parameters including the gain factor which is constant and equals unity. Since $K_2 > 1$, the network realizes complex poles only ($Q_p > 0{\cdot}5$). It is clear that K_2 must be less than 2 for the circuit to be stable.

The circuit is capable of realizing zeros anywhere in the right half plane. Complex zeros result if $K_1 < 1$ and positive real axis zeros are realizable if $K_1 > 1$. The design formulas are

$$K_2 = 2 - \frac{1}{2Q_p} \tag{34}$$

$$CR = \frac{1}{\omega_0} \tag{35}$$

$$K_1 = \frac{1}{2Q_0} \tag{36}$$

For an all-pass characteristic

$$K_1 = \frac{1}{2Q_p} = 2 - K_2$$

If $K_1 = 0$, and the output is taken from terminal 4, the circuit reduces to the well-known configuration[23] of a notch filter.

Sensitivity analysis

The effect of the finite gain of the OAs is considered. Assuming A_i to be real and equal to A_{0i} ($i = 1, 2$), the actual values of ω_0, Q_0 and Q_p are given by

$$\omega_{0a} = \omega_0 \tag{37}$$

$$Q_{0a} = Q_0\left(1+\frac{K_1+1}{A_{01}}\right) \tag{38}$$

$$Q_{p_a} \simeq \frac{Q_p}{1+(4K_2Q_p/A_{02})} \tag{39}$$

Thus the circuit is absolutely ω_0 invariant to the OA gain.

$$S_{A_{01}}^{Q_0} \simeq -\frac{1+(1/2Q_0)}{A_{01}} \tag{40}$$

$$S_{A_{02}}^{Q_p} \simeq \frac{4K_2Q_p}{A_{02}} < \frac{8Q_p}{A_{02}} \quad \text{for } A_{02} \gg 4K_2Q_p \tag{41}$$

Effect of the rolloff of the OA gain

The frequency limitation equations are

$$\omega_{0a} \simeq \omega_0\left(1-\frac{4\omega_0}{GB}\right) \quad \text{for } Q_p \gg 1 \tag{42}$$

$$Q_{p_a} \simeq Q_p\left(1+\frac{4\omega_0}{GB}\right) \quad \text{for } Q_p \gg 1 \tag{43}$$

Comparing the above equations with equations (25) and (26) it is seen that for $Q_p < 4$ the first network is less sensitive to the rolloff of the OA gain. On the other hand, for $Q_p > 4$, this circuit has better sensitivities to the rolloff of the OA gain.

CONCLUSIONS

Two configurations are given for realizing nonminimum phase transfer functions. The first realization is canonic, is always stable and uses a single OA. The second realization uses two OAs and has a constant gain factor of unity. The frequency limitation equations for both networks are given.

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