Express Letters

Simplified Formulas for $\frac{\Delta\omega_o}{\omega_o}$ and $\frac{\Delta Q}{Q}$ Based on Budak-Petrela's Method

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***Abstract*—This letter presents two simplified expressions to determine the fractional shifts in ω_o and Q of the active filters due to the finite gain-bandwidth of the op amps. The expressions are functions of the coefficients of the characteristic equation of the filter. An example is included to illustrate the simplicity and usefulness of the derived expressions.**

I. Introduction

Over 20 years ago, Budak and Petrela [1] proposed a method to calculate $\Delta\omega_o/\omega_o$ and $\Delta Q/Q$ of the active filters due to the finite gain-bandwidth of the op amps. Their method remains the best method to determine the frequency limitations of the active filters, and some textbooks have included the details of their analysis [2], [3].

The purpose of this letter is to introduce simplified formulas to calculate $\Delta\omega_o/\omega_o$ and $\Delta Q/Q$ in terms of the coefficients of the denominator of the transfer function $D(s)$ of the active filter. The importance of these formulas is not only to simplify the calculations of the fractional shifts in ω_o and Q due to the finite gain-bandwidth of the op amps, but also to give insight into the quality of the active filter by observing the coefficients of $D(s)$.

II. The Simplified Expressions

Consider an active filter in which several op amps are used and assume that each op amp is characterized by a single pole model with a unity gain-bandwidth ω_t. Thus the open loop gain A of the op amp is represented by

$$A(s) \cong \frac{\omega_t}{s}. \tag{1}$$

By direct analysis of the circuit and after neglecting the higher order terms ($1/\omega_t^2$ and $1/\omega_t^3$), the denominator of the transfer function $D(s)$ can be written in the following form

$$D(s) = P_1(s) + \frac{1}{\omega_t}P_2(s) \tag{2}$$

where

$$P_1(s) = s^2 + \frac{\omega_o}{Q}s + \omega_O^2 \tag{3}$$

and

$$P_2(s) = s(as^2 + b\omega_o s + c\omega_o^2). \tag{4}$$

Applying Budak-Petrela's method,

$$\frac{dp}{d\left(\frac{1}{\omega_t}\right)} = \left.\frac{-P_2(s)}{\frac{d}{ds}\left[P_1(s) + \frac{1}{\omega_t}P_2(s)\right]}\right|_{s=p} \tag{5}$$

Manuscript received October 27, 1994. This paper was recommended by Associate Editor H.-D. Chiang.

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IEEE Log Number 9409313.

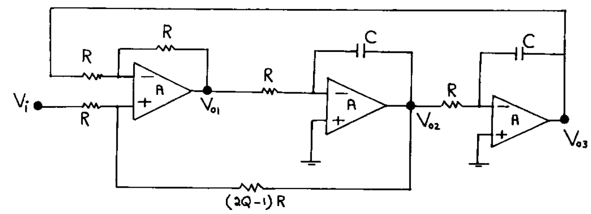

Fig. 1. The KHN biquad filter [3].

where p is the root of $P_1(s)$.

$$p = -\alpha + j\beta = -\frac{\omega_o}{2Q} + j\omega_o\sqrt{1 - \frac{1}{4Q^2}} \tag{6}$$

$$\frac{d\omega_o}{d\left(\frac{1}{\omega_t}\right)} = \frac{1}{\omega_o}\left(\alpha\frac{d\alpha}{d\left(\frac{1}{\omega_t}\right)} + \beta\frac{d\beta}{d\left(\frac{1}{\omega_t}\right)}\right) \tag{7}$$

$$\frac{dQ}{d\left(\frac{1}{\omega_t}\right)} = \frac{Q}{\omega_o}\left[\left(\frac{\alpha}{\omega_o} - \frac{\omega_o}{\alpha}\right)\frac{d\alpha}{d\left(\frac{1}{\omega_t}\right)} + \frac{\beta}{\omega_o}\frac{d\beta}{d\left(\frac{1}{\omega_t}\right)}\right]. \tag{8}$$

Substituting from (3), (4), and (6) in (5), (7), and (8), we can easily obtain the following expressions for $\Delta\omega_o/\omega_o, \Delta Q/Q$ as functions of the $P_2(s)$ coefficients a, b and c

$$\boxed{\frac{\Delta\omega_o}{\omega_o} = -\frac{1}{2}\left(b - \frac{a}{Q}\right)\frac{\omega_o}{\omega_t}}$$
$$\boxed{\frac{\Delta Q}{Q} = \left[(a-c)Q + \frac{1}{2}\left(b - \frac{a}{Q}\right)\right]\frac{\omega_o}{\omega_t}.} \tag{9}$$

From the above two formulas, the performance of the active filter can be predicted immediately after obtaining its characteristic equation and observing the coefficients a, b, and c.

III. Application

To illustrate the usefulness of the above expressions, the KHN circuit is taken as an example.

A. The KHN Circuit

The universal filter, usually referred to in the literature as the Kerwin-Huelsman-Newcomb (KHN) biquad [4], or the state variable filter, which is shown in Fig. 1, is taken as an example to illustrate the use of the simplified formulas.

By direct analysis of the circuit, $D(s)$ is obtained as

$$D(s) = s^2 + \frac{\omega_o}{Q}s + \omega_o^2 + \frac{s}{\omega_t}\left(4s^2 + \left(2 + \frac{1}{Q}\right)\omega_o s + \frac{1}{Q}\omega_o^2\right). \tag{10}$$

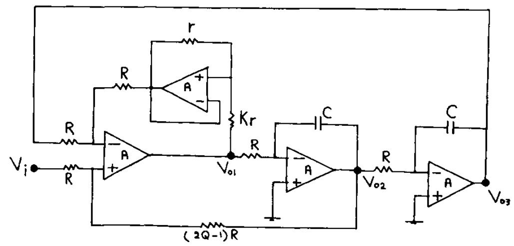

Fig. 2. The improved KHN biquad filter [5].

Thus

$$a = 4, \quad b = 2 + \frac{1}{Q} \quad \text{and} \quad c = \frac{1}{Q}. \tag{11}$$

From the simplified formulas,

$$\frac{\Delta\omega_o}{\omega_o} = \left(-1 + \frac{3}{2Q}\right)\frac{\omega_o}{\omega_t} \tag{12}$$

$$\frac{\Delta Q}{Q} = \left(4Q - \frac{3}{2Q}\right)\frac{\omega_o}{\omega_t}. \tag{13}$$

From (9), we can predict this large shift in Q, since for $Q \gg 1$, the c coefficient is very small compared to the a coefficient, resulting in a term which is multiplied by $Q(\omega_o/\omega_t)$.

B. The Compensated KHN Circuit:

Fig. 2 represents the modified KHN circuit [5] which employs a phase corrector to be adjusted to minimize the shift in Q. The large $\Delta Q/Q$ obtained in the circuit of Fig. 1 can be minimized if the coefficients a and c of $D(s)$ can be made equal.

Analysis of the circuit of Fig. 2 results in the following expression for $D(s)$.

$$D(s) = \left(s^2 + \frac{\omega_o}{Q}s + \omega_o^2\right) + \frac{s}{\omega_t}\left[4s^2 + \left(2 + \frac{K+2}{Q}\right)\omega_o s + \left(K + 1 + \frac{1}{Q}\right)\omega_o^2\right]. \tag{14}$$

To minimize $\Delta Q/Q$, the design value of K is obtained as

$$K = 3 - \frac{1}{Q} \cong 3 \text{ for } Q \gg 1 \tag{15}$$

which is the same result given in [5] based on the phase cancellation criteria.

IV. Conclusions

Simplified expressions to calculate the fractional shifts in ω_o and Q of the active filters are given. The formulas are functions of the characteristic equation coefficients, the nominal ω_o and Q, and they give accurate results as long as the first-order approximation is valid. It is worth noting that although several other methods are available to calculate $\Delta\omega_o/\omega_o$ and $\Delta Q/Q$ of the active filters [6], [7], the objective here is to give simplified expressions for $\Delta\omega_o/\omega_o$ and $\Delta Q/Q$ based specifically on Budak-Petrela's method.

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A Novel Approach to the Convergence of Neural Networks for Signal Processing

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***Abstract*—A novel deterministic approach to the convergence analysis of (stochastic) learning algorithms is presented. The link between the two is the new concept of *time-average invariance*, which is a property of deterministic signals but resembles that of stochastic signals which are ergodic and stationary.**

I. Introduction

Consider an unsupervised learning algorithm of the form

$$\frac{d}{dt}x = bf(x, s) \tag{1}$$

or its counter part in time-discrete form,

$$x(k+1) = x(k) + bf(x(k), s(k)) \tag{2}$$

where $b > 0$ is called the learning rate, $x(t) \in R^n$ and $s(t) \in R^m$ are the state and the input signal respectively. For signal processing purposes, the objective of such learning algorithms is usually to abstract some information from the input signal s at a time when the state x converges. Examples are [1]–[3]. The conventional method for the convergence analysis of such algorithms is a stochastic approach, in which signal s is modeled as a stochastic process [4]–[6].

In this paper, signals are viewed as deterministic functions of t, but satisfy a property called *time-average invariance*. This is a property of deterministic signals but resembles that of stochastic signals which are stationary and ergodic. As such, deterministic-based analysis can be applied to stochastic-like signals. Consequently, the complexity of

Manuscript received August 31, 1994; revised November 23, 1994. This work was supported in part by the Office of Naval Research, N00014-91-J-1461. This paper was recommended by Associate Editor H.-D. Chiang.

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IEEE Log Number 9409324.

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