Canonical high-selectivity parallel resonator, using a single operational amplifier, and its applications in filters

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Abstract: The canonical circuit introduced is an active *RC* circuit using a single operational amplifier, and its input admittance is composed of a frequency-dependent negative resistance (f.d.n.r.) in parallel with a resistor and a capacitor. The quality factor of the proposed circuit can be independently controlled by means of a single grounded resistor. Passive sensitivities and the effects of the nonideal op-amps are given. Applications of the parallel resonator in lowpass and bandpass filters are discussed. Finally, experimental results are presented.

1 Introduction

According to Bruton's transformation,[1] any *LCR* network can be transformed to an equivalent *RDC* network without changing the voltage (current) transfer function. The ideal frequency-dependent negative resistance (f.d.n.r.) can be realised by using at least two capacitors and two operational amplifiers, together with a number of resistors.[2, 3] The proposed circuit realises a grounded parallel resonator formed from a resistance, capacitance (which is responsible for the finite Q of the resonator) and an f.d.n.r. using two capacitors and one operational amplifier. The proposed resonator is used in circuits where an *RDC* parallel combination is needed to save some elements. Also, since the quality factor for the resonator can be independently controlled by means of a single resistor, then the resonator is used to realise tunable lowpass and bandpass filters. The above resonator has several advantages over that which was recently given by Molo.[4] In Molo's resonator the quality factor cannot be controlled independently; besides, it is not canonic since it uses three capacitors. It is also sensitive since it is based on the cancellation of the Y-parameters of two circuits.

2 Basic configuration

The realisation of the *RDC* resonator is based on the circuit shown in Fig. 1. Assuming an ideal operational amplifier, it can be easily shown that

$$Y_i = \{1/(R_1 + R_2)\} + s[(1-K)C_1 + \{1-K(1+M)\}C_2] + s^2(1-K)C_1C_2R_4 \qquad (1)$$

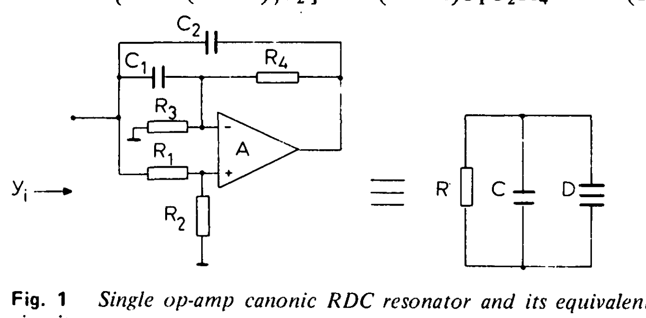

Fig. 1 *Single op-amp canonic RDC resonator and its equivalent circuit*

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where

$$K = R_2/(R_1 + R_2), M = R_4/R_3 \qquad (2)$$

From eqn. 1,

$$R = R_1 + R_2 \qquad (3)$$

$$D = (1-K)C_1C_2R_4 \qquad (4)$$

$$C = \{(1-K)C_1 + [1-K(1+M)]C_2\} \qquad (5)$$

since $\omega_0 = 1/\sqrt{RD}$ and $Q = \omega_0 D/C$, then

$$\omega_0 = 1/\sqrt{(R_1R_4C_1C_2} \qquad (6)$$

$$Q = \frac{\sqrt{\left(\frac{C_2}{C_1}\cdot\frac{R_4}{R_1}\right)}}{1 + \frac{C_2}{C_1}\left(1 - \frac{R_2}{R_1}\cdot\frac{R_4}{R_3}\right)} \qquad (7)$$

It is clear from eqns. 6 and 7 that Q is a function of R_3 while ω_0 is not. Hence, the quality factor of the above resonator can be independently controlled by means of R_3.

3 Passive sensitivities and design equations

The ω_0 and the Q sensitivities to all passive circuit components are given by

$$\left.\begin{aligned}
S^{\omega_0}_{C_1} &= S^{\omega_0}_{C_2} = S^{\omega_0}_{R_1} = S^{\omega_0}_{R_4} = -0.5\\
S^{\omega_0}_{R_2} &= S^{\omega_0}_{R_3} = 0\\
S^{Q}_{R_1} &= -0.5 - \frac{QMK}{1-K}\sqrt{(\alpha/\beta)}\\
S^{Q}_{R_2} &= \frac{QKM}{1-K}\sqrt{(\alpha/\beta)}\\
S^{Q}_{R_3} &= -\frac{QMK}{1-K}\sqrt{(\alpha/\beta)}\\
S^{Q}_{R_4} &= 0.5 + \frac{QMK}{1-K}\sqrt{(\alpha/\beta)}\\
S^{Q}_{C_1} &= -S^{Q}_{C_2} = -0.5 + Q\left\{1 - \frac{KM}{1-K}\right\}\sqrt{(\alpha/\beta)}
\end{aligned}\right\} \qquad (8)$$

where $\alpha = R_1/R_4, \beta = C_1/C_2$

To limit the spread in the circuit components, choose

$$\begin{aligned} C_1 &= C_2 \quad \text{i.e.} \quad \beta = 1 \\ R_1 &= R_2 \quad \text{i.e.} \quad K = 1/2 \end{aligned} \tag{9}$$

Substituting eqn. 9 in eqns. 6 and 7, thus

$$Q = \frac{\sqrt{(R_4/R_1)}}{2-M} \tag{7a}$$

$$\omega_0 = 1/C_1\sqrt{(R_1R_4)} \tag{6a}$$

From eqn. 7*a* it is seen that M has an upper bound, which is two, above which the circuit is unstable. Also, from eqn. 8 it is obvious that the Q sensitivities approach infinity as M approaches two.

Choosing $M = 1{\cdot}5$ will lead to the following design equations:

$$\left.\begin{aligned} R_1 &= R_2 \\ C_1 &= C_2 = 2/\omega_0 Q R_1 \\ R_3 &= Q^2R_1/6 \\ R_4 &= Q^2R_1/4 \end{aligned}\right\} \tag{10}$$

4 Nonideal conditions

The analysis, to this point, has assumed that the operational amplifier has infinite gain-bandwidth product. In reality, the gain of the operational amplifier can be given by

$$A = GB/s \tag{11}$$

Using eqn. 11 it can be shown that

$$\begin{aligned} Y_i = {} & \{\{1/(R_1+R_2)\} + s[(1-K)C_1 \\ & + \{1-K(1+M)\}C_2] + (1-K)C_1C_2R_4s^2\} \\ & + \frac{s}{GB}\left\{\left(\frac{1+M}{R_1+R_2}\right) + s\left\{\frac{R_4}{R_1+R_2}C_1\right.\right. \\ & \left.\left. + (1+M)(C_1+C_2)\right\} + s^2C_1C_2R_4\right\} \\ & \div \left[1 + \frac{s}{GB}\{(1+M) + sC_1R_4\}\right] \end{aligned} \tag{12}$$

Substituting eqn. 10 in eqn. 12 and assuming that $Q \gg 1$, then

$$\begin{aligned} Y_i \simeq {} & \left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right) + \frac{2s}{GB}\left(s^2 + s\frac{\omega_0 Q}{4} + 1{\cdot}25\omega_0^2\right) \\ & \div 2\omega_0^2R_1\left\{1 + \frac{s}{GB}\left(2{\cdot}5 + \frac{sQ}{2\omega_0}\right)\right\} \end{aligned} \tag{12a}$$

The numerator of eqn. 12*a* contains a 3rd-order polynomial in addition to the ideal numerator expressed in eqn. 1. The zeros of Y_i are of interest to us since they appear in the denominator of the voltage transfer functions which are realised by the resonator. The actual values of ω_0 and Q are calculated using a Budak-Petrela[5] analysis with the assumption that $Q \gg 1$. Hence

$$\frac{\omega_{0act}}{\omega_0} \simeq [1 - Q\omega_0/4GB] \tag{13}$$

$$\frac{Q_{act}}{Q} \simeq [1 - Q\omega_0/4GB] \tag{14}$$

From eqns. 13 and 14, it is clear that the actual values of ω_0 and Q are less than the ideal-case values.

5 Applications of the proposed resonator

5.1 *Tunable 2nd-order lowpass filter*

In Fig. 2 a lowpass filter is presented using the proposed resonator. A second operational amplifier is added to act as a buffer, and to provide the desired gain.

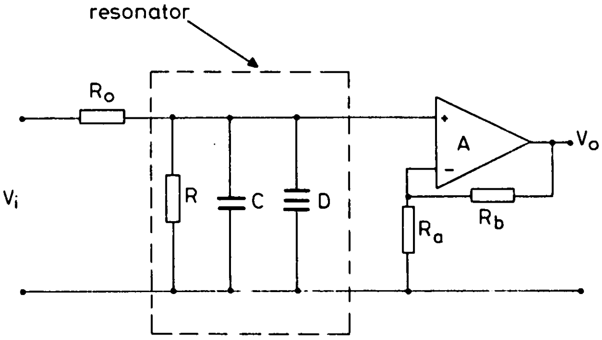

Fig. 2 *Canonic lowpass filter*

It can be easily shown that

$$\frac{V_0}{V_i} = K'/1 + R_0Y_i \tag{15}$$

where

$$K' = 1 + \frac{R_b}{R_a} \tag{16}$$

Substituting eqn. 1 in eqn. 15:

$$\frac{V_0}{V_i} = K'/\{1 + R_0\{1/R + sC + s^2D\}\} \tag{15a}$$

Equation 15*a* is the transfer function of a lowpass filter with positive d.c. gain. Hence,

$$\text{d.c. gain} = K'/\left(1 + \frac{R_0}{R_1+R_2}\right) \tag{17}$$

$$\omega_{0lpf} = \omega_{0pr}\sqrt{1 + \frac{(R_1+R_2)}{R_0}} \tag{18}$$

$$Q_{lpf} = Q_{pr}\sqrt{1 + \frac{(R_1+R_2)}{R_0}} \tag{19}$$

where ω_{pr} and Q_{pr} are those of the parallel resonator given by eqns. 6 and 7.

Since the quality factor Q_{pr} of the resonator can be independently controlled by means of R_3, then consequently Q_{lp} can also be controlled by R_3.

Hence, the quality factor and the gain of the above l.p.f. are independently controlled.

Experimental results: The lowpass filter was constructed in the laboratory using LM operational amplifiers of type LM 741 with $V_{cc} = \pm 15$ V. Taking the amplifier's gain $K' = 1$, for simplicity, and choosing

$R_0 = R_1 = R_2 = 39\,\mathrm{k\Omega}$

$R_4 = 27\,\mathrm{k\Omega}$

Two cases are considered

(i) for $f_{0lp} = 849$ Hz, $C_1 = C_2 = 10$ nF

If R_3 is open circuit then $Q_{lp} = 0{\cdot}72$

and

If $R_3 = 36\,\mathrm{k\Omega}$ then $Q_{lp} = 1{\cdot}15$

(ii) for $f_{0lp} = 3{\cdot}86$ kHz,

$C_1 = C_2 = 2{\cdot}2$ nF

$R_3 = 36\,\mathrm{k\Omega}$

$Q = 1{\cdot}15$

The measured responses of the above cases are shown in Figs. 3*a* and 3*b*, respectively.

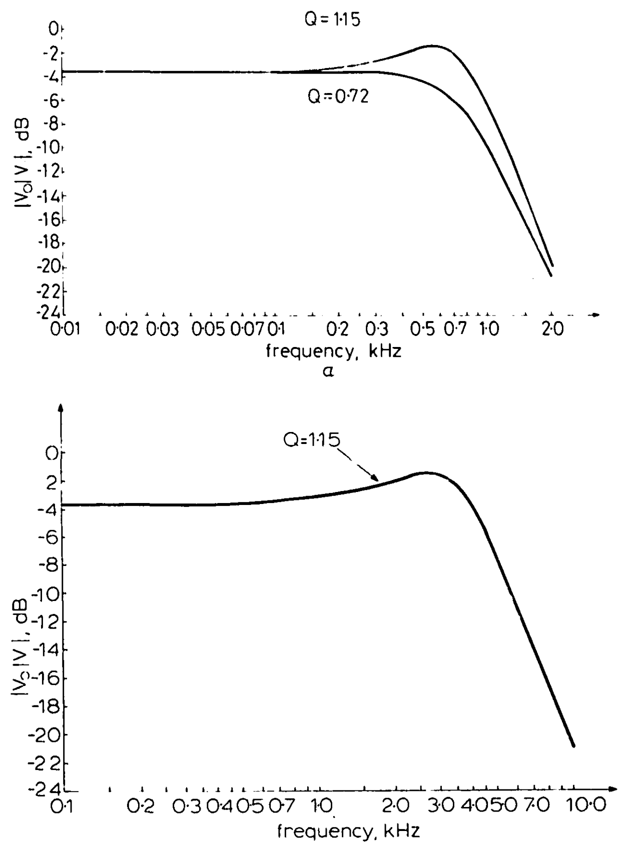

Fig. 3 *Experimental results of lowpass filter*

5.2 A tunable 2nd-order bandpass filter:

A 2nd-order bandpass filter is constructed using the parallel resonator as shown in Fig. 4. It can be shown that

$$\frac{V_0}{V_i} = K'sC_0/(sC_0 + Y_i)$$

$$= K'sC_0/\{1/R + s(C_0 + C) + s^2 D\} \qquad (20)$$

From eqn. 20 it is clear that

$$\omega_{0bpf} = \omega_{0pr} \qquad (21)$$

$$Q_{bpf} = Q_{pr}\frac{1}{1 + C_0/[(1-K)C_1 + C_2\{1-K(1+M)\}]}$$

As in the lowpass case the quality factor Q_{bpf} can be independently controlled by means of R_3. The disadvantage of the above bandpass filter is that it uses three capacitors (noncanonic).

Experimental results: The bandpass filter shown in Fig. 4 was constructed in the laboratory for a design $f_0 = 820$ Hz and $Q = 11{\cdot}5$. K' was taken as 1 for simplicity, and the component values used are listed as follows:

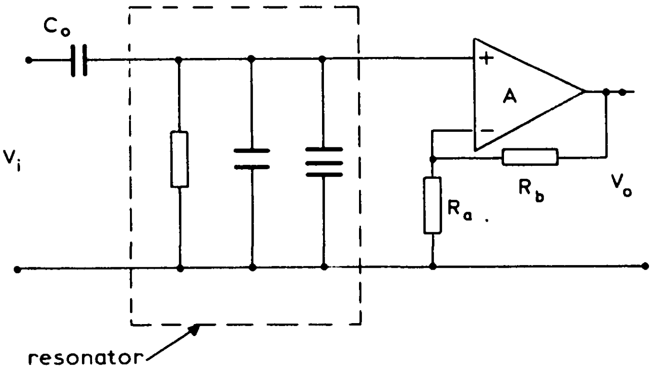

Fig. 4 *Bandpass filter*

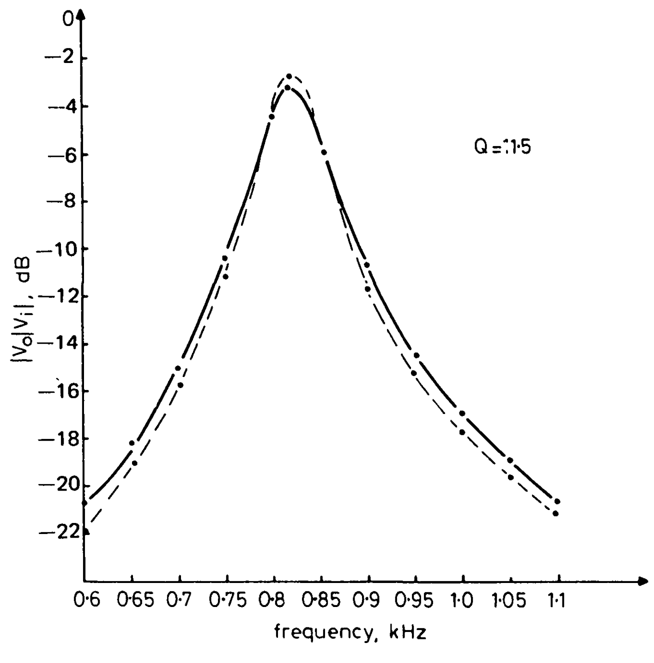

Fig. 5 *Experimental results of bandpass filter*

——— theoretical
– – – experimental

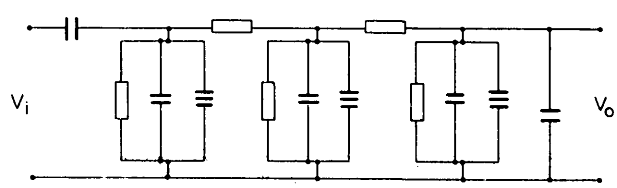

Fig. 6 *High-order bandpass filter*

$$
\begin{aligned}
C_0 &= 10\,\text{nF} \\
C_1 &= C_2 = 15\,\text{nF} \\
R_1 &= R_2 = 510\,\Omega \\
R_3 &= 220\,\text{k}\Omega,\ R_4 = 330\,\text{k}\Omega
\end{aligned}
\tag{23}
$$

The experimental and theoretical results are shown in Fig. 5.

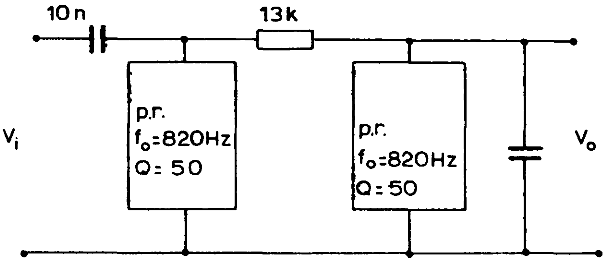

Fig. 7 *4th-order bandpass filter*

5.3 Bandpass filters of higher order

Bandpass filters of higher order can be realised with resistively coupled *RCD* resonators as shown in Fig. 6.

Experimental results: A 4th-order bandpass filter was constructed using two identical *RCD* resonators as shown in Fig. 7. The design values of the resonator are those given in eqn. 23. The measured frequency response of the filter is shown in Fig. 8.

6 Conclusion

A canonical and tunable *RC* resonator using the f.d.n.r. concept is made by a single operational amplifier. Its theoretical inherent finite quality factor can be controlled by a single grounded resistor. The proposed resonator can be used to realise tunable 2nd-order lowpass and bandpass filters. Also, higher-order bandpass fliters can be realised by resistively coupled parallel resonator sections.

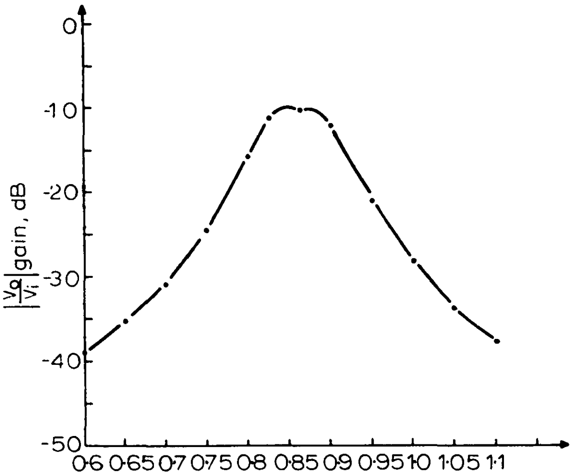

Fig. 8 *Experimental results of 4th-order bandpass filter*

7 References

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