Design of odd nth-order elliptic high-pass filters employing OTRAs

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Abstract: Two new analytical synthesis methods are proposed for synthesising an nth odd-order elliptic high-pass (HP) filter using operational trans-resistance amplifiers (OTRAs). The analytical synthesis scheme is based on the assumption of infinite trans-resistance Rm for the OTRA. The H-space simulations of the realised elliptic third-order HP filters show that the realisation using less number of OTRAs has a performance better than that which uses more number of OTRAs, as is to be expected in view of the assumption that Rm is infinite, when in fact it is finite in practice. However, it is shown that by slightly adjusting a single resistor, the amplitude–frequency response can be made close to the theoretical response.

1 Introduction

Operational trans-conductance amplifier (OTA) is one of the most important active elements in the field of analogue circuit design. Its input–output characteristics are (i) the two null + and – terminal input currents, i.e. \( I_+ = I_- = 0 \), (ii) the output current \( I_o = (V_+ - V_-)g_m \) \( g_m \) being the trans-conductance of the OTA, and (iii) very high input and output impedances. On the other hand, the operational trans-resistance amplifier (OTRA) has the following input–output relationships: (i) the two null + and – terminal input voltages, i.e. \( V_+ = V_- = 0 \), (ii) the output voltage \( V_o = (I_+ - I_-)R_m \), \( R_m \) being the trans-resistance of the OTRA, and (iii) very low input and output impedances. Thus, OTRA can be considered as the dual of OTA [1, 2] and vice-versa. Further, there is an additional advantage of eliminating the parasitics at the input port of an OTRA due to two null voltage input terminals (note that an OTA does not enjoy this benefit). Hence, we may predict that OTRA-based structures may have more advantages than OTA-based ones and therefore very attractive for circuit designers.

Several distinct OTRA realisations have been proposed in the literature [3–8], and used to design many distinct types of analogue circuits, such as Schmitt trigger [9], phase detector [10], receiver [11], low-voltage reference [12], integrators [13], immittance simulators [14–16], oscillators [17–22], square or triangular waveform generators [23, 24], monostable [25–27] and bi-stable [28] multi-vibrators, first- [29–31] and second-order [32] all-pass filters, hearing aid filter [33], shadow filter [34], band-pass (BP) filters [35, 36], second-order single- [37, 38] and multi-function [39–42] filters, and third-order Butterworth and Chebyshev filters [43, 44].

It is well known that the very narrow transition band is a particularly strong point of elliptic filters. It would take a maximally flat (Butterworth) function of order 25 and a Chebyshev filter of order 8 to achieve the same narrow transition band of an elliptic fourth-order filter. Hence, an elliptic filter is a better choice than other kinds of filters for meeting the stringent cut-off rate of a very narrow transition band under the restriction of a finite order.

The transfer function of an elliptic nth-order low-pass (LP) filter is of the form

\[
\frac{V_{out}}{V_{in}} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_n}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0} \quad (1)
\]

when \( n \) is an odd integer, and

\[
\frac{V_{out}}{V_{in}} = \frac{b_0 s^n + b_1 s^{n-2} + \cdots + b_n}{a_n s^n + a_{n-1} s^{n-2} + \cdots + a_0} \quad (2)
\]

when \( n \) is an even integer. It can be seen that when we transform the even order LP filter given by (2) to high-pass (HP), LP, BP or band reject (BR) filter using the standard frequency transformations, the transformed transfer functions are of the same format as (2) itself.

In other words, Equation (2) can represent an elliptic even-order LP, HP, BP or BR filter depending on the values of the coefficients \( a_i \)'s and \( b_j \)'s. Thus, any structure that realises (2) can realise an LP, HP, BP or BR elliptic filter depending on the values of the coefficients. However, this is not the case when \( n \) is odd. The transformed transfer functions will not have either the format of (1) or (2). Thus, any structure that realises (1) can only realise a Cauer LP filter, but not the other types of filters when \( n \) is odd. Hence, the structures proposed in [43] using OTRAs can realise even-order LP, HP, BP or BR elliptic Cauer filters, but can only realise an odd-order LP elliptic filter.

Replacing \( s \) by \( 1/s \) in (1), we obtain the following HP elliptic transfer function:

\[
\frac{V_{out}}{V_{in}} = \frac{b_0 s^n + b_1 s^{n-2} + \cdots + b_n s}{a_n s^n + a_{n-1} s^{n-2} + \cdots + a_0} \quad (3)
\]

where \( n \) is an odd integer. Note that though both (2) and (3) can realise elliptic HP signals, yet they are different from each other because (2) is for even-nth-order filters, (3) is for odd nth-order ones. There is no realisation that has been advanced for the odd-nth-order elliptic HP transfer function (3), though some elliptic odd-even-nth-order filters for realising (1) and (2) have been reported [2, 45].

In recent years, analytical synthesis methods have effectively demonstrated the realisation of even-odd-nth-order voltage/current-mode filters and oscillator [1, 2, 45–53]. In the synthesis process, a complicated nth-order transfer function is decomposed by a series of iterative algebraic operations, which meet the necessity of synthesis with respect to the input–output characteristics of active components used, until a set of simple equations are generated and then synthesised using various types of simple integrators and a
order HP filter. One employs \( n + 1 \) OTRAs and \( n \) capacitors and the other uses \( n \) OTRAs and \( n + 1 \) capacitors. Since the synthesis scheme assumes the OTRA trans-resistance \( R_{\text{in}} \rightarrow \infty \), but is finite in practice, we expect the former structure to result in a larger deviation in the response due to the usage of more OTRAs.

2 Analytical synthesis methods

In this section, we shall realise the following odd-\( n \)-th-order elliptic HP filter transfer function

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{a_{0}s^n + a_{-3}s^{n-2} + \cdots + b_0s + b_{-3}s^{-2} + \cdots + a_1s + a_0}{a_0s^n + a_{-3}s^{n-2} + \cdots + b_0s + b_{-3}s^{-2} + \cdots + a_1s + a_0} \tag{4}
\]

where \( n \) is an odd integer. Equation (4) is the same as (3) but for the way the coefficients are named. Applying the input-and-output characteristics of an OTRA, and assuming that \( R_{\text{in}} \rightarrow \infty \), we obtain that \( I_c \) must equal \( I \) for the concept of realisation. Two analytical synthesis methods for realising (4) using OTRAs are given below.

2.1 Analytical synthesis method I

Cross multiplying (4), dividing it by \( a_0s^n \), and re-arranging yields

\[
V_{\text{out}}(1) - V_{\text{in}}(\frac{b_j}{a_j}) = V_{\text{out}}\left(\frac{b_{n-2}}{a_{n-2}} + \cdots + b_1a_{n-3} + b_0a_{n-1}\right) \\
- V_{\text{out}}(\frac{a_{n-1}}{a_0s^n} + \frac{a_{n-3}}{a_0s^{n-2}} + \cdots + \frac{a_1}{a_0s^2} + \frac{a_0}{a_0}) \tag{5}
\]

since

\[
\left(\frac{a_j}{a_0}\right) = \prod_{j=0}^{n-1} \left(\frac{a_j}{a_j+1}\right) \quad \text{for } j = 0, 1, \ldots, n - 1; \tag{6}
\]

and

\[
\left(\frac{b_j}{a_j}\right) = \prod_{j=1}^{n-2} \left(\frac{a_j}{a_j+1}\right) \left(\frac{b_j}{a_j}\right) \quad \text{for } j = 1, 3, \ldots, n - 2. \tag{7}
\]

Taking the first \( V_{\text{in}} \) term and the first two \( V_{\text{out}} \) terms from the right side in (5) and rearranging them, we obtain

\[
V_{\text{out}}\left(\frac{b_{n-2}}{a_{n-2}} + \cdots + b_1a_{n-3} + b_0a_{n-1}\right) = \left(\frac{a_{n-1}}{a_0s^n} + \frac{a_{n-3}}{a_0s^{n-2}} + \cdots + \frac{a_1}{a_0s^2} + \frac{a_0}{a_0}\right) \tag{8}
\]

Taking one more \( V_{\text{in}} \) term and two more \( V_{\text{out}} \) terms out from the right side in (5) and rearranging them, we obtain

\[
V_{\text{out}}\left(\frac{b_{n-2}}{a_{n-2}} + \cdots + b_1a_{n-3} + b_0a_{n-1}\right) = \left(\frac{a_{n-1}}{a_0s^n} + \frac{a_{n-3}}{a_0s^{n-2}} + \cdots + \frac{a_1}{a_0s^2} + \frac{a_0}{a_0}\right) \tag{9}
\]

We continue iterative combinations similar to (8) and (9). Finally, taking one more \( V_{\text{in}} \) term and three more \( V_{\text{out}} \) terms out from the right side in (5) and rearranging them, we obtain (see (10)). Consequently, (5) may be decomposed as shown below: (see (11)). Therefore, we obtain the following \( n + 1 \) simple equations decomposed from (11) by introducing new node voltages, \( V_i \), where \( i = 1, 2, \ldots, n \).

\[
V_i = \frac{1}{s}(-V_{\text{out}}(1)) \tag{12}
\]
\[ V_2 = \frac{1}{s} - V_{\text{out}(1)} + V_{\text{in}} \left( \frac{b_1}{a_1} \right) + V \left( \frac{a_0}{a_1} \right) \quad (13) \]

\[ V_3 = \frac{1}{s} - V_{\text{out}(1)} + V_{\text{in}} \left( \frac{a_0}{a_1} \right) \quad (14) \]

\[ V_{n-1} = \frac{1}{s} - V_{\text{out}(1)} + V_{\text{in}} \left( \frac{a_{n-1}}{a_{n-2}} \right) + V_{n-2} \quad (15) \]

\[ V_n = \frac{1}{s} - V_{\text{out}(1)} + V_{\text{in}} \left( \frac{a_{n-1}}{a_{n-2}} \right) \quad (16) \]

\[ V_{\text{out}(1)} - V_{\text{in}} \left( \frac{b_1}{a_1} \right) = V \left( \frac{a_{n-1}}{a_{n-2}} \right) \quad (17) \]

Applying the input-and-output characteristics of OTRA, we can easily realise (12)–(17), each of which employs a single OTRA and several passive elements. The superposition of all the sub-circuitries realised from (12)–(17) is shown in Fig. 1 in which the six dotted block sub-circuitries represent the realisations of (12)–(17), respectively. The structure of Fig. 1 is the first elliptic OTRA-based odd-nth-order HP filter structure.

To illustrate the above realisation for \( n = 3 \), using (12), (13), (16) and (17), we have

\[ V_1 = \frac{1}{s} - V_{\text{out}(1)} \quad (18) \]

\[ V_2 = \frac{1}{s} - V_{\text{out}(1)} + V_{\text{in}} \left( \frac{b_1}{a_1} \right) + V \left( \frac{a_0}{a_1} \right) \quad (19) \]

\[ V_3 = \frac{1}{s} - V_{\text{out}(1)} + V_{\text{in}} \left( \frac{a_0}{a_1} \right) \quad (20) \]

\[ V_{\text{out}(1)} - V_{\text{in}} \left( \frac{b_1}{a_1} \right) = V \left( \frac{a_0}{a_1} \right) \quad (21) \]

The superposition of the four OTRA-based sub-circuitries synthesised from (18) to (21) is presented in Fig. 2, which realises the elliptic third-order HP filter I.

The transfer function of Fig. 2 is

\[ \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{s^3C_1C_2C_3G_2 + sC_2G_3G_5 + sC_2G_1G_4 + sG_1G_3G_5}{s^3C_1C_2C_3G_2 + sC_2G_1G_4 + sC_2G_3G_5 + sG_1G_3G_5} \quad (22) \]

As a matter of fact, the four OTRA-based sub-circuitries synthesised from (18) to (21) have the other realisation with a resistor in the feedback loop (the former one is with a capacitor in the feedback loop). Their superposition is shown in Fig. 3, which is the other form of the realised elliptic third-order HP filter I.

Since an OTRA is with two input terminals both of which are with null voltage, the resistor shown in Fig. 2 may be replaced by two parallel and matched NMOS transistors \( M_1 \) and \( M_2 \), shown in Fig. 3, operating in the deep triode region. The current flowing through an NMOS transistor is given by

\[ I = K_N(V_G - V_T)(V_D - V_S) + \sum \alpha_i(V_D - V_i) \quad (23) \]

The same source and drain voltages of the transistors \( M_1 \) and \( M_2 \) remove the odd and even non-linearities. The current difference presented in Fig. 4 is

\[ I_1 - I_2 = G(V_1 - V_2) \quad (24) \]

where

\[ G = \mu_S C_{\text{in}} \frac{W}{L}(V_1 - V_6) \quad (25) \]

Replacing the resistor with two parallel NMOS transistors, Fig. 2 may be transformed into the corresponding third-order elliptic-OTA-MOS-C filter I shown in Fig. 5.

### 2.2 Analytical synthesis method 2

Different from the above analytical synthesis, another approach for synthesising (4) is given below.

Cross multiplying (4) and dividing it by \( s^{n-1} \) yields

\[ \frac{V_{\text{out}}}{\text{in}} \left( \frac{a_0s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0}{s^{n-1}} \right) = V_{\text{in}} \left( \frac{a_0s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0}{s^{n-1}} \right) \quad (26) \]

\[ \frac{b_1}{s^{n-1}} + \frac{b_{n-2}}{s^{n-2}} + \cdots + \frac{b_1}{s^{n-1}} \quad (26) \]

The analytical synthesis of Fig. 2 is

\[ \frac{V_{\text{out}}}{\text{in}} \left( \frac{a_0s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0}{s^{n-1}} \right) = V_{\text{in}} \left( \frac{a_0s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0}{s^{n-1}} \right) \quad (10) \]

\[ V_{\text{out}}(1) - V_{\text{in}} \left( \frac{b_1}{a_1} \right) = \frac{a_{n-1}}{a_0s^n} \quad (11) \]

\[ \left[ -V_{\text{out}}(1) + V_{\text{in}} \left( \frac{b_1}{a_1} \right) - V_{\text{in}} \left( \frac{a_0}{a_1} \right) - V_{\text{out}} \left( \frac{a_0}{a_1} \right) \right] \]

\[ \times \left[ \begin{array}{c} -V_{\text{out}}(1) + V_{\text{in}} \left( \frac{a_{n-1}}{a_{n-2}a_0} \right) + V_{\text{in}} \left( \frac{a_{n-2}}{a_{n-1}} \right) + \cdots + V_{\text{in}} \left( \frac{a_0}{a_1} \right) \\ -V_{\text{out}}(1) + V_{\text{in}} \left( \frac{a_0}{a_1} \right) + \cdots + V_{\text{in}} \left( \frac{a_{n-1}}{a_{n-2}a_0} \right) + V_{\text{in}} \left( \frac{a_{n-2}}{a_{n-1}} \right) \end{array} \right] \quad (11) \]
Fig. 4 Non-linearity cancellation in two parallel and matched-NMOS transistor circuit

Fig. 5 Third-order elliptic OTRA-MOS-C HP filter I

Fig. 6 Odd-nth-order elliptic OTRA-based HP filter structure II

Combine the last two terms of the left side and the last term of the right side in (26) to obtain

\[ V_{\text{in}} \left( \frac{b_1}{s^{n-2}} \right) - V_{\text{out}} \left( a_1, \frac{a_2}{s^{n-1}} \right) = \left( \frac{1}{s^{n-2}} \right) \]

Add the last third term of the left side in (26) to (27) and arrange them as

\[ \left( \frac{1}{s^{n-2}} \right) V_{\text{in}}(b_1) - V_{\text{out}} \left( a_1, \frac{a_2}{s^{n-1}} \right) \]

\[ = \left( \frac{1}{s^{n-2}} \right) V_{\text{in}}(b_1) - V_{\text{out}} \left( a_1 + \frac{a_2}{s} \right) - V_{\text{out}} \left( a_2 \right) \]

(28)

Add the last fourth terms of the left side and the last second term of the right side in (26) to (28) and re-arrange them as

\[ \left( \frac{1}{s^{n-2}} \right) V_{\text{in}}(b_1) - V_{\text{out}} \left( a_1 + \frac{a_2}{s} \right) - V_{\text{out}} \left( a_2 \right) + V_{\text{in}} \left( \frac{b_1}{s^{n-2}} \right) \]

\[ - V_{\text{out}} \left( \frac{a_1}{s} \right) = \left( \frac{1}{s^{n-2}} \right) V_{\text{in}}(b_1) - V_{\text{out}} \left( a_1 + \frac{a_2}{s} \right) + V_{\text{in}} \left( \frac{b_1}{s^{n-2}} \right) \]

(29)

Continue the iterative combinations, (27) to (29), for the other terms in (26) to decompose as

\[ V_{\text{out}}(a_2, a_{n-1}) = V_{\text{in}}(\frac{b_1}{s}) \]

\[ \left( \frac{1}{s^{n-2}} \right) \left[ V_{\text{in}}(b_1) - V_{\text{out}} \left( a_1 + \frac{a_2}{s} \right) \right] \]

\[ - V_{\text{out}} \left( \frac{a_1}{s} \right) \]

\[ + V_{\text{in}}(b_1) - V_{\text{out}} \left( a_1 \right) \]

\[ + \cdots \]

\[ - V_{\text{out}} \left( a_{n-1} \right) \]

\[ + V_{\text{in}}(b_{n-1}) - V_{\text{out}}(a_{n-1}) \]

(30)

Therefore, we obtain the following \( n \) simple equations decomposed from (30) by introducing the new node voltages, \( V_i \), where \( i = 1, 2, \ldots, \) and \( n-1 \).

\[ V_i = - \left( \frac{a_i}{s} \right) V_{\text{out}}, \text{ i.e., } V_i \left( \frac{s}{a_i} \right) = - V_{\text{out}}(1) \]

(31)

\[ V_i \left( \frac{s}{a_i} \right) = V_{\text{in}}(\frac{b_i}{a_i}) - V_{\text{out}}(1) + V_i \left( \frac{1}{a_i} \right) \]

(32)

\[ V_i \left( \frac{s}{a_i} \right) = - V_{\text{out}}(1) + V_i \left( \frac{1}{a_i} \right) \]

(33)

\[ V_i \left( \frac{s}{a_i} \right) = V_{\text{in}}(\frac{b_i}{a_i}) - V_{\text{out}}(1) + V_i \left( \frac{1}{a_i} \right) \]

(34)

\[ V_{n-1} \left( \frac{s}{a_{n-1}} \right) = - V_{\text{out}}(1) + V_{n-1} \left( \frac{1}{a_{n-1}} \right) \]

(35)

\[ V_{n-1} \left( \frac{s}{a_{n-1}} \right) = V_{\text{in}}(\frac{b_{n-1}}{a_{n-1}}) - V_{\text{out}}(1) + V_{n-1} \left( \frac{1}{a_{n-1}} \right) \]

(36)

and \( V_{\text{out}}(a_2, a_{n-1}) = V_{\text{in}}(a_2, a_{n-1}) + V_{\text{out}}(1), \) i.e.

\[ V_{\text{out}}(a_2, a_{n-1}) + 1 = V_{\text{in}}(a_2, a_{n-1}) + V_{\text{out}}(1) \]

(37)

Each of the simple (31)–(37) can be easily realised using a single OTRA, several resistors and one or two capacitor(s). The superposition of all the realised OTRA-based sub-circuits yields the second odd-nth-order elliptic OTRA-based HP filter structure and is shown in Fig. 6, in which six dotted block sub-circuits represent the realisations of (31)–(37), respectively. Note that this structure employs \( n \) OTRs (one fewer OTRA) and \( n+1 \) capacitors (one more capacitor) compared to the first structure, which uses \( n+1 \) OTRs (one more OTRA) and \( n \) capacitors (one fewer capacitor). It is very interesting for designers to know which of the two structures is better in terms of the output performance.

Based upon (31), (36) and (37), when \( n = 3 \), we obtain the following three simple equations:

\[ V_i \left( \frac{s}{a_i} \right) = - V_{\text{out}}(1) \]

(38)

\[ V_i \left( \frac{s}{a_i} \right) = V_{\text{in}}(\frac{b_i}{a_i}) - V_{\text{out}}(1) + V_i \left( \frac{1}{a_i} \right) \]

(39)
The superposition of the three OTRA-based sub-circuitries realised from the above three equations is the second third-order elliptic OTRA-based HP filter structure, which is shown in Fig. 7. 

The transfer function of Fig. 7 is

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{s^3 C C G_1 s + s C G G_m}{s^3 C C G_1 s + s C C G_1 + s C G G_m + G G G_m + G G G_m}
\]

Replacing each resistor by two parallel and matched NMOS transistors operating in the deep triode region, Fig. 7 may be transformed to the corresponding third-order elliptic-OTA-MOS-C HP filter II shown in Fig. 8.

3 Non-ideal analysis

In order to evaluate as to which of the two structures give more accurate output, we consider the above two third-order elliptic HP filters. Since in practice, the trans-resistance of the terminals, and no influence of the output parasitics due to the zero order elliptic HP filters shown in Figs. 2 and 7 can be found. These are considered (the input one can be neglected for MOS transistor build devices). The non-ideal transfer functions of the two third-order elliptic HP filters shown in Figs. 2 and 7 can be found. These are given below.

For Fig. 2, the non-ideal numerator, \( \Delta_N \), and the non-ideal denominator, \( \Delta_D \), are

\[
\Delta_N = s^3 C C C G_1 + s^3 C C G G_m + C C G G_m + C C G G_m + G G G_m G_m (G + G_m)
\]

\[
\Delta_D = s^3 C C C G_1 + s^3 C C C G_1 + s^3 C C G G_m + C C G G_m G_m + C C G G_m G_m + C C G G_m G_m + G G G_m G_m + G G G_m G_m + G G G_m G_m + G G G_m G_m
\]

The corresponding expressions for Fig. 7 are

\[
\Delta_N = s^3 C C C G_1 + s^3 C C G G_m + s^3 C C G G_m + G G G_m
\]

\[
\Delta_D = s^3 C C C G_1 + s^3 C C C G_1 + s^3 C C G G_m + C C G G_m G_m + C C G G_m G_m + G G G_m G_m + G G G_m G_m + G G G_m G_m + G G G_m G_m + G G G_m G_m
\]

In (42)–(45), \( G_m = 1/\beta_m \) for \( i = 1, 2, 3, \) and 4 where \( \beta_m \) represents the trans-resistance of the \( i \)th OTRA. If \( G_m \to 0 \) or \( R_m \to \infty \), both non-ideal (42) and (43) (resp. (44) and (45)) reduce to the idealistic (22) (resp. (41)). Comparing the theoretical and the non-ideal ones, we find that (i) none of the parasitics appear in the above non-ideal transfer functions due to the null voltage input terminals, and no influence of the output parasitics due to the zero output impedance of an OTRA, (ii) the first circuit has more non-ideal terms than the second leading to larger output deviations. Since the design scheme assumes the trans-resistance \( R_m \to \infty \), but \( R_m \) is finite in practice, we may conclude that the larger deviation in the first circuit results due to more ORAs being employed.

In this paper, the CMOS implementation [6] of an OTRA is employed and shown in Fig. 9 with \( V_{\text{BS}} = -0.97 \) V in which the W/L’s of the MOSFETs are given by 70/1.75 µm for M1–M3 and

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{G (G + G_m)}{1 + (G G_m + G + G_m)}
\]

Fig. 9 CMOS implementation [6] of the OTRA

M12–M13, 7/1.75 µm for M4 and M7, 21/1.75 µm for M5–M6, 35/1.75 µm for M8–M11, and 35/0.35 µm for M14, and ±1.65 V supply voltages are employed. Note that the frequency characteristics of the open loop trans-resistance gain are shown in Fig. 9 of [6] which is like an LP output curve with an upper 3 dB frequency at 10^5 Hz. Substituting the upper 3 dB frequency into (42)–(45), It is obvious that the dominant frequency limiting of the elliptic third-order HP filters is 10^5 Hz.

4 Simulation results

The third-order elliptic OTRA-based HP filters shown in Figs. 2 and 6 are used for validating filter feasibility using H-Spice with TSMC 0.35 µm process. The CMOS implementation [6] of the
are 100.84, 82.14 kHz, 0.8312, 0.128353, and 0.9208, thus having $R_{53.683/20.789} = 2.58$ for the resistances, and $40/40 = 1$ for the tolerance component values (whether precise or tolerant component values are used) for the circuit of Fig. 2, and $R_{53.683/20.789} = 2.58$ for the resistances, and $40/40 = 1$ for the capacitances, for simulating the following third-order elliptic HP transfer function:

$$V_{out} = \frac{s^4 + 0.58836s}{s^4 + 1.91391s^3 + 1.41856s^2 + 1.67029}$$

with ideal $f_p = 100$ kHz, $f_c = 83.33$ kHz, $A_1 = 0.85$, $A_2 = 0.127519$, and peak $P_k = 1$.

Since the fabrication of a resistor in an integrated circuit is difficult to meet delicate values such as 20.789, 53.683, 33.792, and 50.123 kΩ, we replace the above resistances by 21, 54, 34, and 50 kΩ. The simulated amplitude–frequency responses using exact and approximated values along with the theoretical response are shown in Figs. 10 and 11 for the filters of Figs. 2 and 7, respectively.

It is evident from these figures that the responses using precise and tolerance component values almost overlap. Further, the filter parameters, $f_p, f_c, A_1, A_2$, and $P_k$ (whether precise or tolerance component values are used) for the circuit of Fig. 2, are 100.84, 82.14 kHz, 0.8312, 0.128353, and 0.9208, thus having errors of 0.84, −1.43 −2.21, 0.65, and −7.92%, respectively. The corresponding values for the circuit of Fig. 7 are 101.30, 82.95 kHz, 0.8619, 0.127877 and 0.9654, respectively, with corresponding errors of 1.30, −0.46, 1.40, 0.28, and −3.46%. It is clear that the amplitude–frequency response of Fig. 7 is more accurate than that of Fig. 2. Therefore, we may conclude that (1) an integrated circuit with precise or tolerant component values have very close output responses, and (2) since the design scheme assumes the trans-resistance $R_{in} \rightarrow \infty$, which in fact is finite in practice, the larger deviation in the circuit of Fig. 2 is a consequence of more OTRAs being employed than in the case of Fig. 7.

In order to examine if the errors in the values of the five filter parameters $f_p, f_c, A_1, A_2$, and $P_k$ in the responses of Figs. 2 and 7 can be reduced by appropriately tuning one of the resistors, we change each of the resistor values, one at a time, by a small percentage and observe the effect on these parameters. Tables 1 and 2 list the % errors in the parameter values when the resistor values are changed by 1% in Figs. 2 and 7, respectively. Let us first consider the circuit of Fig. 2. It is seen from Table 1 that there are 11 errors that are larger than 1%, but 24 of them are smaller than 1%, the largest error is +3% in the value of $P_k$ when $R_1$ is changed. It is noted the errors in the values of the parameters $f_p, f_c, A_1, A_2$, and $P_k$ are +, −, −, +, and +, respectively, when precise values of resistances are used, with the largest error being −7.92% in the value of $P_k$. If we choose $R_1$ to tune the circuit, it is seen from Table 1 that the tendency for the errors in the parameters to be −, +, − and +, respectively, which are exactly opposite to that of the tendencies without tuning. It is observed from Table 2 that in the case of

### Table 1 Variation tendency (±) and amount (%) of the parameters when the value of the resistance is changed by 1% in the circuit of Fig. 2 ($R$ and $P$ denote resistance and parameter, respectively)

<table>
<thead>
<tr>
<th>$P$</th>
<th>$R_1 +1%$</th>
<th>$R_2 +1%$</th>
<th>$R_3 +1%$</th>
<th>$R_5 +1%$</th>
<th>$R_6 +1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_p$</td>
<td>−0.7</td>
<td>−0.7</td>
<td>+0.8</td>
<td>−0.4</td>
<td>+0.8</td>
</tr>
<tr>
<td>$f_s$</td>
<td>+0.1</td>
<td>+0.1</td>
<td>−0.1</td>
<td>−0.4</td>
<td>−0.1</td>
</tr>
<tr>
<td>$A_1$</td>
<td>+1.2</td>
<td>−0.3</td>
<td>−0.4</td>
<td>−0.4</td>
<td>−0.4</td>
</tr>
<tr>
<td>$A_2$</td>
<td>−0.3</td>
<td>+0.3</td>
<td>+1.1</td>
<td>−0.4</td>
<td>+1.1</td>
</tr>
<tr>
<td>$P_k$</td>
<td>+3.0</td>
<td>+0.8</td>
<td>−2.0</td>
<td>−0.4</td>
<td>−2.0</td>
</tr>
</tbody>
</table>

Tables 1 and 2 list the % errors in the parameter values when the resistor values are changed by 1% in Figs. 2 and 7, respectively. Let us first consider the circuit of Fig. 2. It is seen from Table 1 that there are 11 errors that are larger than 1%, but 24 of them are smaller than 1%, the largest error is +3% in the value of $P_k$ when $R_1$ is changed. It is noted the errors in the values of the parameters $f_p, f_c, A_1, A_2$, and $P_k$ are +, −, −, +, and +, respectively, when precise values of resistances are used, with the largest error being −7.92% in the value of $P_k$. If we choose $R_1$ to tune the circuit, it is seen from Table 1 that the tendency for the errors in the parameters to be −, +, − and +, respectively, which are exactly opposite to that of the tendencies without tuning. It is observed from Table 2 that in the case of
Fig. 7, there are 6 errors that are larger than 1%, but 29 of them are equal to or lower than 1%. In this case, we choose $R_2$ to do the tuning, and the reason for this is as follows: (i) Though the errors of the five parameters of Fig. 7 are $+, -, +, +, -$ respectively, when precise values are used for the resistances, yet none of the tendencies due to changes in any of the six resistances is just opposite to the above tendencies. (ii) Since the three parameters, $f_p$, $A_1$, and $P_k$, are more important than other two parameters, $f_s$ and $A_2$, we only consider to compensate the three variation tendencies, $+, +, -$ of $f_p$, $A_1$, and $P_k$, for Fig. 7. In such a case, we find that the three variation tendencies of a change in $R_2$ value are $-, -, +$, exactly opposite to the aforementioned three variation tendencies, $+, +, -$ of the original circuit of Fig. 7. By adjusting $R_1$ in Fig. 2 by $+1.9\%$ and $R_2$ in Fig. 7 by $+3.4\%$, the accuracy of the output parameters is very much improved and are shown in Figs. 12 and 13, respectively. Table 3 gives the values of the parameters for the circuits of Figs. 2 and 7, after tuning only one resistance. It is seen from this table that the errors in the five elliptic filtering parameters, $f_p$, $f_s$, $A_1$, $A_2$, and $P_k$ are reduced to $0.11$, $0.96$, $0.01$, $1.89$, and $2.43\%$, respectively, in the case of Fig. 2 and $0.02$, $0.35$, $0.53$, $0.79$, and $0.54\%$, respectively, for that of Fig. 7. The second part of simulations focuses on the comparison of the performance between Figs. 2 and 3, the former one with $C$ in

### Table 2
Variation tendency ($\pm$) and amount (%) of the parameters when the value of the resistance is changed by 1% in the circuit of Fig. 7 ($R$ and $P$ denote resistance and parameter, respectively)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$R_1 +1%$</th>
<th>$R_2 +1%$</th>
<th>$R_3 +1%$</th>
<th>$R_4 +1%$</th>
<th>$R_5 +1%$</th>
<th>$R_6 +1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_p$</td>
<td>$-0.5$</td>
<td>$-0.6$</td>
<td>$+0.5$</td>
<td>$-0.5$</td>
<td>$+0.5$</td>
<td>$-0.3$</td>
</tr>
<tr>
<td>$f_s$</td>
<td>$+0.1$</td>
<td>$-0.1$</td>
<td>$-0.1$</td>
<td>$-0.5$</td>
<td>$-0.1$</td>
<td>$-0.3$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>$+1.0$</td>
<td>$-0.3$</td>
<td>$-0.3$</td>
<td>$-0.4$</td>
<td>$-0.4$</td>
<td>$+0.3$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$-0.3$</td>
<td>$+0.3$</td>
<td>$+1.1$</td>
<td>$-0.4$</td>
<td>$+1.1$</td>
<td>$-1.7$</td>
</tr>
<tr>
<td>$P_k$</td>
<td>$+2.9$</td>
<td>$+0.8$</td>
<td>$-2.0$</td>
<td>$-0.4$</td>
<td>$-2.0$</td>
<td>$+0.9$</td>
</tr>
</tbody>
</table>

### Table 3
Output parameters of the third-order elliptic filter after adjusting one resistance in the circuits of Figs. 2 and 7

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case</th>
<th>Theory</th>
<th>Fig. 2 after tuning $R_1$</th>
<th>Fig. 7 after tuning $R_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_p$</td>
<td></td>
<td>100 kHz</td>
<td>100.11 kHz ($0.11%$)</td>
<td>99.98 kHz ($0.02%$)</td>
</tr>
<tr>
<td>$f_s$</td>
<td></td>
<td>83.33 kHz</td>
<td>82.53 kHz ($0.96%$)</td>
<td>83.04 kHz ($0.35%$)</td>
</tr>
<tr>
<td>$f_p-f_s$</td>
<td></td>
<td>16.67 kHz</td>
<td>17.57 kHz ($5.40%$)</td>
<td>16.95 kHz ($1.68%$)</td>
</tr>
<tr>
<td>$A_1$</td>
<td></td>
<td>0.85</td>
<td>0.8499 ($0.01%$)</td>
<td>0.8545 ($0.53%$)</td>
</tr>
<tr>
<td>$A_2$</td>
<td></td>
<td>0.127519</td>
<td>0.125113 ($1.89%$)</td>
<td>0.126516 ($0.79%$)</td>
</tr>
<tr>
<td>peak</td>
<td></td>
<td>1</td>
<td>0.9757 ($2.43%$)</td>
<td>0.9946 ($0.54%$)</td>
</tr>
</tbody>
</table>

Fig. 12 Amplitude–frequency response of Fig. 2 before and after tuning $R_1$ by $+1.9\%$

Fig. 13 Amplitude–frequency response of Fig. 7 before and after tuning $R_2$ by $+3.4\%$

Fig. 14 Amplitude–frequency response of the filter shown in Fig. 3

Fig. 15 Transient response of a square wave of the filter with $R$ in a feedback loop
the feedback loop and the latter one with $R$ in a feedback loop in structure.

The amplitude–frequency response for the circuit of Fig. 3 is shown in Fig. 14 with the filter parameters, $f_p$, $f_s$, $A_1$, $A_2$, and $P_k$ to be 102.82, 84.89 kHz, 0.8527, 0.1280, and 0.9749, respectively. When compared to those of Fig. 2, the former two parameters, $f_p$ and $f_s$, of Fig. 3 are worse in accuracy than those of Fig. 2, but the latter three parameters, $A_1$, $A_2$, and $P_k$ of Fig. 3 are more precise than those of Fig. 2.

The transient responses of the HP filters shown in Figs. 2 and 3 to an input square wave are presented in Figs. 15 and 16, respectively. These clearly ensure the stability of operation of the two HP filters, with the first overshoot and the PB ripple of the LP response.

The component sensitivities of $f_p$ and $f_s$, with respect to each passive element of the two HP filters shown in Figs. 2 and 3 are presented in Figs. 17–20, respectively. It can be seen that the

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**Fig. 16**  Transient response of a square wave of the filter shown in Fig. 2

**Fig. 17**  Component sensitivities of $f_p$ of the filter with R in a feedback loop

**Fig. 18**  Component sensitivities of $f_s$ of the filter with R in a feedback loop

**Fig. 19**  Component sensitivities of $f_p$ of the filter shown in Fig. 2

**Fig. 20**  Component sensitivities of $f_s$ of the filter shown in Fig. 2

**Fig. 21**  Input–output relationship with dynamic range 0.6684 V

**Fig. 22**  Input–output relationship with dynamic range 0.4867 V
maximum component sensitivities are 0.8631 for the $f_p$ and 0.4859 for the $f_s$ of the HP filter with $R$ in the feedback loop (Fig. 3) and 0.9414 for the $f_p$ and 0.4839 for the $f_s$ of the HP filter with $C$ in the feedback loop (Fig. 2). All the above component sensitivities are lower than unity, leading to a benefit of low sensitivity. On the other hand, we also notice that the component sensitivities of the $f_p$ of Fig. 3 (resp. Fig. 2) are getting larger (resp. smaller) from 10 kHz varying to 100 kHz. A linear relationship between input and output signals cannot always be kept in a linear system as the input signal gets larger. As the input signal gets larger, the slope of the input–output relationship starts to get lower and lower and finally, saturation sets in. Hence, the dynamic ranges have been investigated and shown in Fig. 21 for Figs. 3 and 22 for Fig. 2; the values are 0.6684 and 0.4867 V, respectively. The larger is the better. We define that a linear range to be the range in which all the output sinusoidal signals have the total harmonic distortion (THD) no more than unity. Under this standard, the linear ranges of the elliptic third-order HP filters are shown in Figs. 23 and 24; the ranges are 0.1110 V for Fig. 3 and 0.0811 V for Fig. 2. The linear range of Fig. 3 with $R$ in the feedback loop is larger than that of Fig. 2 with $C$ in a feedback loop. Noise simulations have been carried out and shown in Figs. 25 and 26. The noises at 0.1 Hz of the elliptic HP filters are 2.606 and 2.612 mV for Figs. 2 and 3, respectively. Both are nearly the same with a small difference. Two or more distinct frequencies of a system with non-linearity produce undesirable intermodulation leading to adjacent channel interference, which can reduce audio clarity. Two-tone tests for intermodulation linearity simulations of the two elliptic third-order HP filters are carried out, and the results shown in Fig. 27 (for the one with $R$ in the feedback loop) and Fig. 28 (for the other one with $C$ in the feedback loop). As can be seen, the frequency spectrum of the intermodulation resulting from two linear tones is much clearer than that resulting from two non-linear tones.
Finally, the results simulation for the power consumption of Figs. 2 and 3 have the same values, 973 μW due to the usage of the same component numbers.

5 Conclusions
In addition to the traditional even-th-order elliptic HP response function, odd-th-order elliptic HP filter obtained using the transformation s to 1/s from the traditional odd-th-order elliptic LP transfer function is realised in this paper. A couple of analytical synthesis methods for generating n odd-order elliptic HP filter structures using OTRAs have been proposed. It is shown that the realisation that uses a less number of OTRAs exhibits a performance that is better than the one that uses more OTRAs. It is due to the fact that the synthesis scheme assumes that the OTRA trans-resistance $R_m \rightarrow \infty$, but in fact, $R_m$ is finite in practice. The H-spice simulations have been conducted on the resulting two third-order HP filters, confirming the above result. It is shown that by slightly adjusting a single resistor, the amplitude–frequency response can be made close to the theoretical response.

6 Acknowledgments
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References
