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ORIGINAL PAPER

Stability of cylindrical thin shell wormholes supported by MGCG in f(R) gravity

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Abstract: In the framework of $f(R)$ modified theory of gravity, the dynamical equations of motion of a cylindrical thin shell wormholes supported by a modified generalized Chaplygin gas are constructed, using the cut and paste scheme (Darmois Israel formalism). The mechanical stability analysis of a cylindrical thin shell wormhole is discussed using a linearized radial perturbation around static solutions at the wormhole throat. The presence of stable static solutions depends on the suitable values of some parameters of dynamical shell.

Keywords: General relativity; Astrophysics; f(R) gravity; Exotic matter; Cosmology

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1. Introduction

Within the context of general relativity, the observed accelerated expansion of the universe during the matter dominated epoch requires the presence of dark energy. Instead of this nonstandard fluid, modifications of general relativity were proposed in order to solve both the problems of dark matter and dark energy, required by the ΛCDM-model. One of the simplest possible modifications called $f(R)$ gravity [1] in which the Einstein–Hilbert Lagrangian is replaced by a function of the Ricci scalar R. The stability of this sort of exotic matter is being analyzed in extended gravity theories [2]. Thibeault et al. [3] investigated 5D thin-shell wormhole in the scheme of modified theory. Rahaman et al. [4] constructed the TSW in the framework of string theory and investigated its stability against perturbation. Lobo and Oliveria [5] studied the static wormhole solutions for traceless matter supported by barotropic EoS in f(R) gravity. Sharif and Yousaf [6] have also studied the stability of the collapsing models in f(R) gravity theories.

The stability analysis of thin shell wormholes (TSW) with a linearized equation of state at the throat under radial perturbations has been discussed by several authors [7–9]. Halilsoy et al. [10]. studied the stability of regular Hayward TSW with different equation of state (EoS). Varela [11] has analyzed the linearized stability of Schwarzschild thin shell wormholes with variable equations of state. Eiroa and Simeone [12] have discussed the stability of plane and cylindrical thin shell wormholes.

In this paper, we will investigate the stability analysis of a cylindrical thin shell wormhole with a modified generalized Chaplygin gas (MGCG) in the framework of $f(R)$ gravity.

2. Dynamics of cylindrical thin shell wormhole in $f(R)$ gravity

The Einstein-Hilbert action of general relativity, in $f(R)$ theories, is modified by using a general analytic function $f(R)$ instead of R; the action is given by

$$A = \int \left(\frac{f(R)}{16\pi G} + L_m \right) \sqrt{-g} d^4x, \quad (1)$$

where L_m is the Lagrangian for the matter distribution. The variation of the action (1) with respect to the metric tensor

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leads to the following fourth order partial differential equation as the field equation,

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\nabla_\alpha\nabla^\alpha F(R) = -8\pi G T^m_{\mu\nu}, \tag{2}$$

where $F(R) = \frac{df}{dR}$. Writing this equation in the form of Einstein tensor, one obtains

$$G_{\mu\nu} = \frac{k}{F}\left(T^m_{\mu\nu} + T^D_{\mu\nu}\right), \tag{3}$$

where

$$T^D_{\mu\nu} = \frac{1}{2k}\{(f(R) - F(R)R)g_{\mu\nu} + 2\nabla_\mu\nabla_\nu F(R) - 2g_{\mu\nu}\nabla_\alpha\nabla^\alpha F(R)\}, \tag{4}$$

$T^D_{\mu\nu}$ represents the contribution of the curvature in addition to Einstein tensor (called effective stress-energy tensor with a purely geometrical origin) while $T^m_{\mu\nu}$ is an usual stress-energy tensor. The metric of static cylindrically thin shell wormhole is given by [13, 14],

$$ds^2 = -A(r)dt^2 + A^{-1}(r)dr^2 + B(r)(d\varphi^2 + \alpha^2 dz^2), \tag{5}$$

where,

$$A(r) = \alpha^2 r^2 - \frac{4m}{r\alpha} + \frac{4Q^2}{r^2\alpha^2}, \quad B(r) = r^2, \tag{6}$$

where Q, m and Λ are the charge density, the ADM mass and the cosmological constant ($\Lambda = 3\alpha^2$), respectively. The metric (5) becomes singular for $g_{00} = 0$ and at $r = 0$. The outer and inner charged black hole string horizons are given by

$$r_{h\pm} = \frac{(4m)^{\frac{1}{3}}}{2\alpha}\left\{(\Upsilon)^{\frac{1}{2}} \pm \left[-\Upsilon + 2\left(\Upsilon^2 - Q^2\left(\frac{2}{m}\right)^{\frac{4}{3}}\right)\right]^{\frac{1}{2}}\right\}$$

where

$$\Upsilon = \left[\frac{1}{2} + \frac{1}{2}\left(1 - \frac{(2Q)^6}{27m^4}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} + \left[\frac{1}{2} - \frac{1}{2}\left(1 - \frac{(2Q)^6}{27m^4}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}.$$

The outer and inner horizons merge into each other, for $Q^2 = \frac{3}{4}m^{\frac{4}{3}}$, representing extremal black hole string. The given spacetime does not possess event horizon for $Q^2 > \frac{3}{4}m^{\frac{4}{3}}$, while for $Q^2 \le \frac{3}{4}m^{\frac{4}{3}}$ it possess event horizon. According to the cut and paste scheme, two copies M^+ and M^- with radius $r \ge a$ are selected and pasted at the boundary surface Σ defined by $r - a = 0$. Thus giving a complete geodesically new manifold $M = M^+ \cup M^-$. If the geometry is open at Σ, this resulted in a cylindrical TSW with two parts correlated by a throat at Σ (flair-out condition) [15]. It is assumed here that radius $a > r_h$ in such a way that there are no singularity and horizons in M.

To discuss the stability of f(R) gravity, a familiar form of $f(R)$, given by Starobinsky [16] as

$$f(R) = R + \delta R^2. \tag{7}$$

Einstein theory of general relativity is recovered if $\delta = 0$. Cembranos [17] discussed, by the given $f(R)$ model, both as the dark matter model in R^2 gravity with $\delta = \frac{1}{6m^2}$. In the framework of Darmois-Israel formalism, the Lanczos equations [18], with $f(R)$ terms [19], take the form

$$\frac{-k}{\alpha F(R)}\left(\alpha t^i_j + \overset{D}{t^i_j}\right) = \left[K^i_j\right] - g_{ij}[K] \tag{8}$$

where K_{ij} is the extrinsic curvature of the hypersurface Σ, $[K]$ is the trace of $[K_{ij}]$ and $[K_{ij}] = K^+_{ij} - K^-_{ij}$. The surface stress-energy tensor in terms of the surface energy density σ, and surface pressure p is:$t^i_j = diag(-\sigma, p, p)$. Let the equation of the shell be $r = a(\tau)$, the throat radius $a(\tau)$ describes the time evolution of the shell. The Lanczos equations are given by

$$\sigma = \frac{-4F(R)}{ak}\sqrt{\dot{a}^2 + A(a)} + \frac{1}{k\alpha}\left\{\frac{1}{2}(f(R) - RF(R)) + A(a)F'' + AF'\left(\frac{B'(a)}{B(a)} + \frac{A'(a)}{2A(a)}\right)\right\} \tag{9}$$

$$p = \frac{F(R)}{ak}\left(\frac{2\dot{a}^2 + 2a\ddot{a} + 2A(a) + aA'}{\sqrt{\dot{a}^2 + A(a)}}\right) - \frac{1}{k\alpha}\left\{\frac{1}{2}(f(R) - RF(R)) + A(a)F'' + AF'\left(\frac{B'(a)}{2B(a)} + \frac{A'(a)}{A(a)}\right)\right\} \tag{10}$$

The equation of state for MGCG at the throat is defined as [20],

$$p = \eta\sigma - \frac{\zeta}{\sigma^\beta} \tag{11}$$

where η, ζ are positive constants and $0 < \beta \le 1$. The dynamical evolution of the wormhole throat can be obtained by replacing Eqs. (7), (9) and (10) into the Eq. (11):

$$2\dot{a}^2 + 2a\ddot{a} + 2A(a) + aA' = \frac{a}{F(R)}\sqrt{\dot{a}^2 + A(a)}\left\{\mathrm{H}_p + \eta\mathrm{H}_\sigma - \frac{4F}{a}\eta\sqrt{\dot{a}^2 + A(a)} - \frac{(ak)^\beta k\zeta}{(a\mathrm{H}_\sigma - 4F\sqrt{\dot{a}^2 + A(a)})^\beta}\right\} \tag{12}$$

where

$$\mathrm{H}_\sigma = \frac{\delta}{\alpha}\left\{-\frac{1}{2}R^2(a) + 2A(a)R''(a) + 2A(a)R'(a)\left(\frac{B'(a)}{B(a)} + \frac{A'(a)}{2A(a)}\right)\right\}$$

$$\mathrm{H}_p = \frac{\delta}{\alpha}\left\{-\tfrac{1}{2}R^2(a) + 2A(a)R''(a) + 2A(a)R'(a)\left(\frac{B'(a)}{2B(a)} + \frac{A'(a)}{A(a)}\right)\right\}$$

$$R(a) = A'' + \frac{2BA''}{A} + \frac{A'}{A}R'(a)\left(2A' - \frac{AB'}{2B}\right).$$

This differential equation should be satisfied by the throat's radius of cylindrical thin shell wormholes threaded by modified the generalized Chaplygin gas equation of state. The surface energy density and pressure, satisfy the conservation equation

$$\frac{d}{d\tau}(\Pi\sigma) + p\frac{d}{d\tau}\Pi = 0, \tag{13}$$

where $\Pi = 4\pi a^2$ is the area of the wormhole throat. This equation describes the continuity equation for the energy–momentum tensor (i.e. the change in internal energy of the throat plus the work done by the throat's internal forces), and can be written in the form

$$\frac{d}{d\tau}\sigma = -\frac{2}{a}(\sigma + p)\frac{da}{d\tau}, \tag{14}$$

which can rewritten as

$$\frac{d\sigma}{da} = -\frac{2}{a}(p + \sigma). \tag{15}$$

Differentiation of Eq. (11) gives:

$$\sigma' + 2p' = \sigma'\left[1 + 2\eta(\beta + 1) - \frac{2\beta}{\sigma}p\right] \tag{16}$$

where $\chi^2 = \frac{dp}{d\sigma}$ is the squared of sound velocity. The prime denotes a derivative with respect to a. Rearranging Eqs. (9) and (7) in order to obtain the dynamical equation of motion of the cylindrical thin shell wormhole,

$$\dot{a}^2 + \Psi(a) = 0 \tag{17}$$

where $\Psi(a)$ is known as the effective potential function given by

$$\Psi(a) = A(a) - \left(\frac{ak}{4(1 + 2\delta R)}\right)^2\left(\frac{\mathrm{H}_\sigma}{k} - \sigma\right)^2 \tag{18}$$

where, $1 + 2\delta R = F(R) \equiv \frac{df}{dR}$.

3. Results and discussion of stability

The Taylor series expansion of $\Psi(a)$ up to second order around $a_\circ$, is given by

$$\Psi(a) = \sum_{n=0}^{2} c_n(a - a_\circ)^2,\ c_n = \frac{\Psi^{(n)}(a_\circ)}{n!} \tag{19}$$

The first and the second derivatives of $\Psi(a)$ are given, using Eq. (16), by

$$\Psi'(a) = A' - \frac{1}{2}\left(\frac{ak}{2(1+2\delta R)}\right)^2\left(\frac{\mathrm{H}_\sigma}{k} - \sigma\right) \left\{\frac{\mathrm{H}'_\sigma}{k} + \frac{1}{a}\left(2p + \sigma + \frac{\mathrm{H}_\sigma}{k}\right) - \frac{2\delta R'}{(1+2\delta R)}\left(\frac{\mathrm{H}_\sigma}{k} - \sigma\right)\right\}, \tag{20}$$

$$\begin{aligned}\Psi''(a) = A'' &- \frac{a}{2}\left(\frac{k}{2(1+2\delta R)}\right)^2\left[\frac{\mathrm{H}'_\sigma}{k} + \frac{1}{a}\left(2p + \sigma + \frac{\mathrm{H}_\sigma}{k}\right)\right.\\ &\left.- \frac{2\delta R'}{(1+2\delta R)}\left(\frac{\mathrm{H}_\sigma}{k} - \sigma\right)\right]\\ &\left[\frac{a\mathrm{H}'_\sigma}{k} + 2(p+\sigma) + 2\left(1 - \frac{2a\delta R'}{(1+2\delta R)}\right)\left(\frac{\mathrm{H}_\sigma}{k} - \sigma\right)\right]\\ &- \frac{1}{2}\left(\frac{ak}{2(1+2\delta R)}\right)^2\left(\frac{\mathrm{H}_\sigma}{k} - \sigma\right)\left\{\frac{\mathrm{H}''_\sigma}{k} - \frac{1}{a^2}\left(2p + \sigma + \frac{\mathrm{H}_\sigma}{k}\right)\right.\\ &+ \frac{\mathrm{H}'_\sigma}{ak} - \frac{2}{a^2}(p+\sigma)\left(1 + 2\eta(\beta+1) - \frac{2\beta p}{\sigma}\right)\\ &- \frac{2\delta}{(1+2\delta R)}\left[R'\left(\frac{\mathrm{H}'_\sigma}{k} + \frac{2}{a}(p+\sigma)\right)\right.\\ &\left.\left.+\left(R'' - \frac{2R'^2}{(1+2\delta R)}\right)\left(\frac{\mathrm{H}_\sigma}{k} - \sigma\right)\right]\right\}.\end{aligned} \tag{21}$$

The dynamical Eq. (12) for the static configuration of wormhole (where $\ddot{a} = \dot{a} = 0$), becomes

$$\begin{aligned}&2A(a_\circ) + a_\circ A'(a_\circ)\\ &= \frac{a_\circ}{(1+2\delta R_\circ)}\sqrt{A(a_\circ)}\left\{\mathrm{H}_{p\circ} + \eta\mathrm{H}_{\sigma\circ} - \frac{4\eta}{a_\circ}(1+2\delta R_\circ)\sqrt{A(a_\circ)}\right.\\ &\left.- \frac{(a_\circ k)^\beta k\zeta}{(a_\circ\mathrm{H}_{\sigma\circ} - 4(1+2\delta R_\circ)\sqrt{A(a_\circ)})^\beta}\right\}.\end{aligned} \tag{22}$$

The surface energy density and pressure in the static case are given by

$$\sigma_\circ = \frac{-4}{a_\circ k}(1 + 2\delta R_\circ)\sqrt{A(a_\circ)} + \frac{1}{k}\mathrm{H}_{\sigma\circ} \tag{23}$$

$$p_\circ = \frac{1}{a_\circ k}(1 + 2\delta R_\circ)\left(\frac{2A(a_\circ) + a_\circ A'(a_\circ)}{\sqrt{A(a_\circ)}}\right) - \frac{1}{k}\mathrm{H}_{p\circ} \tag{24}$$

where $\mathrm{H}_{\sigma\circ}$, $\mathrm{H}_{p\circ}$ and $R_\circ$ are evaluated at $a = a_\circ$. Evaluate Eq. (21) by using (23) and (24) at $a = a_\circ$, it follows that

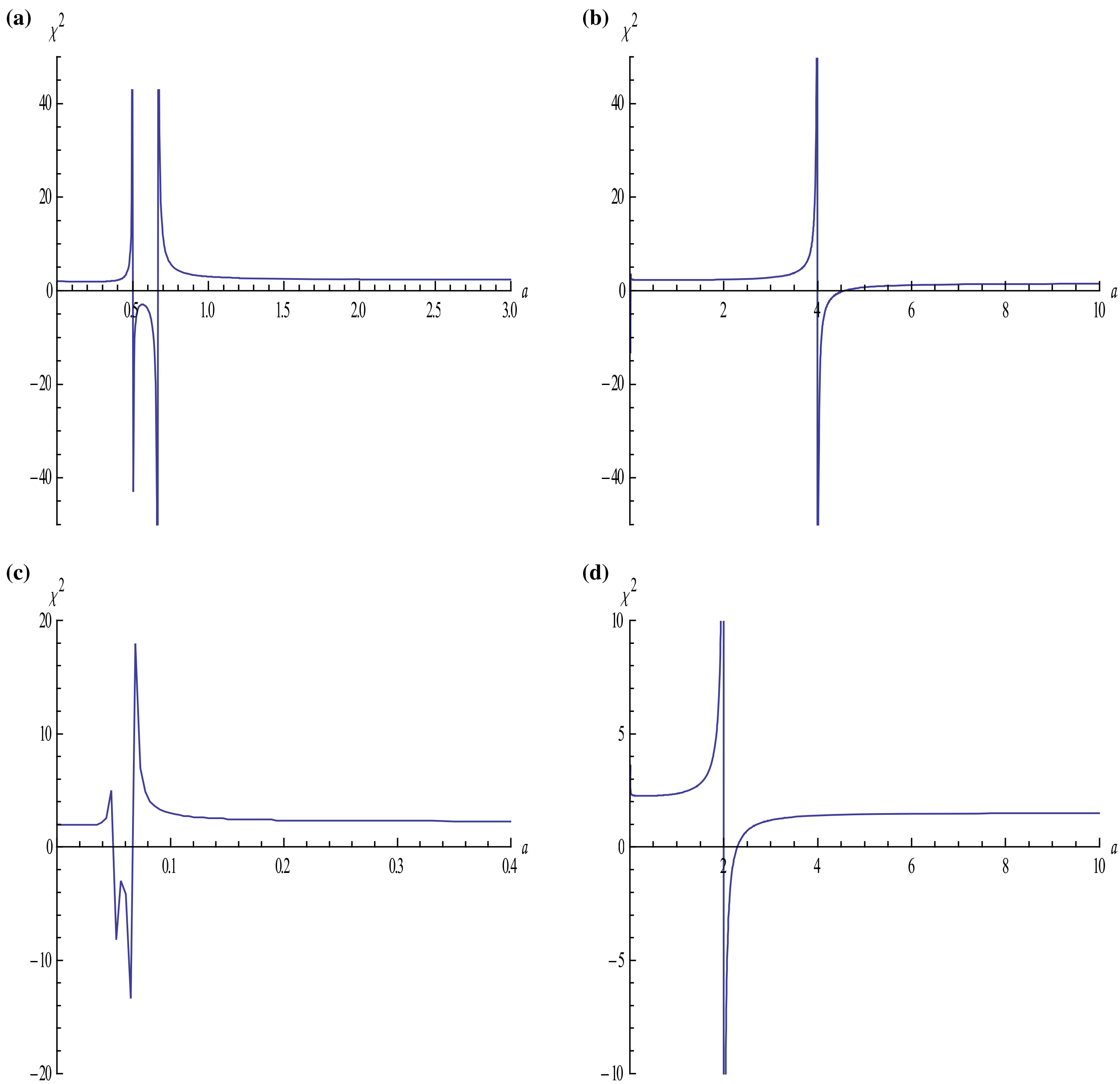

Fig. 1 The squared of sound velocity $\chi_{\circ}^{2}$ for m = 2, $\eta = 1$, Q = 0.1 and $\beta = 1$ with different values of α: (a) $\alpha = 0.01$, (b) $\alpha = 0.5$, (c) $\alpha = 0.1$, (d) $\alpha = 1$

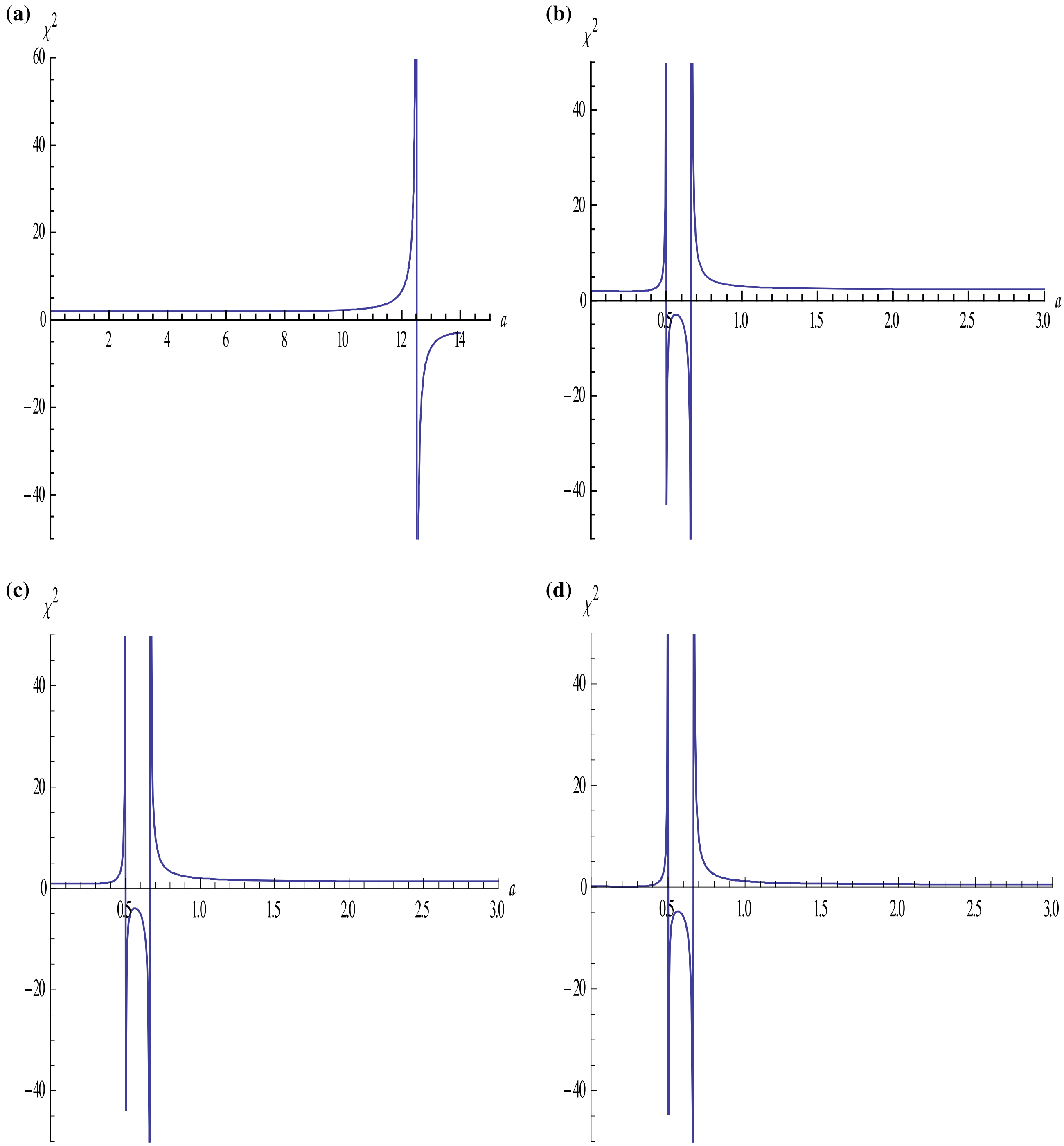

Fig. 2 The squared of sound velocity $\chi_\circ^2$ for m = 2, $\alpha = 0.01$ and $\beta = 1$ with different values of Q and η: (a) $\eta = 1$, Q = 0.5, (b) $\eta = 1$, Q = 0.1, (c) $\eta = 0.5$, Q = 0.1, (d) $\eta = 0.1$, Q = 0.1

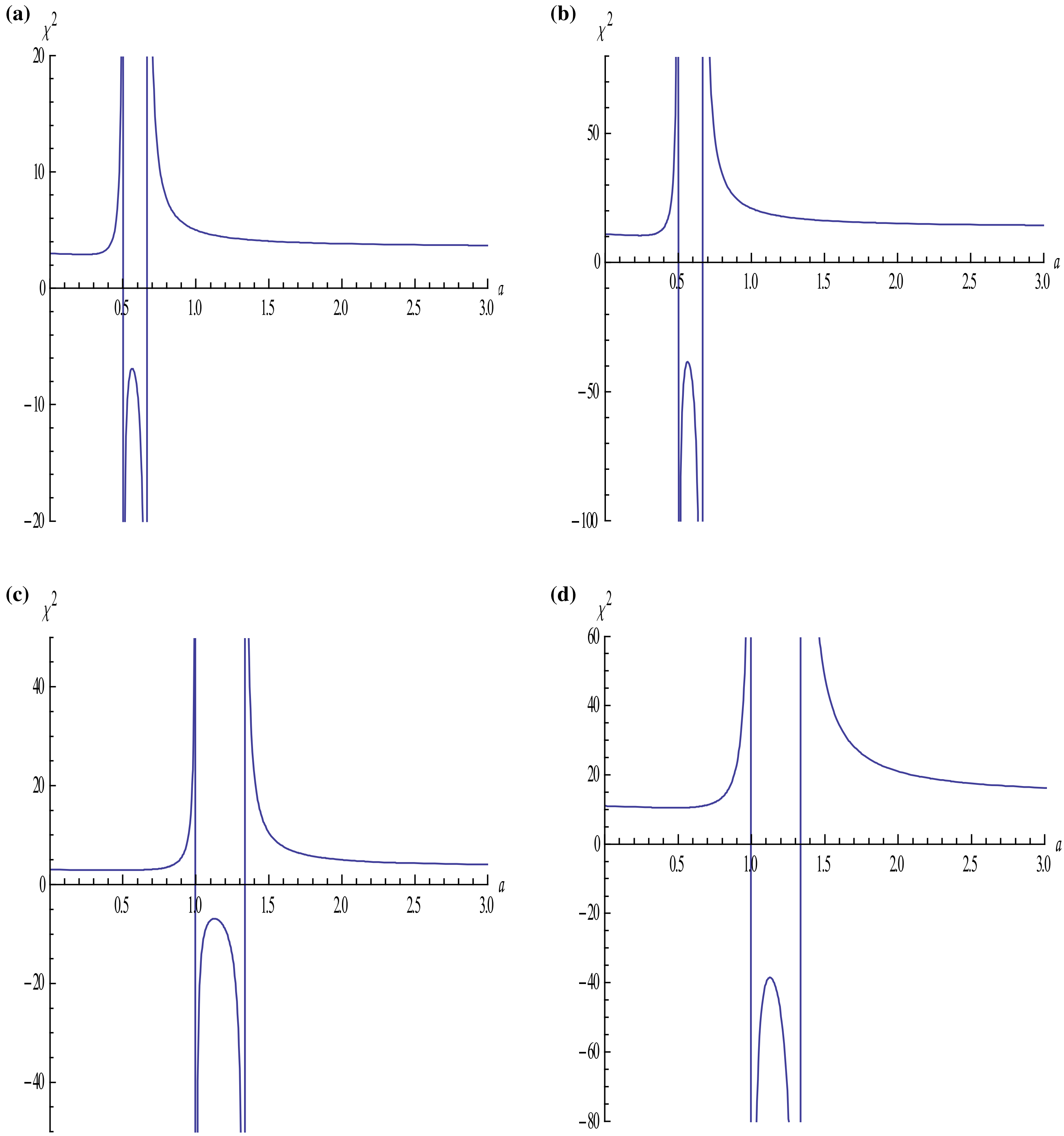

Fig. 3 The squared of sound velocity $\chi_{\circ}^{2}$ for $\alpha = 0.01$ and $\eta = 1$ with different values of m, β and Q: (a) m = 2, Q = 0.1, $\beta = 0.5$, (b) m = 2, Q = 0.1, $\beta = 0.1$, (c) m = 1, Q = 0.1, $\beta = 0.5$, (d) m = 1, Q = 0.1, $\beta = 0.1$

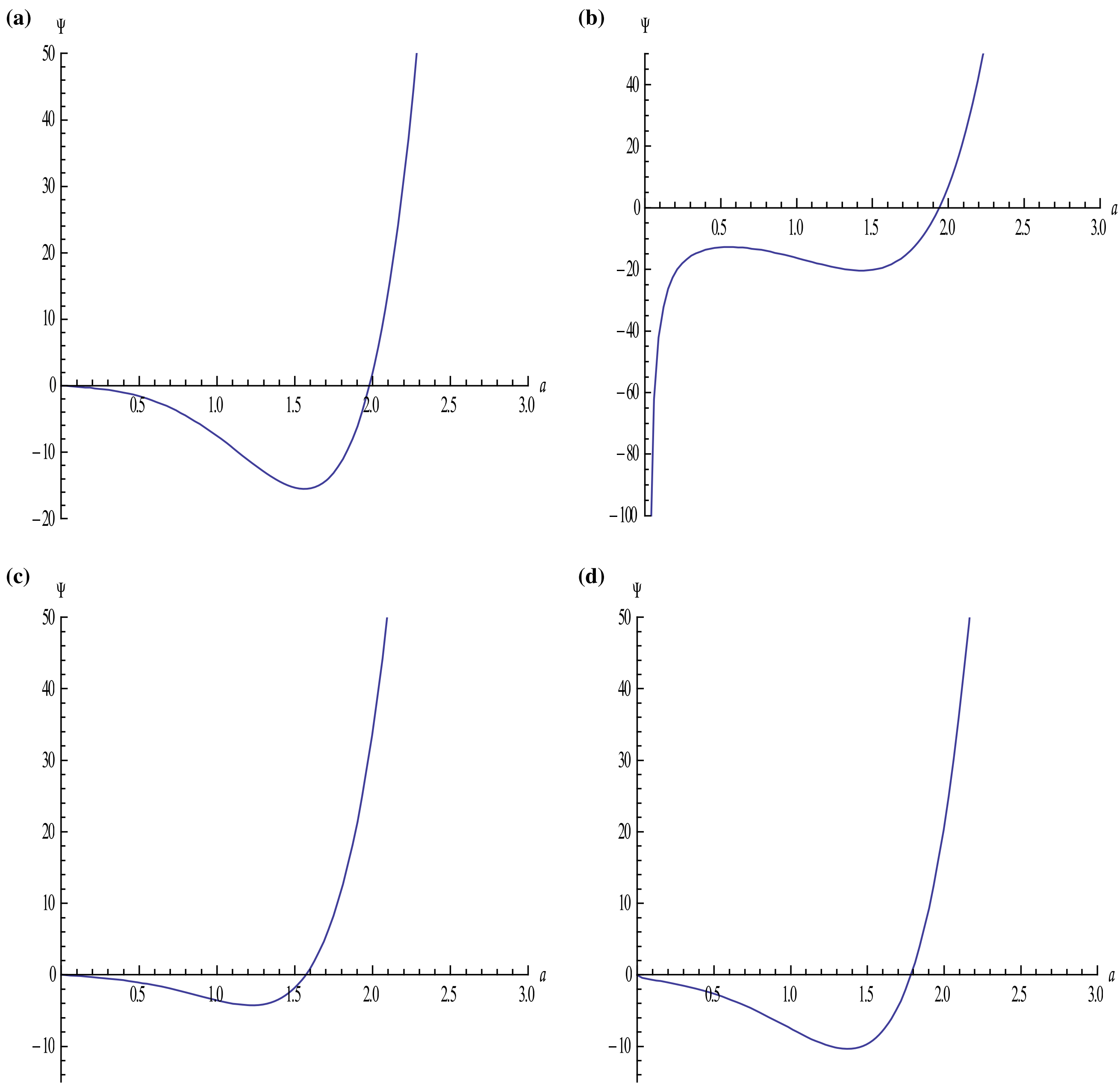

Fig. 4 The effective potential Ψ for $\delta = 1$, $\zeta = 1$, $\alpha = 1$, $c = 1$ and $R_0 = \mathbf{1}$ with different values of Q, m, η and β: (a) $m = 2$, $\eta = 1$, $\beta = 0.5$, $Q = 1$, (b) $m = 1$, $\eta = 0.1$, $\beta = 0.1$, $Q = 0.1$, (c) $m = 1$, $\eta = 1$, $\beta = 0.5$, $Q = 1$, (d) $m = 1$, $\eta = 1$, $\beta = 0.1$, $Q = 0.1$

$$
\begin{aligned}
\Psi''(a_\circ) = A''(a_\circ) - \frac{a_\circ}{2}\left(\frac{k}{2(1+2\delta R_\circ)}\right)^2 \Bigg[& \frac{\delta}{\alpha k}\mathrm{N1} + \frac{2}{a_\circ k}\left(\frac{2\delta}{\alpha}A_\circ R'_\circ\left(\frac{1}{a_\circ} - \frac{A'_\circ}{2A_\circ}\right) + (1+2\delta R_\circ)\frac{A'_\circ}{\sqrt{A_\circ}}\right) \\
& - \frac{8\delta R'}{k a_\circ}\sqrt{A_\circ}\Bigg]\left[\frac{a_\circ \delta}{\alpha k}\mathrm{N1} + \frac{2}{a_\circ k}\left(a_\circ(1+2\delta R_\circ)\frac{A'_\circ}{\sqrt{A_\circ}} + \frac{2\delta}{\alpha}a_\circ A_\circ R'_\circ\left(\frac{1}{a_\circ} - \frac{A'_\circ}{2A_\circ}\right) + 2(1+2\delta R_\circ)\sqrt{A_\circ}\right)\right. \\
& \left. - \frac{16\delta R'}{k}\sqrt{A_\circ}\right] - \left(\frac{a_\circ k\sqrt{A_\circ}}{2(1+2\delta R)}\right)\Bigg\{\frac{\delta}{\alpha k}\mathrm{N2} - \frac{2}{a_\circ^2 k}\left((1+2\delta R_\circ)\frac{A'_\circ}{\sqrt{A_\circ}} + \frac{2\delta}{\alpha}A_\circ R'_\circ\left(\frac{1}{a_\circ} - \frac{A'_\circ}{2A_\circ}\right)\right) \\
& + \frac{\delta}{\alpha k a_\circ}\mathrm{N1} - \frac{8\delta}{k a_\circ}\sqrt{A_\circ}\left(R''_\circ - \frac{2R'^2_\circ}{1+2\delta R_\circ}\right) - \left(\frac{2\delta R'_\circ}{(1+2\delta R)}\right)\frac{\delta}{\alpha k}\mathrm{N1} \\
& - \left(\frac{4\delta R'_\circ}{a_\circ(1+2\delta R)}\right)\left((1+2\delta R_\circ)\frac{A'_\circ}{k\sqrt{A_\circ}} + \frac{2\delta}{\alpha k}A_\circ R'_\circ\left(\frac{1}{a_\circ} - \frac{A'_\circ}{2A_\circ}\right) - \frac{2}{a_\circ k}(1+2\delta R_\circ)\sqrt{A_\circ}\right) \\
& - \frac{2}{a_\circ^2}\left((1+2\delta R_\circ)\frac{A'_\circ}{k\sqrt{A_\circ}} + \frac{2\delta}{\alpha k}A_\circ R'_\circ\left(\frac{1}{a_\circ} - \frac{A'_\circ}{2A_\circ}\right) - \frac{2}{a_\circ k}(1+2\delta R_\circ)\sqrt{A_\circ}\right)\left(1+2\eta(\beta+1) - 2\beta\chi^2\right)\Bigg\}
\end{aligned} \tag{25}
$$

where

$$
\begin{aligned}
\mathrm{N1} = & -R_\circ R'_\circ + 2\left(A'_\circ R''_\circ + A_\circ R'''_\circ\right) \\
& + 4\left(\frac{1}{a_\circ} + \frac{A'_\circ}{4A_\circ}\right)\left(A'_\circ R'_\circ + A_\circ R''_\circ\right) \\
& + 2A_\circ R'_\circ\left(\frac{-2}{a_\circ^2} - \frac{A'^2_\circ}{2A_\circ^2} + \frac{A''_\circ}{2A_\circ}\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{N2} = & -R_\circ R''_\circ + R'^2_\circ + 2\left(A''_\circ R''_\circ + A_\circ R''''_\circ + 2A'_\circ R'''_\circ\right) \\
& + 4\left(\frac{-2}{a_\circ^2} - \frac{A'^2_\circ}{2A_\circ^2} + \frac{A''_\circ}{2A_\circ}\right)\left(A'_\circ R'_\circ + 2A_\circ R''_\circ\right) \\
& + 4\left(A''_\circ R'_\circ + A_\circ R'''_\circ + 2A'_\circ R''_\circ\right)\left(\frac{1}{a_\circ} + \frac{A'_\circ}{4A_\circ}\right) \\
& + 2A_\circ R'_\circ\left(\frac{4}{a_\circ^3} + \frac{A'^3_\circ}{A_\circ^3} + \frac{A'''_\circ}{2A_\circ} - \frac{3A'_\circ A''_\circ}{2A_\circ^2}\right)
\end{aligned}
$$

and

$$
\chi_\circ^2 = -\frac{(1+2\delta R_\circ)(2A_\circ + A'_\circ a_\circ) - a_\circ \mathrm{H}_{p\circ}\sqrt{A_\circ}}{-4(1+2\delta R_\circ)A_\circ + a_\circ \mathrm{H}_{\sigma\circ}\sqrt{A_\circ}}.
$$

For $R_\circ = \widehat{R}$, $\widehat{R}$ is constant, Eq. (25) becomes

$$
\begin{aligned}
\Psi''(a_\circ) = & A''(a_\circ) - 2\beta\left(\frac{A'_\circ}{a_\circ\sqrt{A_\circ}} - \frac{2\sqrt{A_\circ}}{a_\circ^2}\right) \\
& \times \frac{2\alpha(1+2\delta\widehat{R}_\circ)(2A_\circ + A'_\circ a_\circ) + \delta\widehat{R}_\circ^2 a_\circ\sqrt{A_\circ}}{-8\alpha(1+2\delta\widehat{R}_\circ)\sqrt{A_\circ} - \delta\widehat{R}_\circ^2 a_\circ} \\
& - \frac{2A_\circ}{a_\circ^2}(1+2\eta(\beta+1)) - A'_\circ\left(\frac{A'_\circ}{2A_\circ} - \frac{1}{a_\circ} - \frac{2}{a_\circ}\eta(\beta+1)\right).
\end{aligned} \tag{26}
$$

Using Eq. (5), the energy density (23) and surface pressure (24), under constant Ricci scalar condition $R_\circ = \widehat{R}$, can be written as

$$
\sigma_\circ = \frac{-4}{\alpha a_\circ^2 k}(1+2\delta\widehat{R}_\circ)\sqrt{\alpha^4 a_\circ^4 - 4m\alpha a_\circ + 4Q^2} - \frac{\delta\widehat{R}_\circ^2}{2\alpha k} \tag{27}
$$

$$
p_\circ = \frac{4(1+2\delta\widehat{R}_\circ)(\alpha^3 a_\circ^3 - m)}{a_\circ k\sqrt{\alpha^4 a_\circ^4 - 4m\alpha a_\circ + 4Q^2}} + \frac{\delta\widehat{R}_\circ^2}{2\alpha k}. \tag{28}
$$

The dynamical Eq. (22) becomes

$$
\begin{aligned}
& \frac{4\eta}{\alpha^2 a_\circ^2}(\alpha^4 a_\circ^4 - 4m\alpha a_\circ + 4Q^2) \\
& + \frac{\delta\widehat{R}_\circ^2}{2(1+2\delta\widehat{R}_\circ)\alpha^2}(\eta+1)\sqrt{\alpha^4 a_\circ^4 - 4m\alpha a_\circ + 4Q^2} \\
& + \frac{k\zeta(2\alpha k a_\circ^2)^\beta\sqrt{\alpha^4 a_\circ^4 - 4m\alpha a_\circ + 4Q^2}}{\alpha(1+2\delta\widehat{R}_\circ)\left[-a_\circ^2\delta\widehat{R}_\circ^2 - 8(1+2\delta\widehat{R}_\circ)\sqrt{\alpha^4 a_\circ^4 - 4m\alpha a_\circ + 4Q^2}\right]^\beta} \\
& + 4\alpha^2 a_\circ^2 - \frac{4m}{\alpha a_\circ} = 0
\end{aligned} \tag{29}
$$

while Eq. (26) leads to

$$\begin{aligned}\Psi''(a_\circ) &= \frac{2}{\alpha^2 a_\circ^4}\left(\alpha^4 a_\circ^4 + 2m\alpha a_\circ + 4Q^2\right) + \frac{8\eta}{\alpha^2 a_\circ^4}(\beta+1)\left(3m\alpha a_\circ - 4Q^2\right)\\ &- \frac{2}{\alpha^2 a_\circ^4\left(\alpha^4 a_\circ^4 - 4m\alpha a_\circ + 4Q^2\right)}\left[-8Q^2\left(\alpha^4 a_\circ^4 + 2m\alpha a_\circ - 2Q^2\right) + 4m\alpha^2 a_\circ^2\left(m + \alpha^3 a_\circ^3\right) + \alpha^8 a_\circ^8\right]\\ &+ \frac{4\beta}{X_\circ \alpha^3 a_\circ^4}\left\{a_\circ \delta \hat{R}_\circ^2\left(3m\alpha a_\circ - 4Q^2\right)\sqrt{\alpha^4 a_\circ^4 - 4m\alpha a_\circ + 4Q^2} + 8\alpha(1 + 2\delta\hat{R}_\circ)\left(3m\alpha a_\circ - 4Q^2\right)\left(\alpha^3 a_\circ^3 - m\right)\right\}\end{aligned} \tag{30}$$

where

$$X_\circ = \frac{1}{\alpha a_\circ}\sqrt{\alpha^4 a_\circ^4 - 4m\alpha a_\circ + 4Q^2}\left(\tfrac{1}{2}\delta\hat{R}_\circ^2 a_\circ^2 + 4(1 + 2\delta\hat{R}_\circ)\sqrt{\alpha^4 a_\circ^4 - 4m\alpha a_\circ + 4Q^2}\right).$$

The charged cylindrical thin shell wormhole is stable under radial perturbations if and only if $\Psi''(a_\circ) > 0$. For $\Psi''(a_\circ) < 0$, the static solution is unstable. From Eq. (26), $\Psi''(a_\circ) = 0$, the squared of sound velocity $\chi_\circ^2$ is given by

$$\chi_\circ^2 = \frac{1}{4\beta(3m\alpha a_\circ - 4Q^2)}\left\{4Q^2 + \alpha^4 a_\circ^4 + 2m\alpha a_\circ + 4\eta(\beta+1)\left(3m\alpha a_\circ - 4Q^2\right) - \frac{\left(\alpha^4 a_\circ^4 + 2m\alpha a_\circ - 4Q^2\right)^2}{\left(\alpha^4 a_\circ^4 - 4m\alpha a_\circ + 4Q^2\right)}\right\}. \tag{31}$$

When $\delta \neq 0$ in Eq. (7), the stability of a cylindrical thin shell wormhole supported by MGCG in $f(R)$ gravity is recovered; while the stability of cylindrical thin shell wormhole supported by MGCG in general relativity is considered when $\delta = 0$. From Eq. (31), the plots of $\chi_\circ^2$ against a with different values of the parameters $(\delta, \eta, \zeta, \beta, m, Q, \alpha, R_\circ)$ are shown in Figs. 1, 2 and 3.

From Eqs. (18) and (15), $\Psi(a_\circ)$ is given by

$$\Psi(a_\circ) = A(a_\circ) - \left(\frac{a_\circ k}{4(1 + 2\delta\hat{R}_\circ)}\right)^2\left(-\frac{\delta\hat{R}_\circ^2}{2\alpha k} - \left[\frac{1}{\eta+1}\left(\varsigma + \left(\frac{c}{a_\circ^2}\right)^{\frac{1}{(\beta+1)(\eta+1)}}\right)\right]^{\frac{1}{\beta+1}}\right)^2. \tag{32}$$

The plots of $\Psi(a_\circ)$ against a with different values of the parameters $(\delta, \eta, \zeta, \beta, m, Q, \alpha, R_\circ, c)$ are shown in Fig. 4.

4. Conclusion

The dynamical equation of a cylindrical thin shell wormholes supported by MGCG within the framework of $f(R)$ gravity is derived, using the cut and paste scheme. This exotic matter is recently considered to be a particular interest in cosmology and explains the observed accelerated expansion of the universe. The stability analysis of cylindrical thin shell wormholes with the equation of state of MGCG, with linearized radial perturbation about a static equilibrium solution, is investigated. The output of a cylindrical thin shell wormholes can be a stable or unstable, depending on the mass m, the parameters η, Q, β, α and the initial position $a_\circ$ of the dynamical shell with constant $\hat{R}$.

The numerical analysis is used to explain Eq. (32) for the squared of sound velocity $\chi_\circ^2$ against a with different values of the parameters $(\delta, \eta, \zeta, \beta, m, Q, \alpha, R_\circ)$. Also, the numerical analysis is used to explain Eq. (31) for the effective potential $\Psi(a_\circ)$ against a with different values of the parameters $(\delta, \eta, \zeta, \beta, m, Q, \alpha, R_\circ, c)$. When $\delta \neq 0$, the stability of a cylindrical thin shell wormhole supported by MGCG in $f(R)$ gravity is recovered; while for $\delta = 0$ the results will reduced to the stability of cylindrical thin shell wormhole supported by MGCG in general relativity.

The presence of the charge seems to enlarge the stability regions for shells around black holes. The cylindrical TSW is stable under radial perturbations if and only if $\Psi''(a_\circ) > 0$, while for $\Psi''(a_\circ) < 0$, the static solution is unstable.

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