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Inference for a step-stress partially accelerated life test model with an adaptive Type-II progressively hybrid censored data from Weibull distribution

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ABSTRACT

In this paper, the maximum likelihood estimators of Weibull distribution parameters and the acceleration factor are discussed based on two different types of progressively hybrid censoring schemes under step-stress partially accelerated life test model. The performances of the estimators of the model parameters using the two progressively hybrid censoring schemes are evaluated and compared in terms of biases and mean squared errors through a Monte Carlo simulation study.

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1. Introduction

In reliability analysis, it is not easy to collect lifetimes on highly reliable products with very long lifetimes since very few or even no failures may occur within a limited testing time under normal operating conditions. To induce early failures an accelerated life test (ALT) or partially accelerated life test (PALT) is often used. If all test units are exposed to higher-than-usual stress levels, then the test is called ALT. But if only some of them are run under severe condition then the test is called PALT. The information obtained from the test performed in the accelerated or partially accelerated environment is used to predict actual product performance in the usual environment.

The stress can be applied in several ways. According to Nelson [1] the common methods are step-stress and constant-stress. Under step-stress PALT (SSPALT), a test item is first run at normal (use) condition and, if it does not fail for a specified time, then it is run at accelerated condition until the test terminates. But the constant-stress PALT runs each item at either use condition or accelerated condition only, i.e. each unit is run at a constant-stress level until it fails or censors.

As indicated by Lin et al. [2], there are many situations in life testing and reliability experiments in which units are lost or removed from test before failure. The experimenter may not obtain complete information on failure times for all test units. Data observed from such experiments are called censored data.

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Nomenclature

ALT	Accelerated life test.
PALT	Partially accelerated life test.
SSPALT	Step-stress partially accelerated life test.
WD	Weibull distribution.
MLEs	Maximum likelihood estimates/estimators.
pdf	Probability density function.
MSEs	Mean squared errors.
n	Sample size (total number of test units in a SSPALT).
n_u	Number of failed units at use condition.
n_a	Number of failed units at accelerated condition.
m	A pre-specified number of failures.
η	A pre-specified time.
y_i	Observed lifetime of unit i tested under SSPALT.
τ	Stress change time.
R_i	Number of removed units at the ith failure.
$y_{(r)}$	The time of the rth failure.
$\wedge$	Denotes maximum likelihood estimate.
α	WD shape parameter ($\alpha > 0$).
λ	WD scale parameter ($\lambda > 0$).
β	Acceleration factor ($\beta > 1$).

The most commonly used censoring schemes are Type-I and Type-II censoring schemes (see, for example, Balakrishnan and Ng [3, Section 2.3]). Briefly, they can be described as follows. Consider n items under observations in a particular experiment. In the conventional Type-I censoring scheme, the experiment continues up to a pre-specified time η. On the other hand, the conventional Type-II censoring scheme requires the experiment to continue until a pre-specified number of failures $m \leq n$ occurs.

Many articles have considered a hybrid censoring scheme, which is a mixture of Type-I and Type-II censoring schemes, and the related statistical inference (see, for example, Childs et al. [4], Gupta and Kundu [5] and Kundu [6]). Lack of flexibility to remove the units from the experiment at any time point other than the terminal point is the major drawback of these censoring schemes. To deal with this problem, more general censoring schemes called progressive Type-II censoring or progressively Type-II hybrid censoring schemes are used.

Progressive Type-II censoring scheme can be described as follows. Suppose that n units are placed on a life test and the experimenter decides earlier the quantity m, the number of units to be failed. Now at the time of the first failure, R_1 of the remaining $n-1$ surviving units are randomly removed from the experiment. Continuing on, at the time of the second failure, R_2 of the remaining $n-2-R_1$ units are randomly withdrawn from the experiment. The test continues until the mth failure. At this time, all the remaining $R_m = n - m - R_1 - R_2 - \cdots - R_m - 1$ surviving units are removed from the experiment. The R_i are fixed prior to the study. Generally, they are determined by the goal of the experiment in controlling the total time on test or having a higher chance to observe some extreme failures which in return have a gain in efficiency for statistical inference. It can be seen that, in progressive Type-II censoring, if $R_1 = R_2 = \cdots = R_m - 1 = R_m = 0$, then $n = m$ which corresponds to the complete sample. But if $n - m - R_1 - \cdots - R_m - 1 = 0$, then $R_m = n - m$ which corresponds to the conventional Type-II right censoring scheme.

Specifically, PALT has been studied under conventional Type-I and Type-II censoring schemes by several authors, for example, see Goel [7], DeGroot and Goel [8], Bhattacharyya and Soejoeti [9], Bai and Chung [10], Bai, Chung and Chun [11], Abdel-Ghaly et al. [12], Abdel-Ghaly et al. [13], Abdel-Ghaly et al. [14], Abdel-Ghani [15], Abdel-Ghani [16], Ismail [17], Aly and Ismail [18], Ismail and Sarhan [19], Ismail and Aly [20], Ismail [21] and Ismail [22]. Also, SSPALT has been studied under hybrid censoring, see Ismail [23]. In addition, Ismail [24] has considered SSPALT using the progressive Type-II censoring scheme.

Based on the progressively Type-II hybrid censoring scheme, there have been some interesting studies that had been made under either ALT or ordinary life testing, for example, see Kundu and Joarder [25], Childs et al. [26], Ng et al. [27], Lin et al. [2] and Mokhtari et al. [28]. But under PALT, there are no previous related studies, in this respect. So, this paper will concentrate on progressively Type-II hybrid censoring scheme under SSPALT. This censoring scheme under SSPALT will be described in the next section.

The rest of this paper is organized as follows. In Section 2 the model and the available data are described. The maximum likelihood estimators (MLEs) of the SSPALT model parameters are provided in Section 3 based on two different progressively hybrid censoring schemes. Section 4 contains the simulation results that demonstrate and evaluate the performance of the estimators based on the proposed censoring schemes. Section 5 concludes the paper and suggests some future ideas in this area.

2. Description of the model

Assume that the random variable Y representing the lifetime of a product has Weibull distribution (WD) with shape and scale parameters as α and λ respectively. So, the probability density function (pdf) of Y is

$$f_Y(y;\alpha,\lambda)=\frac{\alpha}{\lambda}\left(\frac{y}{\lambda}\right)^{\alpha-1}e^{-(y/\lambda)^{\alpha}},\quad y>0,\ \alpha>0,\ \lambda>0. \tag{1}$$

WD is one of the most common distributions which are used to analyze several lifetime data. Its hazard function can be increasing, decreasing and constant depending on the value of the shape parameter. Thus, this distribution has lots of flexibility compared to other distributions.

The survival function of Weibull distribution in (1) takes the form

$$S(y)=\exp\{-(y/\lambda)^{\alpha}\}, \tag{2}$$

and the corresponding failure rate function is given by

$$h(y)=\frac{\alpha}{\lambda}\left(\frac{y}{\lambda}\right)^{\alpha-1}. \tag{3}$$

The pdf of Y under SSPALT model can be given by

$$f(y)=\begin{cases}0, & y\le 0,\\ f_1(y)\equiv f_Y(y;\alpha,\lambda), & 0<y\le\tau\\ f_2(y), & y>\tau,\end{cases} \tag{4}$$

where

$$f_2(y)\equiv f_Y(y;\alpha,\lambda,\beta)=\beta\frac{\alpha}{\lambda}\left(\frac{\tau+\beta(y-\tau)}{\lambda}\right)^{\alpha-1}\exp\{-([\tau+\beta(y-\tau)]/\lambda)^{\alpha}\}, \tag{5}$$

which is obtained by the transformation-variable technique using the density in (1) and the model proposed by DeGroot and Goel [8] which is given by

$$Y=\begin{cases}T & \text{if } \mathrm{T}\le\tau\\ \tau+(\mathrm{T}-\tau)/\beta & \text{if } \mathrm{T}>\tau\end{cases} \tag{6}$$

where T is the lifetime of the unit under normal use condition, τ is the stress change time and β is the acceleration factor; $\beta>1$.

Kundu and Joarder [25] suggested a censoring scheme namely the progressively Type-II hybrid censoring (PHC) scheme, in which the life testing experiment with progressive censoring scheme $(R_1,\ldots,R_m)$ is ended at a random time min $(Y_{m:m:n},\eta)$, where $\eta\in(0,\infty)$ and $1\le m\le n$ are fixed in advance, and $Y_{1:m:n}\le Y_{2:m:n}\le\cdots\le Y_{m:m:n}$ are the ordered failure times resulting from the experiment. Specifically, if the mth progressively censored observed failure happens before time η (i.e., $Y_{m:m:n}<\eta$), the experiment terminates at time $Y_{m:m:n.}$ otherwise, the experiment will be terminated at time η with $Y_{J:m:n}<\eta<Y_{J+1:m:n}$, and all the remaining $\left(n-\sum_{i=1}^{J}R_{\mathrm{i}}-J\right)$ surviving items are censored at time η. Here, the random variable J is the number of failures up to time η.

As indicated by Lin et al. [2], the objective of this censoring scheme is to control the total time on test at a predetermined time. But, the actual sample size under this scheme is random and it can turn out to be a very small number. Thus the normal statistical inference techniques that may result in low efficiency become less valid. For this reason, Ng et al. [27] suggested an adaptive progressively hybrid censoring scheme in which the effective sample size (or the number of observed failures) m is fixed in advance and the experiment time is permitted to run over time η. If $Y_{m:m:n}<\eta$, then the experiment continues along with the prefixed progressive censoring scheme $(R_1,\ldots,R_m)$; else, we will not eliminate any surviving unit from the experiment immediately following the $(J+1)$th, ..., $(m-1)$th observed failure, and finally remove all the remaining units $R_m=n-m-\sum_{i=1}^{J}R_{\mathrm{i}}$ at the time $Y_{m:m:n}$. That is, in this case, $R_{J+1}=\cdots=R_{m-1}=0$. If $\eta\to\infty$, we will have a typical progressive Type-II censoring scheme. But if $\eta\to 0$, the scheme reduces to the case of the traditional Type-II censoring scheme.

According to Lin et al. [2], this new scheme which is called an adaptive progressively Type-II hybrid censoring (APHC) scheme guarantees us not only to acquire m observed failure times for efficiency of statistical inference but also control the total time on test to be not too far away from the ideal time η. The advantage of this censoring scheme is that the experimenter can change the value of η as a compromise between a shorter experimental time and a higher chance to observe many failures.

3. Statistical inference under progressively hybrid censored data

This section discusses the process of obtaining the maximum likelihood estimates (MLEs) of the parameters α, λ and β based on data obtained from PHC and APHC. Suppose n independent units are placed on a life test with corresponding lifetimes $Y_1, Y_2, \ldots, Y_n$ being independent and identically distributed as Weibull with pdf given in Eq. (1). We shall denote the m completely observed (ordered) lifetimes by

$$y_{1:m:n} < \cdots < y_{n_u:m:n} \leq \tau < y_{n_u+1:m:n} < \cdots < y_{J:m:n} \leq \eta < y_{J+1:m:n} < \cdots < y_{m:m:n},$$

where n_u is the number of failed units at use condition.

3.1. Estimation based on PHC

In this subsection, the MLEs of the unknown parameters are given based on the observed data from PHC assuming WD. We provide the likelihood function under SSPALT as follows:

$$L(\alpha, \lambda, \beta) \propto \prod_{i=1}^{J} f_1(y_{i:m:n})\ [s_1(\tau)]^{R_i} f_2(y_{i:m:n})\ [s_2(\eta)]^{R_J^*}, \tag{7}$$

for

$$y_{1:m:n} < \cdots < y_{n_u:m:n} \leq \tau < y_{n_u+1:m:n} < \cdots < y_{J:m:n} \leq \eta,$$

where $R_J^* = \sum_{k=j+1}^{m} (R_k + 1)$ with $J = n_u + n_a < m$, where n_a is the number of failed units at accelerated condition,

$$s_1(y) = \exp\{-(y/\lambda)^\alpha\} \quad \text{and} \quad s_2(y) = \exp\{-([\tau + \beta(y - \tau)]/\lambda)^\alpha\}.$$

To obtain the MLEs of the model parameters, it is usually easier to maximize the natural logarithm of the likelihood function than the likelihood function itself. The natural logarithm of the likelihood function when $J \geq 1$, is given by

$$\begin{aligned}\ln L(\alpha, \lambda, \beta) =\ & 2J \ln \alpha - 2J\alpha \ln \lambda + J \ln \beta + (\alpha - 1) \left[\sum_{i=1}^{J} \ln y_i + \sum_{i=1}^{J} \ln \psi_i\right] \\ & - \frac{1}{\lambda^\alpha} \left[\sum_{i=1}^{J} y_i^\alpha + \sum_{i=1}^{J} \psi_i^\alpha\right] - (\tau/\lambda)^\alpha \sum_{i=1}^{J} R_i - \frac{J}{\lambda^\alpha}\ \psi_\eta^\alpha\ R_J^*,\end{aligned} \tag{8}$$

where $\psi_i = \tau + \beta(y_i - \tau)$ and $\psi_\eta = \tau + \beta(\eta - \tau)$.

By equating the partial derivatives of $\ln L$ to zero with respect to α, λ and β, the resulting three equations are

$$\begin{aligned}\frac{\partial \ln L}{\partial \alpha} =\ & \frac{2J}{\alpha} - 2J \ln \lambda + \sum_{i=1}^{J} \ln y_i + \sum_{i=1}^{J} \ln \psi_i - \frac{1}{\lambda^\alpha} \left\{\sum_{i=1}^{J} y_i^\alpha \ln y_i + \sum_{i=1}^{J} \psi_i^\alpha \ln \psi_i - \left[\sum_{i=1}^{J} y_i^\alpha + \sum_{i=1}^{J} \psi_i^\alpha\right] \ln \lambda\right\} \\ & - (\tau/\lambda)^\alpha \sum_{i=1}^{J} R_i \ln(\tau/\lambda) - (J\ R_J^*\ \psi_\eta^\alpha/\lambda^\alpha)(\ln \psi_\eta - \ln \lambda) = 0,\end{aligned} \tag{9}$$

$$\frac{\partial \ln L}{\partial \lambda} = -\frac{2J\alpha}{\lambda} + \frac{\alpha}{\lambda^{\alpha+1}} \left\{\sum_{i=1}^{J} y_i^\alpha + \sum_{i=1}^{J} \psi_i^\alpha + \tau^\alpha \sum_{i=1}^{J} R_i + J\ R_J^*\ \psi_\eta^\alpha\right\} = 0, \tag{10}$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{J}{\beta} + (\alpha - 1) \sum_{i=1}^{J} (y_i - \tau) - \frac{\alpha}{\lambda^\alpha} \left\{\sum_{i=1}^{J} \psi_i^{\alpha-1} (y_i - \tau) + J\ R_J^*\ \psi_\eta^{\alpha-1} (\eta - \tau)\right\} = 0. \tag{11}$$

Now, we have a system of three non-linear likelihood equations in three unknowns α, λ and β. It cannot be solved analytically. An iterative method such as the Newton–Raphson is used to obtain the MLEs of the unknown parameters.

3.2. Estimation based on APHC

According to APHC scheme, the likelihood function under SSPALT is given by

$$L(\alpha, \lambda,\ \beta) \propto \prod_{i=1}^{m} f_1(y_{i:m:n})\ f_2(y_{i:m:n})\ \prod_{i=1}^{J} [s_1(\tau)]^{R_i} [s_2(y_{m:m:n})]^{\left(n-m-\sum_{i=1}^{J} R_i\right)}, \tag{12}$$

for $y_{1:m:n} < \cdots < y_{n_u:m:n} \leq \tau < y_{n_u+1:m:n} < \cdots < y_{J:m:n} \leq \eta < y_{J+1:m:n} < \cdots < y_{m:m:n}$.

The natural logarithm of the above likelihood function is then

$$\ln L(\alpha,\lambda,\beta) = 2m\ln\alpha - 2m\,\alpha\ln\lambda + m\ln\beta + (\alpha-1)\left[\sum_{i=1}^{m}\ln y_i + \sum_{i=1}^{m}\ln\psi_i\right] - \frac{1}{\lambda^\alpha}\left[\sum_{i=1}^{m} y_i^\alpha + \sum_{i=1}^{m}\psi_i^\alpha\right] - (\tau/\lambda)^\alpha \sum_{i=1}^{J} R_i - \frac{J}{\lambda^\alpha}\,\psi_m^\alpha\left(n-m-\sum_{i=1}^{J}R_i\right),$$

where $\psi_m = \tau + \beta(y_{m:m:n} - \tau)$.

By equating the partial derivatives of $\ln L$ to zero with respect to α, λ and β, the resulting three equations are

$$\frac{\partial \ln L}{\partial\alpha} = \frac{2m}{\alpha} - 2m\ln\lambda + \sum_{i=1}^{m}\ln y_i + \sum_{i=1}^{m}\ln\psi_i - \frac{1}{\lambda^\alpha}\left\{\sum_{i=1}^{m} y_i^\alpha\ln y_i + \sum_{i=1}^{m}\psi_i^\alpha\ln\psi_i - \left[\sum_{i=1}^{m} y_i^\alpha + \sum_{i=1}^{m}\psi_i^\alpha\right]\ln\lambda\right\} - (\tau/\lambda)^\alpha\sum_{i=1}^{J}R_i\ln(\tau/\lambda) - \left[J\left(n-m-\sum_{i=1}^{J}R_i\right)\psi_m^\alpha/\lambda^\alpha\right](\ln\psi_m - \ln\lambda) = 0, \tag{13}$$

$$\frac{\partial \ln L}{\partial\lambda} = -\frac{2m\alpha}{\lambda} + \frac{\alpha}{\lambda^{\alpha+1}}\left\{\sum_{i=1}^{m} y_i^\alpha + \sum_{i=1}^{m}\psi_i^\alpha + \tau^\alpha\sum_{i=1}^{J}R_i + J\left(n-m-\sum_{i=1}^{J}R_i\right)\psi_m^\alpha\right\} = 0, \tag{14}$$

$$\frac{\partial \ln L}{\partial\beta} = \frac{m}{\beta} + \alpha\sum_{i=1}^{m}\frac{(y_i-\tau)}{\psi_i} - \frac{\alpha}{\lambda^\alpha}\left\{\sum_{i=1}^{m}(y_i-\tau)\psi_i^{\alpha-1} + J\left(n-m-\sum_{i=1}^{J}R_i\right)(y_{m:m:n}-\tau)\psi_m^{\alpha-1}\right\} = 0. \tag{15}$$

The above system can be solved numerically using an iterative method such as Newton–Raphson to get the MLEs of the model parameters.

4. Simulation studies

Since the performance of the different methods or schemes of censoring cannot be compared theoretically, Monte Carlo simulations are used to compare different sampling censoring schemes for different parameter values. The term different sampling censoring schemes means for different sets of R_i and for different η values.

Specifically, in this section simulation studies are conducted to compare the performance of the MLEs in terms of their biases and mean squared errors (MSEs) for different choices of n, m, τ and η values and for different parameter values based on two different types of progressively hybrid censoring schemes which are PHC and APHC schemes.

Three progressive censoring schemes are considered:

Scheme 1: $R_1 = \cdots = R_{m-1} = 0$ and $R_m = n - m$;

Scheme 2: $R_1 = n - m$ and $R_2 = \cdots = R_m = 0$; and

Scheme 3: $R_1 = \cdots = R_{m-1} = 1$ and $R_m = n - 2m + 1$.

For each setting, the biases and MSEs based on 10,000 simulations are estimated and reported in Tables 1–7.

The simulation study is carried out according to the following algorithm:

(1) Specify the values of nm, τ and η.
(2) Specify the values of the parameters α, λ and β.
(3) Generate a random sample of size n from the random variable Y given by Eq. (6) and sort it. The Weibull random variable can be easily generated. For example, if U represents a uniform random variable from [0, 1], then $Y = -\lambda\,[\ln(1-U)]^{1/\alpha}$ has Weibull distribution with pdf given by Eq. (1) if $y \le \tau$. But if $y > \tau$ then $Y = \tau + \{-\lambda\,[\ln(1-U)]^{1/\alpha} - \tau\}/\beta$ has Weibull distribution with pdf given by equation (5).
(4) Use the model given by equation (4) to generate progressively hybrid censored data for given n, m, τ, η $(\eta > \tau), \alpha, \lambda$ and β. Two data sets can be considered:
    Set 1(HPC): $y_{1:m:n} < \cdots < y_{n_u:m:n} \le \tau < y_{n_u+1:m:n} < \cdots < y_{J:m:n} \le \eta$,
    and
    Set 2 (AHPC): $y_{1:m:n} < \cdots < y_{n_u:m:n} \le \tau < y_{n_u+1:m:n} < \cdots < y_{J:m:n} \le \eta < y_{J+1:m:n} < \cdots < y_{m:m:n}$.
(5) Use the progressive hybrid censored data to compute the MLEs of the model parameters. The Newton–Raphson method is applied for solving the nonlinear system to obtain the MLEs of the parameter s.
(6) Replicate the steps 3–5 10,000 times.
(7) Compute the average values of biases and MSEs associated with the MLEs of the parameters.
(8) Steps 1–7 are done with different values of n, m, τ, η $(\eta > \tau), \alpha, \lambda$ and β.

Conducting the above algorithm, the average values of biases and MSEs are obtained using 10,000 replications to avoid randomness. The results reported in Tables 1–7 are based on different values of n, m, τ, η $(\eta > \tau)$, α, λ and β to investigate the performance of the MLEs of the model parameters.

It is observed from Tables 1–7 that in all cases the MLEs of the model parameters based on APHC give smaller MSEs compared to those based on PHC. In all cases the MSEs of the MLEs of the three parameters based on APHC decrease as the effective sample size (m) increases. This is also true for PHC except for some cases for Scheme 2 because of the heavy censoring at the early stages of the experiment. In addition, the biases of the MLEs are all smaller under APHC. Generally, the performance of estimation under the APHC scheme is better because we have a larger number of failures to be noticed. Thus, when the time of experiment is not the major concern, the APHC scheme will be a desirable choice in order to improve the quality of the statistical inference about the model parameters.

Moreover, from Tables 1–7 the following observations based on APHC can be made.

(1) For fixed n, τ and η, the MSEs decrease as m increases.
(2) For fixed τ and η, the MSEs considerably decrease as n and m increase at the same time.
(3) For fixed n, m and η, the MSEs decrease as τ decreases. This is also true in the case of PHC.
(4) For fixed n, m and τ, the MSEs decrease as η increases. This is also true in the case of PHC. It is noted that, under PHC, as η gets longer the MSEs decrease unless τ is large.

The same pattern is observed for the biases as shown by the Tables 1–7.

Table 1
Average values of the biases and MSEs of the MLEs based on both PHC and APHC when α, λ, β, τ and η set at 0.4, 0.7, 1.2, 2 and 5, respectively.

(n, m)	Scheme	Bias of $\hat{\alpha}$		MSE of $\hat{\alpha}$		Bias of $\hat{\lambda}$		MSE of $\hat{\lambda}$		Bias of $\hat{\beta}$		MSE of $\hat{\beta}$	
		PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC
(25, 10)	1	0.466	0.316	0.556	0.395	0.564	0.447	0.597	0.488	0.637	0.499	0.657	0.522
	2	0.583	0.352	0.694	0.513	0.705	0.586	0.746	0.633	0.796	0.647	0.821	0.677
	3	0.562	0.321	0.667	0.435	0.678	0.492	0.717	0.537	0.765	0.549	0.789	0.575
(35, 10)	1	0.402	0.227	0.476	0.280	0.484	0.317	0.512	0.346	0.443	0.353	0.563	0.370
	2	0.433	0.303	0.515	0.374	0.524	0.423	0.554	0.462	0.591	0.412	0.609	0.494
	3	0.415	0.257	0.479	0.317	0.487	0.358	0.515	0.391	0.546	0.399	0.567	0.418
(50, 10)	1	0.325	0.143	0.387	0.177	0.393	0.202	0.416	0.218	0.343	0.282	0.439	0.215
	2	0.368	0.194	0.438	0.239	0.445	0.273	0.471	0.295	0.550	0.302	0.518	0.316
	3	0.333	0.181	0.396	0.224	0.403	0.253	0.426	0.276	0.502	0.293	0.458	0.292
(25, 15)	1	0.181	0.121	0.212	0.149	0.219	0.169	0.232	0.184	0.161	0.148	0.255	0.197
	2	0.257	0.147	0.606	0.181	0.311	0.205	0.629	0.224	0.455	0.229	0.692	0.240
	3	0.251	0.146	0.299	0.180	0.304	0.203	0.322	0.222	0.351	0.223	0.354	0.238
(35, 15)	1	0.118	0.082	0.144	0.104	0.143	0.112	0.151	0.121	0.174	0.128	0.166	0.128
	2	0.145	0.117	0.543	0.145	0.176	0.164	0.586	0.179	0.247	0.183	0.635	0.192
	3	0.128	0.084	0.152	0.110	0.154	0.115	0.163	0.125	0.199	0.132	0.179	0.134
(50, 15)	1	0.054	0.034	0.102	0.043	0.071	0.049	0.074	0.053	0.079	0.054	0.081	0.057
	2	0.097	0.075	0.116	0.092	0.118	0.104	0.125	0.114	0.134	0.117	0.138	0.122
	3	0.094	0.063	0.107	0.080	0.111	0.091	0.115	0.099	0.123	0.101	0.127	0.107

Table 2
Average values of the biases and MSEs of the MLEs based on both PHC and APHC when α, λ, β, τ and η set at 0.4, 0.7, 1.2, 2 and 10, respectively.

(n, m)	Scheme	Bias of $\hat{\alpha}$		MSE of $\hat{\alpha}$		Bias of $\hat{\lambda}$		MSE of $\hat{\lambda}$		Bias of $\hat{\beta}$		MSE of $\hat{\beta}$	
		PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC
(25, 10)	1	0.165	0.088	0.251	0.138	0.209	0.134	0.304	0.204	0.326	0.182	0.422	0.246
	2	0.206	0.114	0.314	0.182	0.254	0.174	0.371	0.259	0.408	0.237	0.528	0.319
	3	0.198	0.096	0.302	0.152	0.245	0.147	0.357	0.223	0.391	0.201	0.507	0.271
(35, 10)	1	0.141	0.062	0.216	0.098	0.175	0.097	0.254	0.142	0.280	0.129	0.362	0.174
	2	0.153	0.083	0.233	0.131	0.188	0.127	0.275	0.189	0.302	0.173	0.391	0.233
	3	0.142	0.071	0.217	0.111	0.177	0.107	0.255	0.168	0.281	0.146	0.364	0.197
(50, 10)	1	0.114	0.039	0.175	0.062	0.136	0.064	0.206	0.089	0.227	0.081	0.294	0.110
	2	0.133	0.056	0.198	0.084	0.144	0.081	0.234	0.121	0.257	0.111	0.332	0.156
	3	0.117	0.054	0.179	0.079	0.142	0.075	0.211	0.113	0.233	0.103	0.301	0.139

Table 2 (*continued*)

(n, m)	Scheme	Bias of $\hat{\alpha}$		MSE of $\hat{\alpha}$		Bias of $\hat{\lambda}$		MSE of $\hat{\lambda}$		Bias of $\hat{\beta}$		MSE of $\hat{\beta}$	
		PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC
(25, 15)	1	0.064	0.033	0.098	0.052	0.079	0.043	0.115	0.075	0.126	0.054	0.164	0.065
	2	0.094	0.047	0.138	0.067	0.114	0.062	0.163	0.092	0.182	0.084	0.233	0.113
	3	0.091	0.041	0.135	0.063	0.112	0.051	0.160	0.091	0.176	0.069	0.227	0.093
(35, 15)	1	0.042	0.019	0.063	0.032	0.051	0.026	0.075	0.051	0.082	0.039	0.107	0.063
	2	0.051	0.032	0.079	0.051	0.063	0.049	0.092	0.073	0.102	0.066	0.132	0.112
	3	0.045	0.023	0.069	0.036	0.059	0.034	0.081	0.053	0.089	0.046	0.115	0.094
(50, 15)	1	0.023	0.011	0.031	0.015	0.026	0.017	0.038	0.024	0.044	0.033	0.056	0.024
	2	0.037	0.021	0.053	0.032	0.048	0.031	0.062	0.049	0.069	0.043	0.088	0.059
	3	0.034	0.018	0.049	0.028	0.042	0.029	0.057	0.047	0.063	0.039	0.082	0.052

Table 3
Average values of the biases and MSEs of the MLEs based on both PHC and APHC when α, λ, β, τ and η set at 0.4, 0.7, 1.2, 7 and 10, respectively.

(n, m)	Scheme	Bias of $\hat{\alpha}$		MSE of $\hat{\alpha}$		Bias of $\hat{\lambda}$		MSE of $\hat{\lambda}$		Bias of $\hat{\beta}$		MSE of $\hat{\beta}$	
		PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC
(25, 10)	1	0.210	0.161	0.382	0.184	0.343	0.169	0.476	0.237	0.376	0.253	0.514	0.306
	2	0.259	0.147	0.478	0.167	0.429	0.154	0.596	0.216	0.471	0.232	0.644	0.278
	3	0.249	0.191	0.458	0.22	0.412	0.203	0.571	0.275	0.452	0.299	0.618	0.361
(35, 10)	1	0.178	0.139	0.327	0.159	0.294	0.146	0.408	0.201	0.323	0.218	0.441	0.263
	2	0.193	0.119	0.356	0.134	0.32	0.123	0.444	0.178	0.351	0.184	0.48	0.223
	3	0.179	0.104	0.329	0.119	0.296	0.112	0.413	0.151	0.325	0.163	0.444	0.197
(50, 10)	1	0.144	0.092	0.266	0.096	0.239	0.086	0.332	0.121	0.263	0.133	0.359	0.157
	2	0.163	0.089	0.301	0.102	0.27	0.093	0.375	0.128	0.297	0.142	0.405	0.176
	3	0.148	0.065	0.273	0.075	0.248	0.074	0.341	0.094	0.272	0.102	0.369	0.124
(25, 15)	1	0.213	0.069	0.388	0.076	0.348	0.071	0.484	0.097	0.383	0.106	0.523	0.127
	2	0.299	0.079	0.552	0.077	0.496	0.071	0.688	0.098	0.545	0.106	0.744	0.128
	3	0.252	0.055	0.464	0.063	0.416	0.059	0.579	0.082	0.458	0.087	0.625	0.105
(35, 15)	1	0.182	0.038	0.335	0.044	0.304	0.041	0.418	0.056	0.331	0.062	0.451	0.073
	2	0.254	0.054	0.464	0.062	0.413	0.056	0.573	0.077	0.453	0.083	0.623	0.106
	3	0.185	0.038	0.341	0.044	0.306	0.039	0.425	0.054	0.336	0.058	0.459	0.071
(50, 15)	1	0.147	0.035	0.273	0.039	0.242	0.036	0.336	0.053	0.266	0.054	0.364	0.067
	2	0.175	0.031	0.323	0.034	0.291	0.033	0.403	0.052	0.318	0.049	0.436	0.059
	3	0.155	0.015	0.286	0.018	0.257	0.018	0.357	0.025	0.283	0.024	0.386	0.029

Table 4
Average values of the biases and MSEs of the MLEs based on both PHC and APHC when α, λ, β, τ and η set at 1.4, 0.7, 1.2, 2 and 10, respectively.

(n, m)	Scheme	Bias of $\hat{\alpha}$		MSE of $\hat{\alpha}$		Bias of $\hat{\lambda}$		MSE of $\hat{\lambda}$		Bias of $\hat{\beta}$		MSE of $\hat{\beta}$	
		PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC
(25, 10)	1	0.262	0.143	0.377	0.213	0.301	0.197	0.411	0.278	0.423	0.244	0.486	0.228
	2	0.326	0.186	0.472	0.277	0.366	0.256	0.501	0.361	0.529	0.317	0.608	0.398
	3	0.313	0.157	0.454	0.235	0.353	0.217	0.483	0.306	0.508	0.269	0.584	0.386
(35, 10)	1	0.224	0.101	0.324	0.151	0.251	0.143	0.343	0.197	0.363	0.173	0.417	0.211
	2	0.242	0.135	0.356	0.202	0.271	0.187	0.371	0.263	0.392	0.231	0.451	0.282
	3	0.225	0.115	0.326	0.171	0.262	0.158	0.345	0.223	0.365	0.195	0.420	0.238
(50, 10)	1	0.181	0.064	0.263	0.096	0.204	0.088	0.279	0.124	0.295	0.109	0.339	0.133
	2	0.206	0.086	0.298	0.129	0.231	0.119	0.316	0.168	0.333	0.148	0.383	0.182
	3	0.186	0.081	0.269	0.121	0.212	0.111	0.285	0.157	0.302	0.138	0.347	0.168
(25, 15)	1	0.101	0.054	0.147	0.087	0.113	0.074	0.155	0.105	0.164	0.092	0.189	0.112
	2	0.144	0.066	0.408	0.098	0.161	0.093	0.480	0.128	0.233	0.112	0.518	0.137
	3	0.143	0.062	0.203	0.094	0.158	0.091	0.216	0.123	0.228	0.104	0.262	0.134
(35, 15)	1	0.066	0.032	0.095	0.056	0.074	0.050	0.101	0.071	0.107	0.065	0.123	0.076
	2	0.081	0.052	0.368	0.078	0.091	0.072	0.385	0.102	0.132	0.089	0.474	0.109
	3	0.071	0.037	0.103	0.059	0.085	0.053	0.109	0.074	0.115	0.068	0.132	0.079
(50, 15)	1	0.032	0.015	0.047	0.038	0.038	0.024	0.052	0.034	0.057	0.026	0.065	0.032
	2	0.057	0.034	0.291	0.053	0.061	0.046	0.310	0.065	0.089	0.058	0.394	0.071
	3	0.053	0.029	0.079	0.042	0.058	0.042	0.084	0.054	0.082	0.051	0.102	0.063

Table 5
Average values of the biases and MSEs of the MLEs based on both PHC and APHC when $\alpha, \lambda, \beta, \tau$ and η set at 1.4, 0.7, 1.2, 2 and 5, respectively.

(n, m)	Scheme	Bias of $\hat{\alpha}$		MSE of $\hat{\alpha}$		Bias of $\hat{\lambda}$		MSE of $\hat{\lambda}$		Bias of $\hat{\beta}$		MSE of $\hat{\beta}$	
		PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC
(25, 10)	1	0.344	0.214	0.405	0.293	0.486	0.376	0.528	0.422	0.602	0.461	0.631	0.507
	2	0.426	0.279	0.507	0.381	0.608	0.493	0.661	0.545	0.753	0.598	0.789	0.657
	3	0.412	0.236	0.487	0.323	0.585	0.414	0.634	0.463	0.721	0.507	0.758	0.558
(35, 10)	1	0.293	0.152	0.345	0.208	0.417	0.267	0.453	0.298	0.514	0.326	0.541	0.359
	2	0.316	0.203	0.351	0.278	0.452	0.356	0.491	0.398	0.557	0.436	0.585	0.481
	3	0.303	0.174	0.349	0.236	0.423	0.301	0.456	0.337	0.518	0.369	0.545	0.406
(50, 10)	1	0.237	0.096	0.283	0.132	0.339	0.170	0.368	0.188	0.418	0.206	0.442	0.226
	2	0.269	0.132	0.320	0.178	0.384	0.234	0.417	0.254	0.473	0.279	0.498	0.307
	3	0.243	0.121	0.289	0.166	0.347	0.213	0.377	0.238	0.429	0.261	0.451	0.286
(25, 15)	1	0.132	0.062	0.158	0.102	0.189	0.151	0.205	0.158	0.323	0.174	0.245	0.231
	2	0.188	0.098	0.423	0.134	0.268	0.172	0.291	0.193	0.333	0.212	0.348	0.291
	3	0.183	0.081	0.218	0.111	0.262	0.162	0.285	0.191	0.231	0.202	0.343	0.233
(35, 15)	1	0.086	0.036	0.102	0.047	0.123	0.069	0.134	0.111	0.152	0.118	0.162	0.130
	2	0.106	0.078	0.317	0.108	0.152	0.138	0.165	0.154	0.187	0.169	0.197	0.186
	3	0.093	0.055	0.111	0.075	0.133	0.097	0.144	0.118	0.164	0.122	0.172	0.134
(50, 15)	1	0.042	0.023	0.045	0.032	0.061	0.037	0.065	0.046	0.074	0.043	0.078	0.055
	2	0.092	0.054	0.078	0.068	0.102	0.077	0.112	0.098	0.126	0.111	0.133	0.118
	3	0.071	0.046	0.050	0.039	0.094	0.071	0.102	0.085	0.115	0.108	0.121	0.103

Table 6
Average values of the biases and MSEs of the MLEs based on both PHC and APHC when $\alpha, \lambda, \beta, \tau$ and η set at 1.4, 0.7, 1.2, 3.5 and 5, respectively.

(n, m)	Scheme	Bias of $\hat{\alpha}$		MSE of $\hat{\alpha}$		Bias of $\hat{\lambda}$		MSE of $\hat{\lambda}$		Bias of $\hat{\beta}$		MSE of $\hat{\beta}$	
		PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC
(25, 10)	1	0.381	0.219	0.427	0.241	0.584	0.468	0.612	0.498	0.667	0.518	0.691	0.543
	2	0.492	0.285	0.534	0.313	0.731	0.607	0.765	0.646	0.832	0.672	0.862	0.704
	3	0.458	0.241	0.514	0.265	0.702	0.515	0.735	0.548	0.799	0.574	0.828	0.598
(35, 10)	1	0.326	0.156	0.367	0.171	0.501	0.332	0.525	0.353	0.572	0.367	0.546	0.385
	2	0.483	0.207	0.397	0.228	0.542	0.443	0.568	0.472	0.617	0.492	0.629	0.514
	3	0.329	0.176	0.369	0.193	0.504	0.375	0.528	0.399	0.574	0.415	0.595	0.435
(50, 10)	1	0.266	0.098	0.298	0.108	0.407	0.209	0.426	0.222	0.464	0.231	0.481	0.242
	2	0.427	0.133	0.337	0.146	0.461	0.283	0.483	0.301	0.525	0.314	0.544	0.329
	3	0.301	0.125	0.305	0.137	0.417	0.265	0.437	0.282	0.475	0.293	0.492	0.307
(25, 15)	1	0.149	0.083	0.166	0.091	0.227	0.177	0.393	0.188	0.351	0.196	0.332	0.205
	2	0.512	0.112	0.576	0.112	0.822	0.214	0.787	0.228	0.869	0.246	0.887	0.258
	3	0.205	0.102	0.321	0.110	0.315	0.212	0.438	0.226	0.359	0.237	0.368	0.246
(35, 15)	1	0.096	0.047	0.108	0.053	0.148	0.121	0.355	0.128	0.268	0.137	0.174	0.139
	2	0.503	0.082	0.413	0.088	0.592	0.172	0.591	0.183	0.638	0.193	0.675	0.202
	3	0.104	0.056	0.117	0.062	0.159	0.134	0.367	0.132	0.281	0.143	0.188	0.141
(50, 15)	1	0.047	0.021	0.049	0.026	0.073	0.051	0.076	0.054	0.082	0.056	0.085	0.059
	2	0.431	0.045	0.382	0.056	0.472	0.109	0.497	0.116	0.548	0.129	0.553	0.127
	3	0.074	0.034	0.053	0.039	0.113	0.095	0.128	0.101	0.142	0.121	0.145	0.114

Table 7
Average values of the biases and MSEs of the MLEs based on both PHC and APHC when $\alpha, \lambda, \beta, \tau$ and η set at 1.4, 0.7, 1.2, 3.5 and 10, respectively.

(n, m)	Scheme	Bias of $\hat{\alpha}$		MSE of $\hat{\alpha}$		Bias of $\hat{\lambda}$		MSE of $\hat{\lambda}$		Bias of $\hat{\beta}$		MSE of $\hat{\beta}$	
		PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC
(25, 10)	1	0.345	0.232	0.455	0.286	0.695	0.319	0.740	0.405	0.625	0.349	0.733	0.403
	2	0.431	0.211	0.570	0.260	0.870	0.291	0.926	0.368	0.782	0.317	0.917	0.365
	3	0.414	0.274	0.547	0.344	0.835	0.377	0.890	0.477	0.751	0.412	0.899	0.475
(35, 10)	1	0.297	0.200	0.391	0.246	0.597	0.276	0.634	0.347	0.537	0.301	0.628	0.349
	2	0.322	0.171	0.424	0.209	0.649	0.239	0.690	0.309	0.583	0.257	0.683	0.293
	3	0.297	0.151	0.393	0.184	0.601	0.211	0.642	0.261	0.540	0.225	0.634	0.261
(50, 10)	1	0.241	0.137	0.318	0.149	0.484	0.162	0.516	0.208	0.436	0.179	0.512	0.207
	2	0.275	0.128	0.359	0.159	0.547	0.178	0.582	0.223	0.494	0.193	0.577	0.232
	3	0.254	0.095	0.326	0.117	0.502	0.142	0.531	0.163	0.451	0.141	0.525	0.164

Table 7 (*continued*)

(n, m)	Scheme	Bias of $\hat{\alpha}$		MSE of $\hat{\alpha}$		Bias of $\hat{\lambda}$		MSE of $\hat{\lambda}$		Bias of $\hat{\beta}$		MSE of $\hat{\beta}$	
		PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC	PHC	APHC
(25, 15)	1	0.351	0.100	0.463	0.118	0.706	0.134	0.754	0.167	0.635	0.146	0.747	0.166
	2	0.498	0.113	0.658	0.123	1.006	0.134	1.072	0.169	0.905	0.146	1.059	0.169
	3	0.422	0.082	0.554	0.102	0.845	0.114	0.900	0.137	0.761	0.125	0.890	0.139
(35, 15)	1	0.302	0.061	0.399	0.071	0.618	0.085	0.649	0.097	0.549	0.085	0.642	0.097
	2	0.417	0.078	0.549	0.095	0.837	0.107	0.891	0.137	0.753	0.115	0.882	0.143
	3	0.308	0.061	0.406	0.076	0.621	0.074	0.661	0.094	0.558	0.083	0.654	0.095
(50, 15)	1	0.244	0.054	0.323	0.060	0.491	0.067	0.523	0.087	0.442	0.075	0.518	0.088
	2	0.291	0.043	0.386	0.053	0.590	0.064	0.626	0.094	0.529	0.068	0.622	0.079
	3	0.258	0.022	0.342	0.029	0.521	0.034	0.555	0.051	0.470	0.037	0.553	0.045

5. Concluding remarks and further studies

In this paper, we have considered the likelihood estimation of Weibull distribution parameters and the acceleration factor under SSPALT when the data are coming from two different types of progressively hybrid censoring schemes which are PHC and APHC schemes. The MLEs of the model parameters are obtained numerically using the Newton–Raphson method and their performances are evaluated and discussed in terms of biases and MSEs. A higher efficiency in estimation of parameters is obtained when APHC is supposed because a large number of failures can be observed. In general, if the experimental time is not a major concern, then the APHC is recommended in order to obtain better estimates of model parameters. In contrast, if one needs to have a shorter experimental time or allow only a few experimental units to be damaged during the experiment, then the PHC will be a reasonable alternative to achieve the goal. As a future work, Bayesian inference under SSPALT assuming the same progressively hybrid censoring schemes proposed in this article will be considered.

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