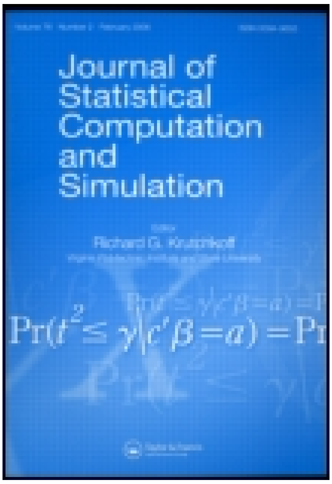

Journal of Statistical Computation and Simulation
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gscs20

Likelihood inference for a step-stress partially accelerated life test model with Type-I progressively hybrid censored data from Weibull distribution

Ali A. Ismail[ab]
[a] King Saud University, College of Science, Department of Statistics and Operations Research, P. O. BOX 2455, Riyadh 11451, Saudi Arabia
[b] Cairo University, Faculty of Economics & Political Science, Department of Statistics, Giza, Egypt
Published online: 10 Oct 2013.

Journal of Statistical Computation and Simulation, 2014
Vol. 84, No. 11, 2486–2494, http://dx.doi.org/10.1080/00949655.2013.836195

Taylor & Francis
Taylor & Francis Group

Likelihood inference for a step-stress partially accelerated life test model with Type-I progressively hybrid censored data from Weibull distribution

Ali A. Ismail[a,b*]

[a]*King Saud University, College of Science, Department of Statistics and Operations Research, P. O. BOX 2455, Riyadh 11451, Saudi Arabia;* [b]*Cairo University, Faculty of Economics & Political Science, Department of Statistics, Giza, Egypt*

(*Received 25 November 2012; accepted 15 August 2013*)

Recently, progressively hybrid censoring schemes have become quite popular in life testing and reliability studies. In this article, the point and interval maximum-likelihood estimations of Weibull distribution parameters and the acceleration factor are considered. The estimation process is performed under Type-I progressively hybrid censored data for a step-stress partially accelerated test model. The biases and mean square errors of the maximum-likelihood estimators are computed to assess their performances in the presence of censoring developed in this article through a Monte Carlo simulation study.

Keywords: reliability; step-stress partially accelerated life test; Weibull distribution; maximum likelihood; confidence interval; Type-I progressively hybrid censored data; Monte Carlo simulation

2000 Mathematics Subject Classification: 62N01; 62N05

1. Introduction

Due to rapid advance in technology and increasing global competition, pressure on manufactures to produce high quality products has increased. Thus, the mean times to failure of those products are too large under typical operating conditions. So, in order to shorten life or accelerate performance degradation, all of test units or some of them are subject to stresses which are more severe than usual like temperature, humidity, pressure, voltage, vibration etc. If the experimenter puts all test units under such stresses, the test is called accelerated life test (ALT), but if he puts some of them then the test is called partially accelerated life test (PALT). The information obtained from the test performed in accelerated environment is used to predict the actual product performance in the usual environment.

As Nelson [1] indicates, the stress can be applied in various ways, the commonly used methods are step-stress and constant-stress. Under step-stress PALT (SSPALT), a test item is first run at use (normal) condition and, if it does not fail for a specified time, then it is run at accelerated condition (stress) until failure occurs or the observation is censored. But the constant-stress PALT runs each item at either the use condition or the accelerated condition only, i.e. each unit is run

*Email: aismail100@yahoo.com
This version has been corrected. Please see Erratum (http://dx.doi.org/10.1080/00949655.2013.856585).

at a constant-stress level until the test is terminated. The objective of a PALT is to collect more failure data in a limited time without necessarily using high stresses to all test units.

PALT has been studied under conventional Type-I and Type-II censoring schemes by several authors, for example, see Goel,[2] DeGroot and Goel,[3] Bhattcharyya and Soejoeti,[4] Bai and Chung,[5] Bai et al.,[6] Abdel-Ghaly et al.,[7–9] Abdel-Ghani,[10,11] Ismail,[12] Aly and Ismail,[13] Ismail and Sarhan,[14] Ismail and Aly,[15] Ismail.[16] Also, SSPALT has been studied under hybrid censoring, see Ismail.[17] In addition, Ismail [18] has considered SSPALT, using the progressive Type-II censoring scheme.

Based on the progressively Type-I hybrid censoring scheme, there are some studies under ALT, for example, see [19,20]. But under PALT there are no studies have taken place before in this respect. So, this article will concentrate on SSPALT under the progressively Type-I hybrid censoring scheme. This scheme under SSPALT will be described in the next section.

The rest of this article is arranged as follows. In Section 2, the model and the available data are described. The maximum-likelihood (ML) estimators of the SSPALT model parameters are provided in Section 3. Also, in Section 3, the asymptotic confidence bounds for the model parameters are constructed based on the asymptotic distribution of ML estimators. Section 4 contains the simulation results. Concluding remarks and further studies are given in Section 5.

2. Description of the model

Assume that the random variable Y representing the lifetime of a product has Weibull distribution (WD) with the shape and scale parameters as α and λ, respectively. So, the probability density function (pdf) of Y is

$$f_Y(y;\alpha,\lambda) = \frac{\alpha}{\lambda}\left(\frac{y}{\lambda}\right)^{\alpha-1} \mathrm{e}^{-(y/\lambda)^{\alpha}}; \quad y>0,\ \alpha>0,\ \lambda>0. \tag{1}$$

WD is one of the most common distributions which are used to analyse several lifetime data. Its hazard function can be increasing, decreasing and constantly depending on the shape parameter value. Thus, this distribution has lots of flexibility compared to other distributions.

The survival function of WD in Equation (1) takes the form as in the following:

$$S(y) = \exp\left\{-\left(\frac{y}{\lambda}\right)^{\alpha}\right\}, \tag{2}$$

and the corresponding failure rate function is given by

$$h(y) = \frac{\alpha}{\lambda}\left(\frac{y}{\lambda}\right)^{\alpha-1}. \tag{3}$$

The pdf of Y under SSPALT model can be given by

$$f(y) = \begin{cases} 0, & y \le 0, \\ f_1(y) \equiv f_Y(y;\alpha,\lambda), & 0 < y \le \tau, \\ f_2(y), & y > \tau, \end{cases} \tag{4}$$

where

$$f_2(y) \equiv f_Y(y;\alpha,\lambda,\beta) = \beta\frac{\alpha}{\lambda}\left(\frac{\tau+\beta(y-\tau)}{\lambda}\right)^{\alpha-1} \exp\left\{-\left(\frac{[\tau+\beta(y-\tau)]}{\lambda}\right)^{\alpha}\right\} \tag{5}$$

which is obtained by the transformation-variable technique using the density in Equation (1) and the model proposed by DeGroot and Goel [3] which is given by

$$Y = \begin{cases} T & \text{if } T \leq \tau, \\ \tau + \beta^{-1}(T - \tau) & \text{if } T > \tau, \end{cases} \tag{6}$$

where T is the lifetime of the unit under use condition, τ is the stress change time and β is the acceleration factor; $\beta > 1$.

Now, the data available under progressively Type-I hybrid censoring scheme can be described as follows. Under this censoring scheme, suppose that n identical and independent units are placed on a life test. Each of the n units is first run under the use condition. This use condition level is changed to an accelerated condition at time τ at which m unfailed units of the remaining units are randomly removed and the test is continued. It is noted that τ and m are prefixed. If the rth failure $(r < n)$ occurs at a time $y_{r:n}$ before a prefixed $\eta > \tau$, the experiment terminates at the time point $y_{r:n}$. But if $y_{r:n} > \eta$, then all the remaining units are removed and the experiment terminates at the time η. This censoring scheme is called the progressively Type-I hybrid censoring scheme. It is noted that compared to the conventional Type-I censoring scheme, the termination time of the progressively Type-I hybrid censoring scheme is at most η. Let n_{u} be the number of units that fail before time τ, n_{a} be the number of units that fail before time η at accelerated condition and n_{f} be the number of units that fail before the experiment terminates. Thus, we have

$$n_{\mathrm{f}} = \begin{cases} n_{\mathrm{u}} + n_{\mathrm{a}} = r, & \text{if } r < y_{r:n} \leq \eta, \\ n_{\mathrm{u}} + n_{\mathrm{a}} < r, & \text{if } y_{r:n} \leq \eta. \end{cases} \tag{7}$$

Under the progressively Type-I hybrid censoring scheme, we can observe the following types of observations:

Set 1: $y_{1:n} < \cdots < y_{n_{\mathrm{u}}:n} \leq \tau < y_{n_{\mathrm{u}}+1:n} < \cdots < y_{r:n} \leq \eta, \quad \text{if } \tau < y_{r:n} \leq \eta$

Set 2: $y_{1:n} < \cdots < y_{n_{\mathrm{u}}:n} \leq \tau < y_{n_{\mathrm{u}}+1:n} < \cdots < y_{n_{\mathrm{u}}+n_{\mathrm{a}}:n} \leq \eta, \quad \text{if } y_{r:n} > \eta$

That is, two main sets of data can be considered. We shall not consider the case that no failures can be observed either at the use condition or at the accelerated condition.

3. Maximum-likelihood estimation

This section discusses the process of obtaining the ML estimates of the parameters α, λ and β based on progressively Type-I hybrid censored data under the SSPALT model. Both point and interval estimations of the parameters are considered.

3.1. *Point estimation*

Here, the ML estimates of the unknown parameters are given. Based on the observed progressively Type-I hybrid censored data from WD, we provide the likelihood function under SSPALT for the two sets of data indicated above as follows.

The likelihood function of the data set 1 is given by

$$L(\alpha, \lambda, \beta) \propto \prod_{i=1}^{n_{\mathrm{u}}} f_1(y_i) \cdot [S_1(\tau)]^m \cdot \prod_{i=n_{\mathrm{u}}+1}^{r} f_2(y_i) \cdot [S_2(y_{r:n})]^{n-r-m}, \tag{8}$$

where

$$s_1(y) = \exp\left\{-\left(\frac{y}{\lambda}\right)^{\alpha}\right\}$$

and

$$s_2(y) = \exp\left\{-\left[\frac{(\tau + \beta(y-\tau))}{\lambda}\right]^{\alpha}\right\},$$

For data set 2, the likelihood function is given by

$$L(\alpha, \lambda,\ \beta) \propto \prod_{i=1}^{n_{\mathrm{u}}} f_1(y_i) \cdot [S_1(\tau)]^m. \prod_{i=n_{\mathrm{u}}+1}^{n_{\mathrm{u}}+n_{\mathrm{a}}} f_2(y_i) \cdot [S_2(\eta)]^{n-(n_{\mathrm{u}}+n_{\mathrm{a}})-m}. \tag{9}$$

To obtain the ML estimates of the model parameters, it is usually easier to maximize the natural logarithm of the likelihood function than the likelihood function itself. The natural logarithm of the likelihood function for both data set 1 and data set 2 is, respectively, as follows:

$$\begin{aligned}\ln L(\alpha,\lambda,\beta) &= r\ln\alpha - r\alpha\ln\lambda + n_{\mathrm{a}}\ln\beta + (\alpha-1)\left\{\sum_{i=1}^{n_{\mathrm{u}}}\ln y_i + \sum_{i=n_{\mathrm{u}}+1}^{r}\ln[\tau+\beta(y_i-\tau)]\right\}\\ &\quad - \frac{\alpha}{\lambda}\left\{\sum_{i=1}^{n_{\mathrm{u}}} y_i + \sum_{i=n_{\mathrm{u}}+1}^{r}[\tau+\beta(y_i-\tau)] + mn_{\mathrm{u}}\tau\right.\\ &\quad \left. +\ (n-r-m)n_{\mathrm{a}}[\tau+\beta(y_{r:n}-\tau)]\right\},\end{aligned} \tag{10}$$

and

$$\begin{aligned}\ln L(\alpha,\lambda,\beta) &= (n_{\mathrm{u}}+n_{\mathrm{a}})\ln\alpha - (n_{\mathrm{u}}+n_{\mathrm{a}})\alpha\ln\lambda + n_{\mathrm{a}}\ln\beta + (\alpha-1)\\ &\quad \times\left\{\sum_{i=1}^{n_{\mathrm{u}}}\ln y_i + \sum_{i=n_{\mathrm{u}}+1}^{n_{\mathrm{u}}+n_{\mathrm{a}}}\ln[\tau+\beta(y_i-\tau)]\right\} - \frac{\alpha}{\lambda}\left\{\sum_{i=1}^{n_{\mathrm{u}}} y_i + \sum_{i=n_{\mathrm{u}}+1}^{n_{\mathrm{u}}+n_{\mathrm{a}}}[\tau+\beta(y_i-\tau)]\right.\\ &\quad \left. + mn_{\mathrm{u}}\tau + [n-(n_{\mathrm{u}}+n_{\mathrm{a}})-m]n_{\mathrm{a}}[\tau+\beta(\eta-\tau)]\right\}.\end{aligned}$$

We shall consider the case of data set 1. In a similar way, the case of data set 2 can be studied.

Considering the case of data set 1 and equating the partial derivatives of ln L to zero with respect to α, λ and β, the resulting three equations are as follows:

$$\begin{aligned}\frac{\partial\ln L}{\partial\alpha} &= \frac{r}{\alpha} - r\ln\lambda + \sum_{i=1}^{n_{\mathrm{u}}}\ln y_i + \sum_{i=n_{\mathrm{u}}+1}^{r}\ln\psi_i\\ &\quad - \frac{1}{\lambda}\left\{\sum_{i=1}^{n_{\mathrm{u}}} y_i + \sum_{i=n_{\mathrm{u}}+1}^{r}\psi_{\mathrm{i}} + mn_{\mathrm{u}}\tau + (n-r-m)n_{\mathrm{a}}\psi_r\right\} = 0,\end{aligned} \tag{11}$$

where $\psi_i = \tau + \beta(y_i - \tau)$ and $\psi_r = \tau + \beta(y_{r:n} - \tau)$.

$$\frac{\partial\ln L}{\partial\lambda} = -\frac{r\alpha}{\lambda} + \frac{\alpha}{\lambda^2}\left\{\sum_{i=1}^{n_{\mathrm{u}}} y_i + \sum_{i=n_{\mathrm{u}}+1}^{r}\psi_i + mn_{\mathrm{u}}\tau + (n-r-m)n_{\mathrm{a}}\psi_r\right\} = 0, \tag{12}$$

$$\frac{\partial\ln L}{\partial\beta} = \frac{n_{\mathrm{a}}}{\beta} + (\alpha-1)\sum_{i=n_{\mathrm{u}}+1}^{r}\frac{y_i-\tau}{\psi_i} - \frac{\alpha}{\lambda}\left\{\sum_{i=n_{\mathrm{u}}+1}^{r}(y_i-\tau) + (n-r-m)n_{\mathrm{a}}\psi_r\right\} = 0, \tag{13}$$

From Equation (11), we can obtain $\hat{\alpha}$ as a function of $\hat{\lambda}$ and $\hat{\beta}$ as

$$\hat{\alpha} = \frac{r}{r \ln \lambda - (\sum_{i=1}^{n_u} \ln y_i + \sum_{i=n_u+1}^{r} \ln \psi_i) + (1/\lambda) \{\sum_{i=1}^{n_u} y_i + \sum_{i=n_u+1}^{r} \psi_i + mn_u\tau + (n-r-m)n_a\psi_r\}}. \tag{14}$$

Moreover, from Equation (12), we can obtain $\hat{\lambda}$ as a function of $\hat{\beta}$ as

$$\hat{\lambda} = \frac{\sum_{i=1}^{n_u} y_i + \sum_{i=n_u+1}^{r} \psi_i + mn_u\tau + (n-r-m)n_a\psi_r}{r}. \tag{15}$$

Now, the system is reduced to one nonlinear likelihood equation of β. It can be solved iteratively using an iterative method such as the Newton–Raphson method to obtain the ML estimate of β. Therefore, the ML estimates of both λ and α can be easily obtained from Equations (15) and (14), respectively.

3.2. *Interval estimation*

In this subsection, the approximate confidence intervals of the parameters are derived based on the asymptotic distribution of the ML estimators of the elements of the vector of unknown parameters $\Omega = (\alpha, \lambda, \beta)$. It is known that the asymptotic distribution of the ML estimators of Ω is given by, see [21]

$$((\hat{\alpha} - \alpha), (\hat{\lambda} - \lambda), (\hat{\beta} - \beta)) \to N(0, \mathbf{I}^{-1}(\alpha, \lambda, \beta)),$$

where $\mathbf{I}^{-1}(\alpha, \lambda, \beta)$ is the variance–covariance matrix of the unknown parameters $\Omega = (\alpha, \lambda, \beta)$. The elements of the 3×3 matrix $\mathbf{I}^{-1}$, $I_{ij}(\alpha, \lambda, \beta)$, $i, j = 1, 2, 3$; can be approximated by $I_{ij}(\hat{\alpha}, \hat{\lambda}, \hat{\beta})$, where

$$I_{ij}(\hat{\Omega}) = - \left. \frac{\partial^2 \ln L(\Omega)}{\partial \Omega_i \partial \Omega_j} \right|_{\Omega = \hat{\Omega}}.$$

Now, we get the following:

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = -\frac{r}{\alpha^2}, \tag{16}$$

$$\frac{\partial^2 \ln L}{\partial \alpha\, \partial \lambda} = -\frac{r}{\lambda} + \frac{1}{\lambda^2} \left\{ \sum_{i=1}^{n_u} y_i + \sum_{i=n_u+1}^{r} \psi_i + mn_u\tau + (n-r-m)n_a\psi_r \right\}, \tag{17}$$

$$\frac{\partial^2 \ln L}{\partial \alpha\, \partial \beta} = \sum_{i=n_u+1}^{r} \frac{y_i - \tau}{\psi_i} - \frac{1}{\lambda} \left\{ \sum_{i=n_u+1}^{r} (y_i - \tau) + (n-r-m)n_a(y_{r:n} - \tau) \right\}, \tag{18}$$

$$\frac{\partial^2 \ln L}{\partial \lambda^2} = \frac{r\alpha}{\lambda^2} - \frac{2\alpha}{\lambda^3} \left\{ \sum_{i=1}^{n_u} y_i + \sum_{i=n_u+1}^{r} \psi_i + mn_u\tau + (n-r-m)n_a\psi_r \right\}, \tag{19}$$

$$\frac{\partial^2 \ln L}{\partial \lambda\, \partial \beta} = \frac{\alpha}{\lambda^2} \left\{ \sum_{i=n_{\mathrm{u}}+1}^{r} (y_i - \tau) + (n - r - m) n_{\mathrm{a}} (y_{r:n} - \tau) \right\}, \tag{20}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -\frac{n_{\mathrm{a}}}{\beta^2} - (\alpha - 1) \sum_{i=n_{\mathrm{u}}+1}^{r} \frac{(y_i - \tau)^2}{\psi_i^2}, \tag{21}$$

Thus, the approximate $100(1-\gamma)\%$ two-sided confidence intervals for α, λ and β are, respectively, given by

$$\hat{\alpha} \pm Z_{\gamma/2} \sqrt{\mathrm{I}_{11}^{-1}(\hat{\alpha}, \hat{\lambda}, \hat{\beta})}, \quad \hat{\lambda} \pm Z_{\gamma/2} \sqrt{\mathrm{I}_{22}^{-1}(\hat{\alpha}, \hat{\lambda}, \hat{\beta})} \quad \text{and} \quad \hat{\beta} \pm Z_{\gamma/2} \sqrt{\mathrm{I}_{33}^{-1}(\hat{\alpha}, \hat{\lambda}, \hat{\beta})}, \tag{22}$$

where $Z_{\gamma/2}$ is the upper $(\gamma/2)$th percentile of a standard normal distribution.

4. Simulation studies

In this section, simulation studies are conducted to discuss the performance of the ML estimators in terms of their biases and mean square errors (MSEs) for different choices of n, m/n, r/n, τ and η values. Also, 95% asymptotic confidence intervals based on the asymptotic distribution of the ML estimators are constructed and their lengths are computed.

The simulation study is carried out according to the following algorithm:

(1) Specify the values of n, m/n, r/n, τ and η.
(2) Specify the values of the parameters α, λ and β.
(3) Generate a random sample of size n from the random variable Y given by Equation (6) and sort it. The Weibull random variable can be easily generated. For example, if U represents a uniform random variable from [0, 1], then $Y = -\lambda\, [\ln(1-U)]^{1/\alpha}$ has WD with pdf given by Equation (1) *if* $y \le \tau$. But *if* $y > \tau$ then $Y = \tau + \beta^{-1}\{-\lambda\, [\ln(1-U)]^{1/\alpha} - \tau\}$ has WD with pdf given by Equation (5).
(4) Use the model given by Equation (4) to generate progressively Type-I hybrid censored data for a given$n$, m/n, r/n, τ, η $(\eta > \tau)$, α, λ and β. Two data sets can be considered as
    Set 1: $y_{1:n} < \cdots < y_{n_u:n} \le \tau < y_{n_u+1:n} < \cdots < y_{r:n} \le \eta$, if $\tau < y_{r:n} \le \eta$,
    and
    Set 2: $y_{1:n} < \cdots < y_{n_u:n} \le \tau < y_{n_u+1:n} < \cdots < y_{n_u+n_a:n} \le \eta$, if $y_{r:n} > \eta$.
(5) Use the progressively Type-I hybrid censored data to compute the ML estimates of the model parameters. The Newton–Raphson method is applied for solving the nonlinear equation given by Equation (13) to obtain the ML estimate of the parameter β. Then obtain the ML estimates of λ and α from Equations (15) and (14), respectively.
(6) Replicate the steps 3–5 10,000 times.
(7) Compute the average values of biases and MSEs associated with the ML estimators of the parameters.
(8) Compute the average values of intervals lengths (ILs) associated with each parameter with confidence level $1 - \gamma = 0.95$.
(9) Steps 1–8 are done with different values of n, m/n, r/n, τ, η $(\eta > \tau)$, α, λ and β.

Conducting the above algorithm, the average values of biases, MSEs and ILs are obtained using 10,000 replications to avoid randomness. The results are reported in Tables 1 and 2 based on different values of n, m/n, r/n, τ, η $(\eta > \tau)$, α, λ and β to investigate the performance of the ML estimators of the model parameters.

It is shown from Table 1 when $n = 30$ for the second combination of $(r/n, \tau, \eta) = (0.40, 5, 8)$ that the biases and MSEs of the estimators are the same as the third combination of $(r/n, \tau, \eta) = (0.40, 5, 11)$. This means that the time of the rth failure $y_{r:n}$ $(r < n)$ occurs before the prefixed $\eta > \tau$ and the experiment terminates at the time point $y_{r:n}$. While Table 2 shows that when $n = 75$, the biases and MSEs of the estimators for the first combination of $(r/n, \tau, \eta) = (0.25, 2, 8)$ are the same as the second combination of $(r/n, \tau, \eta) = (0.40, 2, 8)$. This proposes that the time of the rth failure $y_{r:n}$ is greater than η and the experiment terminates at the time η.

Moreover, from Tables 1 and 2, the following observations can be made.

(1) For fixed n, τ and η, the MSEs decrease as r/n increases.
(2) For fixed r/n, τ and η, the MSEs decrease as n incrases.

The same pattern is observed for the biases and ILs.

In addition, point (s) and 95% confidence interval (s_L, s_U) estimations for the reliability at mission times of $y = 1, 1.5, 2, 3, 4, 6$ and 8 are obtained. The estimations of the true reliability are computed using the following forms:

$$\hat{s}_1(y) = \exp\left\{-\left(\frac{y}{\hat{\lambda}}\right)^{\hat{\alpha}}\right\}, \quad \text{if } y \le \tau,$$

or

$$\hat{s}_2(y) = \exp\left\{-\left[\frac{(\tau + \hat{\beta}(y - \tau))}{\hat{\lambda}}\right]^{\hat{\alpha}}\right\}, \quad \text{if } y \succ \tau.$$

As Soliman [22] indicates, estimation of the reliability function of some equipment is one of the main problems of reliability theory. In most practical applications and life-test experiments, the distributions with positive domain, e.g. Weibull, Burr-XII, Pareto, Beta, and Rayleigh, are quite appropriate models. There have been many papers on estimating the reliability function of these distributions in non-Bayes as well as Bayes contexts, e.g. [23–26].

The results of the true reliability estimations are presented in Tables 3 and 4.

Table 1. Average values of the biases, MSEs (between brackets) and ILs when α, λ, β and m/n set at 1.5, 2, 3.5 and 0.05, respectively.

n	Parameters $(r/n, \tau, \eta)$	(0.25, 5, 8)	(0.40, 5, 8)	(0.40, 5, 11)
20	α	0.2301 (0.1988), 2.8956	0.2074 (0.1765), 2.6132	0.1802 (0.1508), 2.4311
	λ	0.3452 (0.3226), 3.3447	0.3245 (0.2987), 3.1682	0.3078 (0.2679), 2.8753
	β	0.3971 (0.3527), 3.7458	0.3516 (0.3182), 3.4690	0.3249 (0.2873), 3.1433
25	α	0.1944 (0.1567), 2.6859	0.1722 (0.1374), 2.5724	0.1562 (0.1327), 2.2105
	λ	0.3178 (0.2913), 3.1262	0.2949 (0.2617), 2.9821	0.2861 (0.2473), 2.6492
	β	0.3764 (0.3313), 3.3172	0.3181 (0.2751), 2.9575	0.2971 (0.2633), 2.8056
30	α	0.1542 (0.1323), 2.4372	0.1233 (0.1104), 2.0613	0.1233 (0.1104), 2.0602
	λ	0.2961 (0.2485), 2.9068	0.2614 (0.2173), 2.6572	0.2614 (0.2173), 2.4523
	β	0.3208 (0.2813), 3.0411	0.2755 (0.2412), 2.7290	0.2755 (0.2412), 2.7490
50	α	0.1271 (0.1065), 2.1621	0.0941 (0.0715), 1.8748	0.0801 (0.0627), 1.6952
	λ	0.2354 (0.1974), 2.6893	0.1582 (0.1392), 2.3402	0.1137 (0.1049), 2.2106
	β	0.2735 (0.2466), 2.7148	0.1876 (0.1528), 2.4512	0.1479 (0.1103), 2.3476
75	α	0.0638 (0.0410), 1.5215	0.0517 (0.0370), 1.4024	0.0352 (0.0141), 1.3159
	λ	0.1543 (0.1192), 1.7065	0.1131 (0.0751), 1.6835	0.0886 (0.0418), 1.3760
	β	0.1807 (0.1247), 1.7120	0.1364 (0.0801), 1.6055	0.0925(0.0631), 1.5211
100	α	0.0118 (0.0050), 1.3417	0.0073 (0.0041), 1.2811	0.0047 (0.0029), 1.1507
	λ	0.0415 (0.0163), 1.5213	0.0176 (0.0115), 1.4105	0.0124 (0.0066), 1.2358
	β	0.0721 (0.0316), 1.6862	0.0418 (0.0162), 1.4725	0.0275(0.0073), 1.3152

Table 2. Average values of the biases, MSEs (between brackets) and ILs when α, λ, β and m/n set at 0.5, 0.7, 1.6 and 0.08, respectively.

n	Parameters $(r/n, \tau, \eta)$	(0.25, 2, 8)	(0.40, 2, 8)	(0.40, 2, 11)
20	α	0.2683 (0.2286), 3.2341	0.2282 (0.1942), 2.7740	0.1873 (0.1721), 2.6885
	λ	0.4385 (0.3812), 3.4785	0.3827 (0.3521), 3.1543	0.3478 (0.3170), 2.9135
	β	0.4728 (0.4171), 3.5231	0.4189 (0.3749), 3.3478	0.3711 (0.3411), 3.2146
25	α	0.2283 (0.2051), 2.9244	0.1973 (0.1669), 2.6388	0.1592 (0.1322), 2.5720
	λ	0.3916 (0.3355), 3.1076	0.3591 (0.3092), 2.9721	0.3116 (0.2853), 2.7850
	β	0.4287 (0.3642), 3.4106	0.3877 (0.3362), 3.2781	0.3582 (0.3056), 2.9462
30	α	0.1758 (0.1641), 2.7109	0.1485 (0.1205), 2.5730	0.1162 (0.0915), 2.4155
	λ	0.3361 (0.2920), 2.9815	0.3134 (0.2698), 2.7527	0.2811 (0.2513), 2.6418
	β	0.3724 (0.3381), 3.2815	0.3422 (0.3076), 2.8669	0.3252 (0.2735), 2.7867
50	α	0.1382 (0.1124), 2.3520	0.1201 (0.0954), 1.9246	0.1059 (0.0743), 1.8374
	λ	0.2638 (0.2215), 2.7133	0.2261 (0.1782), 2.5491	0.1928 (0.1536), 2.4179
	β	0.3243 (0.2712), 2.8719	0.2477 (0.2055), 2.7426	0.2158 (0.1764), 2.6170
75	α	0.0947 (0.0501), 1.9622	0.0947 (0.0501), 1.9602	0.0478 (0.0276), 1.4854
	λ	0.1937 (0.1485), 2.0711	0.1937 (0.1485), 2.0704	0.1006 (0.0625), 1.5844
	β	0.2473 (0.1876), 2.1352	0.2473 (0.1876), 2.1316	0.1173(0.0744), 1.6329
100	α	0.0325 (0.0085), 1.5964	0.0094 (0.0071), 1.4621	0.0066 (0.0052), 1.3914
	λ	0.0861 (0.0361), 1.7853	0.0326 (0.0156), 1.6027	0.0211 (0.0091), 1.4056
	β	0.1138 (0.0650), 1.8346	0.0582 (0.0213), 1.6415	0.0410(0.0102), 1.5211

Table 3. Point and confidence interval estimations for the reliability when α, λ, β and m/n set at 0.5, 0.7, 1.6 and 0.08, respectively, for a mission time t using $\tau = 2$.

T	s	s_L	s_U
1	0.8371	0.7990	0.8752
1.5	0.8104	0.7692	0.8516
3	0.7723	0.7168	0.8278
4	0.7309	0.6749	0.7869

Table 4. Point and confidence interval estimations for the reliability when α, λ, β and m/n set at 1.5, 2, 3.5 and 0.05, respectively, for a mission time t using $\tau = 5$.

T	s	s_L	s_U
2	0.7864	0.7455	0.8273
3	0.7429	0.7032	0.7826
6	0.6944	0.6577	0.7311
8	0.6431	0.5990	0.6872

5. Concluding remarks and further studies

In this article, we have discussed the likelihood estimation for parameters of WD and the acceleration factor when the data are progressively Type-I hybrid censored under SSPALTs. It is observed that the ML estimates cannot be obtained in a closed form and we have proposed to use the Newton–Raphson as an iterative method to compute them. The approximate confidence intervals of the model parameters are also constructed based on the asymptotic distribution of ML estimators. The performances of the estimators are investigated using the biases and MSEs by Monte Carlo simulations. It is observed that they are quite satisfactory, especially when the sample size is sufficiently large. Finally, as a future work, the Bayesian inference in the case of SSPALT under the same censoring scheme proposed in this article will be considered. Also, another important aspect needed in this direction is to consider the progressively Type-II hybrid censoring scheme.

Acknowledgements

This project was supported by King Saud University, Deanship of Scientific Research, College of Science Research Center.

References

[1] Nelson W. Accelerated life testing: statistical models, data analysis and test plans. New York: John Wiley and Sons; 1990.
[2] Goel PK. Some estimation problems in the study of tampered random variables [Technical Rep. No. 50]. Pittsburgh, Pennsylvania: Department of statistics, Carnegie-Mellon University; 1971.
[3] DeGroot MH, Goel PK. Bayesian and optimal design in partially accelerated life testing. Nav Res Log Quart. 1979;16(2):223–235.
[4] Bhattacharyya GK, Soejoeti Z. A tampered failure rate model for step-stress accelerated life test. Comm Statist Theory Methods. 1989;18(5):1627–1643.
[5] Bai DS, Chung SW. Optimal design of partially accelerated life tests for the exponential distribution under type-I censoring. IEEE Trans Reliab. 1992;41(3):400–406.
[6] Bai DS, Chung SW, Chun YR. Optimal design of partially accelerated life tests for the lognormal distribution under type-I censoring. Reliab Eng Sys Safety. 1993;40:85–92.
[7] Abdel-Ghaly AA, El-Khodary EH, Ismail AA. Maximum likelihood estimation and optimal design in step partially accelerated life tests for Pareto distribution using type-I censoring. Proceedings of the 14th annual conference of statistics and computer modeling in human and social sciences; Egypt: Faculty of Economics and Political Sciences, Cairo, University; 2002, p. 27–40.
[8] Abdel-Ghaly AA, El-Khodary EH, Ismail AA. Estimation and optimal design in step partially accelerated life tests for Pareto distribution using type-II censoring. Proceedings of the 15th annual conference on statistics and computer modeling in human and social sciences; Egypt: Faculty of Economics and Political Science, Cairo University; 2003. p. 16–29.
[9] Abdel-Ghaly AA, El-Khodary EH, Ismail AA. Optimum constant-stress life test plans for Pareto distribution under type-I censoring. J Statist Comput Simul. 2011;81(12):1835–1845.
[10] Abdel-Ghani MM. Investigation of some lifetime models under partially accelerated life tests [Ph.D. thesis]. Egypt: Department of Statistics, Faculty of Economics & Political Science, Cairo University; 1998.
[11] Abdel-Ghani MM. The estimation problem of the log-logistic parameters in step partially accelerated life tests using type-I censored data. Nat Rev Soc Sci. 2004;41(2):1–19.
[12] Ismail AA. The test design and parameter estimation of Pareto lifetime distribution under partially accelerated life tests [Ph.D. thesis]. Egypt: Department of Statistics, Faculty of Economics & Political Science, Cairo University; 2004.
[13] Aly HM, Ismail AA. Optimum simple time-step stress plans for partially accelerated life testing with censoring. Far East J Theor Statist. 2008;24(2):175–200.
[14] Ismail AA, Sarhan AM. Design of step-stress life test with progressively type-II censored exponential data. Int Math Forum. 2009;4(40):1963–1976.
[15] Ismail AA, Aly HM. Optimal planning of failure-step stress partially accelerated life test under type-II censoring. J Statist Comput Simul. 2010;80(12):1335–1348.
[16] Ismail AA. Bayes estimation of Gompertz distribution parameters and acceleration factor under partially accelerated life tests with type-I censoring. J Statist Comput Simul. 2010;80(11):1253–1264.
[17] Ismail AA. Estimating the parameters of Weibull distribution and the acceleration factor from hybrid partially accelerated life test. Appl Math Model. 2012;36(7):2920–2925.
[18] Ismail AA. Inference in the generalized exponential distribution under partially accelerated tests with progressive type-II censoring. Theor Appl Fract Mech. 2012;59(1):49–56.
[19] Lin C-T, Chou C-C, Huang Y-L. Inference for the Weibull distribution with progressive hybrid censoring. Comput. Statist Data Anal. 2012;56:451–467.
[20] Ling L, Xu W, Li M. Parametric inference for progressive type-I hybrid censored data on a simple step-stress accelerated life test model. Math Comput Simul. 2009;79:3110–3121.
[21] Miller RC. Survival analysis. New York: John Wiley & Sons; 1981.
[22] Soliman AA. Reliability estimation in a generalized life-model with application to the burr-XII. IEEE Trans Reliab. 2002;51(3):337–343.
[23] Meeker WQ, Escobar LA. Statistical methods for reliability data. New York: John Wiley & Sons; 1998.
[24] Pensky M, Singh RS. Empirical Bayes estimation of reliability characteristics for an exponential family. Canad J Statist. 1999;27(1):127–136.
[25] Sun D, Berger JO. Bayesian sequential reliability for Weibull and related distribution. Ann Inst Statist Math. 1994;46:221–249.
[26] Zacks S. Introduction to reliability analysis: probability models and statistical models. New York: Springer Verlag; 1992.