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Estimating the parameters of Weibull distribution and the acceleration factor from hybrid partially accelerated life test

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ABSTRACT

This article considers the estimation of parameters of Weibull distribution based on hybrid censored data. The parameters are estimated by the maximum likelihood method under step-stress partially accelerated test model. The maximum likelihood estimates (MLEs) of the unknown parameters are obtained by Newton–Raphson algorithm. Also, the approximate Fisher information matrix is obtained for constructing asymptotic confidence bounds for the model parameters. The biases and mean square errors of the maximum likelihood estimators are computed to assess their performances through a Monte Carlo simulation study.

1. Introduction

The observed lifetimes of high reliability devices tend to be very long. So, the time necessary to test a sample of such devices under use conditions tends to be excessive. Such circumstances call for testing the sample under stresses which are higher than usual conditions. The test data are then extrapolated to estimate the life distribution under use conditions. So, people often use accelerated life tests (ALTs) or partially accelerated life tests (PALTs) to save time and cost.

As Nelson [1] indicates, the stress can be applied in various ways, commonly used methods are step-stress and constant-stress. Under step-stress PALT, a test item is first run at normal use condition and, if it does not fail for a specified time, then it is run at accelerated use condition until failure occurs or the observation is censored. But the constant-stress PALT runs each item at either normal use condition or accelerated use condition only, i.e. each unit is run at a constant-stress level until the test is terminated. Accelerated test stresses involve higher than usual temperature, voltage, pressure, load, humidity,$\ldots$, etc., or some combination of them. The objective of a PALTs is to collect more failure data in a limited time without necessarily using high stresses to all test units. But in ALTs, all test units are run under high stresses.

PALTs have been studied under Type-I and Type-II censoring schemes by several authors, for example, see Goel [2], DeGroot and Goel [3], Bhattacharyya and Soejoeti [4], Bai and Chung [5], Bai et al. [6], Abdel-Ghaly et al. [7–9], Abdel-Ghani [10,11], Ismail [12], Aly and Ismail [13], Ismail and Sarhan [14], Ismail and Aly [15], Ismail [16].

In ALTs or PALTs, tests are often stopped before all units fail. The estimates from the censored data are less accurate than those from complete data. However, this is more than offset by the reduced test time and expense. The most common censoring schemes are Type-I and Type-II censoring. Consider n units placed on life test. In conventional Type-I censoring, the experiment continues up to a prespecified time η. Any failures that occur after that time are not observed. The termination

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Nomenclature

Acronyms and Notations

ALTs	accelerated life tests
PALTs	partially accelerated life tests
SSPALTs	step-stress partially accelerated life tests
WD	Weibull distribution
MLEs	maximum likelihood estimates/estimators
PDF	probability density function
MSE	mean square error
$1-\gamma$	confidence level
n	sample size (total number of test items in a PALT)
R	a pre-specified number of failures
η	a pre-determined time
y_i	observed lifetime of unit i tested under SS-PALT
τ	stress change time
y_R	the time of the Rth failure
δ_{1R}, δ_{2R}	indicator functions: $\delta_{1R} \equiv I(y_R > \eta)$, $\delta_{2R} \equiv I(y_R < \eta)$
$\wedge$	denotes maximum likelihood estimate
α	WD shape parameter ($\alpha > 0$)
λ	WD scale parameter ($\lambda > 0$)
β	acceleration factor ($\beta > 1$)

point η of the experiment is assumed to be s -independent of the failure times. But in conventional Type-II censoring, the experimenter terminates the experiment after a prespecified number of units $m \leqslant n$ fail. In this scenario, only the smallest lifetimes are observed. In Type-I censoring, the number of failures observed is random and the endpoint of the experiment is fixed. While the number of failures is fixed in Type-II censoring and the endpoint is random. Numerous articles in the literature have dealt with inference under Type-I and Type-II censoring for various parametric families of distributions, see Cohen [17] for details.

This paper will concentrate on step-stress PALTs under hybrid censoring. Hybrid censoring scheme can be described as follows. Consider a life testing experiment in which n units are put on test simultaneously. Failure times are observed and the test is terminated either at a pre-determined time η or based on a pre-specified number R of failures whichever is occurred first. Such a mixture of Type-I and Type-II censoring schemes is known as hybrid censoring scheme. So, sampling according to the hybrid censoring scheme is terminated at $min(\eta, Y_R)$. It is clear that Type-I and Type-II censoring schemes can be obtained as special cases of hybrid censoring scheme by taking $R = n$ and $\eta = \infty$ respectively. According to Ismail [18], the main advantage of using hybrid censoring scheme is that it saves time and money through the reduction achieved in the expected testing time and expected number of failures observed in the experiment. Many authors have treated the statistical inference problem about the parameters for sampling schemes Type-I and Type-II censoring. In this work the estimation of parameters is considered under step-stress partially accelerated life tests (SSPALTs) with hybrid censored data assuming two-parameter Weibull life distribution. It is also assumed that the failed items are not replaced.

In spite of the applicability of the hybrid censoring scheme, all of previous works under PALTs were considered using the two most common censoring schemes Type-I and Type-II censoring schemes and no attention has been paid in analyzing hybrid censored lifetime data. All studies made under hybrid censoring were concerned with ordinary or fully accelerated life tests. For example, see Fairbanks et al. [19], Draper and Guttman [20], Chen and Bhattacharya [21], Ebrahimi [22], Gupta and Kundu [23, Kundu [24], Xie [25], Park and Balakrishnan [26] and Zhang et al. [27]. In this article hybrid censoring and step-stress PALT are combined for developing a step-stress PALT model with hybrid censored lifetime data when the lifetime follows a two-parameter Weibull distribution.

The rest of the paper is arranged as follows. In Section 2 the model and the available data are described. The MLEs of the SSPALTs model parameters are provided in Section 3. In Section 4 the asymptotic confidence bounds for the model parameters are constructed based on the asymptotic distribution of MLEs. Section contains the simulation results. Conclusion is made in Section 6.

2. Description of the model

Assume the lifetime random variable Y has a Weibull distribution with the shape and scale parameters as α and λ respectively. So, the probability density function (PDF) of Y is

$$f_Y(y;\alpha,\lambda) = \frac{\alpha}{\lambda}\left(\frac{y}{\lambda}\right)^{\alpha-1} e^{-(y/\lambda)^{\alpha}}, \quad y > 0,\ \alpha > 0,\ \lambda > 0 \tag{1}$$

Weibull distribution is one of the most common distributions which is used to analyze several lifetime data. Its hazard function can be increasing, decreasing and constant depending on the shape parameter. Thus, this distribution has lots of flexibility compared to other distributions.

The reliability function of Weibull distribution in (1) takes the form

$$S(y) = \exp\{-(y/\lambda)^{\alpha}\}, \tag{2}$$

and the corresponding failure rate function is given by:

$$h(y) = \frac{\alpha}{\lambda}\left(\frac{y}{\lambda}\right)^{\alpha-1}. \tag{3}$$

Therefore, the pdf of Y under step-stress PALT model can be given by:

$$f(y) = \begin{cases} 0, & y \leqslant 0, \\ f_1(y) \equiv f_Y(y;\alpha,\lambda), & 0 < y \leqslant \tau \\ f_2(y), & y > \tau, \end{cases} \tag{4}$$

where

$$f_2(y) \equiv f_Y(y;\alpha,\lambda,\beta) = \beta\frac{\alpha}{\lambda}\left(\frac{[\tau+\beta(y-\tau)]}{\lambda}\right)^{\alpha-1}\exp\{-([\tau+\beta(y-\tau)]/\lambda)^{\alpha}\}, \tag{5}$$

which is obtained by the transformation-variable technique using the density in (1) and the model proposed by DeGroot and Goel [3] which is given by

$$Y = \begin{cases} T & \text{if } T \leqslant \tau, \\ \tau + (T-\tau)/\beta & \text{if } T > \tau, \end{cases} \tag{6}$$

where T is the lifetime of the unit under normal use condition, τ is the stress change time and β is the acceleration factor; $\beta > 1$.

Now, the data available under the hybrid censoring scheme can be described. Under the hybrid censoring scheme, it is assumed that R and η are known in advance. Therefore, according to this censoring scheme, the data can be observed as follows.

$$y_1 < y_2 < \cdots < \eta \quad \text{if } y_R > \eta \quad \text{or } y_1 < y_2 < \cdots < y_R \quad \text{if } y_R < \eta.$$

3. Maximum likelihood estimation

The idea behind maximum likelihood parameter estimation is to determine the parameters that maximize the probability (likelihood) of the sample data. From a statistical point of view, the method of maximum likelihood is considered to be more robust and yields estimators with good statistical properties. In other words, maximum likelihood methods are versatile and apply to most models and to different types of data. In addition, they provide efficient methods for quantifying uncertainty through confidence bounds. Since these estimators do not always exist in closed form, numerical techniques are used to compute them such as Newton–Raphson.

This section discusses the process of obtaining the maximum likelihood estimates of the parameters α, λ and β based on hybrid censored data under step-stress PALTs model. Both point and interval estimations of the parameters are derived.

3.1. Point estimations

In this section the MLEs of the unknown parameters are given. Based on the observed hybrid censored data from Weibull distribution, the likelihood function under SSPALTs is given by

$$L(\alpha,\lambda,\beta) = \left(\frac{\alpha}{\lambda}\right)^{2R}\beta^R\prod_{i=1}^{R}\left(\frac{y_i}{\lambda}\right)^{\alpha-1}\left(\frac{\psi_i}{\lambda}\right)^{\alpha-1}e^{-\left\{\sum_{i=1}^{R}\left(\frac{y_i}{\lambda}\right)^{\alpha}+\sum_{i=1}^{R}\left(\frac{\psi_i}{\lambda}\right)^{\alpha}+(n-R)\left[\left(\frac{\eta^{\delta_{1R}}y_R^{\delta_{2R}}}{\lambda}\right)^{\alpha}+\left(\frac{\psi_R}{\lambda}\right)^{\alpha}\right]\right\}}, \tag{7}$$

where $\psi_i = \tau + \beta(y_i - \tau)$, $\psi_R = \tau + \beta\left(\eta^{\delta_{1R}}y_R^{\delta_{2R}} - \tau\right)$, $\delta_{1R} = 1$ if $y_R > \eta$ and 0 otherwise, and $\delta_{2R} = 1$ if $y_R < \eta$ and 0 otherwise.

Using $ln L$ to denote the natural logarithm of $L(\alpha,\lambda,\beta)$, then we have

$$\begin{aligned} ln L(\alpha,\lambda,\beta) &= Rln\beta + 2Rln\alpha - 2Rln\lambda - 2R(\alpha-1)ln\lambda + (\alpha-1)\left[\sum_{i=1}^{R}ln y_i + \sum_{i=1}^{R}ln\psi_i\right] \\ &\quad - R\left\{\sum_{i=1}^{R}\left(\frac{y_i}{\lambda}\right)^{\alpha} + \sum_{i=1}^{R}\left(\frac{\psi_i}{\lambda}\right)^{\alpha} + (n-R)\left[\left(\frac{\eta^{\delta_{1R}}y_R^{\delta_{2R}}}{\lambda}\right)^{\alpha} + \left(\frac{\psi_R}{\lambda}\right)^{\alpha}\right]\right\}. \end{aligned} \tag{8}$$

Equating the partial derivatives of $\ln L$ with respect to α, λ and β to zero, the resulting three equations are:

$$\frac{\partial \ln L}{\partial \alpha} = \frac{2R}{\alpha} - 2R\ln\lambda + \sum_{i=1}^{R} \ln y_i + \sum_{i=1}^{R} \ln \psi_i - R\left\{\sum_{i=1}^{R}\left(\frac{y_i}{\lambda}\right)^{\alpha} \ln\left(\frac{y_i}{\lambda}\right) + \sum_{i=1}^{R}\left(\frac{\psi_i}{\lambda}\right)^{\alpha} \ln\left(\frac{\psi_i}{\lambda}\right) + (n-R)\left[\left(\frac{\eta^{\delta_{1R}} y_R^{\delta_{2R}}}{\lambda}\right)^{\alpha} \ln\left(\frac{\eta^{\delta_{1R}} y_R^{\delta_{2R}}}{\lambda}\right) + \left(\frac{\psi_R}{\lambda}\right)^{\alpha} \ln\left(\frac{\psi_R}{\lambda}\right)\right]\right\} = 0, \tag{9}$$

$$\frac{\partial \ln L}{\partial \lambda} = -\frac{2R(\alpha+1)}{\lambda} + \frac{\alpha R}{\lambda^{\alpha+1}}\left\{\sum_{i=1}^{R} y_i^{\alpha} + \sum_{i=1}^{R} \psi_i^{\alpha} + (n-R)\left[\left(\eta^{\delta_{1R}} y_R^{\delta_{2R}}\right)^{\alpha} + \psi_R^{\alpha}\right]\right\} = 0, \tag{10}$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{R}{\beta} + (\alpha-1)\sum_{i=1}^{R} \frac{(y_i-\tau)}{\psi_i} - \frac{\alpha R}{\lambda}\left\{\sum_{i=1}^{R} (y_i-\tau)\left(\frac{\psi_i}{\lambda}\right)^{\alpha-1} + (n-R)\left(\eta^{\delta_{1R}} y_R^{\delta_{2R}} - \tau\right)\left(\frac{\psi_R}{\lambda}\right)^{\alpha-1}\right\} = 0. \tag{11}$$

From (10) we can obtain $\hat{\lambda}$ as a function of $\hat{\alpha}$ and $\hat{\beta}$ as

$$\hat{\lambda} = \left\{\frac{\hat{\alpha}\left\{\sum_{i=1}^{R} y_i^{\hat{\alpha}} + \sum_{i=1}^{R} \psi_i^{\hat{\alpha}} + (n-R)\left[\left(\eta^{\delta_{1R}} y_R^{\delta_{2R}}\right)^{\hat{\alpha}} + \psi_R^{\hat{\alpha}}\right]\right\}}{2(\hat{\alpha}+1)}\right\}^{1/\hat{\alpha}}. \tag{12}$$

Now, the system is reduced to two non-linear likelihood equations of α and β and can be solved iteratively using the Newton–Raphson to obtain the MLEs of α and β. Hence, the MLE of λ can be easily obtained from (12).

3.2. Interval estimations

Here, the approximate confidence intervals of the parameters are derived based on the asymptotic distribution of the MLEs of the elements of the vector of unknown parameters $\Omega = (\alpha, \lambda, \beta)$. It is known that the asymptotic distribution of the MLEs of Ω is given by, see Miller [28],

$$((\hat{\alpha}-\alpha), (\hat{\lambda}-\lambda), (\hat{\beta}-\beta)) \to N(0, \mathbf{I}^{-1}(\alpha, \lambda, \beta)),$$

where $\mathbf{I}^{-1}(\alpha, \lambda, \beta)$ is the variance–covariance matrix of the unknown parameters $\Omega = (\alpha, \lambda, \beta)$. The elements of the 3×3 matrix $\mathbf{I}^{-1}$, $I_{ij}(\alpha, \lambda, \beta)$, $i, j = 1, 2, 3$; can be approximated by $I_{ij}(\hat{\alpha}, \hat{\lambda}, \hat{\beta})$, where

$$I_{ij}(\hat{\Omega}) = -\left.\frac{\partial^2 \ln L(\Omega)}{\partial \Omega_i \partial \Omega_j}\right|_{\Omega=\hat{\Omega}}.$$

Now, we get the following

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = -\frac{2R}{\alpha^2} - R\left\{\sum_{i=1}^{R}\left(\frac{y_i}{\lambda}\right)^{\alpha}\left[\ln\left(\frac{y_i}{\lambda}\right)\right]^2 - \sum_{i=1}^{R}\left(\frac{\psi_i}{\lambda}\right)^{\alpha}\left[\ln\left(\frac{\psi_i}{\lambda}\right)\right]^2 + \left(n-R\right)\left[\left(\frac{\eta^{\delta_{1R}} y_R^{\delta_{2R}}}{\lambda}\right)^{\alpha}\left[\ln\left(\frac{\eta^{\delta_{1R}} y_R^{\delta_{2R}}}{\lambda}\right)\right]^2 + \left(\frac{\psi_R}{\lambda}\right)^{\alpha}\left[\ln\left(\frac{\psi_R}{\lambda}\right)\right]^2\right]\right\}, \tag{13}$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \lambda} = -\frac{2R}{\lambda} + \frac{R}{\lambda^{\alpha+1}}\left\{\sum_{i=1}^{R}\left[\alpha y_i^{\alpha} \ln\left(\frac{y_i}{\lambda}\right) + y_i^{\alpha}\right] + \sum_{i=1}^{R}\left[\alpha \psi_i^{\alpha} \ln\left(\frac{\psi_i}{\lambda}\right) + \psi_i^{\alpha}\right] + (n-R)\left[\alpha\left(\eta^{\delta_{1R}} y_R^{\delta_{2R}}\right)^{\alpha} \ln\left(\frac{\eta^{\delta_{1R}} y_R^{\delta_{2R}}}{\lambda}\right) + \left(\eta^{\delta_{1R}} y_R^{\delta_{2R}}\right)^{\alpha} + \alpha \psi_R^{\alpha} \ln\left(\frac{\psi_R}{\lambda}\right) + \psi_R^{\alpha}\right]\right\}, \tag{14}$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \beta} = \sum_{i=1}^{R}\left(\frac{y_i-\tau}{\psi_i}\right) - R\left\{\sum_{i=1}^{R}\left[\alpha\left(\frac{\psi_i}{\lambda}\right)^{\alpha-1} \frac{(y_i-\tau)}{\lambda} \ln\left(\frac{\psi_i}{\lambda}\right) + \left(\frac{\psi_i}{\lambda}\right)^{\alpha}\left(\frac{\lambda}{\psi_i}\right)\frac{(y_i-\tau)}{\lambda}\right] + (n-R)\left[\alpha\left(\frac{\psi_R}{\lambda}\right)^{\alpha-1}\left(\frac{\eta^{\delta_{1R}} y_R^{\delta_{2R}} - \tau}{\lambda}\right) \ln\left(\frac{\psi_R}{\lambda}\right) + \left(\frac{\psi_R}{\lambda}\right)^{\alpha}\left(\frac{\lambda}{\psi_R}\right)\left(\frac{\eta^{\delta_{1R}} y_R^{\delta_{2R}} - \tau}{\lambda}\right)\right]\right\}, \tag{15}$$

$$\frac{\partial^2 \ln L}{\partial \lambda^2} = \frac{2R(\alpha+1)}{\lambda^2} - \frac{\alpha(\alpha+1)R}{\lambda^{\alpha+2}}\left\{\sum_{i=1}^{R} y_i^{\alpha} + \sum_{i=1}^{R}\psi_i^{\alpha} + (n-R)\left[\left(\eta^{\delta_{1R}} y_R^{\delta_{2R}}\right)^{\alpha} + \psi_R^{\alpha}\right]\right\}, \tag{16}$$

$$\frac{\partial^2 \ln L}{\partial \lambda \partial \beta} = \frac{\alpha^2 R}{\lambda^{\alpha+1}}\left[\sum_{i=1}^{R}\psi_i^{\alpha-1}(y_i-\tau) + (n-R)\psi_R^{\alpha-1}\left(\eta^{\delta_{1R}} y_R^{\delta_{2R}} - \tau\right)\right], \tag{17}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -\frac{R}{\beta^2} - (\alpha-1)\sum_{i=1}^{R}\frac{(y_i-\tau)^2}{\psi_i^2} - \frac{\alpha(\alpha-1)R}{\lambda^{\alpha-1}}\left[\sum_{i=1}^{R}(y_i-\tau)^2\psi_i^{\alpha-2} + (n-R)\left(\eta^{\delta_{1R}} y_R^{\delta_{2R}} - \tau\right)^2\psi_R^{\alpha-2}\right], \tag{18}$$

Thus, the approximate $100(1-\gamma)\%$ two sided confidence intervals for α, λ and β are, respectively, given by

$$\hat{\alpha} \pm Z_{\gamma/2}\sqrt{I_{11}^{-1}(\hat{\alpha},\hat{\lambda},\hat{\beta})},\ \hat{\lambda} \pm Z_{\gamma/2}\sqrt{I_{22}^{-1}(\hat{\alpha},\hat{\lambda},\hat{\beta})} \quad \text{and} \quad \hat{\beta} \pm Z_{\gamma/2}\sqrt{I_{33}^{-1}(\hat{\alpha},\hat{\lambda},\hat{\beta})}.$$

where $Z_{\gamma/2}$ is the upper $(\gamma/2)$th percentile of a standard normal distribution.

4. Simulation studies

In this section simulation studies are conducted to investigate the performances of the maximum likelihood estimators (MLEs) in terms of their biases and mean square errors (MSEs) for different choices of n, R and η values. Also, the 95 % asymptotic confidence intervals based on the asymptotic distribution of the MLEs are computed and their lengths are given. For a particular set of hybrid censored data, the average values of biases, MSEs, variances and interval lengths are obtained based on 1000 replications and the results are reported in Tables 1–4. In each Table, the first, second and third rows represent the results for α, λ and β, respectively.

From Tables 1–4 the following observations can be made.

(1) For fixed n and R, the MSEs decrease as η increases.
(2) For fixed n and η, the MSEs decrease as R increases.

Table 1
Average values of the biases, MSEs (between brackets), variances and interval lengths, respectively when η = 5 and n = 25.

R = 10	R = 15	R = 20
1.015 (0.043), 0.0018, 1.231	0.921 (0.020), 0.0012, 0.871	0.535 (0.012), 0.0007, 0.563
1.049 (0.052), 0.0042, 1.874	1.022 (0.029), 0.0021, 1.612	0.745 (0.017), 0.0014, 0.986
1.088 (0.074), 0.0065, 2.028	1.046 (0.055), 0.0057, 1.801	0.827 (0.041), 0.0036, 1.375

Table 2
Average values of the biases, MSEs (between brackets), variances and interval lengths, respectively when η = 5 and n = 35.

R = 15	R = 20	R = 25
0.786 (0.014), 0.0013, 0.696	0.414 (0.008), 0.0011, 0.502	0.395 (0.005), 0.0008, 0.472
0.954 (0.023), 0.0016, 1.323	0.733 (0.015), 0.0013, 0.775	0.710 (0.014), 0.0009, 0.659
0.986 (0.047), 0.0051, 1.514	0.799 (0.038), 0.0034, 1.112	0.735 (0.031), 0.0029, 0.982

Table 3
Average values of the biases, MSEs (between brackets), variances and interval lengths, respectively when η = 15 and n = 25.

R = 10	R = 15	R = 20
1.001 (0.027), 0.0012, 1.043	0.879 (0.018), 0.0007, 0.703	0.498 (0.006), 0.0004, 0.488
1.038 (0.045), 0.0039, 1.635	1.006 (0.024), 0.0015, 1.472	0.711 (0.012), 0.0011, 0.924
1.067 (0.062), 0.0058, 2.014	1.034 (0.051), 0.0049, 1.675	0.806 (0.037), 0.0031, 1.112

Table 4
Average values of the biases, MSEs (between brackets), variances and interval lengths, respectively when η = 15 and n = 35.

R = 15	R = 20	R = 25
0.586 (0.009), 0.0009, 0.451	0.391 (0.005), 0.0007, 0.378	0.132 (0.003), 0.0002, 0.283
0.723 (0.012), 0.0014, 0.946	0.634 (0.008), 0.0011, 0.652	0.379 (0.006), 0.0005, 0.414
0.845 (0.039), 0.0048, 1.223	0.676 (0.019), 0.0023, 0.946	0.460 (0.015), 0.0017, 0.758

(3) For fixed R and η, the MSEs decrease as n increases.

The same pattern is observed for bias, variance and interval length.

5. Conclusion

In this paper I have considered the classical inference procedure for the unknown parameters of the Weibull distribution and the acceleration factor when the data are hybrid censored from step-stress partially accelerated life tests. It is observed that the maximum likelihood estimators can not be obtained in closed form and I have proposed to use the Newton–Raphson as an iterative method to compute them. The approximate confidence intervals of the model parameters are also constructed. The performances of the estimators are investigated by Monte Carlo simulations and it is observed that they are quite satisfactory. Finally, as a future work, further studies of finding optimal censoring schemes and another important aspect, namely goodness of fit test from hybrid censored data should be addressed. More work is needed in those directions.

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