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Meson spectra using Nikiforov-Uvarov method

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ABSTRACT

There are many trials for modeling of the quarkonium systems like ($c\bar{c}$, $b\bar{b}$ and $\bar{b}c$). One of them is using phenomenological potentials by different techniques to describe their spectroscopy. In the present work we choose a phenomenological potential (linear, Yukawa and harmonic potentials) in the framework of the nonrelativistic Schrödinger's equation with relativistic corrections using the Nikiforov-Uvarov (NU) method. We obtained the eigen function and eigen value comparing the results with the available experimental data. The results are in good agreement with the available experimental data.

Introduction

Theorists have been trying to explain the behavior and characteristics of the quarkonium systems ($c\bar{c}$, $b\bar{b}$ and $\bar{b}c$) by using different techniques and models like Bethe-Salpeter equation [1–7], lattice quantum chromodynamics techniques [8–11], the relativistic approaches [6,7,12–16]¸ the semi-relativistic approaches [17–19], the nonrelativistic approaches [7,20–28] in which they used different phenomenological potentials such as the Coulomb and linear potentials [29–33], the Woods-Saxon Potential [26,34], power potentials [21,35,36], exponential potentials [32,33], harmonic and anharmonic potentials [12,22,28,37–39]. In the present work, we will explain the behavior the quarkonium systems by choosing specific pheneomenlogical potentials potentials (Yukawa, linear, harmonic) and fitting the spectra with coefficients potentials. We use nonrelativistic Schrödinger equation because c, b quarks are relatively heavy and solve it by the Nikiforov-Uvarov (NU) method [10,22,28–33,37,38,40–50]. The pheneomenlogical potentials (Yukawa, linear, harmonic) have physical importance for understanding Quantitative and qualitative behavior that Yukawa potential describe the coulomb interaction and decay process. Yukawa and linear potentials describe asymptotic freedom and confinement properties. Hamonic potential describe as bound state system and correction (perturbed) term. Additionally, Relativistic corrections [14,20–22,27,31] are considered as spin dependent term for the quarkonium systems. The used method is the Nikiforov-Uvarov (NU) method that is mathematical tools to solve hypergeometric equation, so we used it for Schrödinger equation to get the eigen value and eigen function solutions.

Theory

The masses of the quark and antiquark in the quarkonium system are bigger than the chromodynamics scaling, i.e. $M_{q,\bar{q}} \gg \Lambda_{QCD}$. So, nonrelativistic treatment is suitable for heavy bound systems. Schrödinger equation of two-body system in spherical symmetric potential reads

$$\frac{d^2R}{dr^2} + \left[\frac{2\mu}{\hbar^2}(E - V(r)) - \frac{l(l+1)}{r^2}\right]R = 0 \tag{1}$$

where μ is the reduced mass, E is the energy eigenvalue, l is the orbital quantum number and R(r) is a radial wavefunction solution of the Schrödinger equation. Our radial potential is taken as:

$$V(r) = \frac{-b}{r}e^{-cr} + ar + dr^2 \tag{2}$$

Additionally, we use relativistic corrections (spin-dependent splitting) terms: spin-spin interaction $V_{S-S}(r)$, spin-orbital interaction $V_{s-l}(r)$, and tensor interaction $V_T(r)$ [14,20–22].

$$V_{s-l-T}(r) = V_{S-S}(r) + V_{s-l}(r) + V_T(r) \tag{3}$$

where

$$V_{S-S}(r) = \frac{2}{3m_q m_{\bar{q}}}\nabla^2 V_V(r)\left[\overrightarrow{S_q} \bullet \overrightarrow{S_{\bar{q}}}\right] \tag{4}$$

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$$V_{s-l}(r)=\frac{1}{2m_q m_{\bar{q}} r}\left[3\frac{dV_V(r)}{dr}-\frac{dV_s(r)}{dr}\right]\left[\vec{L}\bullet\vec{S}\right] \tag{5}$$

$$V_T(r)=\frac{1}{12m_q m_{\bar{q}}}\left[\frac{1}{r}\frac{dV_V(r)}{dr}-\frac{d^2V_V(r)}{dr^2}\right]\left[6\left(\vec{S_q}\bullet\frac{\vec{r}}{|r|}\right)\left(\vec{S_{\bar{q}}}\bullet\frac{\vec{r}}{|r|}\right)-2\vec{S_q}\bullet\vec{S_{\bar{q}}}\right] \tag{6}$$

$V_V(r)=\frac{-b}{r}e^{-cr}+a_v r$ is a vector potential term and $V_s(r)=a_S r+dr^2$ is a scalar potential one.

The total potential $V_{tot}(r)$ due to spin splitting terms is

$$V_{tot}(r)=V(r)+V_{s-l-T}(r) \tag{7}$$

Using Eqs. (4), (5), (6) in Eq. (2), we get

$$V_{tot}(r)=\frac{e^{-cr}}{r^3}\left[\frac{3bv(ls)}{2m_q m_{\bar{q}}}+\frac{bv(t)}{4m_q m_{\bar{q}}}\right]+\frac{e^{-cr}}{r^2}\left[\frac{3cbv(ls)}{2m_q m_{\bar{q}}}+\frac{cbv(t)}{4m_q m_{\bar{q}}}\right]+\frac{e^{-cr}}{r}\left[\frac{-2c^2bv(ss)}{3m_q m_{\bar{q}}}+\frac{c^2bv(t)}{12m_q m_{\bar{q}}}-b\right]+\frac{1}{r}\left[\frac{4a_v v(ss)}{3m_q m_{\bar{q}}}+\frac{(3a_v-a_S)v(ls)}{2m_q m_{\bar{q}}}+\frac{a_v v(t)}{12m_q m_{\bar{q}}}\right]-\frac{dv(ls)}{m_q m_{\bar{q}}}+ar+dr^2 \tag{8}$$

where

$$v(ss)=\left[\vec{S_q}\bullet\vec{S_{\bar{q}}}\right],\quad v(ls)=\left[\vec{L}\bullet\vec{S}\right],\quad v(t)=-2\left[\vec{S_q}\bullet\vec{S_{\bar{q}}}-3\left(\vec{S_q}\bullet\frac{\vec{r}}{|r|}\right)\left(\vec{S_{\bar{q}}}\bullet\frac{\vec{r}}{|r|}\right)\right],\quad a_S+a_v=a \tag{9}$$

By using natural units and substitute Eq. (8) into Eq. (1), we obtain

$$\frac{d^2R}{dr^2}+\left[2\mu E-\frac{2\mu f_1 e^{-cr}}{r^3}-\frac{2\mu f_2 e^{-cr}}{r^2}-\frac{2\mu f_3 e^{-cr}}{r}-\frac{2\mu f_4}{r}+2\mu f_5-2\mu ar-2\mu dr^2-\frac{l(l+1)}{r^2}\right]R=0 \tag{10}$$

where

$$f_1=\frac{3bv(ls)}{2m_q m_{\bar{q}}}+\frac{bv(t)}{4m_q m_{\bar{q}}}=\frac{b}{4m_q m_{\bar{q}}}[6v(ls)+v(t)],\qquad f_5=\frac{dv(ls)}{m_q m_{\bar{q}}} \tag{11}$$

$$f_2=\frac{3cbv(ls)}{2m_q m_{\bar{q}}}+\frac{cbv(t)}{4m_q m_{\bar{q}}}=\frac{bc}{4m_q m_{\bar{q}}}[6v(ls)+v(t)]=cf_1 \tag{12}$$

$$f_3=\frac{-2c^2bv(ss)}{3m_q m_{\bar{q}}}+\frac{c^2bv(t)}{12m_q m_{\bar{q}}}-b=\frac{bc^2}{12m_q m_{\bar{q}}}[-8v(ss)+v(t)]-b \tag{13}$$

$$f_4=\frac{4a_v v(ss)}{3m_q m_{\bar{q}}}+\frac{(3a_v-a_S)v(ls)}{2m_q m_{\bar{q}}}+\frac{a_v v(t)}{12m_q m_{\bar{q}}}=\frac{1}{12m_q m_{\bar{q}}}[16a_v v(ss)+6(3a_v-a_S)v(ls)+a_v v(t)] \tag{14}$$

We transform the exponential form into summation by Taylor series around zero

$$e^{-cr}=\sum_{i=0}^{O}\frac{(-1)^i c^i}{i!}r^i \tag{15}$$

Substituting Eq. (15) in Eq. (10) and rearranging it, we get

$$\frac{d^2R}{dr^2}+\left[h_4-\frac{h_1}{r^3}+\frac{h_2}{r^2}+\frac{h_3}{r}-Ar-Dr^2+\sum_{i=0}^{O}M_i r^i\right]R=0 \tag{16}$$

where

$$\epsilon=2\mu E,\quad 2\mu f_1=h_1,\quad 2\mu f_1 c-2\mu f_2-l(l+1)=h_2 \tag{17}$$

$$2\mu f_2 c-\frac{2\mu f_1 c^2}{2}-2\mu f_3-2\mu f_4=h_3,\quad 2\mu f_5+\epsilon=h_4,\quad 2\mu d=D \tag{18}$$

$$2\mu a=A,\quad 2\mu\frac{(-1)^i c^{i+1}}{(i+1)!}\left[\frac{-f_1c^2}{(i+3)}+f_3\right]=M_i \tag{19}$$

Let $x=\frac{1}{r}$ and by substituting in Eq. (16), we get

$$\frac{d^2R}{dx^2}+\frac{2}{x}\frac{dR}{dx}+\frac{1}{x^2}\left[\frac{h_4}{x^2}-\frac{D}{x^4}-\frac{A}{x^3}-h_1x+h_2+\frac{h_3}{x}+\sum_{i=0}^{O}M_i x^{-i-2}\right]R=0 \tag{20}$$

Due to the singularity point in Eq. (20), we put $y+\delta=x$, and by using the Taylor's series to expand to second order terms then change again to equation in x variable, one obtains

$$\frac{d^2R}{dx^2}+\frac{2}{x}\frac{dR}{dx}+\frac{1}{x^2}\left[-q+wx-zx^2\right]R=0 \tag{21}$$

where

$$-q=\sum_{i=0}^{O}\frac{M_i(i+4)(i+3)}{2\delta^{i+2}}+h_2+\frac{3h_3}{\delta}+\frac{6h_4}{\delta^2}-\frac{15D}{\delta^4}-\frac{10A}{\delta^3} \tag{22}$$

$$-w=\sum_{i=0}^{O}\frac{M_i(i+2)(i+4)}{\delta^{i+3}}+h_1+\frac{3h_3}{\delta^2}+\frac{8h_4}{\delta^3}-\frac{24D}{\delta^5}-\frac{15A}{\delta^4} \tag{23}$$

$$-z=\sum_{i=0}^{O}\frac{M_i(i+2)(i+3)}{2\delta^{i+4}}+\frac{h_3}{\delta^3}+\frac{3h_4}{\delta^4}-\frac{10D}{\delta^6}-\frac{6A}{\delta^5} \tag{24}$$

One can use the Nikiforov-Uvarov method (NU) for equation (21) to get the eigenvalue and the eigenfunction equations.

The eigenfunction equation is

$$R(r)=N_{nls}\ r^{1+\sqrt{\left(q+\frac{1}{4}\right)}}\,e^{\frac{\sqrt{z}}{r}}L_n^{1+2\sqrt{\left(q+\frac{1}{4}\right)}}\left(\frac{2\sqrt{z}}{r}\right) \tag{25}$$

where $L_n^{2\sqrt{\left(q+\frac{1}{4}\right)}}\left(\frac{2\sqrt{z}}{r}\right)$ is the Rodrigues's formula of the associated Laguerre polynomial and N_{nls} is a normalization constant.

The eigen value equation is

$$E=E_1-\frac{1}{6\mu}\left[\frac{E_2}{2n+1+2\sqrt{E_3+\left(l+\frac{1}{2}\right)^2}}\right]^2 \tag{26}$$

where

$$E_1=\frac{10d}{3\delta^2}+\frac{2a}{\delta}-f_5-\frac{\left(f_2c-\frac{f_1c^2}{2}-f_3-f_4\right)\delta}{3}-\frac{1}{6}\sum_{i=0}^{i}\frac{(-1)^i c^{i+1}(i+2)(i+3)}{(i+1)!\delta^i}\left[\frac{-f_1c^2}{(i+3)}+f_3\right] \tag{27}$$

Table 1
Parameter values for each system.

Variables Systems	m_q	$m_{\bar{q}}$	r_0	as	av	b	c	d
Units	GeV	GeV	GeV^{-1}	GeV^2	GeV^2	–	GeV	GeV^3
c quark	1.317	1.317	28.6261	0.2572	0.07565	433.7577	0.07217	−0.0056
b quark	4.584	4.584						

Table 2
Charmonia mass spectrum of S and P-states in GeV.

Level	Present work	[1]	[68]	[69]	[27]	[70]	[17]	[71]	[72]	[10]	[58]	PDG [67]
1^1S_0	3.0413	2.93	2.981	2.984	2.989	2.979	2.980	2.980	2.982	3.088	2.979	2.984 (1.92%)
1^3S_1	3.1404	3.11	3.096	3.097	3.094	3.097	3.097	3.097	3.090	3.168	3.096	3.097 (1.39%)
2^1S_0	3.6610	3.68	3.635	3.637	3.602	3.623	3.597	3.633	3.630	3.669	3.600	3.639 (0.61%)
2^3S_1	3.7017	3.68	3.685	3.679	3.681	3.673	3.685	3.690	3.672	3.707	3.680	3.686 (0.43%)
3^1S_0	4.1347	–	3.989	4.004	4.058	3.991	4.014	3.992	4.043	4.067	4.011	–
3^3S_1	4.0502	3.80	4.039	4.030	4.129	4.022	4.095	4.030	4.072	4.094	4.077	4.039 (0.28%)
4^1S_0	4.4136	–	4.401	4.264	4.448	4.250	4.433	4.244	4.384	4.398	4.397	–
4^3S_1	4.4185	–	4.427	4.281	4.514	4.273	4.477	4.273	4.406	4.420	4.454	4.421 (0.06%)
5^1S_0	4.6618	–	4.811	4.459	4.799	4.446	–	4.440	–	–	–	–
5^3S_1	4.6591	–	4.837	4.472	4.863	4.463	–	4.464	–	–	–	–
6^1S_0	4.8825	–	5.155	–	5.124	4.595	–	4.601	–	–	–	–
6^3S_1	4.8801	–	5.167	–	5.185	4.608	–	4.621	–	–	–	–
1^3P_0	3.4137	3.32	3.413	3.415	3.428	3.433	3.416	3.392	3.424	3.448	3.488	3.415 (0.04%)
1^3P_1	3.5036	3.49	3.511	3.521	3.468	3.510	3.508	3.491	3.505	3.520	3.514	3.511 (0.21%)
1^1P_1	3.5180	3.43	3.525	3.526	3.470	3.519	3.527	3.524	3.516	3.536	3.536	3.525 (0.20%)
1^3P_2	3.4888	3.55	3.555	3.553	3.480	3.556	3.558	3.570	3.556	3.564	3.565	3.556 (0.25%)
2^3P_0	3.7646	3.83	3.870	3.848	3.897	3.842	3.844	3.845	3.852	3.870	3.947	3.918 (0.51%)
2^3P_1	3.8072	3.67	3.906	3.914	3.938	3.901	3.940	3.902	3.925	3.934	3.972	–
2^1P_1	3.8239	3.75	3.926	3.916	3.943	3.908	3.960	3.922	3.934	3.950	3.996	–
2^3P_2	3.9151	–	3.949	3.937	3.955	3.937	3.994	3.949	3.972	3.976	4.021	3.927 (0.30%)
3^3P_0	4.0804	–	4.301	4.146	4.296	4.131	–	4.192	4.202	4.214	–	–
3^3P_1	4.1210	3.91	4.319	4.192	4.338	4.178	–	4.178	4.271	4.275	–	–
3^1P_1	4.1368	–	4.337	4.193	4.344	4.184	–	4.137	4.279	4.291	–	–
3^3P_2	4.1514	–	4.354	4.211	4.358	4.208	–	4.212	4.317	4.316	–	–
4^3P_0	4.3621	–	4.698	–	4.653	–	–	–	–	–	–	–
4^3P_1	4.4005	–	4.728	–	4.696	–	–	–	–	–	–	–
4^1P_1	4.4155	–	4.744	–	4.704	–	–	–	–	–	–	–
4^3P_2	4.4298	–	4.763	–	4.718	–	–	–	–	–	–	–
5^3P_0	4.6129	–	–	–	4.983	–	–	–	–	–	–	–
5^3P_1	4.6493	–	–	–	5.026	–	–	–	–	–	–	–
5^1P_1	4.6636	–	–	–	5.034	–	–	–	–	–	–	–

$$E_2=\frac{24D}{\delta^3}+\frac{15A}{\delta^2}-2\mu\sum_{i=0}^{i}\frac{(-1)^i c^{i+1}(i+2)(i+4)}{(i+1)!\delta^{i+1}}\left[\frac{-f_1c^2}{(i+3)}+f_3\right]-3h_3-\frac{8(2\mu f_5+\epsilon)}{\delta}-h_1\delta^2 \tag{28}$$

$$E_3=\frac{15D}{\delta^4}+\frac{10A}{\delta^3}-2\mu\sum_{i=0}^{i}\frac{(-1)^i c^{i+1}(i+4)(i+3)}{2(i+1)!\delta^{i+2}}\left[\frac{-f_1c^2}{(i+3)}+f_3\right]-\frac{3h_3}{\delta}-\frac{6(2\mu f_5+\epsilon)}{\delta^2} \tag{29}$$

n is a principal quantum number

Similarly, l is an orbital quantum number

Let $r_0=\frac{1}{\delta}$, where r_0 is the characteristic radius (the minimum distance between two particles in system). We use equation (15) to change the form of Eqs. (29), (28), (27). Therefore, the mass spectra become

$$M(q\bar{q})=E_1+m_q+m_{\bar{q}}-\frac{1}{6\mu}\left[\frac{E_2}{2n+1+2\sqrt{E_3+\left(l+\frac{1}{2}\right)^2}}\right]^2 \tag{30}$$

where

$$E_1=\frac{10d}{3}r_0^2+2(a_v+a_S)r_0-\frac{dv(ls)}{m_qm_{\bar{q}}}+\frac{1}{36m_qm_{\bar{q}}r_0}[16a_vv(ss)+6(3a_v-a_S)v(ls)+a_vv(t)]-\frac{bce^{-cr_o}}{6}\left[\frac{[v(t)-8v(ss)]c^2}{12m_qm_{\bar{q}}}-1\right]\left[4-cr_o-\frac{2}{cr_o}\right]+\frac{bc^2e^{-cr_o}}{24m_qm_{\bar{q}}}[6v(ls)+v(t)]\left[c-\frac{1}{r_0}\right] \tag{31}$$

$$E_2=2\mu\Bigg[24dr_0^3+15(a_v+a_S)r_0^2+\frac{[16a_vv(ss)+6(3a_v-a_S)v(ls)+a_vv(t)]}{4m_qm_{\bar{q}}}-8\left[E+\frac{dv(ls)}{m_qm_{\bar{q}}}\right]r_0-e^{-cr_o}b\left[\frac{[-8v(ss)+v(t)]c^2}{12m_qm_{\bar{q}}}-1\right]\left[5cr_0-c^2r_0^2-3\right]+\frac{bc^2e^{-cr_o}[6v(ls)+v(t)]}{4m_qm_{\bar{q}}}\left[cr_0-2-\frac{1}{r_0c}-\frac{1}{c^2r_0^2}\right]\Bigg] \tag{32}$$

$$E_3=2\mu\Bigg[15dr_0^4+10(a_v+a_S)r_0^3+\frac{r_0}{4m_qm_{\bar{q}}}[16a_vv(ss)+6(3a_v-a_S)v(ls)+a_vv(t)]-6r_0^2\left[E+\frac{dv(ls)}{m_qm_{\bar{q}}}\right]-\frac{r_0e^{-cr_o}b}{2}\left[\frac{[-8v(ss)+v(t)]c^2}{12m_qm_{\bar{q}}}-1\right]\left[6cr_0-c^2r_0^2-6\right]+\frac{bc^2r_0e^{-cr_o}}{8m_qm_{\bar{q}}}[6v(ls)+v(t)][cr_0-3]\Bigg] \tag{33}$$

The energy eigenvalue Eq. (30) has spin-orbital-tensor coefficients

Table 3
Mass spectrum of the charmonia for D and F-states in GeV.

Level	Present work	[68]	[71]	[72]	[69]	[1]	[27]	[70]	[17]	[10]	[58]
1^3D_3	3.51402	3.813	3.844	3.806	3.808	3.869	3.755	3.799	3.831	3.809	3.798
1^1D_2	3.47795	3.807	3.802	3.799	3.805	3.739	3.765	3.796	3.824	3.803	3.796
1^3D_2	3.46047	3.795	3.788	3.800	3.807	3.550	3.772	3.798	3.824	3.804	3.794
1^3D_1	3.40228	3.783	3.729	3.785	3.792	–	3.775	3.787	3.804	3.789	3.792
2^3D_3	3.863	4.220	4.132	4.167	4.112	3.806	4.176	4.103	4.202	4.167	4.425
2^1D_2	3.82825	4.196	4.105	4.158	4.108	–	4.182	4.099	4.191	4.158	4.224
2^3D_2	3.81161	4.190	4.095	4.158	4.109	–	4.188	4.100	4.189	4.159	4.223
2^3D_1	3.75606	4.105	4.057	4.142	4.095	–	4.188	4.089	4.164	4.143	4.222
3^3D_3	4.17431	4.574	4.351	–	4.340	–	4.549	4.331	–	–	–
3^1D_2	4.14085	3.549	4.330	–	4.336	–	4.553	4.326	–	–	–
3^3D_2	4.12502	4.544	4.322	–	4.337	–	4.557	4.327	–	–	–
3^3D_1	4.07208	4.507	4.293	–	4.324	–	4.555	4.317	–	–	–
4^3D_3	4.45154	4.920	4.526	–	–	–	4.890	–	–	–	–
4^1D_2	4.41933	4.898	4.509	–	–	–	4.892	–	–	–	–
4^3D_2	4.40431	4.896	4.504	–	–	–	4.896	–	–	–	–
4^3D_1	4.35388	4.857	4.480	–	–	–	4.891	–	–	–	–
1^3F_2	3.38961	4.041	–	4.029	–	–	3.990	–	4.068	–	–
1^3F_3	3.46746	4.068	–	4.029	–	3.999	4.012	–	4.070	–	–
1^1F_3	3.48494	4.071	–	4.026	–	4.037	4.017	–	4.066	–	–
1^3F_4	3.54185	4.093	–	4.021	–	–	4.036	–	4.062	–	–
2^3F_2	3.74376	4.361	–	4.351	–	–	4.378	–	–	–	–
2^3F_3	3.81815	4.400	–	3.352	–	–	4.396	–	–	–	–
2^1F_3	3.83479	4.406	–	4.350	–	–	4.400	–	–	–	–
2^3F_4	3.88949	4.434	–	4.348	–	–	4.415	–	–	–	–
3^3F_2	4.06014	–	–	–	–	–	4.730	–	–	–	–
3^3F_3	4.13113	–	–	–	–	–	4.746	–	–	–	–
3^1F_3	4.14695	–	–	–	–	–	4.749	–	–	–	–
3^3F_4	4.1995	–	–	–	–	–	4.761	–	–	–	–

Table 4
Mass spectrum of the bottomonia for S and P-states in GeV.

Level	Present work	[73]	[68]	[69]	[1]	[76]	[62]	[71]	[74]	PDG [67]
1^1S_0	9.43601	9.402	9.398	9.390	9.414	9.389	9.393	9.392	9.455	9.398 (0.41%)
1^3S_1	9.49081	9.465	9.460	9.460	9.490	9.460	9.460	9.460	9.502	9.460 (0.33%)
2^1S_0	9.99146	9.976	9.990	9.990	9.987	9.987	9.987	9.991	9.990	9.999 (0.075%)
2^3S_1	10.01257	10.003	10.023	10.015	10.089	10.016	10.023	10.024	10.015	10.023 (0.1%)
3^1S_0	10.1386	10.336	10.329	10.326	–	10.330	10.345	10.323	10.330	–
3^3S_1	10.32775	10.354	10.355	10.343	10.327	10.351	10.364	10.346	10.349	10.355 (0.26%)
4^1S_0	10.3236	10.523	10.573	10.584	–	10.595	10.623	10.558	–	–
4^3S_1	10.5461	10.635	10.586	10.597	–	10.611	10.643	10.575	10.607	10.579 (0.31%)
5^1S_0	10.4977	10.869	10.851	10.800	–	10.817	–	10.741	–	–
5^3S_1	10.82628	10.878	10.869	10.811	–	10.831	–	10.755	10.818	10.876 (0.46%)
6^1S_0	10.6615	11.097	11.061	10.997	–	11.011	–	10.892	–	–
6^3S_1	10.97061	11.102	11.088	10.988	–	10.988	–	10.904	10.995	11.019 (0.44%)
1^3P_0	9.8432	9.847	9.859	9.864	9.815	9.865	9.861	9.862	9.855	9.859 (0.16%)
1^3P_1	9.87371	9.876	9.892	9.903	9.842	9.897	9.891	9.888	9.874	9.893 (0.195%)
1^1P_1	9.87919	9.882	9.900	9.909	9.806	9.903	9.900	9.896	9.879	9.899 (0.2%)
1^3P_2	9.89083	9.897	9.912	9.921	9.906	9.918	9.912	9.908	9.886	9.912 (0.214%)
2^3P_0	10.19625	10.226	10.233	10.220	10.254	10.226	10.230	10.241	10.221	10.232 (0.35%)
2^3P_1	10.21695	10.246	10.255	10.249	10.120	10.251	10.255	10.256	10.236	10.255 (0.37%)
2^1P_1	10.22153	10.250	10.260	10.254	10.154	10.256	10.262	10.261	10.240	10.260 (0.38%)
2^3P_2	10.22961	10.261	10.268	10.264	–	10.269	10.271	10.268	10.246	10.269 (0.38%)
3^3P_0	10.1342	10.552	10.521	10.490	–	10.502	–	10.511	10.500	–
3^3P_1	10.1378	10.538	10.541	10.515	10.303	10.524	–	10.507	10.513	–
3^1P_1	4.14695	10.541	10.544	10.519	–	10.529	–	10.497	10.516	–
3^3P_2	10.1405	10.550	10.550	10.528	–	10.540	–	10.516	10.521	–
4^3P_0	10.3193	10.775	10.781	–	–	10.732	–	–	–	–
4^3P_1	10.3229	10.788	10.802	–	–	10.753	–	–	–	–
4^1P_1	10.3242	10.790	10.804	–	–	10.757	–	–	–	–
4^3P_2	10.3255	10.798	10.812	–	–	10.767	–	–	–	–
5^3P_0	10.4936	11.004	–	–	–	10.933	–	–	–	–
5^3P_1	10.497	11.014	–	–	–	10.951	–	–	–	–
5^1P_1	10.4983	11.016	–	–	–	10.955	–	–	–	–

$v(ss)$, $v(sl)$, $v(t)$ [14,51–66]. Furthermore, it has potential parameters (a_S, a_v, b, d, c) and r_0 due to the expansion, so we have six parameters of the eigenvalue equation which can be obtained from the experimental data by best fitting.

Results and Discussions

In Table 1, the potential parameters are shown for the systems under consideration. We use spectroscopic notation for the levels ($n^{2S+1}L_J$). S is the total spin of the system, L is the orbital quantum number, n is the principal quantum number, J is the total (orbital + spin) quantum

Table 5
Mass spectrum of the bottomonia for D and F-states in GeV.

Level	Present work	[73]	[68]	[69]	[1]	[76]	[62]	[71]	[74]	PDG [67]
1^3D_3	9.73855	10.115	10.166	10.157	10.232	10.156	10.163	10.177	10.127	–
1^1D_2	9.7355	10.148	10.163	10.153	10.194	10.152	10.158	10.166	10.123	–
1^3D_2	10.1126	10.147	10.161	10.153	10.145	10.151	10.157	10.162	10.122	10.163 (0.5%)
1^3D_1	9.72905	10.138	10.154	10.146	–	10.145	10.149	10.147	10.117	–
2^3D_3	9.94704	10.455	10.449	10.436	–	10.442	10.456	10.447	10.422	–
2^1D_2	9.94405	10.450	10.445	10.432	–	10.439	10.452	10.440	10.419	–
2^3D_2	9.94259	10.449	10.443	10.432	–	10.438	10.450	10.437	10.418	–
2^3D_1	9.93775	10.441	10.435	10.425	–	10.432	10.443	10.428	10.414	–
3^3D_3	10.1435	10.711	10.717	–	–	10.680	–	10.652	–	–
3^1D_2	10.1405	10.706	10.713	–	–	10.677	–	10.646	–	–
3^3D_2	10.1391	10.705	10.711	–	–	10.676	–	10.645	–	–
3^3D_1	10.1344	10.698	10.704	–	–	10.670	–	10.637	–	–
4^3D_3	10.3284	10.939	10.963	–	–	10.886	–	10.817	–	–
4^1D_2	10.3255	10.935	10.959	–	–	10.883	–	10.813	–	–
4^3D_2	10.3241	10.934	10.957	–	–	10.882	–	10.811	–	–
4^3D_1	10.3195	10.928	10.949	–	–	10.877	–	10.811	–	–
1^3F_2	9.72948	10.350	10.343	10.338	–	–	10.353	–	10.315	–
1^3F_3	9.7361	10.355	10.346	10.340	10.302	–	10.356	–	10.321	–
1^1F_3	9.73759	10.355	10.347	10.339	10.319	–	10.356	–	10.322	–
1^3F_4	9.74242	10.358	10.349	10.340	–	–	10.357	–	–	–
2^3F_2	9.93815	10.615	10.610	–	–	–	10.610	–	–	–
2^3F_3	9.94462	10.619	10.614	–	–	–	10.613	–	–	–
2^1F_3	9.94608	10.619	10.647	–	–	–	10.613	–	–	–
2^3F_4	9.9508	10.622	10.617	–	–	–	10.615	–	–	–
3^3F_2	10.1348	10.850	–	–	–	–	–	–	–	–
3^3F_3	10.1411	10.853	–	–	–	–	–	–	–	–
3^1F_3	10.1425	10.853	–	–	–	–	–	–	–	–
3^3F_4	10.1471	10.856	–	–	–	–	–	–	–	–

Table 6
B_c Meson mass spectrum of S and P-states in GeV.

Level	Present work	[27]	[60]	[68]	[75]	[16]	PDG [67]
1^1S_0	6.31528	6.272	6.278	6.272	6.271	6.275	6.275 (0.64%)
1^3S_1	6.62256	6.321	6.331	6.333	6.338	6.314	–
2^1S_0	6.85048	6.864	6.863	6.842	6.855	6.838	6.842 (0.124%)
2^3S_1	6.91694	6.900	6.873	6.882	6.887	6.850	–
3^1S_0	7.18692	7.306	7.244	7.226	7.250	–	–
3^3S_1	7.18587	7.338	7.249	7.258	7.272	–	–
4^1S_0	7.43221	7.684	7.564	7.585	–	–	–
4^3S_1	7.43125	7.714	7.568	7.609	–	–	–
5^1S_0	7.65579	8.025	7.852	7.928	–	–	–
5^3S_1	7.6549	8.054	7.855	7.947	–	–	–
6^1S_0	7.85941	8.340	8.120	–	–	–	–
6^3S_1	7.85861	8.368	8.122	–	–	–	–
1^3P_0	6.60714	6.686	6.748	6.699	6.706	6.672	–
1^3P_1	6.62021	6.705	6.767	6.750	6.741	6.766	–
1^1P_1	6.62531	6.706	6.769	6.743	6.750	6.828	–
1^3P_2	6.6298	6.712	6.775	6.761	6.768	6.776	–
2^3P_0	6.90206	7.146	7.139	7.094	7.122	6.914	–
2^3P_1	6.91462	7.165	7.155	7.134	7.145	7.259	–
2^1P_1	6.91952	7.168	7.156	7.094	7.150	7.322	–
2^3P_2	6.92391	7.173	7.162	7.157	7.164	7.232	–
3^3P_0	7.17152	7.536	7.463	7.474	–	–	–
3^3P_1	7.18358	7.555	7.479	7.510	–	–	–
3^1P_1	7.18829	7.559	7.479	7.500	–	–	–
3^3P_2	7.19258	7.565	7.485	7.524	–	–	–
4^3P_0	7.41742	7.885	–	7.817	–	–	–
4^3P_1	7.42899	7.905	–	7.853	–	–	–
4^1P_1	7.43351	7.908	–	7.844	–	–	–
4^3P_2	7.4377	7.915	–	7.867	–	–	–
5^3P_0	7.64159	8.207	–	–	–	–	–
5^3P_1	7.65268	8.226	–	–	–	–	–
5^1P_1	7.65702	8.230	–	–	–	–	–

Table 7
B_c Meson mass spectrum of D and F-states in GeV.

Level	Present work	[27]	[60]	[68]	[75]	[16]
1^3D_3	6.63884	6.990	7.026	7.029	7.045	6.980
1^1D_2	6.62836	6.994	7.035	7.026	7.041	7.009
1^3D_2	6.62327	6.997	7.025	7.025	7.036	7.154
1^3D_1	6.60632	6.998	7.030	7.021	7.028	7.078
2^3D_3	6.93259	7.399	7.363	7.405	–	–
2^1D_2	6.92242	7.401	7.370	7.400	–	–
2^3D_2	6.91752	7.403	7.361	7,399	–	–
2^3D_1	6.9012	7.403	7.365	7.392	–	–
3^3D_3	7.20091	7.761	–	7.750	–	–
3^1D_2	7.19104	7.762	–	7.743	–	–
3^3D_2	7.18633	7.764	–	7.741	–	–
3^3D_1	7.17061	7.762	–	7.732	–	–
4^3D_3	7.44569	8.092	–	–	–	–
4^1D_2	7.43611	8.093	–	–	–	–
4^3D_2	7.43159	8.094	–	–	–	–
4^3D_1	7.41647	8.091	–	–	–	–
1^3F_2	6.60519	7.234	–	7.273	7.269	–
1^3F_3	6.62785	7.242	–	7.269	7.276	–
1^1F_3	6.63295	7.241	–	7.268	7.266	–
1^3F_4	6.64949	7.244	–	7.277	7.271	–
2^3F_2	6.90002	7.607	–	7.618	–	–
2^3F_3	6.92188	7.615	–	7.616	–	–
2^1F_3	6.92678	7.614	–	7.615	–	–
2^3F_4	6.94281	7.617	–	7.617	–	–
3^3F_2	7.1694	7.946	–	–	–	–
3^3F_3	7.19049	7.954	–	–	–	–
3^1F_3	7.19517	7.953	–	–	–	–
3^3F_4	7.2107	7.956	–	–	–	–

number. By utilizing Eq. (30) and Table 1, we get the mass spectra of the different quantum states (present work or the theoretical states) as shown in Tables 2–7. We notice from the tables that the average of the chi-squared values is 0.000793 for the charmonium system, 0.00012 for the bottomonium system and 0.00013 for the Bc meson system. Beside of the small percentage errors (between two brackets) in the last column the experimental values are given (PDG [67]). Our work is better in comparison with available experimental than other values in the literatures [1,10,16,17,27,58,60,62,69–75].

Conclusions

In the present work, the quarkonium systems are described by solving the nonrelativistic Schrödinger equation with relativistic corrections by the Nikiforov-Uvarov (NU) method to obtain a quantitative description of these systems. Although, the Nikiforov-Uvarov (NU) method is mathematical tool but with physical significant approach give physical equation for Schrodinger equation. We notice from our comparison with experimental data that our theoretical states agree with experimental data in different ranges of spin levels for all quarkonium systems. In case our comparison with other theoretical researches are convenient with different Quantitative and qualitative description. There is an improvement of fitting to the available experimental data in addition to other theoretical states in the present work. In the future it is expected to compare with new experimental data.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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