

The Impairments Effect on The DPD Coefficients Mathematical Relationships

As mentioned previously, u and v are two zero mean Gaussian random variables with variance σ^2 . For any non-negative integer P , the expected value of u^P or v^P are equal ($E(v^P) = E(u^P)$) and is obtained in the following equation:

$$E(u^P) = \begin{cases} 0 & \text{if } P \text{ is odd} \\ \sigma^P (P-1)!! & \text{if } P \text{ is even} \end{cases} \quad (1)$$

where $n!!$ denotes the double factorial, that is, the product of all numbers from n to 1 that have the same parity (even or odd) as n .

$$|y|^2 = a_{0,0}^2 (u^2 + v^2) + 2a_{0,0}a_{2,0} (u^4 + 2u^2v^2 + v^4) + a_{2,0}^2 (u^6 + 3u^4v^2 + 3u^2v^4 + v^6) \quad (2)$$

$$|y|^4 = a_{0,0}^4 (u^4 + 2u^2v^2 + v^4) + 4a_{0,0}^3a_{2,0} (u^6 + 3u^4v^2 + 3u^2v^4 + v^6) + 6a_{0,0}^2a_{2,0}^2 (u^8 + 4u^6v^2 + 6u^4v^4 + 4u^2v^6 + v^8) + 4a_{0,0}a_{2,0}^3 (u^{10} + 5u^8v^2 + 10u^6v^4 + 10u^4v^6 + 5u^2v^8 + v^{10}) + a_{2,0}^4 (u^{12} + 6u^{10}v^2 + 15u^8v^4 + 20u^6v^6 + 15u^4v^8 + 6u^2v^{10} + v^{12}) \quad (3)$$

$$|y|^6 = a_{0,0}^6 (u^6 + 3u^4v^2 + 3u^2v^4 + v^6) + 6a_{0,0}^5a_{2,0} (u^8 + 4u^6v^2 + 6u^4v^4 + 4u^2v^6 + v^8) + 15a_{0,0}^4a_{2,0}^2 (u^{10} + 5u^8v^2 + 10u^6v^4 + 10u^4v^6 + 5u^2v^8 + v^{10}) + 20a_{0,0}^3a_{2,0}^3 (u^{12} + 6u^{10}v^2 + 15u^8v^4 + 20u^6v^6 + 15u^4v^8 + 6u^2v^{10} + v^{12}) + 15a_{0,0}^2a_{2,0}^4 (u^{14} + 7u^{12}v^2 + 21u^{10}v^4 + 35u^8v^6 + 35u^6v^8 + 21u^4v^{10} + 7u^2v^{12} + v^{14}) + 6a_{0,0}a_{2,0}^5 (u^{16} + 8u^{14}v^2 + 28u^{12}v^4 + 56u^{10}v^6 + 70u^8v^8 + 56u^6v^{10} + 28u^4v^{12} + 8u^2v^{14} + v^{16}) + a_{2,0}^6 (u^{18} + 9u^{16}v^2 + 36u^{14}v^4 + 84u^{12}v^6 + 126u^{10}v^8 + 126u^8v^{10} + 84u^6v^{12} + 36u^4v^{14} + 9u^2v^{16} + v^{18}) \quad (4)$$

$$z[n]y^*[n] | y[n] |^2 = a_{0,0}^3 (u^4 + 2u^2v^2 + v^4) + 3a_{0,0}^2a_{2,0} (u^6 + 3u^4v^2 + 3u^2v^4 + v^6) + 3a_{0,0}a_{2,0}^2 (u^8 + 4u^6v^2 + 6u^4v^4 + 4u^2v^6 + v^8) + a_{2,0}^3 (u^{10} + 5u^8v^2 + 10u^6v^4 + 10u^4v^6 + 5u^2v^8 + v^{10}) \quad (5)$$

The third order DPD coefficients mathematical relationships without the system impairments are obtained as the following:

$$\mathbf{d} = \begin{bmatrix} d_0 \\ 0 \\ d_2 \end{bmatrix} = \begin{bmatrix} F_1/\omega_1 \\ 0 \\ G_1/\omega_1 \end{bmatrix} \quad (6)$$

where F_1 , G_1 , and ω_1 are obtained in the following equation.

$$F_1 = a_{0,0}^7 + 72 a_{0,0}^6 a_{2,0} \sigma^2 + 2448 a_{0,0}^5 a_{2,0}^2 \sigma^4 + 49920 a_{0,0}^4 a_{2,0}^3 \sigma^6 + 648576 a_{0,0}^3 a_{2,0}^4 \sigma^8 \\ + 5253120 a_{0,0}^2 a_{2,0}^5 \sigma^{10} + 23592960 a_{0,0} a_{2,0}^6 \sigma^{12} + 40919040 a_{2,0}^7 \sigma^{14} \quad (7)$$

$$G_1 = - (a_{0,0}^4 a_{2,0} + 36 a_{0,0}^3 a_{2,0}^2 \sigma^2 + 504 a_{0,0}^2 a_{2,0}^3 \sigma^4 + 3072 a_{0,0} a_{2,0}^4 \sigma^6 + 5760 a_{2,0}^5 \sigma^8) \quad (8)$$

$$\omega_1 = a_{0,0}^8 + 72 a_{0,0}^7 a_{2,0} \sigma^2 + 2520 a_{0,0}^6 a_{2,0}^2 \sigma^4 + 54528 a_{0,0}^5 a_{2,0}^3 \sigma^6 + 778752 a_{0,0}^4 a_{2,0}^4 \sigma^8 \\ + 7326720 a_{0,0}^3 a_{2,0}^5 \sigma^{10} + 43084800 a_{0,0}^2 a_{2,0}^6 \sigma^{12} + 141557760 a_{0,0} a_{2,0}^7 \sigma^{14} + 212336640 a_{2,0}^8 \sigma^{16} \quad (9)$$

The effect of the system impairments, as additive noise and practical ADC (quantization noise error), on the DPD coefficients mathematical relationships are considered. The DPD coefficients relationships become:

$$\mathbf{d}_m = \begin{bmatrix} d_{0m} \\ 0 \\ d_{2m} \end{bmatrix} = \begin{bmatrix} F_{1m}/\omega_{1m} \\ 0 \\ G_{1m}/\omega_{1m} \end{bmatrix} \quad (10)$$

where F_{1m} , ω_{1m} , and G_{1m} are obtained in equations (11), (12) and (13) respectively on the upper of the next pages. Note that, these equations agree with equations in (7), (8), and (9) at $\alpha = 1$ and $\sigma_w^2 = 0$

$$\begin{aligned}
F_{1m} = & 3a_0^7\alpha^3\sigma^8 - 9a_0^7\alpha^2\sigma^8 + 9a_0^7\alpha\sigma^8 - 2a_0^7\sigma^8 + 84a_0^6a_2\alpha^3\sigma^{10} - 196a_0^6a_2\alpha^2\sigma^{10} + 240a_0^6a_2\alpha\sigma^{10} \\
& - 56a_0^6a_2\sigma^{10} + 1632a_0^5a_2^2\alpha^3\sigma^{12} - 1336a_0^5a_2^2\alpha^2\sigma^{12} + 2872a_0^5a_2^2\alpha\sigma^{12} - 720a_0^5a_2^2\sigma^{12} + 9a_0^5\alpha^3\sigma^6 \\
& \sigma_w^2 - 3a_0^5\alpha^3\sigma^6 - 27a_0^5\alpha^2\sigma^6\sigma_w^2 + 9a_0^5\alpha^2\sigma^6 + 27a_0^5\alpha\sigma^6\sigma_w^2 - 9a_0^5\alpha\sigma^6 - 6a_0^5\sigma^6\sigma_w^2 + 3a_0^5\sigma^6 + 31616 \\
& a_0^4a_2^3\alpha^3\sigma^{14} + 4192a_0^4a_2^3\alpha^2\sigma^{14} + 19552a_0^4a_2^3\alpha\sigma^{14} - 5440a_0^4a_2^3\sigma^{14} + 180a_0^4a_2\alpha^3\sigma^8\sigma_w^2 - 60a_0^4a_2 \\
& \alpha^3\sigma^8 - 484a_0^4a_2\alpha^2\sigma^8\sigma_w^2 + 180a_0^4a_2\alpha^2\sigma^8 + 516a_0^4a_2\alpha\sigma^8\sigma_w^2 - 180a_0^4a_2\alpha\sigma^8 - 120a_0^4a_2\sigma^8\sigma_w^2 + \\
& 60a_0^4a_2\sigma^8 + 471488a_0^3a_2^4\alpha^3\sigma^{16} + 122752a_0^3a_2^4\alpha^2\sigma^{16} + 80320a_0^3a_2^4\alpha\sigma^{16} - 25984a_0^3a_2^4\sigma^{16} + 1584 \\
& a_0^3a_2^2\alpha^3\sigma^{10}\sigma_w^2 - 528a_0^3a_2^2\alpha^3\sigma^{10} - 3296a_0^3a_2^2\alpha^2\sigma^{10}\sigma_w^2 + 1584a_0^3a_2^2\alpha^2\sigma^{10} + 4208a_0^3a_2^2\alpha\sigma^{10}\sigma_w^2 \\
& - 1584a_0^3a_2^2\alpha\sigma^{10} - 1056a_0^3a_2^2\sigma^{10}\sigma_w^2 + 528a_0^3a_2^2\sigma^{10} + 9a_0^3\alpha^3\sigma^4\sigma_w^4 - 6a_0^3\alpha^3\sigma^4\sigma_w^2 - 27a_0^3\alpha^2\sigma^4\sigma_w^4 \\
& + 18a_0^3\alpha^2\sigma^4\sigma_w^2 + 27a_0^3\alpha\sigma^4\sigma_w^4 - 18a_0^3\alpha\sigma^4\sigma_w^2 - 6a_0^3\sigma^4\sigma_w^4 + 6a_0^3\sigma^4\sigma_w^2 + 4336128a_0^2a_2^5\alpha^3\sigma^{18} + \\
& 803328a_0^2a_2^5\alpha^2\sigma^{18} + 192000a_0^2a_2^5\alpha\sigma^{18} - 78336a_0^2a_2^5\sigma^{18} + 7488a_0^2a_2^3\alpha^3\sigma^{12}\sigma_w^2 - 2496a_0^2a_2^3\alpha^3\sigma^{12} \\
& - 6784a_0^2a_2^3\alpha^2\sigma^{12}\sigma_w^2 + 7488a_0^2a_2^3\alpha^2\sigma^{12} + 17536a_0^2a_2^3\alpha\sigma^{12}\sigma_w^2 - 7488a_0^2a_2^3\alpha\sigma^{12} - 4992a_0^2a_2^3\sigma^{12} \\
& \sigma_w^2 + 2496a_0^2a_2^3\sigma^{12} + 108a_0^2a_2\alpha^3\sigma^6\sigma_w^4 - 72a_0^2a_2\alpha^3\sigma^6\sigma_w^2 - 324a_0^2a_2\alpha^2\sigma^6\sigma_w^4 - 72a_0^2a_2\alpha^3\sigma^6\sigma_w^2 \\
& - 324a_0^2a_2\alpha^2\sigma^6\sigma_w^4 + 216a_0^2a_2\alpha^2\sigma^6\sigma_w^2 + 312a_0^2a_2\alpha\sigma^6\sigma_w^4 - 216a_0^2a_2\alpha\sigma^6\sigma_w^2 - 72a_0^2a_2\sigma^6\sigma_w^4 + 72 \\
& a_0^2a_2\sigma^6\sigma_w^2 + 21063168a_0a_2^6\alpha^3\sigma^{20} + 2442240a_0a_2^6\alpha^2\sigma^{20} + 225792a_0a_2^6\alpha\sigma^{20} - 138240a_0a_2^6\sigma^{20} + \\
& 19008a_0a_2^4\alpha^3\sigma^{14}\sigma_w^2 - 6336a_0a_2^4\alpha^3\sigma^{14} + 22272a_0a_2^4\alpha^2\sigma^{14}\sigma_w^2 + 19008a_0a_2^4\alpha^2\sigma^{14} + 36288a_0a_2^4 \\
& \alpha\sigma^{14}\sigma_w^2 - 19008a_0a_2^4\alpha\sigma^{14} - 12672a_0a_2^4\sigma^{14}\sigma_w^2 + 6336a_0a_2^4\sigma^{14} + 504a_0a_2^2\alpha^3\sigma^8\sigma_w^4 - 336a_0a_2^2\alpha^3\sigma^8 \\
& \sigma_w^2 - 1512a_0a_2^2\alpha^2\sigma^8\sigma_w^4 + 1008a_0a_2^2\alpha^2\sigma^8\sigma_w^2 + 1336a_0a_2^2\alpha\sigma^8\sigma_w^4 - 1008a_0a_2^2\alpha\sigma^8\sigma_w^2 - 336a_0a_2^2\sigma^8 \\
& \sigma_w^4 + 336a_0a_2^2\sigma^8\sigma_w^2 + 3a_0\alpha^3\sigma^2\sigma_w^6 - 3a_0\alpha^3\sigma^2\sigma_w^4 - 9a_0\alpha^2\sigma^2\sigma_w^6 + 9a_0\alpha^2\sigma^2\sigma_w^4 + 9a_0\alpha\sigma^2\sigma_w^6 - 9a_0 \\
& \alpha\sigma^2\sigma_w^4 - 2a_0\sigma^2\sigma_w^6 + 3a_0\sigma^2\sigma_w^4 + 37988352a_2^7\alpha^3\sigma^{22} + 2985984a_2^7\alpha^2\sigma^{22} + 55296a_2^7\alpha\sigma^{22} - 110592a_2^7 \\
& \sigma^{22} + 20736a_2^5\alpha^3\sigma^{16}\sigma_w^2 - 6912a_2^5\alpha^3\sigma^{16} + 82944a_2^5\alpha^2\sigma^{16}\sigma_w^2 + 20736a_2^5\alpha^2\sigma^{16} + 25344a_2^5\alpha\sigma^{16}\sigma_w^2 - \\
& 20736a_2^5\alpha\sigma^{16} - 13824a_2^5\sigma^{16}\sigma_w^2 + 6912a_2^5\sigma^{16} + 864a_2^3\alpha^3\sigma^{10}\sigma_w^4 - 576a_2^3\alpha^3\sigma^{10}\sigma_w^2 - 2592a_2^3\alpha^2\sigma^{10}\sigma_w^4 + \\
& 1728a_2^3\alpha^2\sigma^{10}\sigma_w^2 + 1824a_2^3\alpha\sigma^{10}\sigma_w^4 - 1728a_2^3\alpha\sigma^{10}\sigma_w^2 - 576a_2^3\sigma^{10}\sigma_w^4 + 576a_2^3\sigma^{10}\sigma_w^2 + 12a_2\alpha^3\sigma^4\sigma_w^6 - \\
& 12a_2\alpha^3\sigma^4\sigma_w^4 - 36a_2\alpha^2\sigma^4\sigma_w^6 + 36a_2\alpha^2\sigma^4\sigma_w^4 + 36a_2\alpha\sigma^4\sigma_w^6 - 36a_2\alpha\sigma^4\sigma_w^4 - 8a_2\sigma^4\sigma_w^6 + 12a_2\sigma^4\sigma_w^4
\end{aligned} \tag{11}$$

$$\begin{aligned}
\omega_{1m} = & 3a_0^8 \alpha^3 \sigma^8 - 9a_0^8 \alpha^2 \sigma^8 + 9a_0^8 \alpha \sigma^8 - 2a_0^8 \sigma^8 + 96a_0^7 a_2 \alpha^3 \sigma^{10} - 248a_0^7 a_2 \alpha^2 \sigma^{10} + 288a_0^7 a_2 \alpha \sigma^{10} \\
& - 64a_0^7 a_2 \sigma^{10} - 128a_0^6 a_2^2 \alpha^4 \sigma^{12} + 2088a_0^6 a_2^2 \alpha^3 \sigma^{12} - 2800a_0^6 a_2^2 \alpha^2 \sigma^{12} + 4320a_0^6 a_2^2 \alpha \sigma^{12} - 960a_0^6 \\
& a_2^2 \sigma^{12} + 12a_0^6 \alpha^3 \sigma^6 \sigma_w^2 - 3a_0^6 \alpha^3 \sigma^6 - 36a_0^6 \alpha^2 \sigma^6 \sigma_w^2 + 9a_0^6 \alpha^2 \sigma^6 + 36a_0^6 \alpha \sigma^6 \sigma_w^2 - 9a_0^6 \alpha \sigma^6 - 8a_0^6 \\
& \sigma^6 \sigma_w^2 + 3a_0^6 \sigma^6 - 5632a_0^5 a_2^3 \alpha^4 \sigma^{14} + 42624a_0^5 a_2^3 \alpha^3 \sigma^{14} - 12928a_0^5 a_2^3 \alpha^2 \sigma^{14} + 39168a_0^5 a_2^3 \alpha \sigma^{14} - \\
& 8704a_0^5 a_2^3 \sigma^{14} + 288a_0^5 a_2 \alpha^3 \sigma^8 \sigma_w^2 - 72a_0^5 a_2 \alpha^3 \sigma^8 - 784a_0^5 a_2 \alpha^2 \sigma^8 \sigma_w^2 + 216a_0^5 a_2 \alpha^2 \sigma^8 + 864a_0^5 \\
& a_2 \alpha \sigma^8 \sigma_w^2 - 216a_0^5 a_2 \alpha \sigma^8 - 192a_0^5 a_2 \sigma^8 \sigma_w^2 + 72a_0^5 a_2 \sigma^8 - 111104a_0^4 a_2^4 \alpha^4 \sigma^{16} + 679104a_0^4 a_2^4 \alpha^3 \\
& \sigma^{16} + 28864a_0^4 a_2^4 \alpha^2 \sigma^{16} + 233856a_0^4 a_2^4 \alpha \sigma^{16} - 51968a_0^4 a_2^4 \sigma^{16} + 3816a_0^4 a_2^2 \alpha^3 \sigma^{10} \sigma_w^2 - 792a_0^4 a_2^2 \\
& \alpha^3 \sigma^{10} - 7104a_0^4 a_2^2 \alpha^2 \sigma^{10} \sigma_w^2 + 2376a_0^4 a_2^2 \alpha^2 \sigma^{10} + 9504a_0^4 a_2^2 \alpha \sigma^{10} \sigma_w^2 - 2376a_0^4 a_2^2 \alpha \sigma^{10} - 2112 \\
& a_0^4 a_2^2 \sigma^{10} \sigma_w^2 + 792a_0^4 a_2^2 \sigma^{10} + 18a_0^4 \alpha^3 \sigma^4 \sigma_w^4 - 9a_0^4 \alpha^3 \sigma^4 \sigma_w^2 - 54a_0^4 \alpha^2 \sigma^4 \sigma_w^4 + 27a_0^4 \alpha^2 \sigma^4 \sigma_w^2 + 54 \\
& a_0^4 \alpha \sigma^4 \sigma_w^4 - 27a_0^4 \alpha \sigma^4 \sigma_w^2 - 12a_0^4 \sigma^4 \sigma_w^4 + 9a_0^4 \sigma^4 \sigma_w^2 - 1247232a_0^3 a_2^5 \alpha^4 \sigma^{18} + 7147008a_0^3 a_2^5 \alpha^3 \sigma^{18} \\
& + 695808a_0^3 a_2^5 \alpha^2 \sigma^{18} + 940032a_0^3 a_2^5 \alpha \sigma^{18} - 208896a_0^3 a_2^5 \sigma^{18} + 44352a_0^3 a_2^3 \alpha^3 \sigma^{12} \sigma_w^2 - 4992a_0^3 a_2^3 \\
& \alpha^3 \sigma^{12} - 28544a_0^3 a_2^3 \alpha^2 \sigma^{12} \sigma_w^2 + 14976a_0^3 a_2^3 \alpha^2 \sigma^{12} + 59904a_0^3 a_2^3 \alpha \sigma^{12} \sigma_w^2 - 14976a_0^3 a_2^3 \alpha \sigma^{12} - 13312 \\
& a_0^3 a_2^3 \sigma^{12} \sigma_w^2 + 4992a_0^3 a_2^3 \sigma^{12} + 288a_0^3 a_2 \alpha^3 \sigma^6 \sigma_w^4 - 144a_0^3 a_2 \alpha^3 \sigma^6 \sigma_w^2 - 824a_0^3 a_2 \alpha^2 \sigma^6 \sigma_w^4 + 432 \\
& a_0^3 a_2 \alpha^2 \sigma^6 \sigma_w^2 + 864a_0^3 a_2 \alpha \sigma^6 \sigma_w^4 - 432a_0^3 a_2 \alpha \sigma^6 \sigma_w^2 - 192a_0^3 a_2 \sigma^6 \sigma_w^4 + 144a_0^3 a_2 \sigma^6 \sigma_w^2 - 8368128 \\
& a_0^2 a_2^6 \alpha^4 \sigma^{20} + 45646848a_0^2 a_2^6 \alpha^3 \sigma^{20} + 3870720a_0^2 a_2^6 \alpha^2 \sigma^{20} + 2488320a_0^2 a_2^6 \alpha \sigma^{20} - 552960a_0^2 a_2^6 \sigma^{20} \\
& + 466560a_0^2 a_2^4 \alpha^3 \sigma^{14} \sigma_w^2 - 19008a_0^2 a_2^4 \alpha^3 \sigma^{14} - 11136a_0^2 a_2^4 \alpha^2 \sigma^{14} \sigma_w^2 + 57024a_0^2 a_2^4 \alpha^2 \sigma^{14} + 228096 \\
& a_0^2 a_2^4 \alpha \sigma^{14} \sigma_w^2 - 57024a_0^2 a_2^4 \alpha \sigma^{14} - 50688a_0^2 a_2^4 \sigma^{14} \sigma_w^2 + 19008a_0^2 a_2^4 \sigma^{14} + 2016a_0^2 a_2^2 \alpha^3 \sigma^8 \sigma_w^4 - 1008 \\
& a_0^2 a_2^2 \alpha^3 \sigma^8 \sigma_w^2 - 5168a_0^2 a_2^2 \alpha^2 \sigma^8 \sigma_w^4 + 3024a_0^2 a_2^2 \alpha^2 \sigma^8 \sigma_w^2 + 6048a_0^2 a_2^2 \alpha \sigma^8 \sigma_w^4 - 3024a_0^2 a_2^2 \alpha \sigma^8 \sigma_w^2 \\
& - 1344a_0^2 a_2^2 \sigma^8 \sigma_w^4 + 1008a_0^2 a_2^2 \sigma^8 \sigma_w^2 + 12a_0^2 \alpha^3 \sigma^2 \sigma_w^6 - 9a_0^2 \alpha^3 \sigma^2 \sigma_w^4 - 36a_0^2 \alpha^2 \sigma^2 \sigma_w^6 + 27a_0^2 \alpha^2 \sigma^2 \\
& \sigma_w^4 + 36a_0^2 \alpha \sigma^2 \sigma_w^6 - 27a_0^2 \alpha \sigma^2 \sigma_w^4 - 8a_0^2 \sigma^2 \sigma_w^6 + 9a_0^2 \sigma^2 \sigma_w^4 - 31850496a_0 a_2^7 \alpha^4 \sigma^{22} + 159916032a_0 a_2^7 \\
& \alpha^3 \sigma^{22} + 10395648a_0 a_2^7 \alpha^2 \sigma^{22} + 3981312a_0 a_2^7 \alpha \sigma^{22} - 884736a_0 a_2^7 \sigma^{22} + 3290112a_0 a_2^5 \alpha^3 \sigma^{16} \sigma_w^2 - \\
& 41472a_0 a_2^5 \alpha^3 \sigma^{16} + 285696a_0 a_2^5 \alpha^2 \sigma^{16} \sigma_w^2 + 124416a_0 a_2^5 \alpha^2 \sigma^{16} + 497664a_0 a_2^5 \alpha \sigma^{16} \sigma_w^2 - 124416 \\
& a_0 a_2^5 \alpha \sigma^{16} - 110592a_0 a_2^5 \sigma^{16} \sigma_w^2 + 41472a_0 a_2^5 \sigma^{16} + 6912a_0 a_2^3 \alpha^3 \sigma^{10} \sigma_w^4 - 3456a_0 a_2^3 \alpha^3 \sigma^{10} \sigma_w^2 \\
& - 13056a_0 a_2^3 \alpha^2 \sigma^{10} \sigma_w^4 + 10368a_0 a_2^3 \alpha^2 \sigma^{10} \sigma_w^2 + 20736a_0 a_2^3 \alpha \sigma^{10} \sigma_w^4 - 10368a_0 a_2^3 \alpha \sigma^{10} \sigma_w^2 - 4608 \\
& a_0 a_2^3 \sigma^{10} \sigma_w^4 + 3456a_0 a_2^3 \sigma^{10} \sigma_w^2 + 96a_0 a_2 \alpha^3 \sigma^4 \sigma_w^6 - 72a_0 a_2 \alpha^3 \sigma^4 \sigma_w^4 - 288a_0 a_2 \alpha^2 \sigma^4 \sigma_w^6 + 216a_0 \\
& a_2 \alpha^2 \sigma^4 \sigma_w^4 + 288a_0 a_2 \alpha \sigma^4 \sigma_w^6 - 216a_0 a_2 \alpha \sigma^4 \sigma_w^4 - 64a_0 a_2 \sigma^4 \sigma_w^6 + 72a_0 a_2 \sigma^4 \sigma_w^4 - 53747712a_2^8 \alpha^4 \\
& \sigma^{24} + 251817984a_2^8 \alpha^3 \sigma^{24} + 11943936a_2^8 \alpha^2 \sigma^{24} + 2985984a_2^8 \alpha \sigma^{24} - 663552a_2^8 \sigma^{24} + 10616832a_2^6 \alpha^3 \\
& \sigma^{18} \sigma_w^2 - 41472a_2^6 \alpha^3 \sigma^{18} + 746496a_2^6 \alpha^2 \sigma^{18} \sigma_w^2 + 124416a_2^6 \alpha^2 \sigma^{18} + 497664a_2^6 \alpha \sigma^{18} \sigma_w^2 - 124416a_2^6 \alpha \sigma^{18} \\
& - 110592a_2^6 \sigma^{18} \sigma_w^2 + 41472a_2^6 \sigma^{18} + 10368a_2^4 \alpha^3 \sigma^{12} \sigma_w^4 - 5184a_2^4 \alpha^3 \sigma^{12} \sigma_w^2 - 5184a_2^4 \alpha^2 \sigma^{12} \sigma_w^4 + 15552a_2^4 \\
& \alpha^2 \sigma^{12} \sigma_w^2 + 31104a_2^4 \alpha \sigma^{12} \sigma_w^4 - 15552a_2^4 \alpha \sigma^{12} \sigma_w^2 - 6912a_2^4 \sigma^{12} \sigma_w^4 + 5184a_2^4 \sigma^{12} \sigma_w^2 + 288a_2^2 \alpha^3 \sigma^6 \sigma_w^6 - \\
& 216a_2^2 \alpha^3 \sigma^6 \sigma_w^4 - 864a_2^2 \alpha^2 \sigma^6 \sigma_w^6 + 648a_2^2 \alpha^2 \sigma^6 \sigma_w^4 + 864a_2^2 \alpha \sigma^6 \sigma_w^6 - 648a_2^2 \alpha \sigma^6 \sigma_w^4 - 192a_2^2 \sigma^6 \sigma_w^6 + \\
& 216a_2^2 \sigma^6 \sigma_w^4 + 3\alpha^3 \sigma_w^8 - 3\alpha^3 \sigma_w^6 - 9\alpha^2 \sigma_w^8 + 9\alpha^2 \sigma_w^6 + 9\alpha \sigma_w^8 - 9\alpha \sigma_w^6 - 2\sigma_w^8 + 3\sigma_w^6
\end{aligned} \tag{12}$$

$$\begin{aligned}
G_{1m} = & 4608a_2^5 \sigma^{16} + 3a_0^4 a_2 \sigma^8 + 2592a_0 a_2^4 \sigma^{14} - 10368a_2^5 \alpha \sigma^{16} + 68a_0^3 a_2^2 \sigma^{10} + \\
& 616a_0^2 a_2^3 \sigma^{12} + 192a_2^3 \sigma^{10} \sigma_w^2 - 104a_0^3 a_2^2 \alpha \sigma^{10} - 1120a_0^2 a_2^3 \alpha \sigma^{12} + \\
& 3a_0^2 a_2 \sigma^6 \sigma_w^2 + 44a_0 a_2^2 \sigma^8 \sigma_w^2 - 4a_0^4 a_2 \alpha \sigma^8 - 5664a_0 a_2^4 \alpha \sigma^{14}
\end{aligned} \tag{13}$$