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New Formula for the Effective Width of Slender Plate Elements

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Abstract: A new alternative formula for the effective width factor of slender plate elements, which accounts for the local plate buckling of these elements, has been proposed. The new formula was derived to take into account the presence of a stress gradient in the plate element, as in the case of slender webs of plate girders. The new formula is more consistent with the minimum limiting width-thickness ratios for Class 4 slender plate elements and ensures a smooth transition from a stress distribution of pure compression to a stress distribution of pure bending. A study of the application of this formula with the maximum width-thickness limits of Class 3 plate elements as given in the Canadian Standard CAN/CSA S16-01 is made. The background of the classical formula for the effective width factor, which was derived from a modification made by the AISI to Winter's formula, is stated and a comparison between these two formulas is presented.

1. Introduction

Slender plate elements in compression, which are components of a steel cross section, often reach their local plate buckling before the overall capacity of the cross section is reached. Such elements include outstanding flanges, webs, and stiffeners commonly found in cold-formed steel members or plate girders. As plates in compression still have a post-buckling capacity, they are typically designed using an effective width concept. The Canadian Standard CAN/CSA-S16-01 "*Limit States Design of Steel Structures*" [1] in Clause 11.1.1 designates structural steel sections as Class 1, 2, 3, or 4 depending on the maximum width-to-thickness ratios of the elements subject to compression. These Classes are defined as follows:

1. Class 1 sections permit attainment of the plastic moment and subsequent redistribution of the bending moment;
2. Class 2 sections permit attainment of the plastic moment but need not allow for subsequent moment redistribution;
3. Class 3 sections permit attainment of the yield moment; and
4. Class 4 sections generally have local buckling of elements in compression as the limit state of structural resistance.

In addition to this Clause 11.1.2 requires that Class 1 sections, when subject to flexure, shall have an axis of symmetry in the plane of loading and, when subject to axial compression, shall be doubly symmetric. Also, Clause 11.1.3 requires that Class 2 sections, when subject to flexure, shall have an axis of symmetry in the plane of loading unless the effects of asymmetry of the section are included in the analysis. The effective width concept comes into effect for Class 4 sections, which have elements with width-to-thickness ratios exceeding the maximum allowable for Class 3 sections. These sections are subjected to a reduction in the width of the element to account for the nonlinear stress distribution that occurs when the compression stress in the plate element exceeds the critical buckling stress.

In this paper a new formula is introduced for the effective width factor, ρ, of slender plate elements (*i.e.,* with width-to-thickness ratios exceeding those of Class 3 sections) based on a new formula introduced into the Egyptian Code of Practice for Steel Construction and Bridges (Allowable Stress Design), ECP (2001) [2]. This new formula is different from the older more popular formula used in other standards and specifications such as the AISI Specification for the Design of Cold-Formed Steel Structural Members [3] and CAN/CSA-S136-01 [4] in that it takes into account the stress distribution in the plate element especially for the cases of pure compression and pure bending in stiffened compression elements from which it was derived. This is particularly suitable for the case of plate girders with slender elements.

2. New Formula for Effective Width

The elastic critical stress, σ_c, of a long plate segment is determined by the plate width-to-thickness ratio *b/*t, by the restraint conditions along the longitudinal boundaries, and by the elastic material properties, and can be expressed as

$$\sigma_c = \frac{k\pi^2 E}{12(1-\nu^2)(b/t)^2} \tag{1}$$

where k is a plate buckling coefficient, which depends on the plate's boundary conditions and the distribution of axial stress in the plate; E is the modulus of elasticity; ν is Poisson's ratio taken as 0.3 for structural steel; b is the width of the plate element; and t is the thickness of the plate element. For uniformly stressed plates Table 1 shows the values of k for different edge conditions. However, other values of k apply for other combinations of edge conditions and stress distributions in the plate element. Refering to Tables 2.3 and 2.4 of the new ECP [2], or Tables 5.3.2 and 5.3.3 of the Eurocode, ENV 1993 Eurocode 3: Design of Steel Structures [5], expressions of other values of k can be taken as shown in Table 2.

Table 1. Values of k for uniform compression and different edge conditions in a long plate [6].

Boundary condition of longitudinal edges	***k***
1. Both edges simply supported.	4.00
2. One edge simply supported the other fixed.	5.42
3. Both edges fixed.	6.97
4. One edge simply supported the other free.	0.43
5. One edge fixed the other free.	1.28

Table 2. Expressions for buckling factor k for variable values of stress gradient $\psi = \sigma_2/\sigma_1$.

<table>
<tr><td colspan="6">For $1 > \psi \geq -1$ $k = \dfrac{16}{\left[(1+\psi)^2 + 0.112(1-\psi^2)\right]^{1/2} + (1+\psi)}$</td><td rowspan="2">$-1 > \psi \geq -2$</td></tr>
<tr><td>ψ</td><td>1</td><td>$1 > \psi > 0$</td><td></td><td>$0 > \psi > -1$</td><td>-1</td></tr>
<tr><td>k</td><td>4.0</td><td>$\dfrac{8.2}{1.05+\psi}$</td><td>7.81</td><td>$7.81 - 6.29\psi + 9.78\psi^2$</td><td>23.9</td><td>$5.98(1-\psi)^2$</td></tr>
</table>

Local buckling in plates causes a loss of stiffness and a redistribution of stresses. Uniform edge compression in the longitudinal direction results in a nonuniform stress distribution after buckling, and the buckled plate derives almost all of its stiffness from the longitudinal edge supports [6]. Figure 1 shows the nonlinear stress distribution of a buckled plate subject to uniform axial compression where the edge stress is σ_e and the average stress is σ_{av}.

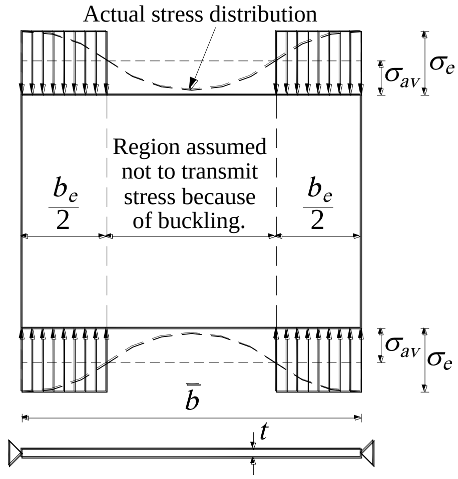
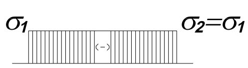
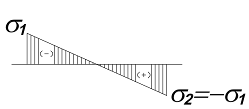


Fig. 1. Nonlinear stress distribution of a buckled plate [6].

Table 3. Comparison between the lower bound limits of λ_p at $\rho=b_e/b=1.0$ and the requirements of Class 3 sections.

Stress distribution in element	Type of element	Width-to-thickness ratio for AISI formula	Width-to-thickness ratio for proposed formula	Maximum width-to-thickness ratio for Class 3 elements [1]
<u>I. Uniform Compression</u> $\psi=1.0$ $\psi=\sigma_2/\sigma_1$	<u>(i) Stiffened</u>: (a) Webs: k= 4.0	(a) $\frac{h}{w} \le \frac{529}{\sqrt{\sigma_y}}$	(a) $\frac{h}{w} \le \frac{646}{\sqrt{\sigma_y}}$	(a) $\frac{h}{w} \le \frac{670}{\sqrt{\sigma_y}}$
 	<u>(ii) Unstiffened:</u> (b) Outstanding flanges: k= 0.43	(b) $\frac{b}{t} \le \frac{106}{\sqrt{\sigma_y}}$	(b) $\frac{b}{t} \le \frac{129}{\sqrt{\sigma_y}}$	(b) $\frac{b}{t} \le \frac{200}{\sqrt{\sigma_y}}$ (1)
<u>II. Stress Gradient</u> $\psi=-1.0$	<u>(i) Stiffened</u>: (a) Webs: k= 23.9	(a) $\frac{h}{w} \le \frac{1399}{\sqrt{\sigma_y}}$	(a) $\frac{h}{w} \le \frac{1844}{\sqrt{\sigma_y}}$	(a) $\frac{h}{w} \le \frac{1900}{\sqrt{\sigma_y}}$
 	<u>(ii) Unstiffened:</u> (b) Outstanding flanges: k= 0.85 - tip in compression. k= 23.8 - tip in tension.	(b) $\frac{b}{t} \le \frac{264}{\sqrt{\sigma_y}}$ $\frac{b}{t} \le \frac{1396}{\sqrt{\sigma_y}}$	(b) $\frac{b}{t} \le \frac{348}{\sqrt{\sigma_y}}$ $\frac{b}{t} \le \frac{1840}{\sqrt{\sigma_y}}$	(b) $\frac{b}{t} \le \frac{315 \rightarrow 288}{\sqrt{\sigma_y}}$ (2) $\frac{b}{t} \le \frac{1666 \rightarrow 1524}{\sqrt{\sigma_y}}$ (2)

(1) Taken as $b/t \le 340/(\sigma_y)^{½}$ for stems of T sections.

(2) These values are derived from a formula in the EPC [1] and adapted for SI units giving $b/t \le 342/(k/\sigma_y)^{½}$ for outstanding flanges of rolled sections and $b/t \le 312/(k/\sigma_y)^{½}$ for outstanding flanges of welded sections.

Sections with slender plate elements are designed using the effective width concept. That is the maximum edge stress acts uniformly over two strips of the plate and the central region is unstressed. Many standards and specifications (AISI, AISC, CSA-S136-01) [3, 7, 4] permit the use of an effective width in the design of members having plate elements with width-to-thickness ratios greater than the limits for full effectiveness (i.e., Class 4 sections according to CAN/CSA-S16-01 [1]).

The expression for the effective width, b_e, of a simply supported slender plate element originated from von Kármán's [8] approximate formula derived from Eq. (1) taking k=4.0, such that

$$\frac{b_e}{t} = \frac{\pi}{\sqrt{3(1-\nu^2)}} \sqrt{\frac{E}{\sigma_e}} \tag{2a}$$

or, using the relationship for σ_c in Eq. (1)

$$\frac{b_e}{b} = \sqrt{\frac{\sigma_c}{\sigma_e}} \tag{2b}$$

Winter [8] modified this on the basis of experimental results from cold-formed sections to include the effect of various imperfections such that

$$\frac{b_e}{t} = 1.9\sqrt{\frac{E}{\sigma_e}}\left(1 - 0.475\sqrt{\frac{E}{\sigma_e}}\frac{t}{b}\right) \tag{3a}$$

or

$$\frac{b_e}{b} = \sqrt{\frac{\sigma_c}{\sigma_e}}\left(1 - 0.25\sqrt{\frac{\sigma_c}{\sigma_e}}\right) \tag{3b}$$

As this expression was reached on the basis of experimental results from tests conducted on cold-formed sections, it does not give a general picture of the effective width of other steel elements such as slender plate girders.

In the 1968 and later editions of the AISI Specification [2] for cold-formed steel member Eqs. (3a) and (3b) were further modified to give the following expressions for the effective width

$$\frac{b_e}{t} = 1.9\sqrt{\frac{E}{\sigma_e}}\left(1 - 0.415\sqrt{\frac{E}{\sigma_e}}\frac{t}{b}\right) \tag{4a}$$

$$\frac{b_e}{b} = \sqrt{\frac{\sigma_c}{\sigma_e}}\left(1 - 0.22\sqrt{\frac{\sigma_c}{\sigma_e}}\right) \tag{4b}$$

Taking the normalised plate slenderness as

$$\lambda_p = \sqrt{\frac{\sigma_e}{\sigma_c}} = \sqrt{\frac{12(1-\nu^2)}{\pi^2 k}}\,\frac{b}{t}\sqrt{\frac{\sigma_e}{E}} \tag{5}$$

gives the final expression for the effective width as

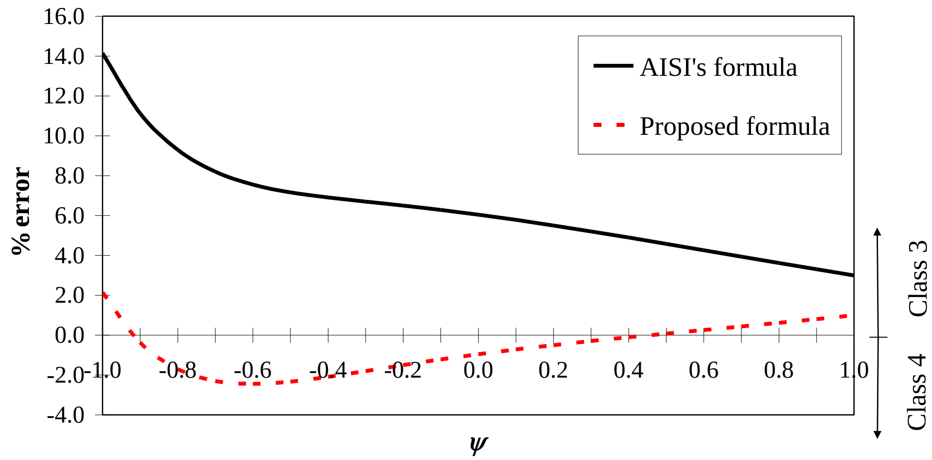
$$\frac{b_e}{b} = \frac{\lambda_p - 0.22}{\lambda_p^2} \tag{6}$$

Comparing the limiting slenderness ratios for non-compact sections with respect to the limiting slenderness ratios at which no reduction in width is required (i.e., at $\rho = b_e/b = 1.0$, where ρ is the effective width factor or the width reduction factor) for stiffened compression elements it is found to be inconsistent for the cases of pure compression and pure bending as shown in Table 2. In order to avoid these contradictions the new Egyptian Code of Practice is proposing to change Eq. (6) to include the effect of the stress gradient on the plate element such that

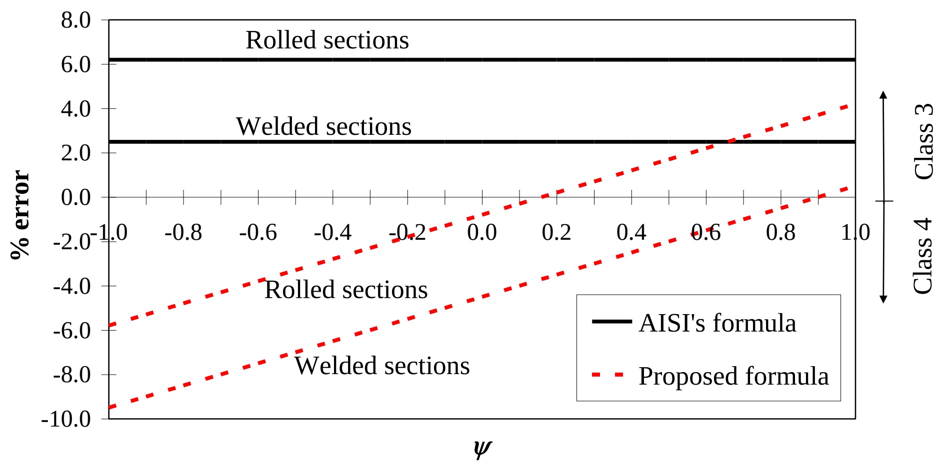
$$\frac{b_e}{b} = \frac{\lambda_p - 0.15 - 0.05\psi}{\lambda_p^2} \tag{7}$$

where ψ is the stress ratio and is the ratio of the smaller end compressive stress (or end tensile stress), σ_2, to the larger end compressive stress, σ_1. Equation (7) was derived for the cases of pure compression ψ=1.0 and pure bending ψ=-1.0 for stiffened compression elements such that at the limiting slenderness ratio between Class 3 and Class 4 sections no reduction in width is required. Table 3 shows the width-to-thickness ratios at ρ=1.0 for this proposed formula.

Figures 2a and 2b show how Eqs. (6) and (7) compare with the maximum width-to-thickness limits of Class 3 non-compact plate elements



(a) Stiffened compression elements

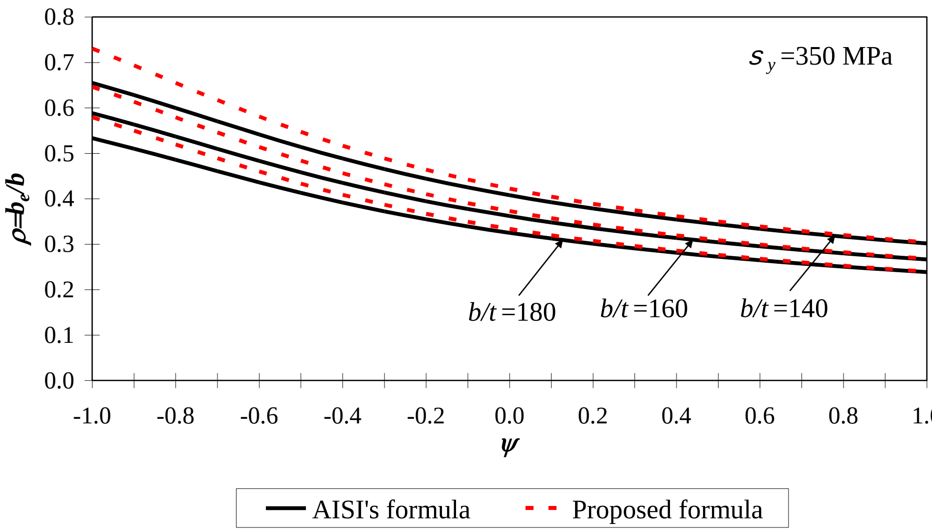


(b) Unstiffened compression elements

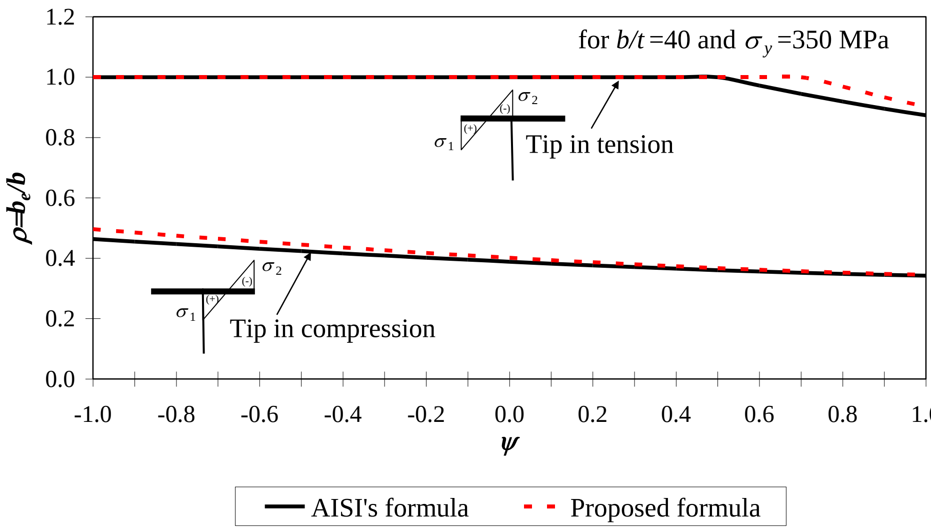
Fig. 2. Error curves for stiffened and unstiffened compression elements.

Figure 2a shows the results of the AISI's formula, Eq. (6), for the width reduction factor, $\rho=b_e/b$, as well as the results of the proposed formula, Eq. (7), for stiffened compression elements. As both formulas are supposed to apply for Class 4 sections and not Class 3 sections, they should give a value of ρ=1.0 at the Class 3 maximum limiting width-to-thickness value. Any value above or below this is the % error. Any point above the 0% error is in the Class 3 non-compact range and any point below the 0% error is in the Class 4 slender range. It can be seen from Fig. 2a that the proposed formula is more consistent with the boundary between Class 4 and Class 3 sections especially near the points ψ=1.0 (pure compression) and ψ=-1.0 (pure bending) for which it was derived. However, most of the range between ψ=1.0 and ψ=-1.0 is in the Class 3 non-compact range leaving a narrow range of elements with width-to-thickness ratios falling in the Class 4 slender range but not requiring a reduction in width, which is slightly unconservative. Whereas, the classical AISI's formula conservatively lies in the Class 3 non-compact range and hence predicts a conservative reduction in width for Class 4 slender elements, but this criterion is inconsistent. Figure 2b shows the same error curves for the AISI's formula and the proposed formula for the case of unstiffened welded and rolled compression elements. The cases of tip in compression and tip in tension yield the same curves as the term *k* cancels out. For welded compression elements, it can be seen that for most values of ψ between 1.0 and -1.0 the proposed formula is in the Class 4 slender range, and the error is especially large for elements under pure bending, reaching -9.5%. This means that elements in the Class 4 range with width-to-thickness ratios close to the maximum Class 3 limit will not undergo a reduction in width. Whereas, the AISI's formula lies entirely in the Class 3 range with a uniform error of +2.5% ensuring that all elements in the Class 4 slender range are bound to have a conservatively reduced effective width. Figure 2b also shows the error curves for unstiffened rolled compression elements. The proposed formula gives an error ranging from -5.8% for pure bending to +4.2% for pure compression. On the other hand, the AISI's formula conservatively gives an error of +6.2% for all values of ψ such that all elements in the Class 4 slender range are bound to undergo a reduction in width.

Figures 3a and 3b show the width reduction factor according to Eqs. (6) and (7), $\rho=b_e/b$, plotted for values of ψ between 1.0 and –1.0 for both stiffened and unstiffened compression elements. Figure 3a illustrates the difference in the reduction in effective width for stiffened compression elements which have a width-to-thickness ratio of b/t=140, 160, and 180 and a yield stress of σ_y=350 MPa. The reduction in width is nearly equal at ψ=1.0 but is slightly less for the proposed formula than for the AISI's formula at ψ=-1.0. Figure 3b illustrates this relationship for unstiffened compression elements where the compression stress at the tip of the element is greater than the stress at the supported edge of the element, which could be in tension (tip in compression). For this case the width-to-thickness ratio was taken as b/t=40 and the yield stress as σ_y=350 MPa. Again, the reduction in width is nearly equal at ψ=1.0 but is slightly less for the proposed formula than for the AISI's formula at ψ=-1.0. This figure also shows the same curves for unstiffened compression elements where the compression stress at the supported edge of the element is greater than the stress at the tip of the element, which could be in tension (tip in tension). For lower values of ψ no reduction in width is required, and for ψ=1.0 the reduction in width is nearly equal for the two formulas.



(a) Stiffened compression elements



(b) Unstiffened compression elements

Fig. 3. Width reduction factor for stiffened and unstiffened compression elements.

3. Plate Buckling Curves

Figure 4 shows the nondimensional buckling curves for plates under uniform edge compression. The horizontal axis represents the normalised plate slenderness given in Eq. (5) but using the yield stress, σ_y, to represent the maximum edge stress, σ_e. The vertical axis represents the buckling stress parameter, which is the ratio of the critical stress to the yield stress, σ_c/σ_y, or the ratio of the average stress to the yield stress, σ_{av}/σ_y. The average stress, shown in Fig. 1, is defined as the uniform stress acting on the plate element if the whole width is resisting the load, such that

$$\sigma_{av} = \frac{b_e \sigma_y}{b} \tag{8a}$$

where σ_e is replaced by σ_y, or using the relationship in Eq. (2b) the average stress can be expressed as

$$\sigma_{av} = \sqrt{\sigma_c \sigma_y} \tag{8b}$$

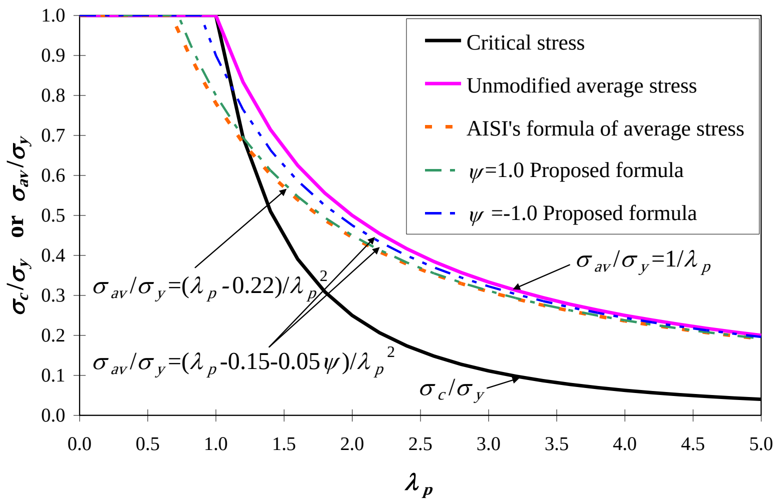


Fig. 4. Nondimensional plate buckling curves.

The first curve in Fig. 4 represents the idealised critical stress curve given by Eq. (1). This curve is plotted for values of $\lambda_p \geq 1.0$ down to $\lambda_p = 1.0$, below which the ratio of σ_c/σ_y is constant and equal to 1.0. The second curve represents the unmodified average stress parameter

$$\frac{\sigma_{av}}{\sigma_y} = \sqrt{\frac{\sigma_c}{\sigma_y}} = \frac{1}{\lambda_p} \tag{9}$$

which is the average stress parameter of von Kármán's approximate formula, Eq. (2b), before any modifications were made by Winter. This curve lies above the curve for the critical stress and also has a constant value of $\sigma_{av}/\sigma_y = 1.0$ for values of $\lambda_p \leq 1.0$. The third curve represents the average stress parameter given by the AISI's formula, which is Eq. (4b) or Eq. (6). It can be seen that this curve is more conservative than the unmodified average stress curve, as it lies below it, and has a constant value of $\sigma_{av}/\sigma_y = 1.0$ for values of $\lambda_p \leq 0.673$. This formula accounts for the effect of various imperfections on the average stress. The last two curves represent the average stress parameter for the proposed formula, Eq. (7), for the cases of $\psi = 1.0$ (pure compression) and $\psi = -1.0$ (pure bending). It is obvious that the AISI's formula is the lower bound, most conservative, of these modified average stresses.

4. Application to Plate Girders

Plate girders with flanges and webs in the Class 1, 2, and 3 plastic and compact ranges can achieve their plastic moment capacity. Plate girders with flanges and webs in the Class 3 non-compact range can achieve their elastic moment capacity. However, plate girders with flanges or webs in the Class 4 slender range require special provisions to prevent failure due to buckling. When designing slender plate girders without stiffeners for flexure, the modes of failure due to buckling that must be considered are: web buckling under pure flexure, vertical buckling of the web, vertical buckling (or local plate buckling) of the compression flange, and lateral-torsional buckling of the unsupported compression flange. This paper only applies to web buckling under pure flexure, shown in Fig. 5, and vertical buckling due to local plate buckling of the compression flange. The gross and effective cross sections of a plate girder with a slender web and a slender compression flange are shown in Fig. 6.

For plate girder I-sections in flexure with slender webs in the Class 4 range, but non-compact or compact flanges in the Class 1, 2, or 3 ranges, the web is expected to buckle at higher loads, and will throw off part of its load onto the stiffer flange. Thus, the web will be less effective than expected and the flange will receive a higher stress than that calculated using ordinary beam theory as shown in Fig. 5. This web buckling under pure bending is dealt with by removing a part of the compression web. Certain standards and specifications such as the AISC [7] and the CSA-S16-01 [1] have adopted a formula developed [10] to reduce the allowable bending stress in the compression flange for plate girders for buildings with webs in the Class 4 slender range and flanges in at least the Class 3 non-compact range. The AISC [7] and CAN/CSA-S16-01 [1] requires that the allowable bending moment for non-hybrid girders be reduced by the factor, R_{PG}, such that

$$R_{PG} = 1 - 0.0005\left(\frac{A_w}{A_f}\right)\left(\frac{h}{w} - \frac{1900}{\sqrt{M_f / \phi S}}\right) \leq 1.0 \tag{10}$$

where A_w is the area of the web at the section under investigation; A_f is the area of the compression flange; h is the height of the web; w is the thickness of the web; M_f is the bending moment in the member under factored loads; ϕ is the resistance factor; and S is the elastic section modulus.

Both web buckling under pure flexure, as well as, local plate buckling of the compression flange can be dealt with by using the effective width concept to derive effective sectional properties.

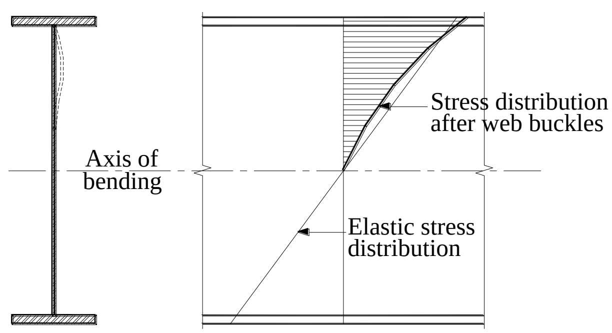


Fig. 5. Web buckling under pure flexure.

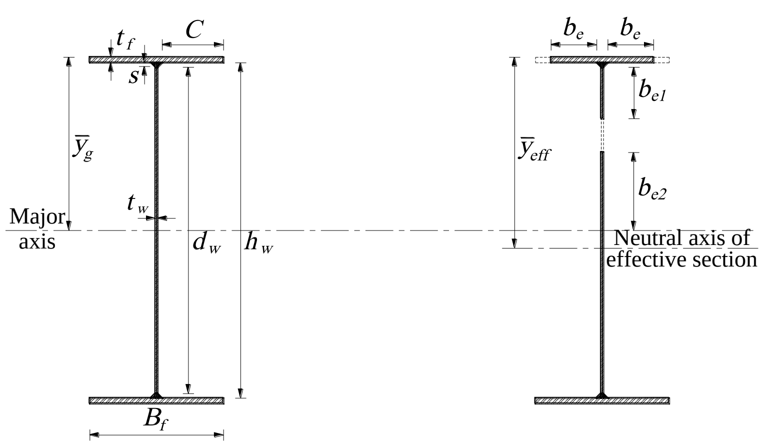


Fig. 6 Gross and Effective Cross Sections of a Plate Girder.

5. Conclusion

It can be concluded that the limits of the proposed formula at $\rho=b_e/b=1.0$ are more consistent with the maximum width-to-thickness ratios of Class 3 non-compact elements than the limits of the effective width factor given by the traditional formula derived from Winter's formulas and modified by the AISI. The proposed formula predicts a similar width reduction to the traditional AISI's formula for slender plate elements under pure compression, but a slightly smaller width reduction for other stress ratios. Therefore, this formula is an appropriate replacement for the old formula.

6. References

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