LOCAL BUCKLING OF SLENDER PLATE GIRDERS IN COMPOSITE BRIDGES

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ABSTRACT

The effect of local plate buckling on the design of slender plate girders is considered in design codes either by reducing the design compressive strength or by reducing the cross section according to the effective width concept. The effective width of slender plate elements used in steel plate girder bridges is usually calculated in some international design codes without reference to the applied stress gradient. This paper presents a comparative study between AASHTO and EC3 codes which shows that considerations of the post-buckling strength by using the effective width approach results in a higher design strength than obtained by the traditional buckling stress approach. The buckling strength of plate girders used in un-shored composite construction of highway bridges is considerably affected by the ratio between top and bottom flange stresses and also the ratio between dead load and total load stresses. For slender webs, the unfavorable effect of the first ratio is balanced by the favorable effect of the second ratio. The effect is always favorable for sections in positive bending with slender compression flanges.

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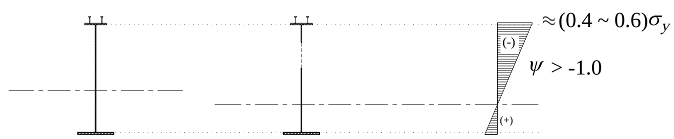
1. INTRODUCTION

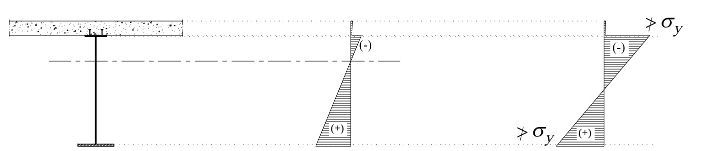
Plate girders are often used in combination with a concrete slab deck in highway bridges to take advantage of the compressive strength of the deck slab in the regions of positive bending. Other advantages include the prevention of compression flange local buckling and lateral-torsional buckling of the steel section in the composite stage.

In un-shored composite construction, the behavior of the girder section is non-composite during the construction stage before the concrete slab has hardened. In this stage the steel section alone is designed to carry the effect of the construction part of the dead load which includes the weight of the steel girder plus the concrete slab and formwork, as shown in Fig. 1(a). In the composite stage after the concrete slab has fully hardened, Fig. 1(b), the additional dead loads and live loads are carried by the composite section.



Gross steel section   Effective steel section   Stress distribution (DL_1)

(a) Un-shored non-composite section with ineffective concrete slab (DL_1 only)



Composite section   Stress distribution (DL_2+LL)   Final stress distribution

(b) Un-shored composite section after hardening of concrete slab (DL_1+DL_2+LL)

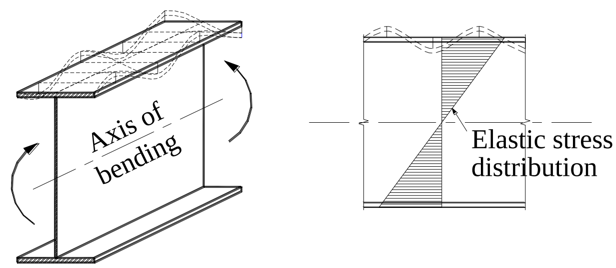
Fig. 1 Stress distribution in un-shored composite construction

The steel section is often designed as mono-symmetrical by reducing the size of the compression flange and increasing the size of the tension flange. First, depending on the ratio of the dead load to the total load, both the compression flange and the web are subjected, in the construction stage, to stresses much less than the yield stress. Second, the depth of the web plate in compression is larger than half its depth. The first factor reduces the possibility of local buckling in both the compression flange and the web, while as the second factor increases the possibility of web bend-buckling also increases. The effect of these two factors on the buckling strength of composite sections is studied in the following sections.

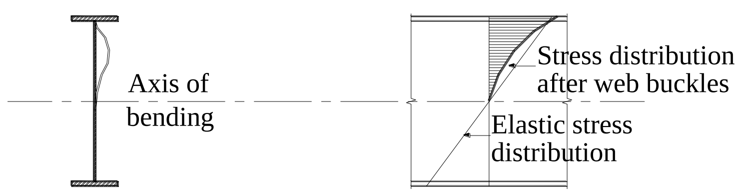
2. LOCAL BUCKLING OF NON-COMPOSITE SECTION

Depending on the width-to-thickness ratio of the plate girder components, the European Eurocode EC3 *Design of Steel Structures – Part 1.1 General Structural Rules* (2005) and the Canadian Standard CAN/CSA-S16-01 *Limit States Design of Steel Structures* (2001) designate structural steel sections into four classes, namely Class 1, 2, 3, or 4. On the other hand, the American Association of State Highway and Transportation Officials (AASHTO) *LRFD Bridge Design Specifications* (2004) and the *Egyptian Code of Practice for Steel Construction and Bridges (Allowable Stress Design)* ECP (2001) combine Class 1 and 2 sections into one class designated as "compact sections", designate Class 3 sections as "non-compact sections", and Class 4 sections are considered "slender sections". This paper deals with the effect of local buckling on the design of composite bridge plate girder sections with width-to-thickness ratios greater than the limiting slenderness ratios given for Class 4 sections in the Canadian Standard S16-01 and the Eurocode EC3, or alternatively, with width to thickness ratios greater than the limiting slenderness ratios given for non-compact sections in the AASHTO LRFD Specification and the ECP, and are henceforth referred to as *slender sections*.

Local plate buckling of composite bridge plate girder sections subject to positive bending can occur in slender compression flanges, as well as, in the compression portion of slender webs, as shown in Fig. 2.



(a) Local buckling of compression flange



(b) Local bend-buckling of web

Fig. 2 Local buckling of non-composite section

The design of slender plate girder sections is considered in some international codes, such as the AASHTO LRFD *Bridge Design Specifications* (2004), by reducing the design bending stress to the critical buckling stress. On the other hand, other codes, such as the European Eurocode EC3 *Design of Steel Structures – Part 1.5 Plated Structural Elements* (2006) and the Egyptian Code of Practice ECP (2001), consider the effect of local buckling by using the effective section properties. The Canadian Standard CAN/CSA-S16-01 (2001) allows for the use of an effective section for plate girders with slender webs and also specifies a reduced design stress approach.

2.1 Critical Stress Approach

The elastic buckling stress of a plate in compression, σ_{cr}, as given by Galambos (1998) is

$$\sigma_{cr} = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \tag{1}$$

where E is the modulus of elasticity, ν is Poisson's ratio, t is the thickness of the plate, b is the width of the plate, and k is the plate buckling factor, which depends on the type of stress distribution and the edge support conditions. Expressions for k for different edge support conditions and stress gradients, $\psi = \sigma_2/\sigma_1$, where σ_1 is the larger end compressive stress and σ_2 is the smaller end compressive stress (or tensile stress), can be found in the EC3 (2006) or the ECP (2001). It should be noted that ψ = +1.0 corresponds to the case of pure compression, while ψ = -1.0 corresponds to the case of pure bending.

2.2 Effective Width Approach

Plates have an additional post-buckling strength due to the nonlinear distribution of the stresses towards the edge supports after buckling. The edge stress, σ_e, increases from the critical stress, σ_{cr}, up to the yield stress, σ_y, as shown in Fig. 3. This nonlinear stress distribution may be considered in the design by using an effective width concept.

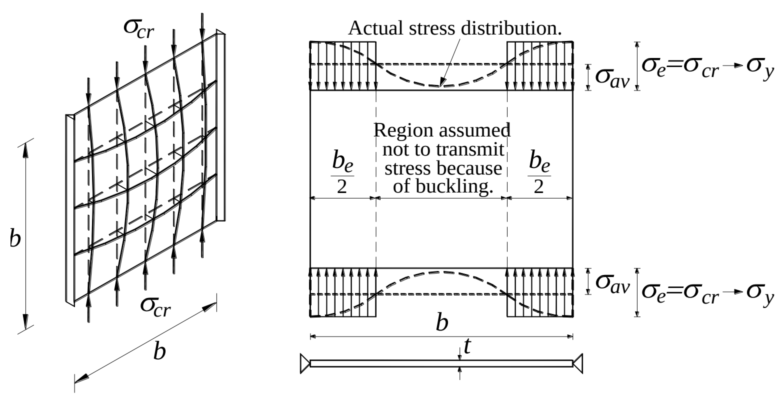


Fig. 3 Post-buckling behavior of compressed plates

The effective width factor for slender plate elements, ρ, according to Winter (1947) as given in the American Iron and Steel Institute's (AISI) *Specification for the Design of Cold-Formed Steel Structural Members* (2007) is

$$\rho = \frac{b_e}{b} = \frac{\lambda_n - 0.22}{\lambda_n^2} \tag{2}$$

where λ_n is the normalized plate slenderness and is equal to

$$\lambda_n = \sqrt{\frac{\sigma_e}{\sigma_{cr}}} = \sqrt{\frac{12(1-\nu^2)}{\pi^2 k}}\,\frac{b}{t}\sqrt{\frac{\sigma_e}{E}} \tag{3}$$

and σ_e is the post-buckling edge stress of the slender plate element after the occurrence of local plate buckling, which can be any value from the critical stress, σ_{cr}, to an upper bound value of the yield stress, σ_y, but is usually taken as the yield stress, σ_y.

Equation (2) defines the effective width without reference to the applied stress ratio $\psi = \sigma_2/\sigma_1$. A new formula for the effective width, taking into consideration the applied stress gradient defined by ψ, was derived for the ECP (2001) by Abu-Hamd and Elmahdy (2003). The same methodology was used to derive the formula using the limiting width-to-thickness ratios of stiffened slender plate elements, for the cases of pure compression and pure bending, given in the Canadian Standard S16-01 (2001) by Elmahdy and Abu-Hamd (2008).

To include a modification for the stress gradient in the effective width formula, Eq. (2) is modified assuming the following formulation

$$\rho = \frac{b_e}{b} = \frac{\lambda_n - x - y\psi}{\lambda_n^2} \tag{4}$$

The values of x and y are derived by substituting the limiting width-to-thickness ratios for the cases of pure compression, $\psi = +1.0$, and pure bending, ψ = -1.0, from different design codes into the expression for λ_n then the value of ρ is taken as 1.0. In this manner the values of *x* and *y* can be derived according to each individual code.

As an example, this method is applied to the limiting slenderness ratios for *stiffened* slender elements specified by the Eurocode EC3 (2005), given as

$$\frac{b}{t} = 42\varepsilon \qquad \text{for} \qquad \psi = +1.0$$
$$\frac{b}{t} = 124\varepsilon \qquad \text{for} \qquad \psi = -1.0 \tag{5}$$

where $\varepsilon = \sqrt{235/\sigma_y}$. The modulus of elasticity, *E*, is taken as 210,000 MPa as given in the EC3, the yield stress, σ_y, conveniently cancels out, and the buckling factor, *k*, is taken as 4.0 for the case of pure compression (ψ = +1.0) and as 23.9 for the case of pure bending (ψ = -1.0). Hence, Eqs. (3), (4), and (5) give the value of *x* as 0.144 and the value of *y* as 0.048 and the formula for the effective width ratio is approximated as

$$\rho = \frac{b_e}{b} = \frac{\lambda_n - 0.14 - 0.05\psi}{\lambda_n^2} \tag{6a}$$

or

$$\rho = \frac{b_e}{b} = \frac{\lambda_n - 0.05(3+\psi)}{\lambda_n^2} \tag{6b}$$

The European Code EC3 in Clause 4.4 of Part 1-5 *Plated structural elements* (2006) has specified a similar equation to Eq. (6b) for the ratio of the effective width factor, ρ, for internal compression elements (webs), which is

$$\rho = \frac{b_e}{b} = \frac{\lambda_n - 0.055(3+\psi)}{\lambda_n^2} \leq 1.0 \qquad \text{for } \lambda_n > 0.673 \qquad (7)$$

As well as, the following expression for the effective width of outstanding slender compression elements (flanges)

$$\rho = \frac{b_e}{b} = \frac{\lambda_n - 0.188}{\lambda_n^2} \leq 1.0 \qquad \text{for } \lambda_n > 0.748 \qquad (8)$$

Figure 4 shows a plot of $\rho = b_e/b$ given by Eqs. (2), (6a), and (7) for values of stress gradient ranging from pure compression to pure bending for a stiffened internal element (web) with a width-to-thickness ratio of 150 calculated using σ_y = 350 MPa. For the AISI effective width factor given in Eq. (2) a value of $k = 4 + 2(1-\psi)^3 + 2(1-\psi)$ is substituted in the expression for λ_n as specified by the AISI (2007), but the value of ψ is not taken as an absolute value (*i.e.*, ψ = +1.0 for pure compression and ψ = -1.0 for pure bending, *etc*.). Equations (6a) and (7) agree well with each other.

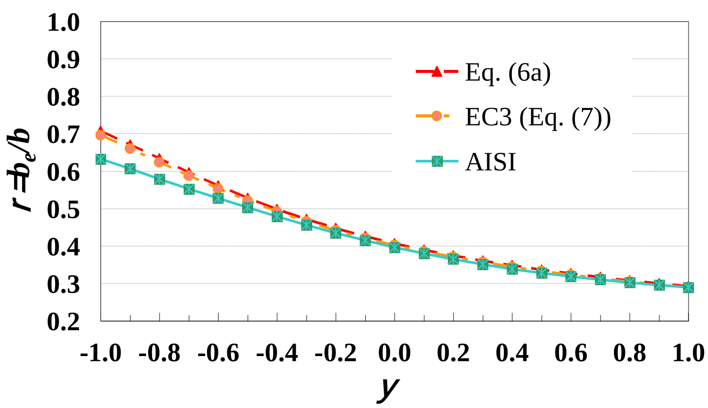


Fig. 4 Effective width ratio for a width-to-thickness ratio (*b/t*) of 150 for a stiffened internal element and σ_y = 350 MPa

Figure 5 shows the normalized plate buckling curves that correspond to the formula in the EC3 given in Eq. (7), as well as, the critical buckling stress curve, the unmodified average buckling stress curve, and the buckling stress curve according to the AISI given in Eq. (2) that does not contain a factor for the stress gradient, ψ. The EC3 curves for pure compression and pure bending are well situated between the AISI curve and the unmodified average stress buckling curve.

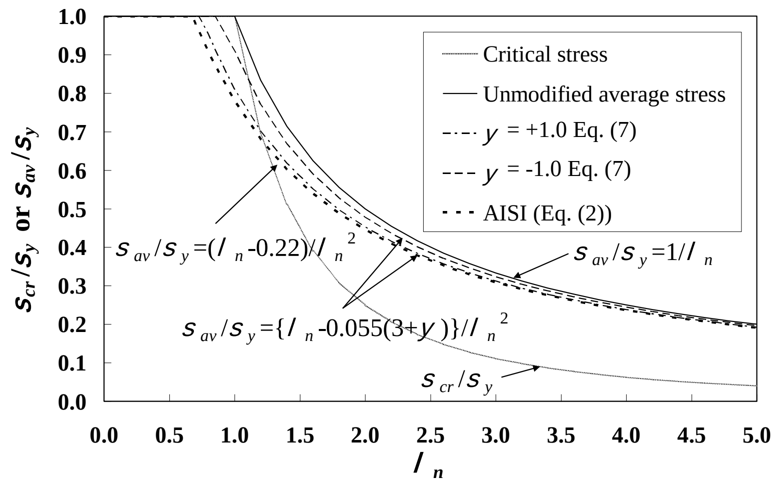


Fig. 5 Normalized plate buckling curves

3. DESIGN STRENGTH IN INTERNATIONAL CODES

The design strength governing the local buckling of slender plate girders according to AASHTO LRFD which uses a reduced stress method and EC3 which uses the effective width method are presented and compared in this section.

3.1 AASHTO LRFD (2004)

The AASHTO LRFD specifies that the nominal web bend-buckling strength be taken as

$$F_{crw} = \frac{0.9Ek}{\left(\frac{D}{t_w}\right)^2} \tag{9}$$

where D is the depth of the web and k is the bend-buckling coefficient assuming partially fixed-end edge conditions at the web to flange connections and is taken as

$$k = \frac{9}{(D_c / D)^2} = 9(1-\psi)^2 \tag{10}$$

and D_c is the depth of the web in compression.

3.2 Eurocode (EC3) EN 1993-1-5:2006 (2006)

The elastic moment strength is calculated from $M_n = S_{eff}\sigma_y$ where S_{eff} is the effective elastic section modulus of the cross section calculated by applying an effective width ratio as defined in Eq. (7) for stiffened elements (web) and in Eq. (8) for unstiffened elements (flange). Accordingly, the ratio σ_{cr}/σ_y cannot be compared to the ratio S_{eff}/S_g, where S_g is the gross elastic section modulus, as the edge stresses in the effective section are assumed to increase from σ_{cr} to σ_y.

3.3 Comparison of AASHTO LRFD and EC3

To compare the design strength of AASHTO LRFD and EC3, a set of mono-symmetrical plate girder sections were chosen having web heights, h_w, of 1000, 1500, and 2000 mm. The thickness of these webs was varied to vary the slenderness of the web, $\lambda_w = h_w/t_w$. The top compression flange was taken as 200x9 mm for the 1000 mm webs, 250x12 mm for the 1500 mm webs, and 300x13 mm for the 2000 mm webs. All of these compression flanges are in the non-compact or compact range with no reduction in the flange width. The size of the bottom tension flange was varied to achieve a stress ratio in the web of ψ = -1.0, -0.8, -0.6, and -0.4, as shown in Fig. 6. The effective web of all the sections was calculated according to EC3 using Eqs. (3) and (7) and the expression $k = 7.81 - 6.29\psi + 9.78\psi^2$ for the plate buckling factor, as specified in EC3, which is valid for values of $0 \geq \psi \geq -1.0$ and was derived on the assumption of simply supported edge conditions. Two values for the yield stress were chosen, σ_y = 350 MPa and σ_y = 250 MPa and the value of E was taken as 210,000 MPa as specified in EC3. Hence, the ratio of the effective section modulus to the gross section modulus, S_{eff}/S_g, was calculated. It was found that a good approximation for the expression S_{eff}/S_g is

$$\frac{S_{eff}}{S_g} = -5.28\gamma^2 + 3.58\gamma + 0.42 \qquad \text{where} \quad \gamma = \frac{\sqrt{-\psi E / \sigma_y}}{\lambda_w} \tag{11}$$

having a deviation of less than ±6.6% from the exact value for both values of σ_y.

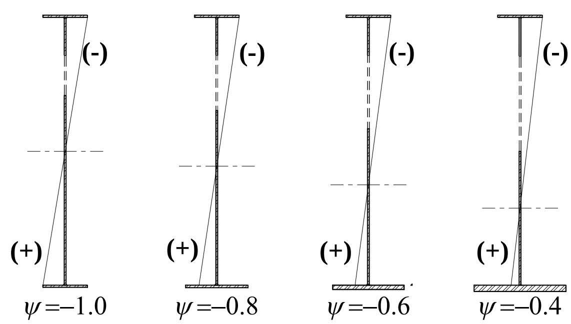


Fig. 6 Effective steel plate girder cross sections with variable web stress gradient

The ratios of S_{eff}/S_g calculated according to EC3 were plotted with the AASHTO LRFD normalized web bend-buckling curve given in Eqs. (9) and (10), as well as, the normalized elastic buckling stress curve according to Eq. (1) using the expression for k with simply supported edge conditions, $k = 7.81 - 6.29\psi + 9.78\psi^2$, rather than partially fixed edge conditions as given in the AASHTO LRFD curve. Figures 7-10 show these curves for the cases of ψ = -1.0, -0.8, -0.6, and -0.4 and the EC3 S_{eff}/S_g results for σ_y = 350 MPa.

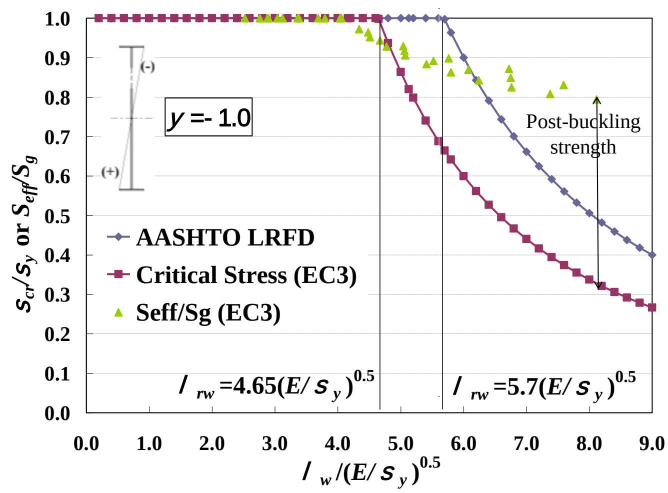


Fig. 7 Comparison of AASHTO LRFD web bend-buckling strength with EC3 results for ψ = -1.0

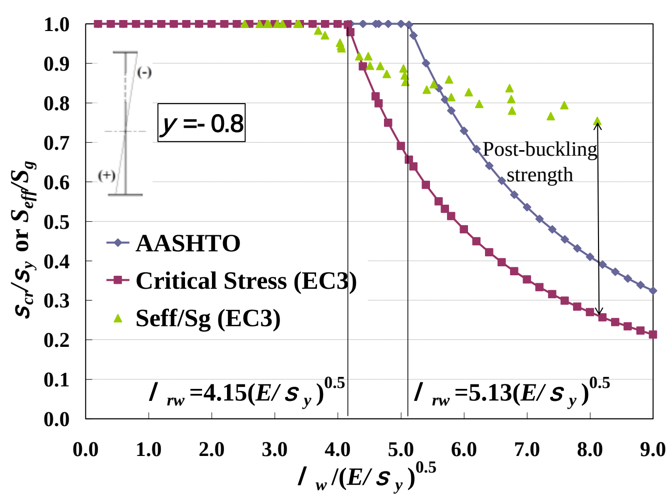


Fig. 8 Comparison of AASHTO LRFD web bend-buckling strength with EC3 results for ψ = -0.8

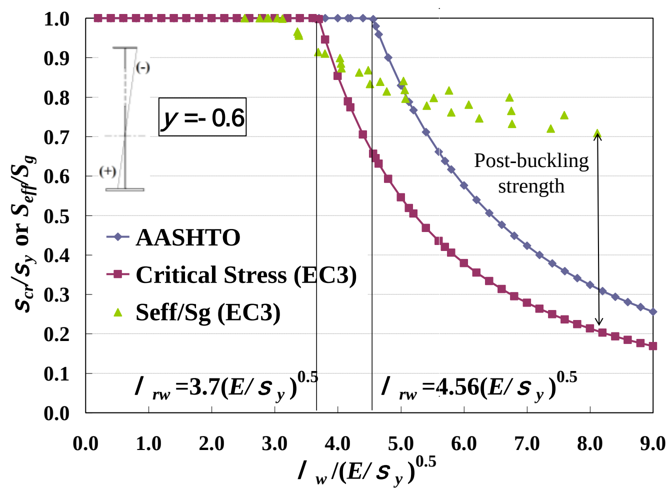


Fig. 9 Comparison of AASHTO LRFD web bend-buckling strength with EC3 results for ψ = -0.6

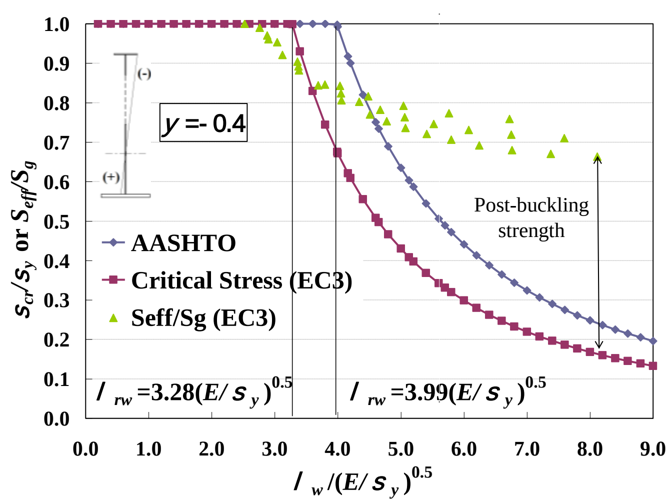


Fig. 10 Comparison of AASHTO LRFD web bend-buckling strength with EC3 results for ψ = -0.4

These figures show that the consideration of post-buckling strength in EC3 results in a higher design strength than the AASHTO LRFD critical stress approach. This post-buckling reserve capacity decreases with the decrease in web slenderness.

3.4 Limiting Slenderness Ratios

It can be seen from Figs. 7-10 that the limiting slenderness ratio for slender web elements, λ_{rw}, decreases as the value of ψ increases from -1.0 to -0.4. Hence the value of the limiting slenderness ratio for slender web elements corresponding to the AASHTO LRFD web bend-buckling curve can be expressed as a function of ψ in the form of

$$\lambda_{rw} = \frac{5.7(1-\psi)}{2}\sqrt{E/\sigma_y} \qquad \text{for} \qquad \psi \leq 0 \qquad (12)$$

Also, the EC3 (2005) specifies a minimum web slenderness limit for slender webs of

$$\begin{aligned} \lambda_{rw} &= 42\varepsilon/(0.67+0.33\psi) && \text{for} \quad \psi > -1.0 \\ \lambda_{rw} &= 62\varepsilon(1-\psi)\sqrt{-\psi} && \text{for} \quad \psi \leq -1.0 \end{aligned} \qquad (13)$$

where $\varepsilon = \sqrt{235/\sigma_y}$. Figure 11 shows the variation of the web slenderness limits for non-compact webs with the stress gradient, ψ, according to the AASHTO LRFD as given in Eq. (12), and the EC3 as given in the first part of Eq. (13). It is clear from Fig. 11 that the limiting slenderness ratios corresponding to doubly symmetric sections, with ψ = -1.0, are not conservative for the steel section in the construction stage. This effect, however, is balanced by the fact that the stress level due to the construction dead loads in the steel section only is not allowed to reach σ_y and usually lies in the range of 40% – 60% of σ_y. Accordingly, a modified value of the slenderness limit can be calculated for this case depending on the ratio of the dead load stresses to the steel yield stress, σ_y, as follows

$$\lambda_{rw\beta} = \frac{\lambda_{rw}}{\sqrt{\beta}} \qquad \text{where} \qquad \beta = \left|\frac{\sigma_1}{\sigma_y}\right| \tag{14}$$

This would give expressions for the modified limiting slenderness ratios for non-compact webs with reduced construction stresses, according to AASHTO LRFD and EC3 with $\psi > -1.0$, as

$$\lambda_{rw\beta} = \frac{5.7(1-\psi)}{2\sqrt{\beta}} \qquad \text{where} \qquad \beta = \left|\frac{\sigma_1}{\sigma_y}\right| \tag{15}$$

$$\lambda_{rw\beta} = \frac{42\varepsilon}{(0.63+0.33\psi)\sqrt{\beta}} \qquad \text{where} \qquad \beta = \left|\frac{\sigma_1}{\sigma_y}\right| \tag{16}$$

where $\varepsilon = \sqrt{235/\sigma_y}$.

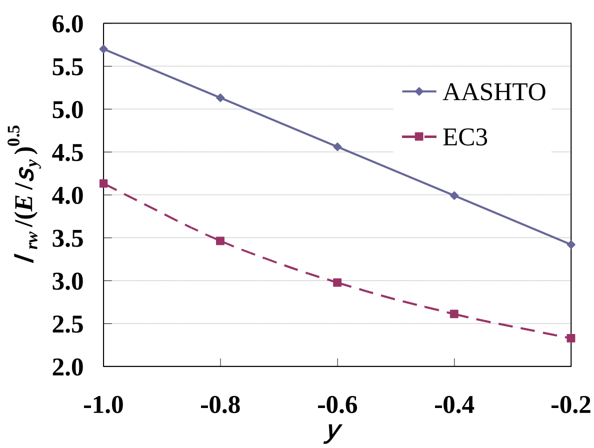
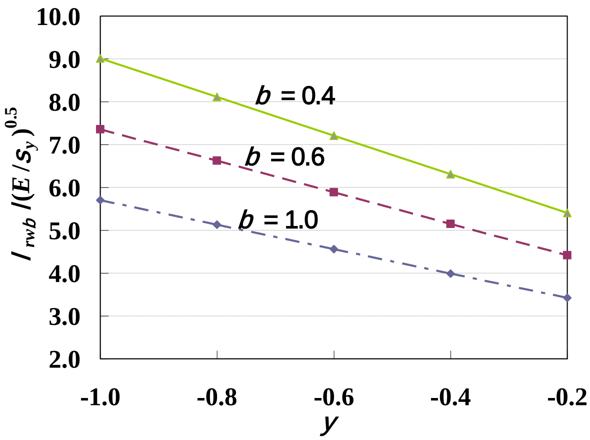
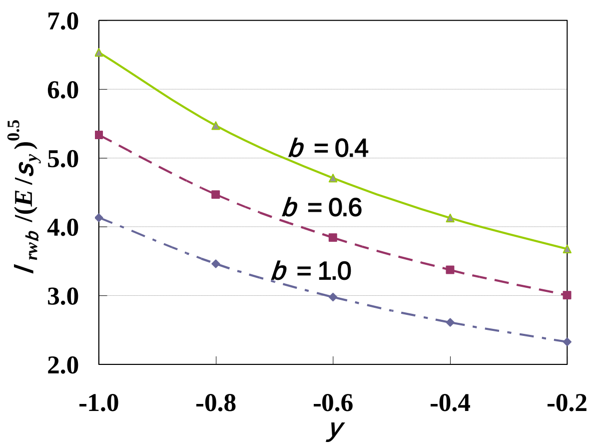


Fig. 11 Modified limiting slenderness ratios for non-compact webs

The modified slenderness limits according to Eqs. (15) and (16) for different values of β are shown on Figs. 12(a) and (b) for the AASHTO LRFD and the EC3, respectively.



(a) AASHTO LRFD



(b) EC3

Fig. 12 Variation of modified web slenderness limits for reduced construction stresses

Similar results are obtained for the variation of slenderness limits of the compression *flange* during the construction stage as shown in Fig. 13. Unlike the web, this effect is always favorable since in the composite stage the compression flange local buckling is prevented by the shear stud attachments to the concrete slab.

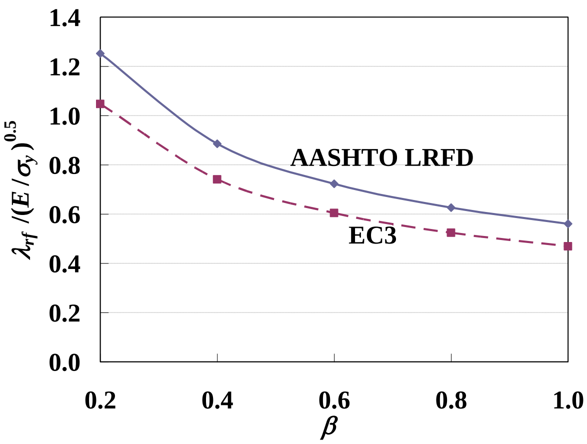


Fig. 13 Variation of modified flange slenderness limits for reduced construction stresses

4. CONCLUSIONS

The addition of the stress ratio factor in the effective width ratio of slender webs of un-shored plate girder composite sections, in positive bending, has beneficial effects on the design of these sections. It has the effect of increasing the effective width of slender webs in the during construction non-composite stage. Not only does the effective section of the plate girder have an excessive amount of post-buckling capacity, the effective section only has to resist the during construction part of the dead load. The composite action of the section has the effect of increasing the stiffness of the section, reducing local plate buckling of slender webs, and preventing local plate buckling of slender compression flanges.

5. NOMENCLATURE

b = width of a plate
b_e = effective width
D = depth of web plate
D_c = depth of part of web in compression
E = modulus of elasticity taken as 210,000 MPa
F_{crw} = web bend-buckling stress
h_w = height of web plate
k = plate buckling factor or web bend-buckling coefficient
M_n = elastic moment strength of a section
S_{eff} = effective elastic section modulus
S_g = gross elastic section modulus
t = thickness of a plate
t_w = thickness of web plate
x, y = variables
β = stress level = $|\sigma_1/\sigma_y|$
γ = parameter = $\sqrt{-\psi E/\sigma_y}\,/\,\lambda_w$
ε = factor = $\sqrt{235/\sigma_y}$
λ_n = normalized plate slenderness
λ_{rf} = modified limiting slenderness parameter for flange plates
λ_{rw} = modified limiting slenderness parameter for web plates
$\lambda_{rw\beta}$ = slenderness parameter for web plates at specified stress level β
λ_w = slenderness of web plate = h_w/t_w
ν = Poisson's ratio taken as 0.3
ρ = effective width factor
σ_1 = larger end compressive stress on a plate
σ_2 = smaller end compressive stress (or tensile stress) on a plate
σ_{av} = average stress
σ_{cr} = elastic buckling stress of a plate
σ_e = edge stress
σ_y = yield stress
ψ = stress gradient = σ_2/σ_1

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