

Effect of Local Buckling on the Design of Steel Plate Girders

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ABSTRACT

This paper presents a study of the effect of local plate buckling on the design of plate girder sections in three internationally recognized codes, the American AISC and AASHTO, and the European EC3. The design provisions related to the local buckling of flange plates under uniform compression, web plates under uniform bending, and web plates under shear according to the three codes are compared over a wide range of design parameters. The results show that considerable differences exist between the American and European codes when the design is governed by elastic buckling. Numerical solutions of the elastic buckling of plate girder slender sections under uniform bending using the finite strip method are used in a parametric study to evaluate the effect of actual plate edge conditions on the elastic buckling strength of these sections. The results of the parametric study show that the idealized edge conditions are always conservative for compression flange buckling and not always conservative for web bend buckling.

1. INTRODUCTION

The design of plate girder sections is usually governed by flexural strength and shear strengths limit states. Local plate buckling affects the calculation of the cross section resistance related to compression flange local buckling, web bend buckling in the flexural strength limit state and web shear buckling in the shear strength limit state. Other limit states such as lateral torsional buckling, tension flange yielding, and fatigue are not covered in this paper.

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Generally, a three-range design format is followed depending on the value of a slenderness parameter, λ , which equals the width-to-thickness ratio of the plate component considered. When the slenderness ratio λ is less than a value λ_p , the section can reach its plastic moment capacity and is classified as compact in the American codes AISC [2005] and AASHTO [2004], and as class 2 in the European Code EC3 [2005]. When $\lambda_p < \lambda < \lambda_r$, the section strength is limited by its yield moment and is called non-compact in the American codes AISC and AASHTO, and class 3 in the European code EC3. When $\lambda > \lambda_r$, the section strength is governed by elastic buckling and the section is slender in AISC/AASHTO and class 4 in EC3. Details of the governing equations used to calculate the cross section resistances in each case are given in the respective codes and several papers such as White [2008] and White and Barker [2008]. Summary of code provisions related to plate buckling in the three considered codes is given in the Appendix to this paper.

Most design codes use basically the same approach to determine the design strength for compact and non-compact sections. As a result, design codes give comparable results for these sections. On the other hand, different approaches are used in present codes to determine the slender section design strength.

For the flexural strength limit states, AISC and AASHTO use a reduced stress which is based on the theoretical elastic buckling solution of the plate buckling problem. EC3 uses a linear stress distribution over an effective width to replace the actual nonlinear stress distribution over the buckled plate. These two approaches for handling local flexural buckling of slender plates are very distinct and therefore give different results. Generally, the reduced stress method is much easier to apply but does not benefit from the additional post buckling strength considered in the effective width method.

For the shear strength limit state, most codes use the same approach to calculate the cross section strength based on the theoretical shear buckling resistance with allowance made for post buckling due to tension field action. The post buckling strength in AISC and AASHTO is based on Basler model which can only be applied to girders having closely spaced transversal stiffeners. On

the other hand, the post buckling strength in EC3 is based on Cardiff and Hoglund models which can be applied to both stiffened and unstiffened girders. Differences between codes exist because different shear failure models are used.

In the following sections, the buckling strength determined according to the American and European codes are compared over a wide range of web and flange slenderness ratios for the three limit states of compression flange local buckling, web bend buckling, and shear buckling. Since each code uses a different format for the flexural strength limit states, the AISC and EC3 equations have been expressed in terms of the nominal flexural strength F_n instead of the nominal moment strength M_n by dividing the moment equations by the elastic section modulus S_x . The resulting equations were then used to plot the relation between the normalized stress (F_n/F_y) against the respective slenderness ratio λ . The limiting slenderness ratios defining compact, non-compact limits were calculated using the values $F_y = 345$ MPa and $E = 2.04 \times 10^5$ MPa.

2. COMPRESSION FLANGE LOCAL BUCKLING

Figure 1 shows the comparison of the compression flange local buckling provisions according to the three codes. The AISC results are presented for the two case of compact web (CW: $\lambda_w = 80$) and slender web (SW: $\lambda_w = 160$). The theoretical elastic buckling stress obtained as the solution to the plate buckling problem assuming simple-free edge conditions ($k_f = 0.43$) is also shown on the Figure. The comparison among the three codes reveals the following:

- 1- The limiting slenderness ratios defining compact and non-compact limits vary considerably between American and European codes as shown in Table 1.
- 2- AASHTO neglects web plastification effect for compact flanges.
- 3- For slender flanges, only AISC considers the effect of web slenderness on compression flange buckling.
- 4- For slender flanges, both AISC and AASHTO do not consider post buckling strength so that the results of applying EC3, which considers post buckling, are much larger, especially at larger slenderness ratios as shown in Figure 5a.

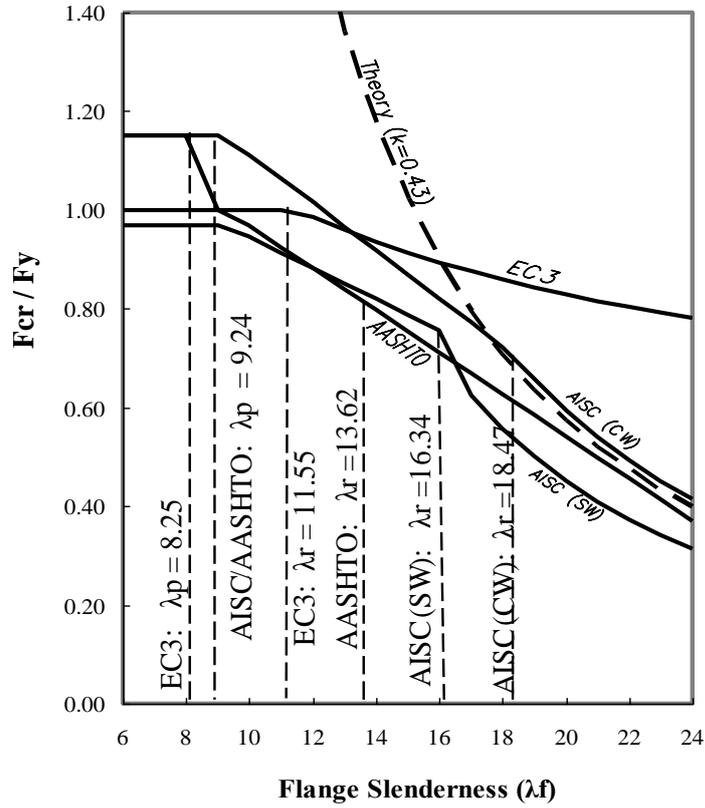


Fig. 1 Compression Flange Buckling Stress

Table 1: Slenderness Limits in Different Codes

Code	Compact	Non-compact
1- Compression Flange Local Buckling:		
AISC/AASHTO	9.24	16.33
EC3	8.25	11.55
2- Web Bend Buckling:		
AISC/AASHTO	91.43	138.61
EC3	68.50	102.34
3- Web Shear Buckling:		
AISC/AASHTO	59.81/60.90	74.49/76.12
EC3	49.32	77.01

3. WEB BEND BUCKLING

The relation between the normalized stress (F_n/F_y) and the web slenderness ratio λ_w according to different codes is shown in Fig. 2. The theoretical elastic buckling stresses obtained as the solution to the plate buckling problem assuming simply supported edges ($k_w=23.9$) and partially fixed edges ($k_w=36$) are also shown on the Figure. The comparison among the three codes reveals the following:

- 1-The limiting slenderness ratios defining compact and non-compact limits vary considerably between American and European codes as shown in Table 1.
- 2- For slender webs, AASHTO does not consider post buckling strength so that the results of applying EC3, which considers post buckling, are much larger, especially at larger slenderness ratios as shown in Figure 5b.

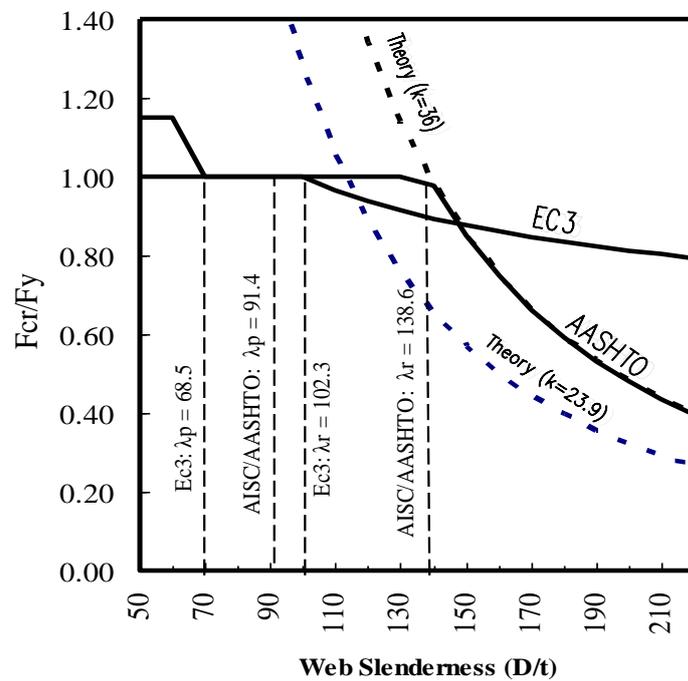


Fig. 2 Web Bend Buckling Stress

4. SHEAR BUCKLING

4.1 Unstiffened Webs

The relation between the normalized shear strength (V_n/V_p) and the web slenderness ratio λ_w for unstiffened webs according to different codes is shown in Fig. 3. The theoretical elastic buckling strength obtained as the solution of the plate buckling problem assuming simply supported edges ($k_q=5.34$) and $k_q=8.25$ are also shown on the Figure. The second k_q value is based on the expression suggested by Lee et al. [1996] for the real edge condition at the web-to-flange connection.

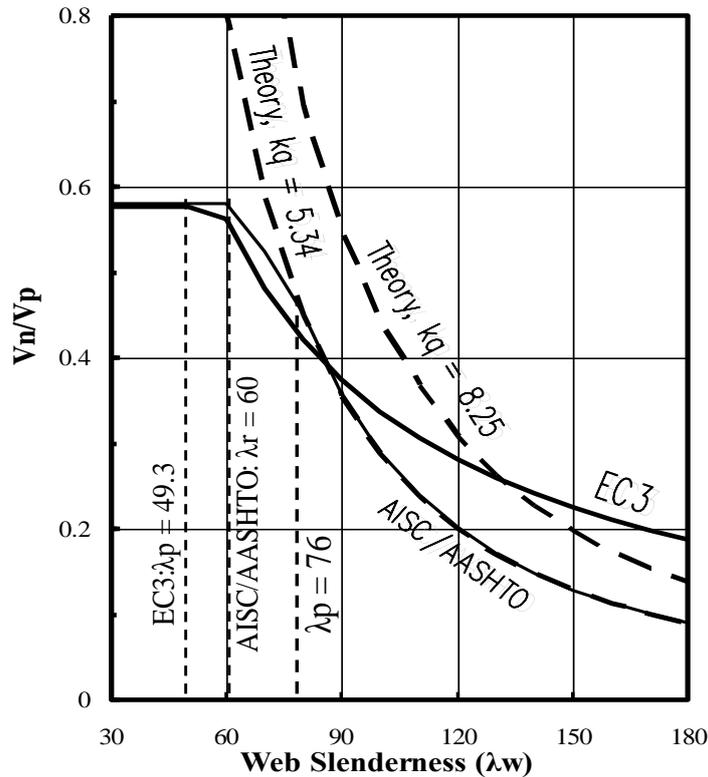


Fig. 3 Shear Buckling Stress for Unstiffened Webs

4.2 Stiffened Webs

The relation between the normalized shear strength (V_n/V_p) and the web slenderness ratio λ_w for stiffened webs, with an aspect ratio equal to 1, according to different codes is shown in Fig. 4. The theoretical elastic buckling stress obtained as the solution to the plate buckling problem assuming simply supported edges ($k_q=9.34$) and $k_q=11.95$ are also shown on the Figure. The second k_q value is based on the expression suggested by Lee et al. [1996] for the real edge condition at the web-to-flange connection.

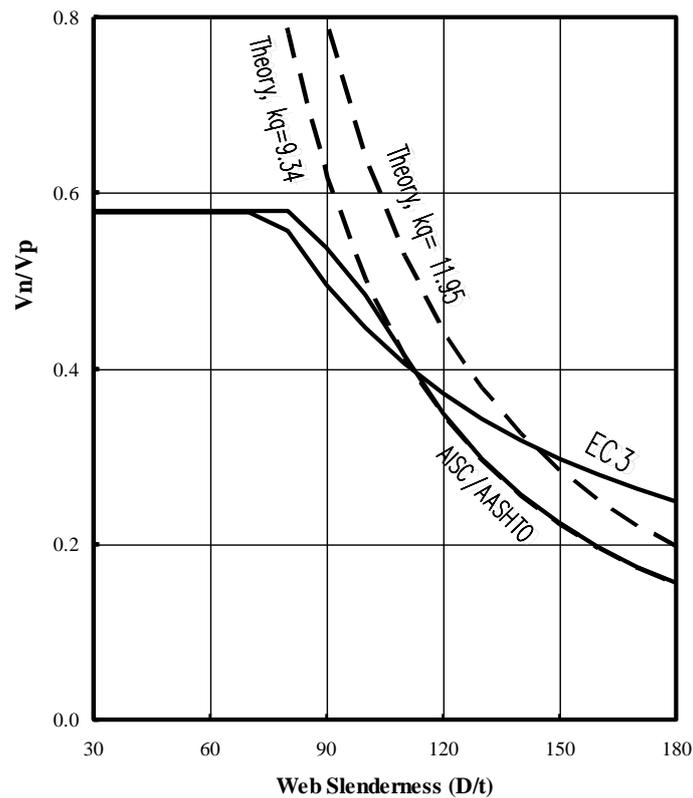


Fig. 4 Shear Buckling Stress for Stiffened Webs ($\alpha = 1$)

The comparison among the three codes reveals the following:

- 1- The limiting slenderness ratios defining compact and non-compact sections vary considerably for compact section limits but nearly equal for non-compact section limits as shown in Table 1.
- 2- For slender webs, both AISC and AASHTO do not consider postbuckling strength for unstiffened webs so that the results of applying EC3, which permits postbuckling, are larger as shown in Figure 5c. Lee et al. [2008] performed an analytical study on the shear strength of long web panels and concluded that the present provisions underestimate both the elastic shear strength and also the postbuckling strength.

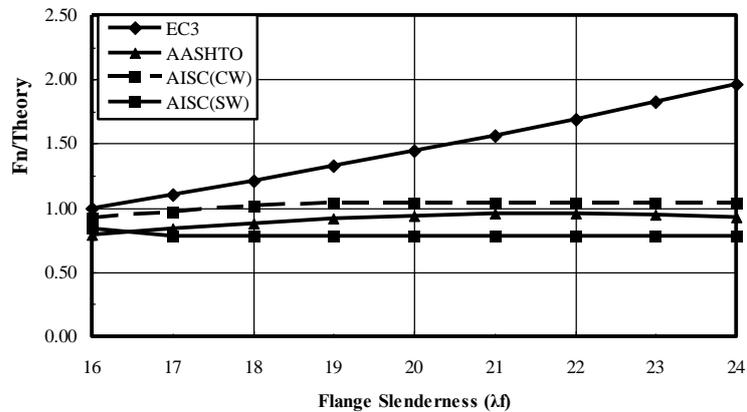


Fig. 5a Ratio of Compression Flange Elastic Buckling Stress to Theory

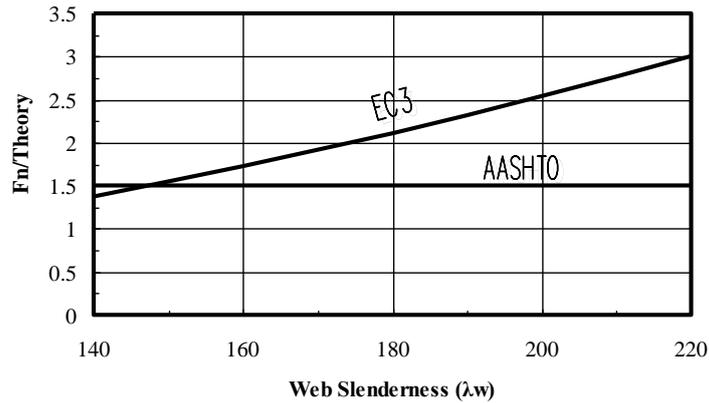


Fig. 5b Ratio of Web Elastic Bend Buckling Stress to Theory

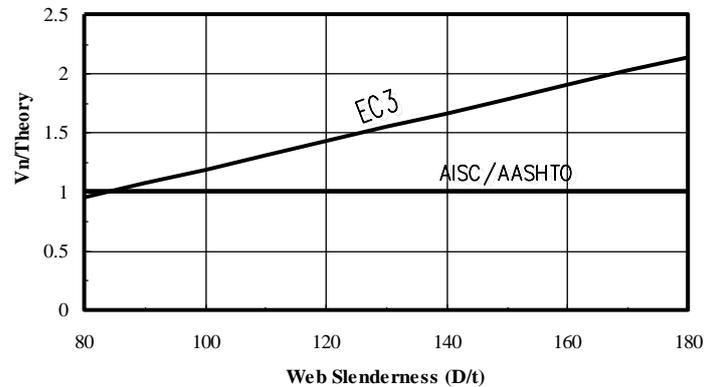


Fig. 5c Ratio of Web Elastic Shear Buckling Stress to Theory

5- EFFECT OF EDGE CONDITIONS

All the previous approaches for calculating the effect of local plate buckling on both the flexural and shear strengths have the same simplification of treating the buckling of individual plate elements in the cross section separately thus ignoring the interaction between flange and web buckling. In addition, the supported edges of the plates at the web-to-flange connection are usually idealized as simple. This idealization does not accurately represent the real

strength of the cross section which can only be determined from physical tests. A practical alternative is to use numerical analysis techniques, such as finite element and finite strip methods, to arrive at a better approximation to the buckling strength of slender sections. Presently, the European code EC3: EN 1993-1-5 [2005] includes the possibility of using finite element analysis as a reliable tool in the verification of buckling limit states.

Available numerical analysis softwares can easily be used to study cross section behavior up to failure including post buckling. These softwares can also handle the effect of initial imperfections, residual stresses, and material nonlinearities as well as geometrical nonlinearities. These additional factors have minor effects on the strength of slender sections since they fail by elastic buckling as shown by Maiorana et al [2009]. For this reason, the finite strip method software CUFSM developed by Schafer and Adany [2006] is used in the present paper to study the elastic buckling strength of slender plate girder section. The program was used to conduct a parametric study of the effect of actual plate edge conditions on the elastic buckling strength of slender plate girder sections. The parameters varied in the study are:

- 1) Web plate height of 1000, 1500, and 2000 mm, and
- 2) Flange plate width of 250, 300,400,500 mm.

The corresponding web and flange plate thicknesses were selected to cover the following combinations:

- 1) Slender flange with compact, non-compact, and slender web,
- 2) Slender web with compact, non-compact, and slender flange.

The steel used has a nominal yield stress F_y of 345 MPa and a modulus of elasticity E of 204000 MPa.

The elastic buckling stress for the compression flange local buckling and web bend buckling were selected from various buckling modes determined by the program and then used to calculate the corresponding elastic buckling coefficients for flange buckling, k_f , and for web buckling, k_w .

5.1 Compression Flange Local Buckling

The theoretical values of the buckling coefficient k_f are 0.43 for a simply supported edge and 1.28 for a fixed edge. The AISC value

ranges from 0.76 if the web is compact to 0.35 if the web is slender. AASHTO uses a value of 0.35 for all cases. Figure 6 shows the results of the parametric study conducted over a wide range of plate girder sections of practical proportions using the finite strip analysis software CUFSM. The results show that the compression flange local buckling coefficient k_f lie in the range 1.10 to 1.20 regardless of the web slenderness. This indicates that the flange support at the web connection is close to being fixed. This result shows that the elastic buckling strength of slenderness flanges is underestimated by AISC and AASHTO. The results of EC3, being larger due to the consideration of post buckling, are closer to the analytical results.

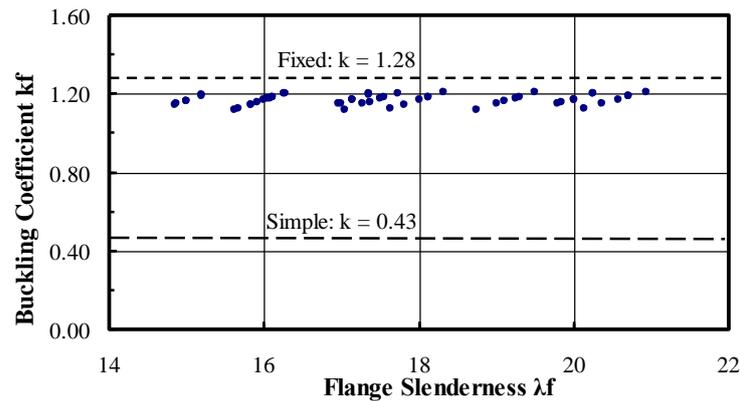


Figure 6 Variation of Flange Buckling Coefficient k_f

5.2 Web Bend Buckling

The theoretical values of the buckling coefficient k_w are 23.9 for simply supported edges and 39.6 for fixed edges. AASHTO uses a value of 36, see White [2008], indicating that the web edges are close to being fixed at the flange connections. Figures 6a, 6b, and 6c show the results of the parametric study conducted over a wide range of plate girder sections of practical proportions using the finite strip analysis software CUFSM. The results show that k_w varies considerably between the two theoretical limits, depending on both the web slenderness λ_w (Fig. 7a) and the flange slenderness λ_f (Fig. 7b). The value of k_w ranges between 33.7 and 38.3 for

compact flanges, between 29.2 and 37.4 for non-compact flanges, and between 21.6 and 32 for slender flanges. This indicates that the value used by AASHTO ($k_w = 36$), is suitable only for sections with compact flanges ($\lambda_f \leq 12$ according to AASHTO), otherwise it is not conservative for sections with slender and non compact flanges. These results are similar to those presented by Schafer and Seif [2008] for the local buckling of AISC rolled W-sections used as axially loaded columns. Based on the results of the girder range considered in the present study, a lower bound on the value of the k_w can be represented by the straight line shown in Fig. 7c as:

$$k_w = 16 + 0.8 (\lambda_w/\lambda_f) \quad (1)$$

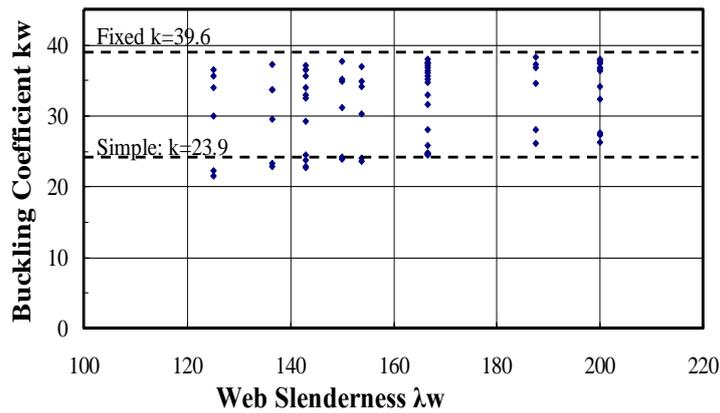


Fig. 7a Variation of Buckling Coefficient k_w with λ_w

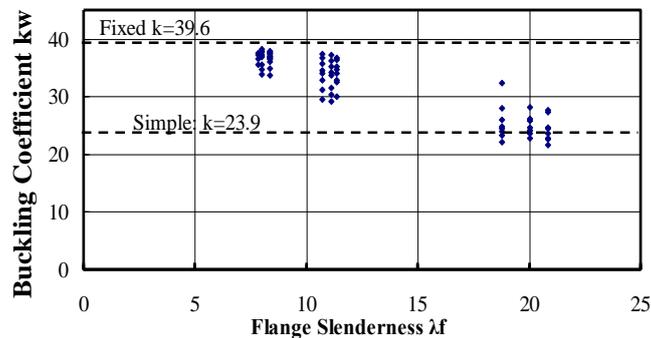


Fig. 7b Variation of Buckling Coefficient k_w with λ_f

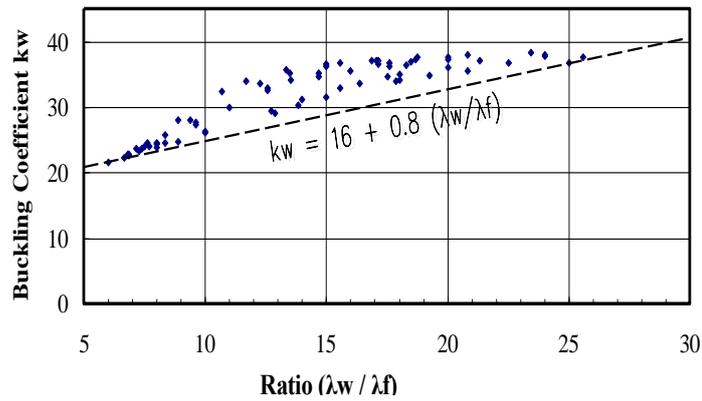


Fig. 7c Variation of Buckling Coefficient k_w with (λ_w/λ_f)

6.3 Web Shear Buckling

The theoretical values of the shear buckling coefficient for unstiffened webs are 5.34 for simply supported edges and 8.98 for fixed edges. The corresponding values for a stiffened web having an aspect ratio of 1 are 9.34 for simple edges and 12.6 for fixed edges. The real boundary condition at the web-to-flange connection is somewhat between simple and fixed supports. Lee et al. [1996] suggest the following expression to better represent the real boundary condition:

$$k = k_{ss} + 0.8(k_{sf} - k_{ss}) \quad \text{for} \quad t_f/t_w \geq 2 \quad (2)$$

$$k = k_{ss} + 0.8(k_{sf} - k_{ss})[1 - 2 \{2 - (t_f/t_w)\}/3] \quad \text{for} \quad 0.5 \leq t_f/t_w \leq 2 \quad (3)$$

The results of applying these expressions are shown in Figures 3 and 4.

6. CONCLUSION

The design provisions related to local buckling of plate girder sections in the three international codes AISC, AASHTO, and EC3 are compared over a wide range of slenderness ratios. The three codes give comparable results for compact and non-compact sections but differ considerably for slender sections depending on the consideration of postbuckling behavior in both the flexural strength and the shear strength limit states. The effect of actual plate edge conditions on the elastic buckling strength of these sections was evaluated through a parametric study using the finite strip method. The results show that the idealized edge conditions are always conservative for compression flange buckling and not always conservative for web bend buckling.

APPENDIX:

DESIGN PROVISIONS FOR LOCAL BUCKLING

This Appendix presents a summary of the local buckling strength provisions in the American codes AISC and AASHTO and the European code EC3 for the three cases of compression flange buckling, web bend buckling, and shear buckling. Since the two American codes use essentially the same approach, they are presented together.

1- COMPRESSION FLANGE LOCAL BUCKLING PROVISIONS

1. 1) AISC 2005 / AASHTO 2004:

a) Slenderness Limits:

AISC: The flange is compact when $\lambda < \lambda_p = 0.38 \sqrt{E / F_y}$ and slender when $\lambda > \lambda_r = 0.95 \sqrt{E k_c / 0.7 F_y}$. The buckling coefficient k_c equals $4 / \sqrt{\lambda w}$ which represents a transition from a maximum

value of 0.76 corresponding to rolled I-shapes to a minimum value of 0.35 corresponding to slender webs.

AASHTO: The flange is compact when $\lambda < \lambda_p = 0.38 \sqrt{E / F_y}$ and slender when $\lambda > \lambda_r = 0.56 \sqrt{E / F_y}$. These limits are the same as used in AISC 2005 with k_c taken equal to 0.35 corresponding to slender webs usually used in bridges.

b) Strength:

i) Compact Flange:

AISC: The nominal moment strength M_n is equal to $R_{pg} R_{pc} M_{yc}$, where the R_{pg} is the flange-strength reduction factor due to bend buckling of slender webs. It takes a value < 1 for slender webs and taken equal to 1 for compact and non-compact web. The factor R_{pc} is the web plastification factor which is equal to the section shape factor when the web is compact and taken equal to 1 when the web is slender. For non-compact webs, R_{pc} varies linearly between 1 and the section shape factor.

AASHTO: The nominal flexural strength F_n is equal to $R_b F_{yc}$, where the R_b is the flange-strength reduction factor due to bend buckling of slender webs which is the same as R_{pg} in AISC 2005. The additional strength due to web plastification as reflected by R_{pc} in AISC 2005 is neglected.

ii) Slender Flange:

AISC: The nominal moment strength M_n is based on the theoretical expression for elastic buckling given by:

$$M_n = 0.9 R_{pg} k_c S_x / \lambda^2.$$

AASHTO limits the flange slenderness ratio to 12 which makes the flange always compact. If this limit is exceeded, the nominal flexural strength for both non-compact and slender flanges is given by: $F_n = [1.0 - 0.30 * (\lambda - \lambda_p) / (\lambda_r - \lambda_p)] * R_b * F_y$.

iii) Non-compact Flange (AISC): The nominal moment strength is based on a linear transition between compact and slender flange.

1.2) EUROCODE EC3:

a) **Slenderness Limits:** The flange is compact when $\lambda < \lambda_p = 10 \sqrt{235/F_y}$ and slender when $\lambda > \lambda_r = 14 \sqrt{235/F_y}$.

b) Strength:

i) **Compact Section:** The nominal moment strength M_n is equal to the plastic moment $M_p = Z_x * F_y$.

ii) **Non-compact Section:** The nominal moment strength M_n is equal to the yield moment $M_y = S_x * F_y$.

iii) **Slender Section:** The nominal moment strength is calculated from $M_n = S_{eff} * F_y$ where S_{eff} is the effective elastic section modulus of the cross section calculated by applying a reduction factor ρ to slender plate components. The reduction factor ρ is expressed in terms of the normalized plate slenderness parameter $\lambda_n = \lambda_f \sqrt{F_y} / 285$ as: $\rho = (\lambda_n - 0.188) / \lambda_n^2 \leq 1$.

2. WEB BEND BUCKLING PROVISIONS:

2.1) AISC 2005 / AASHTO 2004:

a) Slenderness Limits:

The web is compact in both AISC and AASHTO when $\lambda < \lambda_p = 3.76 \sqrt{E / F_y}$ and slender when $\lambda > \lambda_r = 5.7 \sqrt{E / F_y}$.

b) Strength:

AISC: The web bend buckling is not covered in AISC 2004. It only affects the limit state of compression flange buckling. This is explained by the fact that most plate girders used in buildings have non-slender webs.

AASHTO:**i) Compact and Non-compact Web:**

The nominal flexural strength F_n is equal to the yield stress F_y .

ii) Slender Web: The nominal flexural strength is given by: $F_n = 0.9 E k / \lambda^2 \leq F_y$, where k = buckling factor for web bend buckling taken equal to 36. This value is based on assuming the edge restraint at the flange web joint to be almost fixed and is calculated from the expression [xx]: $k = k_{ss} + 0.8*(k_{sf} - k_{ss})$, where k_{ss} = bend buckling coefficient for simply supported edge = 23.9, and k_{sf} = bend buckling coefficient for fully restrained edge = 39.6.

2.2) EUROCODE EC3:**a) Slenderness Limits:**

The web is compact when $\lambda < \lambda_p = 83 \sqrt{235 / F_y}$ and slender when $\lambda > \lambda_r = 124 \sqrt{235 / F_y}$.

b) Strength:

i) Compact Sections: The nominal moment strength M_n is equal to the plastic moment : $M_p = Z_x * F_y$.

ii) Non-compact Sections: The nominal moment strength M_n is equal to the yield moment $M_y = S_x * F_y$.

iii) Slender Sections: The nominal moment strength is calculated from $M_n = S_{eff} * F_y$ where S_{eff} is the effective elastic section modulus of the cross section calculated by applying a reduction factor ρ to slender plate components. The reduction factor ρ is expressed in terms of the normalized plate slenderness parameter $\lambda_n = \lambda_w \sqrt{F_y} / 2125$ as: $\rho = (\lambda_n - 0.11) / \lambda_n^2 \leq 1$.

3. WEB SHEAR BUCKLING PROVISIONS:**3.1) AISC 2005 / AASHTO 2004:****a) Slenderness Limits:**

AISC: The web is compact when $\lambda < \lambda_p = 1.10 \sqrt{E k_q / F_y}$ and slender when $\lambda > \lambda_r = 1.37 \sqrt{E k_q / F_y}$. The buckling coefficient k_q equals $5 + 5 / \alpha^2$ where α = plate aspect ratio.

AASHTO: The web is compact when $\lambda < \lambda_p = 1.12 \sqrt{E k_q / F_y}$ and slender when $\lambda > \lambda_r = 1.40 \sqrt{E k_q / F_y}$. The buckling coefficient k_q equals $4 + 5.34 / \alpha^2$ for $\alpha < 1$ and $5.34 + 4 / \alpha^2$ for $\alpha > 1$, where α = plate aspect ratio = 5.34 for un-stiffened webs.

b) Strength:

i) Compact Web:

AISC: The nominal shear strength V_n is equal to the plastic shear capacity given by $V_p = 0.6 A_w F_y$, where A_w = web area.

AASHTO: The nominal shear strength V_n is equal to plastic shear capacity given by $V_p = 0.58 A_w F_y$.

ii) Slender Web:

AISC: The nominal shear strength V_n is based on the theoretical expression for elastic buckling given by $V_n = C^* V_p$ where C = buckling reduction factor = $(1.51 E k_q / (\lambda^2 F_y))$.

AASHTO: The nominal shear strength V_n is based on the theoretical expression for elastic buckling given by $V_n = C^* V_p$ where C = buckling reduction factor = $(1.57 E k_q / \lambda^2 F_y)$.

iv) Non-compact Web:

AISC: The nominal shear strength is equal to $V_n = C^* V_p$ where C = buckling reduction factor = $(1.1 \sqrt{E k_q / F_y} / \lambda)$ for un-stiffened webs, webs with aspect ratio $\alpha > 3$, and end panels. For stiffened webs with $\alpha < 3$, tension field action is allowed giving:

$$V_n = V_p [C + (1-C) / (1.15 \sqrt{(1+\alpha^2)})].$$

AASHTO: The nominal shear strength is equal to $V_n = C^* V_p$ where $C =$ buckling reduction factor $= (1.12 \sqrt{E k_q / F_y} / \lambda)$ for unstiffened webs, webs with aspect ratio $\alpha > 3$, and end panels. For stiffened webs with $\alpha < 3$, tension field action is allowed giving:
 $V_n = V_p [C + 0.87*(1-C) / \sqrt{(1+\alpha^2)}]$.

3.2) EUROCODE EC3:

a) Slenderness Limits:

The flange is compact when $\lambda < \lambda_p = 25.88 \sqrt{235 k_q / F_y}$ and slender when $\lambda > \lambda_r = 40.39 \sqrt{235 k_q / F_y}$, where $k_q = 4 + 5.34 / \alpha^2$ for $\alpha < 1$ and $5.34 + 4 / \alpha^2$ for $\alpha > 1$.

b) Strength:

- i) **Compact Web:** The nominal shear strength V_n is equal to plastic shear capacity given by $V_p = 0.58 A_w F_y$, where $A_w =$ web area.
- ii) **Non-compact and slender Web:** The nominal shear strength V_n is based on the theoretical expression for elastic buckling given by $V_n = \chi * V_p$ where $\chi =$ buckling reduction factor $= 0.83 / \lambda_o$, where $\lambda_o =$ normalized slenderness parameter in shear $= 0.76 \sqrt{F_y / \tau_{cr}}$, $\tau_{cr} =$ critical shear buckling stress.

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