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SKEW - TYPE I GENERALIZED LOGISTIC DISTRIBUTION AND ITS PROPERTIES

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ABSTRACT

In this paper, we derive, the probability density function (pdf) and cumulative distribution function (CDF) of the skew type I generalized logistic distribution SGLD(α,β,λ).The general statistical properties of the SGLD(α,β,λ) such as: the moment generating function (mgf), characteristic function (ch.f), Laplace and fourier transformations are obtained in explicit form. Expressions for the n^{th} moment, skewness and kurtosis coefficients are discussed. Mean deviation about the mean and about the median, Renye entropy and the order statistics are also given. We consider the general case by inclusion of location and scale parameters. The results of Nadarajah [11] are obtained as special cases. Graphically illustration of some results have been represented. Further we present a numerical example to illustrate some results of this paper.

Keywords: *skew type I generalized logistic distribution, moment generating function, skewness, kurtosis, mean deviation, order statistics.*

MSC 2010 code: *60E10, 62E10.*

1. INTRODUCTION

The fundamental properties and characterization of symmetric distribution about origin are widely used in many applications as stated in Johnson *et al.* [8]. The logistic distribution has important uses in describing growth and as a substitute for the normal distribution. It has several important applications in population modeling, biological, geological issues, psychological issues, actuarial, industrial and engineering felids. Different properties and applications of logistic distribution are studied extensively by, Ojo [15], [16], [17], Olapade [19], Alvarez *et al.* [10], and Nassar and Elmasry [14].

In the recent years, there has been quite an intense activity connected to abroad class of continuous probability distribution which is generated starting from a symmetric distribution and applying a suitable form perturbation of the symmetry. As general result, Azzalini [2] showed that any symmetric distribution was viewed as a member of more general class of skewed distribution. The skew-symmetric models defined by different researchers based on the skew-normal distribution of Azzalini [2] having pdf.

$$f(x,\lambda) = 2h(x).\ H(\lambda x), \quad -\infty < x < \infty, \tag{1.1}$$

Where $\lambda \in R$ is the skewness parameter, h(x) and H(x) are respectively the pdf and CDF of N (0, 1).

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In fact, when $\mathrm{h}(.)$ and $\mathrm{H}(.)$ are the pdf and CDF of the standard logistic distribution respectively, then (1.1) is called skew-logistic distribution with skewness parameter λ (SLD (λ)). The SLD (λ) like other skew symmetric distribution can be used to model both positively and negatively skewed data due to one or more reasons, such as presence of out liars, small number of observations presence of other process, etc. The SLD (λ) and its statistical properties was first studied by Wahed and Ali [22].They considered the pdf and CDF of standard logistic distribution as:

$h(x) = \frac{e^{-x}}{(1+e^{-x})^2}, \ -\infty < x < \infty$, and
$H(x) = \frac{1}{1+e^{-x}}, \quad -\infty < x < \infty$

Hence, the pdf of SLD (λ) is obtained as follows:
$f(x,\lambda) = \frac{2e^{-x}}{(1+e^{-x})^2\,(1+e^{-\lambda x})}, \ -\infty < x < \infty$

Nadarajah [11] extended this SLD (λ) by introducing a scale parameter and studied its statistical properties. Gupta and Kundu [7] used this distribution as generalized of logistic distribution. They showed that the SLD (λ) is more flexible than the skew normal distribution for data analysis purposes. Many properties of SLD (λ) and skew normal distribution are quite similar. The SLD (λ) and its statistical properties have been considered by many authors, see, for example, Gupta *et al.*[6], Olapade [18], Nadarajah *et al.* [12],[13], Chakraborty *el at*.[4] ,and Asgharzadeh *et al.* [1]

Balakrishnan and leung [3] defined the Type I generalized logistic distribution (Type I GLD) as one of three generalized forms of the standard logistic distribution. The two-parameter Type I $\mathrm{GLD}(\alpha,\beta)$, of a random variable X has probability density function (pdf) given by

$$h(x,\alpha,\beta) = \alpha\beta e^{-\beta x}\left(1+e^{-\beta x}\right)^{-(\alpha+1)}, \quad -\infty < x < \infty \tag{1.2}$$

and has the cumulative distribution function (CDF) given by

$$H(x,\alpha,\beta) = \left(1+e^{-\beta x}\right)^{-\alpha}, \quad -\infty < x < \infty \tag{1.3}$$

Where $\beta > 0$ is the scale parameter and $\alpha > 0$ is the shape parameter. Olapade [20] obtained some properties for TypeI $\mathrm{GLD}(\alpha,\beta)$. The pdf (1.2) has been obtained by compounding of extreme value distribution with a gamma distribution. It is observed by Balakrishnan and leung [3] that this distribution is skewed and kourtosis coefficient is greater than the standard logistic distribution.

In this paper, we are using Type I $\mathrm{GLD}(\alpha,\beta)$ given by Balakrishnan and leung [3] to study the Skew Type-I $\mathrm{GLD}(\alpha,\beta)$, with skeweness parameter $\lambda \in R$, and will be denoted as $\mathrm{SGLD}(\alpha,\beta,\lambda)$. This paper is organized as follows In the next section we derive the pdf and CDF of $\mathrm{SGLD}(\alpha,\beta,\lambda)$ in explicit forms. In Section 3, we obtained the moment generating function (mgf), characteristic function (ch. f), Laplace and Fourier transformations. Expressions for $n^{\underline{th}}$ moment including the first four moments, skewness and kurtosis coefficients are given in section 4. In Section 5, the mean deviation about the mean and median are discussed. Sections 6 and 7 are displayed the Entropy and order statistics for $\mathrm{SGLD}(\alpha,\beta,\lambda)$. Some important properties are discussed in Section 8. Numerical example is given in Section 9. Finally, Conclusion remarks are provided at Section 10. Graphical illustration of pdf, CDF of $\mathrm{SGLD}(\alpha,\beta,\lambda)$, skewness and kurtosis have been represented. It should be noted that some known results of Nadarajah and Kotz [12], Nadarajah [11] are obtained as special cases.

2. CUMULATIVE DISTRIBUTION FUNCTION OF SGLD$(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\lambda})$

In this section, we derive a form of pdf and CDF of $\mathrm{SGLD}(\alpha,\beta,\lambda)$, depending on the Type I $\mathrm{GLD}(\alpha,\beta)$ given by Balakrishnan and leung [3] in (1.2), main feature of the Skew-Symmetric distribution in (1.1) is that a new parameter λ is introduced to control skewness and kurtosis.

The pdf of $\mathrm{SGLD}(\alpha,\beta,\lambda)$ is constructed using formula (1.1), and the pdf $\mathrm{h}(.)$ and the CDF $\mathrm{H}(.)$ given in (1.2) and (1.3) as follows

$$f(x,\alpha,\beta,\lambda) = 2\alpha\beta e^{-\beta x}\left(1+e^{-\beta x}\right)^{-(\alpha+1)}\left(1+e^{-\beta\lambda x}\right)^{-\alpha}, \ -\infty < x < \infty \tag{2.1}$$

The pdf of $\mathrm{SGLD}(\alpha,\beta,\lambda)$, given by (2-1) can be expressing in other forms: (a) a single series representation (b) a double series representation, as given in the following lemma.

Lemma: 2.1 If X be a random variable having SGLD(α, β, λ), then its pdf is given by

$$(a)\ f(x,\alpha,\beta,\lambda) = \begin{cases} 2\alpha\beta\left(1+e^{-\beta x}\right)^{-(\alpha+1)} \sum_{j=0}^{\infty}\binom{-\alpha}{j} e^{-\beta x(\lambda j+1)} & x>0 \\ \\ 2\alpha\beta\left(1+e^{\beta x}\right)^{-(\alpha+1)} \sum_{j=0}^{\infty}\binom{-\alpha}{j} e^{\beta x(\alpha+\lambda\alpha+\lambda j)} & x<0 \end{cases} \tag{2.2}$$

$$(b) f(x,\alpha,\beta,\lambda) = \begin{cases} 2\alpha\beta \sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k} e^{-\beta x(k+\lambda j+1)} & x>0 \\ \\ 2\alpha\beta \sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k} e^{\beta x(k+\lambda j+\alpha+\lambda\alpha)} & x<0 \end{cases} \tag{2.3}$$

Proof:
(a) Utilizing binomial expansion for $(1+z)^{-1}$ in the CDF $H(\lambda x)$ given in (1.2), we get

$$H(\lambda x) = \left(1+e^{-\beta\lambda x}\right)^{-\alpha} = \begin{cases} \sum_{j=0}^{\infty}\binom{-\alpha}{j} e^{-\beta j\lambda x}, x>0 \\ \\ e^{\beta\alpha\lambda x} \sum_{j=0}^{\infty}\binom{-\alpha}{j} e^{\beta j\lambda x}, x<0 \end{cases} \tag{2.4}$$

Substituting from (2.4) into (2.1), the result is obtained.

(b) Also by expanding$\left(1+e^{-\beta x}\right)^{-(\alpha+1)}$, one can obtain the double series representation given in (2.3).

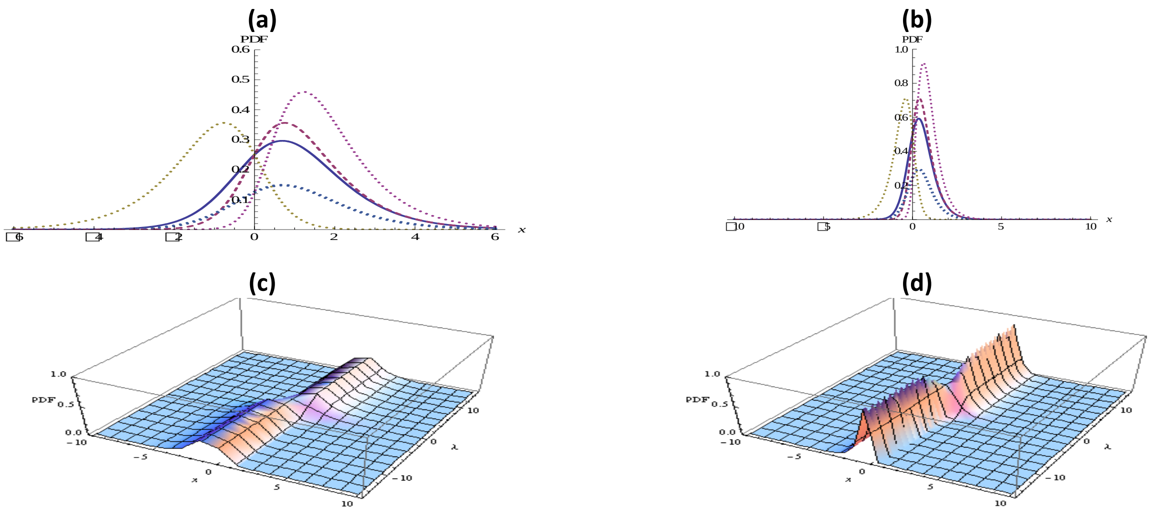


Figure - 1: The shapes of pdf of SGLD(α, β, λ) (a) when $\beta = 1$ with $\alpha = 1, \lambda = 1$ (Thick line), $\alpha = 1, \lambda = 2$ (Dashed line), $\alpha = 1, \lambda = -2$ (Dotted line),$\alpha = 2, \lambda = -2$ (Dashed &Thick line) and $\alpha = 2, \lambda = 2$ (Dashed & Dotted line). (b) when β=2 with $\alpha = 1, \lambda = 1$ (Thick line),$\alpha = 1, \lambda = 2$ (Dashed line), $\alpha = 1, \lambda = -2$ (Dotted line), $\alpha = 2, \lambda = -2$ (Dashed & Thick line) and $\alpha = 2, \lambda = 2$ (Dashed & Dotted line). (c) when $\alpha = 1, \beta = 1$ and $\lambda \in (-15, 15)$. (d) when $\alpha = 1, \beta = 2$ and $\lambda \in (-15, 15)$.

The calculations through this paper are based on the generalized hypergeometric function mF_n defined by $mF_n(\tau_1,\tau_2,\ldots,\tau_m;\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_n;x)=\sum_{k=0}^{\infty}\frac{(\tau_1)_k(\tau_2)_k\ldots(\tau_m)_k.\ x^k}{(\mathcal{V}_1)_k(\mathcal{V}_2)_k\ldots(\mathcal{V}_n)_k.\ k!}$

and $(c)_k = c(c+1)\ldots\ldots(c+k-1)$ denotes the ascending factorial. The properties of this special functions being used can be found in Gradshteyn and Ryzhik [5]

Theorem (2.1): If X is a SGLD(α, β, λ) random variable, then its CDF is given by

$$(a) F(x,\alpha,\beta,\lambda) = \begin{cases} 2\alpha \sum_{j=0}^{\infty}\binom{-\alpha}{j} \left\{\frac{A_1}{\alpha+\lambda\alpha+\lambda j} + \left[2\alpha\frac{1}{(\lambda j+1)}\left[A_2 - A_3 e^{-\beta x(\lambda j+1)}\right]\right]\right\}, x>0 \\ 2\alpha \sum_{j=0}^{\infty}\binom{-\alpha}{j} A_4 \frac{e^{\beta x(\alpha+\lambda\alpha+\lambda j)}}{\alpha+\lambda\alpha+\lambda j}, x<0 \end{cases} \tag{2.5}$$

$$(b)F(x,\alpha,\beta,\lambda)=\begin{cases}1-\left[2\alpha\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}\frac{e^{-\beta x\,(k+\lambda j+1)}}{(k+\lambda j+1)}\right], x>0\\ \\ 2\alpha\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}\frac{e^{\beta x\,(k+\lambda j+\alpha+\lambda\alpha)}}{(k+\lambda j+\alpha+\lambda\alpha)}, x<0\end{cases} \quad (2.6)$$

Where,
$A_1=2F_1(\alpha+1,\alpha+\lambda\alpha+\lambda j,\alpha+\lambda\alpha+\lambda j+1,-1)$, $A_2=2F_1(\alpha+1,\lambda j+1,\lambda j+2,-1)$,

$A_3=2F_1\left(\alpha+1,\lambda j+1,\lambda j+2,-e^{-\beta x}\right)$, and $A_4=2F_1\left(\alpha+1,\alpha+\lambda\alpha+\lambda j,\alpha+\lambda\alpha+\lambda j+1,-e^{\beta x}\right)$

Proof: The proof of this theorem has two cases separately, when $X\geq 0$, and $X<0$, as follows: firstly, when $X\geq 0$ employing the single series form of the pdf given in (2.2), the CDF of $SGLD(\alpha,\beta,\lambda)$, can be written as

$$F(x,\alpha,\beta,\lambda)=P(X\leq x)=\int_{-\infty}^{x}f(x)\,dx=F(0)+\ 2\alpha\beta\sum_{j=0}^{\infty}\binom{-\alpha}{j}I_{(x)},\ \ x>0 \quad (2.7)$$

Where $I_{(x)}$ is the integral given by

$$I_{(x)}=\int_{0}^{x}\frac{e^{-\beta x\,(\lambda j+1)}}{(1+e^{-\beta x})^{\alpha+1}}\,dx \quad (2.8)$$

Substituting $y=e^{-\beta x}$, the integral $I_{(x)}$ reduces to

$$=\frac{1}{\beta}\left[\int_{0}^{1}\frac{y^{\lambda j}}{(1+y)^{\alpha+1}}\,dy-\int_{0}^{e^{-\beta x}}\frac{y^{\lambda j}}{(1+y)^{\alpha+1}}\,dy\right]=\frac{1}{\beta}\left[I_1-I_2\right] \quad (2.9)$$

To evaluate the integrals I_1 and I_2, we use the formula (3.194.1) in Gradshteyn and Ryzhik [5], which stated as follows

$$\int_{0}^{u}\frac{x^{\mu-1}}{(1+bx)^{\mathcal{V}}}=\frac{u^{\mu}}{\mu}\,2F_1(\mathcal{V},\mu,1+\mu,-bu) \quad (2.10)$$

Then, the integrals I_1 and I_2 can be calculated as

$$I_1=\frac{A_2}{\lambda j+1},\quad I_2=\frac{A_3e^{-\beta x\,(\lambda j+1)}}{\lambda j+1} \quad (2.11)$$

Using equations (2.11) and (2.9) we get

$$I_{(x)}=\frac{1}{\beta(\lambda j+1)}\left[A_2-A_3e^{-\beta x\,(\lambda j+1)}\right] \quad (2.12)$$

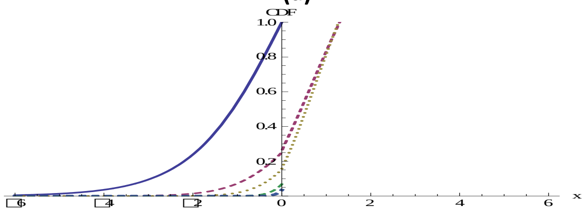
Similarly, $F(0)$ can be calculated as above and we can evaluate it as follows

$$F(0)=2\alpha\sum_{j=0}^{\infty}\binom{-\alpha}{j}\frac{A_1}{\alpha+\lambda\alpha+\lambda j} \quad (2.13)$$

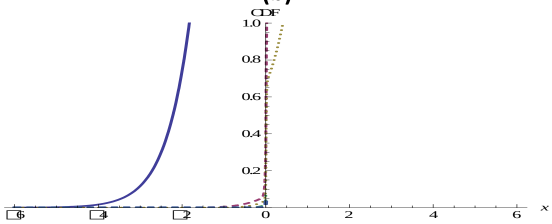
Combining (2.12) and (2.13) into (2.7), the result is yielded.

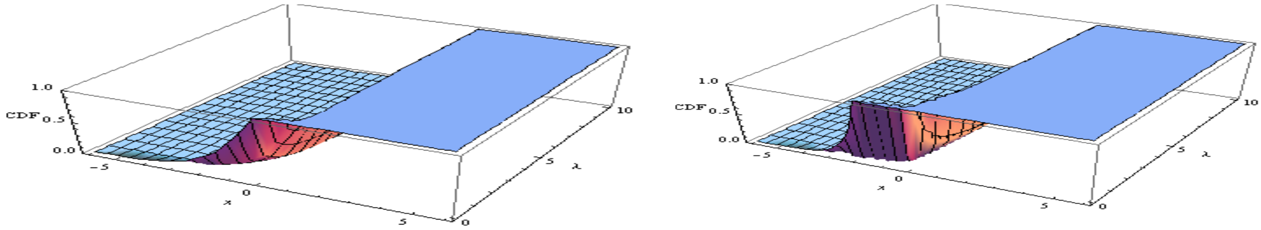
Second case: when X<0, by similar calculation and using (2.10), we get the result. Utilizing the double series representation (2.3), we can also obtain the double series form for the CDF of $SGLD(\alpha,\beta,\lambda)$ given in (2.6). It is clear that the class of the $SGLD(\alpha,\beta,\lambda)$ contains the standard logistic distribution (take $\lambda=0$, $\alpha=1$ and $\beta=1$) and the skew logistic distribution (take $\alpha=1$ and $\beta=1$). Consequent if we put($\alpha=1$ and $\beta=1$), the results of Nadarajah and kotz [12] and Nadarajah [11] are obtained as special cases. We shall assume that $\lambda\geq 0$, since the corresponding, results for $\lambda<0$ can be obtained using the fact that – X has the pdf given in (2.1).

(a)



(b)





(c) **(d)**

Figure - 2: Illustrates the shapes of CDF of SGLD(α, β, λ) (a) when λ= 0 (Thick line), λ= 1 (dashed line), λ= 2 (Dotted line), λ= 5 (Dotted & Dashed line) and λ= 10 (Dashed & Dotted line) when α =1 and β = 1. (b) when λ= 0 (Thick line), λ= 1 (dashed line), λ= 2 (Dotted line),λ= 5 (Dotted & Dashed line) and λ= 10 (Dashed & Dotted line) when α =2 and β = 1. (c) when $\alpha = 1$, $\beta = 1$ and λ ϵ (0, 10). (d) when $\alpha = 2$, $\beta = 1$ and λ ϵ (0, 10).

3. MOMENT GENERATING FUNCTION AND CHARACTERISTIC FUNCTION

Here, we derive the Moment generating function (mgf) and characteristic function (ch.f) of the random variable X with SGLD(α, β, λ). We consider two forms of the pdf, $f(x, \alpha, \beta, \lambda)$ when it is taken the one series and double series given by (2.2) and (2.3) respectively.

Theorem: 3.1 The mgf of the random variable X with SGLD(α, β, λ) is given by
(a) By using one series representation of pdf given in (2.2)

$$M_X(t) = 2\alpha \sum_{j=0}^{\infty} \binom{-\alpha}{j} \left[\left[\frac{A_5}{\alpha+\lambda\alpha+\lambda j+\frac{t}{\beta}}\right] + \left[\frac{A_6}{\lambda j+1-\frac{t}{\beta}}\right] \right] \tag{3.1}$$

(b) By employing double series representation of pdf given in (2.3)

$$M_X(t) = 2\alpha\beta \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \binom{-\alpha}{j} \binom{-\alpha-1}{k} \left[\frac{1}{\beta(k+\lambda j+\lambda\alpha+\alpha)+t} + \frac{1}{\beta(k+\lambda j+1)-t}\right] \tag{3.2}$$

Where,
$A_5 = {}_2F_1(\alpha+1, \alpha+\lambda\alpha+\lambda j+\frac{t}{\beta}, \alpha+\lambda\alpha+\lambda j+\frac{t}{\beta}+1, -1)$, and $A_6 = {}_2F_1(\alpha+1, \lambda j+1-\frac{t}{\beta}, \lambda j+2-\frac{t}{\beta}, -1)$

Proof:
(a) Let $f(x, \alpha, \beta, \lambda)$ given in (2.2), then

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} \; f(x, \alpha, \beta, \lambda) dx = 2\alpha\beta \sum_{j=0}^{\infty} \binom{-\alpha}{j} \; [I_3 + I_4] \tag{3.3}$$

Where, $I_3 = \int_{-\infty}^{0} \frac{e^{\beta x\left(\alpha+\lambda\alpha+\lambda j+\frac{t}{\beta}\right)}}{(1+e^{\beta x})^{\alpha+1}} dx$, and, $I_4 = \int_{0}^{\infty} \frac{e^{-\beta x\left(\lambda j+1-\frac{t}{\beta}\right)}}{(1+e^{-\beta x})^{\alpha+1}} dx$

Substituting $y = e^{\beta x}$, the integral I_3 reduces to

$$I_3 = \frac{1}{\beta} \int_0^1 \frac{y^{\alpha+\lambda\alpha+\lambda j+\frac{t}{\beta}-1}}{(1+y)^{\alpha+1}} dy$$

and the integral I_4 take the following form by using $y = e^{-\beta x}$

$$I_4 = \frac{1}{\beta} \int_0^1 \frac{y^{\lambda j-\frac{t}{\beta}}}{(1+y)^{\alpha+1}} dy$$

Using the formula given in (2.10), the integrals I_3 and I_4 can be calculated as

$$I_3 = \frac{1}{\beta}\left[\frac{A_5}{\alpha+\lambda\alpha+\lambda j+\frac{t}{\beta}}\right] \text{ and } I_4 = \frac{1}{\beta}\left[\frac{A_6}{\lambda j+1-\frac{t}{\beta}}\right] \tag{3.4}$$

From equations (3.1) and (3.4) we get the result (a).

(b)The double series representation of pdf given in (2.3) gives

$$M_X(t) = 2\alpha\beta \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \binom{-\alpha}{j} \binom{-\alpha-1}{k} \; [I_5 + I_6]$$

$where,\ I_5 = \int_{-\infty}^{0} e^{\beta x\left(k+\lambda j+\alpha+\lambda\alpha+\frac{t}{\beta}\right)}\, dx, and, \qquad I_6 = \int_{0}^{\infty} e^{-\beta x\left(k+\lambda j+1-\frac{t}{\beta}\right)}\, dx$

In the same manner, we can obtain the result (b), which completes the proof.

Remark: 3.1 the characteristic function of the SGLD(α, β, λ) random variable is given by
(a) one series representation of pdf given in (2.2)

$$\emptyset_x(t) = 2\alpha \sum_{j=0}^{\infty}\binom{-\alpha}{j}\left[\left[\frac{A_7}{\alpha+\lambda\alpha+\lambda j+\frac{it}{\beta}}\right]+\left[\frac{A_8}{\lambda j+1-\frac{it}{\beta}}\right]\right] \qquad (3.5)$$

(b) double series representation of pdf given in (2.3)

$$\emptyset_x(t) = 2\alpha\beta \sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}\left[\frac{1}{\beta(k+\lambda j+\lambda\alpha+\alpha)+it}+\frac{1}{\beta(k+\lambda j+1)-it}\right]$$

Where,
$A_7 = 2F_1\left(\alpha+1, \alpha+\lambda\alpha+\lambda j+\frac{it}{\beta}, \alpha+\lambda\alpha+\lambda j+\frac{it}{\beta}+1, -1\right)$, $A_8 = 2F_1\left(\alpha+1, \lambda j+1-\frac{it}{\beta}, \lambda j+2-\frac{it}{\beta}, -1\right)$,
and $i = \sqrt{-1}$ is the complex imaginary unit.

The resulting of Nadarajah and kotz [12] and Nadarajah [11] for the mgf $M_X(t)$, and ch.f. $\emptyset_x(t)$ are obtained as special cases by taking ($\alpha = 1$ and $\beta = 1$) in (3.1) and (3.5).

Other useful properties of SGLD(α, β, λ) are the Laplace and Fourier Transforms, which are given by the following remark (3.2) and remark (3.3), for two representations of pdf given in (2.2) and (2.3).

Remark: 3.2 The Laplace Transform of the random variable X having SGLD(α, β, λ) is:

(a) $L(t) = E(e^{-tx}) = 2\alpha \sum_{j=0}^{\infty}\binom{-\alpha}{j}\left[\left[\frac{A_9}{\alpha+\lambda\alpha+\lambda j-\frac{t}{\beta}}\right]+\left[\frac{A_{10}}{\lambda j+1+\frac{t}{\beta}}\right]\right]$

(b) $L(t) = 2\alpha\beta \sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}\left[\frac{1}{\beta(k+\lambda j+\lambda\alpha+\alpha)-t}+\frac{1}{\beta(k+\lambda j+1)+t}\right]$

Where,
$A_9 = 2F_1(\alpha+1, \alpha+\lambda\alpha+\lambda j-\frac{t}{\beta}, \alpha+\lambda\alpha+\lambda j-\frac{t}{\beta}+1, -1)$, and $A_{10} = 2F_1(\alpha+1, \lambda j+1+\frac{t}{\beta}, \lambda j+2+\frac{t}{\beta}, -1)$

Remark: 3.3 The Fourier Transform of the random variable X having SGLD(α, β, λ) is:

(a) $Fo(t) = E(e^{-itx}) = 2\alpha \sum_{j=0}^{\infty}\binom{-\alpha}{j}\left[\left[\frac{A_{11}}{\alpha+\lambda\alpha+\lambda j-\frac{it}{\beta}}\right]+\left[\frac{A_{12}}{\lambda j+1+\frac{it}{\beta}}\right]\right]$

(b) $Fo(t) = 2\alpha\beta \sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}\left[\frac{1}{\beta(k+\lambda j+\lambda\alpha+\alpha)-it}+\frac{1}{\beta(k+\lambda j+1)+it}\right]$

Where,
$A_{11} = 2F_1\left(\alpha+1, \alpha+\lambda\alpha+\lambda j-\frac{it}{\beta}, \alpha+\lambda\alpha+\lambda j-\frac{it}{\beta}+1, -1\right)$, and $A_{12} = 2F_1\left(\alpha+1, \lambda j+1+\frac{it}{\beta}, \lambda j+2+\frac{it}{\beta}, -1\right)$

4. SKEWNESS AND KURTOSIS COEFFICIENTS

The skewness coefficient is measured by $\gamma_1 = \frac{M_3}{\sigma^3}$, and the kurtosis coefficient is measured by $\gamma_2 = \frac{M_4}{\sigma^4}$, where M_3 and M_4 are the third and fourth moments about the mean and, σ^2 is the variance of the random variable X having SGLD(α, β, λ).

We firstly derive the nth moment of the random variable X having pdf given in (2.3) by the following theorem.

Theorem: 4.1 The n$^{\underline{th}}$ moment of the SGLD(α, β, λ) random variable X is
(a) If n is even order

$$E(X^n) = \frac{4\alpha\, n!}{\beta^n}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}.\frac{1}{(k+\lambda j+1)^{n+1}}$$

(b) If n is odd order

$$\mathrm{E}(X^n) = \frac{2\alpha\, n!}{\beta^n}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}.\left[\frac{1}{(k+\lambda j+1)^{n+1}}+\frac{(-1)^{\mathrm{n}}}{(k+\alpha+\lambda j+\lambda\alpha)^{n+1}}\right]$$

Proof:

$$\mathrm{E}(X^n) = \int_{-\infty}^{\infty} x^n\ f(x,\alpha,\beta,\lambda)\,dx$$

(a) If n is even order

$$\mathrm{E}(X^n) = 2\int_{0}^{\infty} x^n\ f(x,\alpha,\beta,\lambda)\,dx$$

Using (2.3), we get

$$\mathrm{E}(X^n) = 4\alpha\beta\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}\ .\frac{\Gamma(n+1)}{\beta^{n+1}(k+\lambda j+1)^{n+1}}$$

Applying the properties of gamma function the result is obtained.

(b) If n is odd order

$$\mathrm{E}(X^n) = \int_{-\infty}^{0} x^n\ f(x,\alpha,\beta,\lambda)\,dx + \int_{0}^{\infty} x^n\ f(x,\alpha,\beta,\lambda)\,dx$$

Employing (2.3), and the properties of gamma function we get the result. Consequently, the first four moments of X can be obtained as

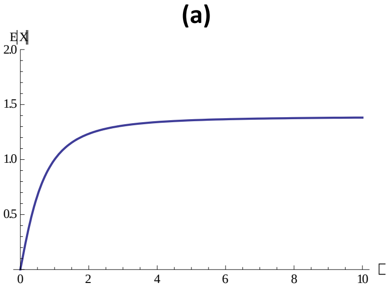
$$\mathrm{E}(X\ ) = \frac{2\alpha}{\beta}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}.\left[\frac{1}{(k+\lambda j+1)^{2}}-\frac{1}{(k+\alpha+\lambda j+\lambda\alpha)^{2}}\right]$$

$$\mathrm{E}(X^2) = \frac{8\alpha}{\beta^2}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}.\frac{1}{(k+\lambda j+1)^{3}}$$

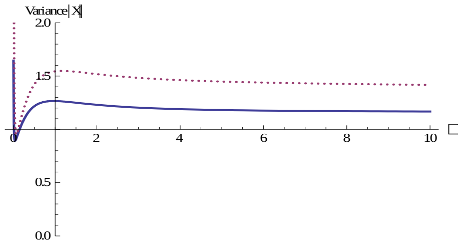
$$\mathrm{E}(X^3) = \frac{12\alpha}{\beta^3}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}.\left[\frac{1}{(k+\lambda j+1)^{4}}-\frac{1}{(k+\alpha+\lambda j+\lambda\alpha)^{4}}\right]$$

$$\mathrm{E}(X^4) = \frac{96\alpha}{\beta^4}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\binom{-\alpha}{j}\binom{-\alpha-1}{k}.\frac{1}{(k+\lambda j+1)^{5}}$$

(a)



(b)



(c) **(d)**

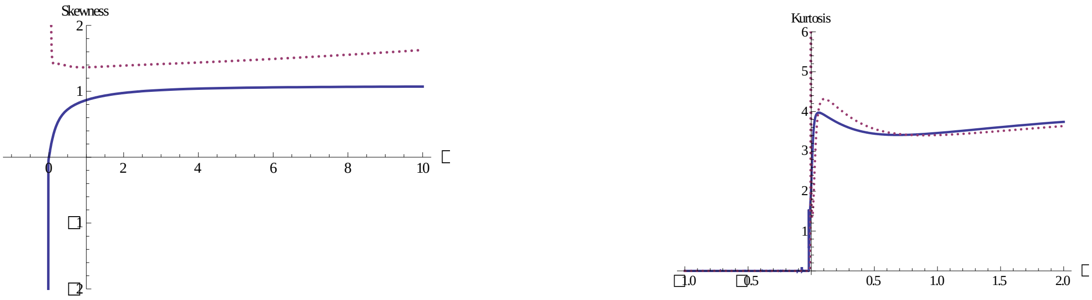


Figure - 2: Illustrates the shapes of (a) the expectation value E(X) of S'GLD(α, β, λ), when λ ϵ (0, 10), β = 1 and α =2 (Thick line) and α =2 (dashed line). (b) the variance var(X) , when λ ϵ (0, 10), β = 2 and α =1 (Thick line) and α =1.5 (dashed line). (c) the skewness coefficient, when λ ϵ (-1, 10), β = 1 and α =1 (Thick line) and α =2 (dashed line). (d) the kurtosis coefficient, when λ ϵ (-1, 10), β = 1 and α =1 (Thick line) and α =1.5 (dashed line).

Therefore, using this four moments, we can easily obtain the variance σ^2, the skewness coefficient γ_1 and kurtosis coefficient γ_2, which will be illustrated here numerically in section(9). If ($\alpha = 1$ and $\beta = 1$) the results of Nadarajah and kotz [12], and Nadarajah [11] are obtained as special cases.

5. MEAN DEVIATION

The amount of scatter in a population is evidently measured to some extent by the totality of deviation from the mean and median. These are known as the mean deviation about the mean, μ and the mean deviation about the median, M defined by

$\delta_1(X) = \int_{-\infty}^{\infty} |x-\mu| \; f(x,\alpha,\beta,\lambda)\, dx$ and,

$$\delta_2(X) = \int_{-\infty}^{\infty} |x-M| \; f(x,\alpha,\beta,\lambda)\, dx$$

Respectively, where $\mu = E(X)$ = expectation of SGLD(α,β,λ) random variable X.

Theorem: 5.1 The mean deviation about the mean μ of a random variable X having SGLD(α,β,λ) is giving as:

$$\delta_1(X) = \begin{cases} \delta_{11}(\mu), & if\ \mu \le 0 \\ \delta_{12}(\mu), & if\ \mu \ge 0 \end{cases}$$

Where,

$$\delta_{11}(\mu) = 2\alpha\beta \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \binom{-\alpha}{j}\binom{-\alpha-1}{k}\left[{B_1}^2\left(2e^{\beta\mu(k+\lambda j+\lambda\alpha+\alpha)}-1\right)+{B_2}^2\right]-\mu,$$

$$\delta_{12}(\mu) = 2\alpha\beta \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \binom{-\alpha}{j}\binom{-\alpha-1}{k}\left[{B_2}^2\left(2e^{\beta\mu(k+\lambda j+1)}-1\right)+{B_1}^2\right]+\mu,$$

$B_1 = \frac{1}{\beta\,(k+\lambda j+\lambda\alpha+\alpha)}$, and, $B_2 = \frac{1}{\beta(k+\lambda j+1)}$

Proof: $\delta_1(X)$ can be written in following form given by Nadarajah [11] as:

$$\delta_1(X) = 2\mu F(\mu) - \mu - \int_{-\infty}^{\mu} x\; f(x,\alpha,\beta,\lambda)\,dx + \int_{\mu}^{\infty} x\; f(x,\alpha,\beta,\lambda)\,dx \qquad (5.1)$$

depending on two cases, when $\mu \le 0$ and $\mu \ge 0$, and the calculations are using the double series form given in (2.3).

When $\mu \le 0$ we have

$$F(\mu) = 2\alpha \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \binom{-\alpha}{j}\binom{-\alpha-1}{k}\beta B_1 e^{\beta\mu(k+\lambda j+\lambda\alpha+\alpha)} \qquad (5.2)$$

The integrals in (5.1) can be obtained by elementary calculation as following

$$\int_{-\infty}^{\mu} x\; f(x,\alpha,\beta,\lambda)\,dx = 2\alpha\beta \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \binom{-\alpha}{j}\binom{-\alpha-1}{k}\left[B_1(\mu-B_1)e^{\beta\mu(k+\lambda j+\lambda\alpha+\alpha)}\right] \qquad (5.3)$$

and

$$\int_{\mu}^{\infty} x\; f(x,\alpha,\beta,\lambda)\,dx = 2\alpha\beta \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \binom{-\alpha}{j}\binom{-\alpha-1}{k}\left[B_1(B_1-\mu)e^{\beta\mu(k+\lambda j+\lambda\alpha+\alpha)}+{B_2}^2-{B_1}^2\right] \qquad (5.4)$$

Substituting from (5.2), (5.3) and (5.4) in (5.1) the result of $\delta_{11}(\mu)$ is obtained.

Now, when $\mu \geq 0$, we have

$$F(\mu) = 1 - 2\alpha \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \binom{-\alpha}{j} \binom{-\alpha-1}{k} \frac{e^{-\beta\mu(k+\lambda j+1)}}{(k+\lambda j+1)} \tag{5.5}$$

The integrals in (5.1) are evaluating as:

$$\int_{-\infty}^{\mu} x\ f(x,\alpha,\beta,\lambda)\ dx = 2\alpha\beta \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \binom{-\alpha}{j} \binom{-\alpha-1}{k} \left[{B_2}^2 + {B_1}^2 - B_2(\mu + B_2) e^{-\beta\mu(k+\lambda j+1)}\right] \tag{5.6}$$

and

$$\int_{\mu}^{\infty} x\ f(x,\alpha,\beta,\lambda)\ dx = 2\alpha\beta \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \binom{-\alpha}{j} \binom{-\alpha-1}{k} \left[B_2(\mu + B_2)\left(e^{-\beta\mu(k+\lambda j+1)}\right)\right] \tag{5.7}$$

Employing (5.5), (5.6) and (5.7) in (5.1) the result of $\delta_{12}(\mu)$ yields.

In the same manner, we can get the mean deviation about the median $\delta_2(X)$ by replacing M by μ. We note that. If ($\alpha = 1$ and $\beta = 1$) in $\delta_{11}(\mu)$ and $\delta_{12}(\mu)$, the results of Nadarajah [11] are obtained as special cases.

6. ENTROPY

The concept of entropy has been successfully applied in a variety of fields including statistical mechanics, statistic, stock market analysis, queuing theory, image analysis and reliability estimation. (see, e. g., Kapur [9]). The entropy is a measure the variation of the uncertainty associated with distribution of a random variable X. Notice however that entropy is no longer positive, in fact it can become arbitrary large negative.

The Renyi entropy is defined by

$$J_R(\gamma) = \frac{1}{1-\gamma} \log\left[\int_{-\infty}^{\infty} f^{\gamma}(x,\alpha,\beta,\lambda)\ dx\right] \tag{6.1}$$

Where $\gamma > 0$, and $\gamma \neq 1$ (Renyi [21]). Renyi's entropies are more flexible than Shannon entropies and include Shannon entropies as special case.

Theorem: 6.1 If X is a random variable having SGLD(α, β, λ) with pdf given in (2.2). The Renyi entropy is

$$J_R(\gamma) = \log\left(\frac{1}{2\alpha\beta}\right) + \frac{1}{1-\gamma}\left\{2\alpha \sum_{j=0}^{\infty} \binom{-\gamma\alpha}{j} \left[\frac{C_1}{\gamma + \lambda j} + \frac{C_2}{\lambda j + \gamma\alpha + \lambda\gamma\alpha}\right]\right\}$$

Where,

$C_1 = 2F_1(\gamma(\alpha+1), \gamma + \lambda j, \gamma + \lambda j + 1, -1)$, and $C_2 = 2F_1(\gamma(\alpha+1), \lambda j + \gamma\alpha + \lambda\gamma\alpha, \lambda j + \gamma\alpha + \lambda\gamma\alpha + 1, -1)$

Proof: Using the single series representation of $f(x,\alpha,\beta,\lambda)$ given in (2.2), we get

$$\int_{-\infty}^{\infty} f^{\gamma}(x,\alpha,\beta,\lambda)\ dx = (2\alpha\beta)^{\gamma} e^{-\beta x} \sum_{j=0}^{\infty} \binom{-\gamma\alpha}{j} [I_7 + I_8] \tag{6.2}$$

Where

$$I_7 = \int_0^{\infty} \frac{e^{-\beta x(\gamma+\lambda j)}}{(1+e^{-\beta x})^{\gamma(\alpha+1)}}\ dx, \quad \text{and,} \quad I_8 = \int_{-\infty}^{0} \frac{e^{\beta x(\lambda j+\gamma\alpha+\lambda\gamma\alpha)}}{(1+e^{\beta x})^{\gamma(\alpha+1)}}\ dx$$

The integrals I_7 and I_8 can be calculated as above in (2.10)

$$I_7 = \frac{C_1}{\beta(\gamma+\lambda j)}, \quad \text{and} \quad I_8 = \frac{C_2}{\beta(\lambda j+\gamma\alpha+\lambda\gamma\alpha)} \tag{6.3}$$

Substituting from (6.3) in (6.2) we get

$$\int_{-\infty}^{\infty} f^{\gamma}(x,\alpha,\beta,\lambda)\ dx = \frac{(2\alpha\beta)^{\gamma}}{\beta} \sum_{j=0}^{\infty} \binom{-\gamma\alpha}{j} \left[\frac{C_1}{\gamma+\lambda j} + \frac{C_2}{\lambda j+\gamma\alpha+\lambda\gamma\alpha}\right] \tag{6.4}$$

Therefore, using (6.1) and (6.4) the result is obtained.

7. ORDER STATISTICS

Suppose $X_1, X_2, \ldots X_n$ is a random sample of size n from a population with pdf $f(x,\alpha,\beta,\lambda)$ Let $X_{1:n} < X_{2:n} < \cdots < X_{n:n}$ denote the corresponding order statistics it is well known that the pdf and CDF of the k^{th} order statistics say $Y_k = X_{k:n}$ are given respectively by

$$f_k(y_k) = \frac{n!}{(k-1)!(n-k)!}[F(y_k)]^{k-1}[1-F(y_k)]^{n-k} f(y_k), \text{ and}$$

$$F_k(y_k) = \sum_{r=k}^{n} \binom{n}{r} [F(y_k)]^r [1 - F(y_k)]^{n-r}, k = 1, 2, \dots, n$$

If the population X having SGLD(α, β, λ) with pdf (2.1). The pdf of the $k^{\underline{th}}$ order statistic $Y_k = X_{k:n}$ is given as

$$f_k(y_k, \alpha, \beta, \lambda) = \frac{4\alpha^2\beta n!}{(k-1)!(n-k)!} \sum_{u=0}^{\infty} (-1)^u \binom{n-k}{u} e^{-\beta x} \left(1 + e^{-\beta x}\right)^{-(\alpha+1)} \left(1 + e^{-\beta\lambda x}\right)^{-\alpha} \sum_{k=0}^{\infty} S^{u+j-1} \tag{7.1}$$

Where, $S = \left[\frac{A_1 + A_4}{\lambda j + \lambda\alpha + \alpha} + \frac{2\alpha}{\lambda j + 1}\left[A_2 - A_3 e^{-\beta x(\lambda j + 1)}\right]\right]$

The smallest and largest order statistics are of special importance, as are certain functions of order statistics known as the sample median and the range. The sample rang is the difference of the smallest from the largest, $R = Y_n - Y_1$ .Consequently, the pdf of the smallest and largest order statistics are given by

$f_1(y_1, \alpha, \beta, \lambda) = n\, f(y_1, \alpha, \beta, \lambda)\, [1 - F(y_1, \alpha, \beta, \lambda)]^{n-1} = 4n\alpha^2\beta \sum_{u=0}^{\infty} (-1)^u \binom{n-k}{u} e^{-\beta x} \left(1 + e^{-\beta x}\right)^{-(\alpha+1)} (1 +$
$e{-}\beta\lambda x{-}\alpha\, Su$
and

$$f_n(y_n, \alpha, \beta, \lambda) = n\, f(y_n, \alpha, \beta, \lambda)\, [F(y_n, \alpha, \beta, \lambda)]^{n-1} = 4n\alpha^2\beta e^{-\beta x} \left(1 + e^{-\beta x}\right)^{-(\alpha+1)} \left(1 + e^{-\beta\lambda x}\right)^{-\alpha} \sum_{j=0}^{\infty} \binom{-\alpha}{j} S$$

Notice that, as the same manner we can obtain the CDF of the $k^{\underline{th}}$ order statistics $F_k(y_k, \alpha, \beta, \lambda)$.

8. TRANSFORMATION OF VARIABLES

In practice, one often works with the family of distribution generated by linear transformation $Z = \mu + \eta X$, where X has SGLD(α, β, λ). The random variable Z gives the general class of the SGLD(α, β, λ) by inclusion of the location parameter μ and the scale parameter η. It is easy to see that the random variable Z having also SGLD$(\alpha, \beta, \lambda, \mu, \eta)$.

Theorem: 8.1 Let X be a random variable having SGLD(α, β, λ), and $Z = \mu + \eta X$. Then the nth moment of the random variable Z is given by $E(Z)^n = \sum_{j=0}^{n} \binom{n}{j} \mu^{n-j}\, \eta^j\, E(X^j)$

By elementary calculation, we can prove the theorem. Therefore by illustrating the first four moments of a random variable X given in section (4)

(i) $E(Z) = \mu + \eta E(X)$
(ii) $var(Z) = \eta^2\, var(X)$
(iii) $\gamma_1(Z) = \eta^3\, \gamma_1(X)$
(iv) $\gamma_2(Z) = \eta^4\, \gamma_2(X)$

9. NUMERICAL EXAMPLE

In this section, we express the flexibility of the distribution to account for wide ranges of the skewness and the kurtosis coefficients γ_1, γ_2 respectively. The mean deviation about the mean is also given for ($\beta = 1$ and different values of α and λ).

λ / α	-2	-1	0	1	2	3	4	15	20
1	--	---	0	0.86647	0.97563	1.01933	1.04061	1.07755	1.08035
2	---	---	0.00413	1.36448	1.39063	1.41414	1.43734	1.83258	2.06964
3	---	---	5.19146×10^{-7}	3.39053×10^{-7}	3.27000×10^{-6}	0.00002	0.00005	0.00663	0.00897812
4	---	---	1.45082×10^{-12}	3.51833×10^{-14}	1.15715×10^{-13}	3.90487×10^{-13}	1.20543×10^{-12}	9.39809×10^{-10}	4.33379×10^{-9}
15	---	---	1.97785×10^{-53}	7.24111×10^{-53}	1.58275×10^{-52}	2.77360×10^{-52}	4.29666×10^{-52}	4.29751×10^{-51}	7.38439×10^{-51}
20	---	---	1.09192×10^{-65}	4.07240×10^{-65}	8.95432×10^{-65}	1.57374×10^{-64}	2.44217×10^{-64}	2.45423×10^{-63}	4.21924×10^{-63}

Table - 1: Skewness coefficients of SGLD (α,β,λ) for β = 1.

λ \ α	-2	-1	0	1	2	3	4	15	20
1	--	--	2.09904	3.46220	3.74207	3.91780	4.01452	4.19759	4.21212
2	--	--	0.000743179	5.03561	4.90893	4.97688	5.07502	7.00707	8.24078
3	--	--	1.35007×10^{-9}	7.06498×10^{-9}	1.38168×10^{-7}	1.13417×10^{-6}	5.70119×10^{-6}	0.00350366	0.00525244
4	--	--	1.87801×10^{-17}	2.99604×10^{-19}	5.63502×10^{-18}	5.01644×10^{-17}	2.80564×10^{-16}	2.39203×10^{-12}	1.83070×10^{-11}
15	--	--	3.65366×10^{-79}	2.50946×10^{-78}	8.05860×10^{-78}	1.86366×10^{-77}	3.58673×10^{-77}	1.12891×10^{-75}	2.54141×10^{-75}
20	--	--	1.32975×10^{-97}	9.46589×10^{-97}	3.07468×10^{-96}	7.15069×10^{-96}	1.38080×10^{-95}	4.38570×10^{-94}	9.88271×10^{-94}

Table - 2: Kurtosis coefficients of SGLD (α, β, λ) for β = 1.

λ \ α	-2	-1	0	1	2	3	4	15
1	--	--	2.81142	8.11295×10^{41}	5.76309×10^{78}	7.59643×10^{111}	3.91773×10^{143}	1.68200×10^{479}
2	--	--	8.38400×10^{1175}	7.56939×10^{99}	7.16303×10^{159}	1.95846×10^{216}	1.54406×10^{272}	6.71361×10^{928}
3	--	--	2.58245×10^{74879}	3.71719×10^{18946}	2.85677×10^{13656}	6.80111×10^{10758}	2.07035×10^{8982}	2.00770×10^{5932}
4	--	--	$1.62501\times10^{15135430}$	$2.99559\times10^{8300509}$	$1.36044\times10^{5807092}$	$7.21447\times10^{4458748}$	$2.85251\times10^{3617515}$	$4.97957\times10^{1200900}$

Table - 3: Mean deviation about the mean of SGLD (α, β, λ) for β = 1.

10. CONCLUSION

From figure 3, table 1 and table 2, we see that (i) the expectation value E(X) increases as λ increases and decreases as α increases. (ii) var(x) decreases as |λ| increases and decreases as α increases. (iii) Skewness coefficient γ_1 increases as λ increases and decreases as α increase. (iv) kurtosis coefficients γ_2 initially decreases before increasing as |λ| increases and decreases as α increases. (v) From for the standard logistic distribution γ_1 = 0 and γ_2 =2 which means it is symmetric platykurtic (vi) from table 3, it is clear that the mean deviation about the mean increases when λ increases and α increases. (vii) The flexibility of SGLD (λ, α, β) in terms of accommodating more general types of skewness than the ordinary SDL(λ) is illustrated by computing moments and, in particular, skewness and kurtosis coefficients.

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