Design of Multi-Objective Robust Pitch Control for Large Wind Turbines

H. M. Hassan, W. A. Farag, M. S. Saad, Abdel Latif Elshafei
Electrical Power and Machines Department
Faculty of engineering, Cairo University
Cairo, Egypt

Abstract—**This paper proposes a design of sophisticated multi-Objective H_2/H_∞ collective pitch controllers using linear Matrix Inequalities' techniques. The proposed controller's design takes into account model uncertainties by designing a controller based on a polytopic model. The LMI-based approach allows additional constraints to be included in the design (e.g. H_∞ problem, H_2 problem, H_∞/H_2 trade-off criteria, and pole clustering).These constraints are exploited to include requirements for perfect regulation, efficient disturbance rejection, and permissible actuator usage. The proposed controller is compared to an optimal linear quadratic regulator pitch controller based on single model. Both controllers are tested on 5-MW Reference wind turbine model developed by NREL using FAST software code.**

Keywords-component; pitch controller, LMI, H_∞ problem, H_2 problem, polytopic system, pole clustering.

I. INTRODUCTION

The use of wind power is increasing rapidly. At the same time the need for better cost effectiveness of wind power plants has stimulated growth in wind turbines' size and power. In above-rated wind conditions, the goals for turbine operation change from control of generator torque for maximum power tracking to those of regulating power at rated levels by regulating the generator speed using collective pitch controller. A lot of difficulties face the design; model uncertainties reflected in the unstructured dynamics of some turbine parts, such as the blades, the drivetrain and the tower. Further, the turbine has a nonlinear variation of rotor torque with wind speed and the pitch angle. Furthermore, the pitch actuator also has restricted limits on pitch angle and pitch rate [1].

Other challenging problems are the presence of nonlinearities in the system dynamics, and the continuous change of the operating points during operation. All previous reasons motivate the need for robust pitch controller that provides an accepted performance over wide range of different operating points. Moreover, the resulted control action must be within the permissible actuator constraints. In this paper, a multi-objective collective pitch controller will be designed using LMI techniques for generator speed regulation.

Pitch controller Design is discussed in many literatures lately, Pitch controller is designed using H_∞ technique in [2], [3]. In these papers, the controller main objective is to regulate the speed by improving disturbance rejection. The required control effort isn't considered in the design. In [4], it is proposed to design gain scheduled feedback /feed forward CPC for speed regulation. Also in [5], optimal LQG feedback /feed forward CPC is proposed for speed regulation combined. In [4], and [5]; All the proposed controllers is based on a single linearized model, which only reflects one single operating point. A multi objective (H_2/H_∞) pitch controller is proposed in [6], but it doesn't provide (H_2/H_∞) trade off criteria. It also doesn't consider improving the transient response at different operating points.

In our proposed work in this paper, an LMI based CPC is considered. The controller design constraints include H_∞ problem for better speed regulation, and H_2 problem for optimizing control action with performance. The design also addresses H_∞/H_2 trade-off criteria for the optimization of the two previous problems. Pole clustering for improving transient response is also considered. The controller is based on a polytopic model to overcome model uncertainty at different operating points.

In Section II, the turbine model specifications plus the turbine linearized models are discussed. In Section III, the proposed CPC design via LMI techniques. In Section IV, LQR controller is designed. The simulation results showing a comparison between the proposed controller and the LQR controller are shown in Section V. Finally the conclusions are stated in Section VI.

II. MODEL DESCRIPTION

Simulations are performed on a full non-linear turbine model provided by the FAST (Fatigue, Aero-dynamics, Structures, and Turbulence) software code developed at the US National Renewable Energy Laboratory (NREL) [7]. The model used is a 3-bladed, variable speed 5 MW wind turbine model with the specifications given in Table I. More specifications could be found in [8].

TABLE I. WIND TURBINE SPECIFICATIONS

Hub height	90 meter
Blade diameter	126 meter
Cut in, Rated, cut out wind speed	3 m/s , 11.4 m/s, 25 m/s
Cut in, Rated rotor speed	6.9 rpm, 12.1 rpm
Rated generator speed	1173.7 rpm

The pitch actuator is represented as a second order model. It has a pitch angle range from 0 to 90° with maximum rate of 8°/s.

FAST provides many degrees of freedom reflecting whether or not different turbine parts' dynamics are considered. The following degrees of freedom (DOF) are considered in our study:
a- Generator DOF(q_1).
b- Drivetrain rotational-flexibility DOF (q_2).
c- First fore-aft tower bending-mode DOF (q_3).
d- First flapwise blade mode for each blade DOF (q_4 ,q_5,q_6).
Where (q_I) denotes the displacement of the Ith DOF. Each DOF could be presented as a linearized model around certain operating point according to:

$$M\Delta\ddot{q}_I + C\Delta\dot{q}_I + K\Delta q_I = F * u + F_d * u_d \qquad (1)$$

where $M, C, K, F, F_d, u, and\ u_d$ denote mass matrix, stiffness matrix, damping matrix, control input matrix, wind input disturbance matrix, control input vector, and disturbance input vector, respectively. Assume $\Delta x = [\Delta q_I, \Delta\dot{q}_I]^T$, the linearized model takes the form:

$$P(S): \begin{cases} \Delta\dot{x} = A\Delta x + B\Delta u + B_d\Delta u_d \\ \Delta y = C\Delta x + D\Delta u + D_d\Delta u_d \end{cases} \qquad (2)$$

where Δx, Δu, Δu_d, and Δy are the state vector perturbation, perturbation in the control action, perturbation in input disturbance, and perturbation in the output, respectively. Fig. 1 shows the synthesis of FAST model used in simulation:

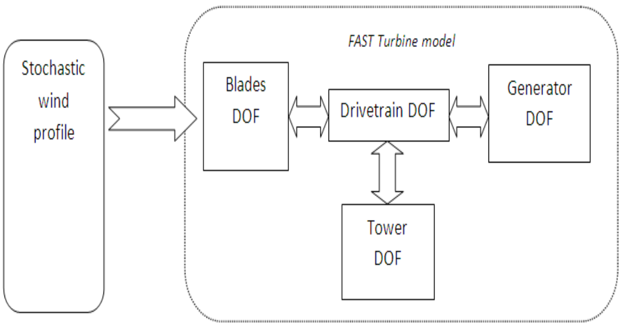

Figure 1. FAST model components

III. DESIGNING AN LMI-BASED COLLECTIVE PITCH CONTROLLER

The proposed technique is to design, state feedback, LMI-based collective pitch controller (CPC) to regulate the generator speed in above rated wind speed. The proposed control strategy is shown in Fig. 2

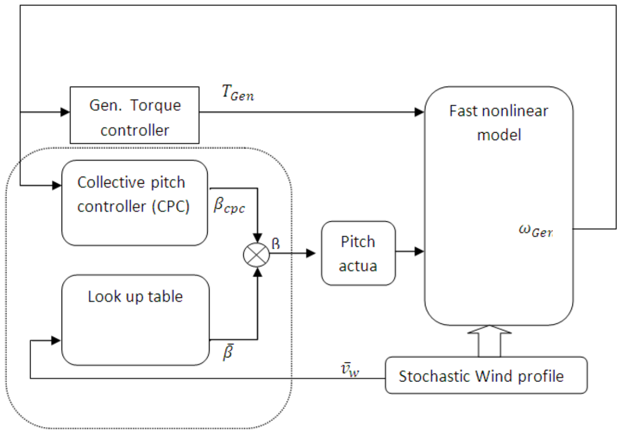

Figure 2. The pitch controller synthesis

T_{gen}, ω_{gen} are the generator torque, speed respectively. $\bar{v}_w$ is the hub height wind speed. The total control action (β) is calculated as follows:

$$\beta = \beta_{cpc} + \bar{\beta} \qquad (3)$$

Where ($\bar{\beta}$) is the pitch angle operating point. It is calculated by changing operating point with wind speed through a look up table. The generator speed is regulated by the control action (β_{cpc}).

In this design, we are looking for a solution that addresses the combination of the following objectives:

1- Efficient disturbance rejection for better speed regulation (H_∞ problem) [9]. This could be achieved by keeping the RMS gain of $T(s)_\infty$ (H_∞ norm) below a predefined value γ_0: ($\gamma_0 > 0$) . $T(s)_\infty$ is the closed loop transfer function from W to Z_∞, where $Z_\infty = [\Delta\omega_{gen}]$ represents the regulation error due to disturbance (W).

2- Trade-off capability between the performance and the exerted control effort through LQG minimization problem. The problem is to minimize a cost function (J) that reflects a weighted sum of the control effort and states' perturbations. Trade-off between the control effort and the performance is represented as a H_2 problem [10]. The minimization is carried by keeping the H_2 norm (LQG Cost) of $T(s)_2$ below a predefined valuev_0: ($v_0 > 0$). $T(s)_2$ is the closed loop transfer function from W to Z_2. Z_2 is defined as:

$$(Z_2 = \lambda * \Delta X + \Psi * \Delta u),$$

where the square matrix λ=diag {$\lambda_1, \lambda_2, \lambda_3$}, and vector Ψ represent the weighting terms of the LQG cost function (J) given in equation 4. Z_2 represents the tradeoff criteria between the perturbations in the states and the control action according to the following objective function:

$$J = \int_0^{\infty} \Delta x^T * \lambda^2 * \Delta x + \Delta u^T * \Psi^2 * \Delta u \quad (4)$$

3-Achieving a desired transient response by maintaining the closed loop poles inside a particular region (D): (pole clustering problem) [10].

FAST can provide a linearized model in the form given in equation 2. This linearized model is calculated at certain operating point $(\bar{\omega}_{gen}, \bar{\beta}, \bar{v}_w)$ [7], where $(\bar{.})$ represents the operating point value of the variable (.). v_w is the hub height wind speed. In our design model, the enabled DOFs are the generator, and the drive train flexibilities DOFs. These are the only DOFs that could be observed of the measured generator speed. They are considered the dominant dynamics in our turbine [8-pp. 30]. As a result, the other enabled DOFs will be considered as unstructured model uncertainty.

The state vector is $\Delta\bar{X}$: $\Delta\bar{X} = [\Delta X_1, \Delta X_2, \Delta X_3]^T$ where:

1) ΔX_1 = Drivetrain rotational-flexibility (perturbations in Drivetrain torsional displacement);(m)
2) ΔX_2=generator DOF(perturbations in rotor speed);(rad/s)
3) ΔX_3 =Drivetrain flexibility (perturbations in Drivetrain torsional velocity);(m/s)

$\Delta u = [\Delta\beta]$, Δu is the perturbation in the collective pitch (control action), $\Delta u_d = [\Delta v_w]$ is the perturbation in the wind speed, $\Delta y = [\Delta\omega_{gen}]$, Δ y is the perturbation in generator speed. The design model p(s) is completely observable and completely controllable, p(s) takes the following form:

$$P(S): \begin{cases} \Delta\dot{X} = A * \Delta X + B_1 * \Delta W + B_2 * \Delta u \\ Z_{\infty} = C_1 * \Delta X + D_{11} * \Delta W + D_{21} * \Delta u \\ Z_2 = C_2 * \Delta X + D_{22} * \Delta u \end{cases} \quad (5)$$

The form is adjusted to make$(Z_{\infty} = \Delta y)$ and $(Z_2 = \lambda * \Delta X + \Psi * \Delta u)$. The state feedback controller takes the form:

$$\Delta u = K * \Delta X \quad (6)$$

In order to design Multi-model controller (Polytopic model), a selection of different operating points' models will be taken into consideration. The different operating points cover the up rated wind speed's region. Six different linearized design models are considered. These models are taken at $(\bar{v}_w = 12\,m/s, 14\,m/s, 16\,m/s, 18\,m/s, 20\,m/s, 22\,m/s)$. Each model represents a different operating point with a unique pitch angle $(\bar{\beta})$. All the models are linearized at the same generator speed equals the rated speed $(\bar{\omega}_{Gen} = \omega_{rated})$. The i^{th} model $P_i(S)$ could be represented in the form:

$$P_i(S): \begin{cases} \Delta\dot{X} = A_i * \Delta X + B_{1i} * \Delta W + B_{2i} * \Delta u \\ Z_{\infty} = C_{1i} * \Delta X + D_{11i} * \Delta W + D_{12i} * \Delta u \\ Z_2 = C_{2i} * \Delta X + D_{2i} * \Delta u \end{cases} \quad (7)$$

In this case the fixed polytope is surrounded by a convex envelope Ω. This envelope has six vertices. Ω is defined as:

$$\Omega = Co\{S_1, S_2, \dots, S_6\} = \left\{ \sum_{i=1}^{6} \alpha_i S_i : \alpha_i \geq 0, \sum_{i=1}^{6} \alpha_i = 1 \right\} \quad (8)$$

Where Co represents the set of vertices defining the set (Ω),and each system matrix is defined as:

$$S_1 = \begin{bmatrix} A_1 & B_{11} & B_{21} \\ C_{11} & D_{111} & D_{121} \\ C_{21} & 0 & D_{21} \end{bmatrix}, \dots, S_6 = \begin{bmatrix} A_6 & B_{16} & B_{26} \\ C_{16} & D_{126} & D_{126} \\ C_{26} & 0 & D_{26} \end{bmatrix}$$

The LMI problem includes the optimization of a cost function *f;* $(H_{\infty}/H_2$ trade off criteria)

$$f = \alpha \|T_{\infty}\|_{\infty}^2 + \beta \|T_2\|_2^2 \quad (9)$$

Where $\alpha, and\ \beta$ are some weighting scalars. The LMI problem formulation is:

$$minimize\ (\alpha * \gamma^2 + \beta * Trace(Q))$$

Subject to:

$$\begin{cases} H_{\infty}\ problem: \left\{ \begin{pmatrix} A_iP + PA_i^T + B_{2i}Y + Y^TB_{2i}^T & B_{1i} & PC_{1i}^T + Y^TD_{12i}^T \\ * & -I & D_{11i}^T \\ * & * & -\gamma^2 I \end{pmatrix} < 0 \right. & (10) \\ H_2\ performance: \begin{cases} \begin{pmatrix} Q & C_{2i}P + D_{22i}Y \\ * & P \end{pmatrix} > 0 \\ Trace(Q) < v_0^2 \\ \gamma^2 < \gamma_0^2 \end{cases} & (11) \\ pole\ clustering: \{\pi \circledast P + \Gamma \circledast (P * A_{cl_i}) + \Gamma^T \circledast (A_{cl_i}^T * P) < 0 & (12) \end{cases}$$

Where (i=1, 2, …, 6), *P* is a Lyapunov matrix that satisfies all the previous constraints, (*) denotes symmetrical element, (⊛) denotes the kroneker product, A_{cl} is the state matrix of the closed loop system. (*Γ, π*) are the parameters' matrices of the desired pole clustering region. Further details and proofs are given in [9], [10]. Once a feasible solution for the LMI framework is reached, an optimal value of the cost function in equation 9 is also reached. The solution yields (*P*, *Y**, γ^*, *Q**),where γ^* is the optimal H_{∞} performance, and *Q** is the optimal H_2 performance. This problem is considered a semi definite problem (SDP). LMI Lab solver in LMI control toolbox [11] is used to solve this problem. The final state feedback controller is calculated as:

$$K_{lmi} = Y^* * (P)^{-1} \quad (13)$$

The values of $\gamma_0\ and\ v_0$ are chosen as:

$$(\gamma_0 = 17.3, v_0 = 1)$$

The criterion behind that choice is that these values should be as small as possible for better performance as long as a feasible solution is obtained.

A. Pole clustering regions

Pole clustering regions are chosen to guarantee improvement in transient response. This could be achieved by specifying regions representing limits on the system's eigenvalues. The first region *(R_1)* guarantees an upper limit on settling time. The second region (*R_2*) guarantees a lower limit on settling time (which prevents excessive control action). Finally region three (*R_3*) is chosen as an upper bound on damping ratio.

The previous regions are convex subsets of the complex plane characterized by [12]:

$$D = \{z \in c : \pi + \Gamma z + \Gamma^T \bar{z} < 0\} \quad (14)$$

R_1, and R_2 regions are defined by a vertical strip between (h_1, h_2) with characteristic function $(F_{D1,2})$ as:

$$F_{D1,2}(Z) = \begin{pmatrix} 2h_1 - (z - \bar{z}) & 0 \\ 0 & (z + \bar{z}) - 2h_2 \end{pmatrix} \quad (15)$$

Region (R_3) is a conic sector centered at the origin with inner angle (2θ) as shown in Figure 3. The characteristic function (F_{D3}) is given as:

$$F_{D3}(Z) = \begin{pmatrix} \sin(\theta) * (z + \bar{z}) & -\cos(\theta) * (z - \bar{z}) \\ \cos(\theta) * (z - \bar{z}) & \sin(\theta) * (z - \bar{z}) \end{pmatrix} \quad (16)$$

The region parameters are taken as $h_1 = -1, h_2 = -4, \theta = 80^{\circ}$(compatible with the system dynamics) [1]. The resulting LMI region (D) is the intersection of the three regions stated above as shown in Fig. 3.

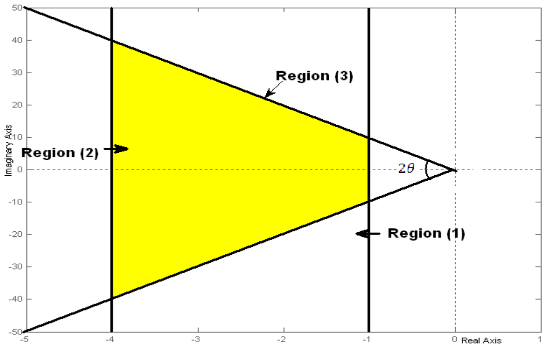

Figure 3. Pole clustering regions

B. (H_2/H_∞)trade off criteria

The relation between (H_∞performance/H_2 performance) is calculated at different weights (β, α) of the trade off criteria given in (9). Figure 4 shows the relation between H_∞performance and H_2 performance

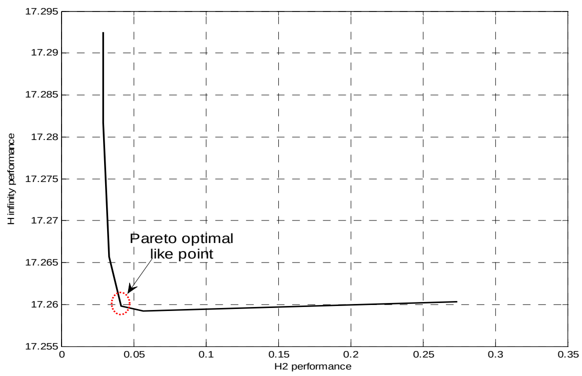

Figure 4. $H_\infty performance/H_2\ performance\ relation$

The optimal performance occurs at the minimum value of the cost function *f*. This value exists as a Pareto-optimal-like point which is circled in Fig. 4.The weights for the cost function *f* that achieve the Pareto-optimal-like point are β=47, and α=0.1.

Now, the LMI problem could be written in this form:

$$minimize\ f: (f = 0.1 * \|T_\infty\|_\infty^2 + 47 * \|T_2\|_2^2),$$

$$subject\ to \begin{cases} \|T_\infty\|_\infty < 17.3 \\ \|T_2\|_2 < 1 \\ closed\ loop\ poles\ (A_{cl}) \in region\ (D) \end{cases} \quad (17)$$

C. Designing kalman observer

CPC is a state feedback controller. As some states are not measurable, an optimal observer (Kalman filter) is designed according to [13]. For designing a Kalman observer, disturbance and noise variance data must be available. According to the linearized model in equation (2), the disturbance in our case is the perturbation in the wind speed around certain operating point. In our study; the perturbation in wind speed ranges from12 to 24 m/s. The observer is based on a linearized model derived at wind speed of 18 m/s (mean wind speed). Assuming a Gaussian probability distribution for the wind speed, then the probability of existence in the previous range equals (99%) if this range represents (6* σ) ; where σ is the standard deviation of the wind speed. As a result the variance in disturbance could be calculated as:

$$Q_{ob} = \sigma^2 = E(WW^T) = 4, R_{ob} = E(NN^T) = 0.01$$

Where W represents the process disturbance, N represents the measurement noise; its variance is taken as 1% of the measured generator speed. The correlation between noise and disturbance is assumed to be zero. The observer model using Kalman observer is:

$$\Delta\hat{\dot{x}} = A\Delta\hat{x} + B\Delta u + K_{ob}(\Delta y - c\Delta\hat{x} - D\Delta u) \quad (18)$$

The covariance matrix of the estimated state error is calculated as:

$$p_{op} = \lim_{t\to\infty}(E((\Delta x - \Delta\hat{x}) * (\Delta x - \Delta\hat{x})^T) \quad (19)$$

K_{ob} is the Kalman filter founded by solving algebraic Riccati equation. The optimal solution could be reached using:

$$K_{ob} = (p_{op}c^T + \bar{N})\bar{R}^{-1} \quad (20)$$

Where: $\bar{R} = R_{ob} + D_d Q_{ob} D_d{}^t$, $\bar{N} = B_d Q_{ob} D_d{}^T$

Kalman observer's structure is depicted in Fig. 5.

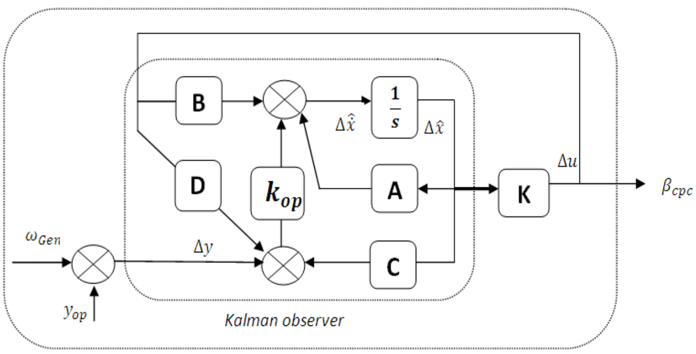

Figure 5. Collective Pitch Regulator

IV. DESIGNING LQR PITCH CONTROLLER BASED ON SINGLE MODEL

This controller is a state feedback controller designed to minimize the following cost function L:

$$L = \int_0^\infty \Delta x^T * q * \Delta x + \Delta u^T * r * \Delta u \qquad (21)$$

Where q, and r are the diagonal weighting matrices for the states perturbations, and the control action perturbations respectively. The LQR controller is full state feedback controller with the following form:

$$\Delta u = K_{lqr} * \Delta X \qquad (22)$$

This controller is based on single design model; this model is derived at certain wind speed (18m/s). The controller K_{lqr} could be calculated as follow:

$$k_{lqr} = r^{-1} * B^T * p_r \qquad (23)$$

Where p_r is the solution of Riccati equation;

$$A^T p_r + p_r A + Q - (p_r B) * R^{-1} * (B^T p_r) = 0 \qquad (24)$$

The weighting matrices (q, r) are chosen according to Bryson's rule [14], this rule calculates the weights depending on the maximum permissible perturbation energy. In case of the control action; r is chosen to be applicable with the actuator's constrain). In case of the states; q is chosen to according to the permissible perturbations in the displacements, and speeds of each DOF). The weigh matrices could be calculated as:

$$\begin{cases} q_{ii} = \dfrac{1}{Max\ (\Delta x_i^2)} \\ r_{ii} = \dfrac{1}{Max\ (\Delta u_i^2)} \end{cases} \qquad (25)$$

Where ii indicates the row, column index respectively in the corresponding weighting matrix,

The same weights are used in the weighting matrices (λ, Ψ) of the H_2 problem to be equivalent to the LQR controller.

V. SIMULATION RESULTS

The proposed controller is tested using FAST nonlinear model. Two uncertainties will be tested. First, the uncertainty is reflected on using a stochastic wind model profile applied to the turbine. This wind profile covers all operating points. Further, It has a specific wind shear. Furthermore, tower shadow effect is also modeled. Moreover, coherent turbulence is presented. Thus, the used profile represents a realistic tough disturbance source capable of testing the controller performance under severe conditions. This was possible by using full field wind profile developed by the NREL TurbSim wind simulator [15].

Second, uncertainty from unstructured model dynamics is represented by enabling more degree of freedoms in FAST model. Although our controller is based on 2 DOFs design model only, it will be tested using a nonlinear model with 6 DOFs (see Section II) to measure the controller robustness.

Fig. 6 compares between the two controllers; the LMI-based controller (K_{LMI}), and the LQR controller (K_{LQR}). The pitch controller objectives is to regulate the generator speed to maintain the rated speed (1173.7 rpm) in order to harvest the Rated generator power (5000 KW).

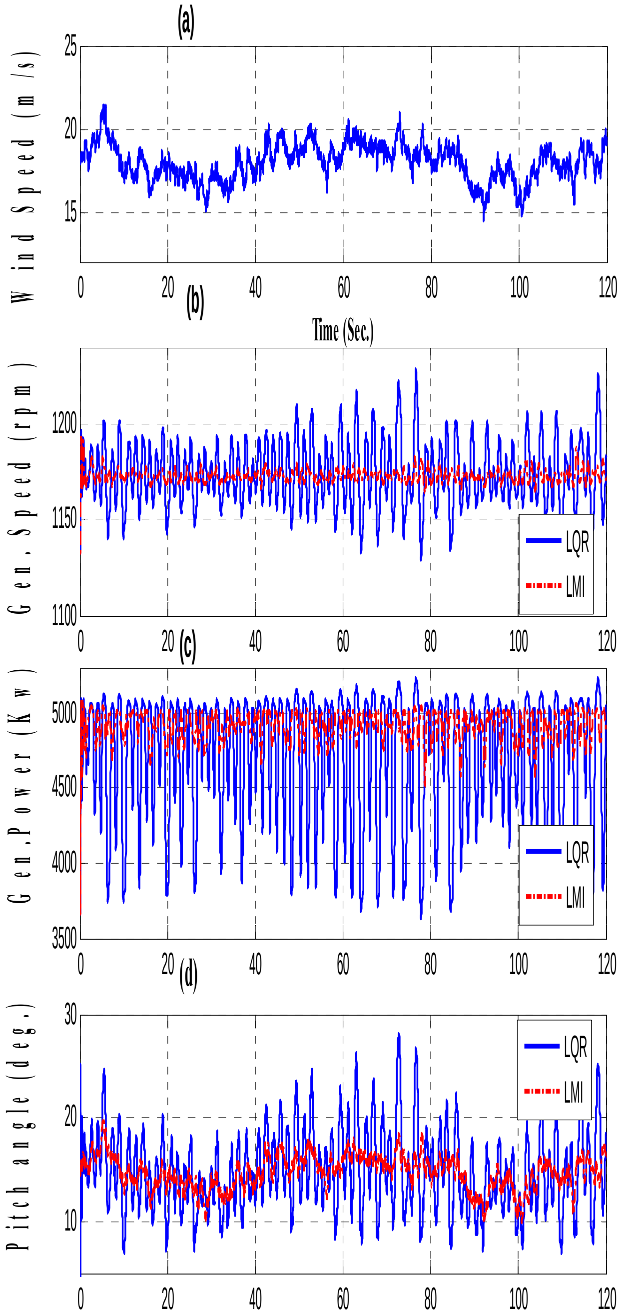

Figure 6. Simulation results of the LMI (dashed line), LQR (solid line) controllers. (a) Hub height wind speed (m/s), (b) Generator speed (rpm) (c) Pitch angle (deg.)

Data analysis of the simulation is shown in Table II. The Table shows the mean, Maximum, and standard deviation for both of the generator speed and the output electrical power.

TABLE II. SPEED, POWER DATA ANALYSIS

		LQR	LMI
Generator speed	Max.	1230 rpm	1195 rpm
	Mean	1174rpm	1174 rpm
	Std.	18 rpm	3 rpm
Electric power	Max.	5235 KW	5090 KW
	Mean	4688 KW	4935 KW
	Std.	440 KW	100 KW

Although the mean generator speed is the same in both cases (LMI, LQR), the LMI has managed to improve the speed regulation, this can be indicated by the standard deviation of the generator speed, which is reduced by six times. As a result, the speed fluctuation has been mitigated significantly; this will reduce the vibrations of the drivetrain and improve the performance of the turbine.

The harvested electrical power is increased significantly in the LMI case. The mean harvested power increased by (250 KW) which represents (5%) increase in the harvested power. Further, the power profile is improved; the standard deviation reduced more than four times. Furthermore, the maximum power is reduced after using the LMI controller. Thus, the generator overloading is mitigated, and the generator is working in the permissible range.

The control action is severe with the LQR controller. As a result, the pitch actuator could reach the saturation level at certain operating points. That is not recommended at all, not only because the Actuator saturation exhausts the actuator and increase the used energy, but also it impairs the system performance significantly. On the contrary, the control action is very smooth with the LMI controller and appropriate for the actuator's constraints mentioned in [8].

VI. CONCLUSION

This paper has studied designing an LMI-based robust CPC for large variable-speed variable-pitch wind turbines. The design constraints have included H_∞ problem, H_2 problem, plus pole clustering. A multi-objective H_∞/H_2 trade-off criteria is used to address a feasible solution for all the previous constraints. A polytopic model is used to represent different operating points to overcome the system uncertainty.

The performance of the proposed controller has been compared to an Optimal LQR controller based on single model. The comparison has shown that the proposed LMI-based controller has achieved improvements in performance in terms of speed regulation (which become six times better), and full load power harvesting (reflected in 5% increase in the harvested power). Unlike, the LQR controller, The LMI controller's progress has been reached without exerting extra actuator effort, and achieved with control action compatible with the actuator's limits .

ACKNOWLEDGEMENT

This work was done in Electrical Power and Machines Department, Cairo University, EGYPT, and supported by the Science and Technology Development Fund (STDF) under project ID. 1411.

REFERENCES

[1] J.M. Jonkman, "Dynamics Modeling and Loads Analysis of an Offshore Floating Wind Turbine", technical Report, NREL/TP-500-41958, November 2007.

[2] M. Geyler,P. Caselitz, "individual blade pitch control for load reduction on large wind turbines", Proceedings of the European Wind Energy Conference, Milan, Italy, 2007.

[3] Jason Laks z, Lucy Paox, Alan Wright," Combined Feed-forward Feedback Control of Wind Turbines to Reduce Blade Flap Bending Moments", *In AIAA/ASME Wind Energy Symp., Orlando, FL, 2009.*

[4] Fiona Dunney, Lucy Y. Paoz, Alan D. Wrightx, Bonnie Jonkman, Neil Kelle, "Combining Standard Feedback Controllers With Feed forward Blade Pitch Control for Load Mitigation in Wind Turbines," *in Proc. 48th AIAA Aerospace Sciences Meeting, Orlando, FL, AIAA-2010-250, Jan. 2010.*

[5] K. Selvam, S. Kanev, J. W. vanWingerden, T. van Engelen, and M. Verhaegen, "Feedback feed forward individual pitch control for wind turbine load reduction," *Int. J. Robust Nonlinear Control*, 2008.

[6] F. Lescher, J. Y. Zhao, and A. Martinez. "Multi objective H2/H∞ control of a pitch regulated wind turbine for mechanical load reduction.". *In Proceedings of the European Wind Energy Conference (EWEC)*, Athens, Greece, 2006.

[7] Jonkman, J., and Buhl, M. L., Jr., "FAST Users Guide, " National Renewable Energy Laboratory, NREL/EL-500-38230, Golden, CO, August 2005.

[8] Jonkman, J., Buttereld, S., Musial, W., and Scott, G., "Definition of a 5-MW Reference Wind Turbine for Offshore System Development, " , National Renewable Energy Laboratory, NREL/TP-500-38060*, Golden, CO*, February 2009.

[9] Kemin zoo, Essentials of Robust Control, *Printice Hall*, 1997.

[10] M. Chilali and P. Gahinet, "H_∞ design with pole placement constrains: an LMI approach", *IEEE Trans. Automat. Contr., pp. 358-367*, 1996.

[11] Pascal Gahinet ,Arkadi Nemirovski ,Alan J. Laub ,Mahmoud Chilali ," LMI Control Toolbox user's guide",1995

[12] Chilali, M., Ghainet, "Robust pole placement in LMI regions.", *IEEE Trans. Automat. Contr., pp. 2257-2269*, Dec. 1999

[13] Lewis, F."Optimal Estimation". John Wiley & Sons, Inc, 1986.

[14] G. F. Franklin, J. D. Powell, and A. Emami-Naeini. "Feedback Control of Dynamic Systems". Prentice Hall, Upper Saddle River, NJ, 4th edition, 2002.

[15] Kelley, N. D., and Jonkman, B. J. Overview of the TurbSim Stochastic In flow Turbulence Simulator. NREL/TP-500-41137, Golden, CO, April 2007.