Induction Machines

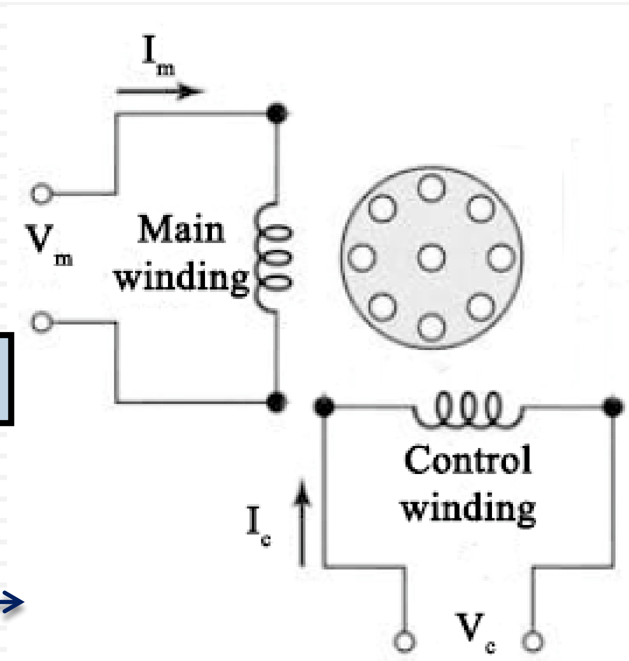
- Three-phase induction motor.
- Three-phase induction generator.
- Two-phase induction motor (AC Servo-Motor).
- Single-phase induction motors.

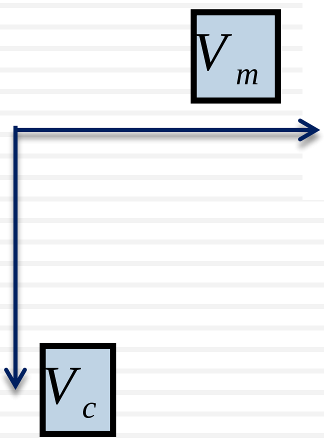
Induction Machines: 2-phase Induction Motors

Balanced Operation

$$V_m = V \angle 0$$

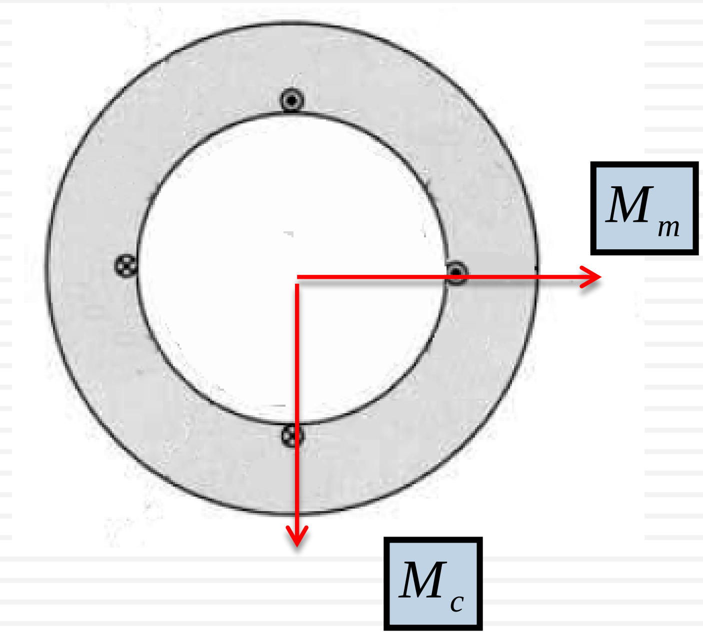
$$I_m = I \angle -\phi$$





$$V_c = V \angle -90^o$$

$$I_c = I \angle -\phi - 90^o$$



Induction Machines: 2-phase Induction Motors

Magnetic Field Production & Distribution

$$i_m = I_m \cos(\omega_e t - \phi)$$

$$i_c = I_m \cos(\omega_e t - \phi - 90^o) \qquad i_c = I_m \sin(\omega_e t - \phi)$$

$$M = \frac{4}{\pi}\frac{N}{2p} I \cos\theta$$

Mechanical Degrees

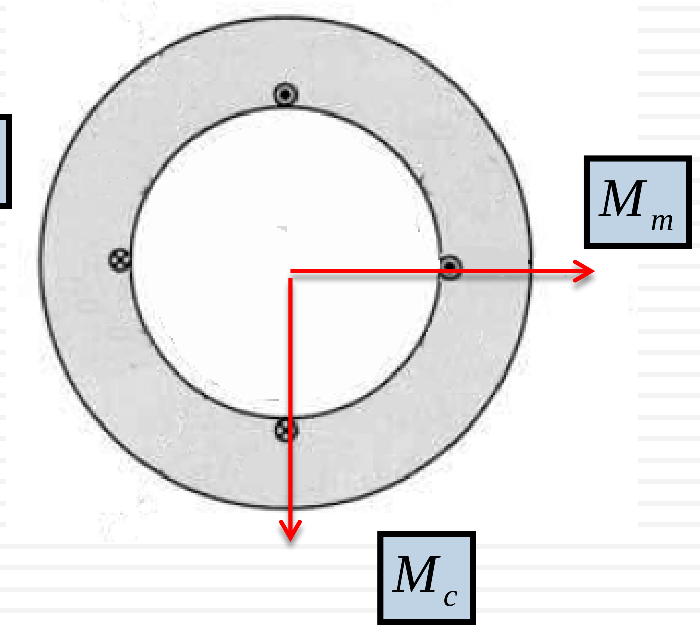
$$M_m = \frac{4}{\pi}\frac{N}{2p} I_m \cos(\omega_e t - \phi)\cos p\theta$$

Electrical Degrees

$$M_c = \frac{4}{\pi}\frac{N}{2p} I_m \sin(\omega_e t - \phi)\sin p\theta$$

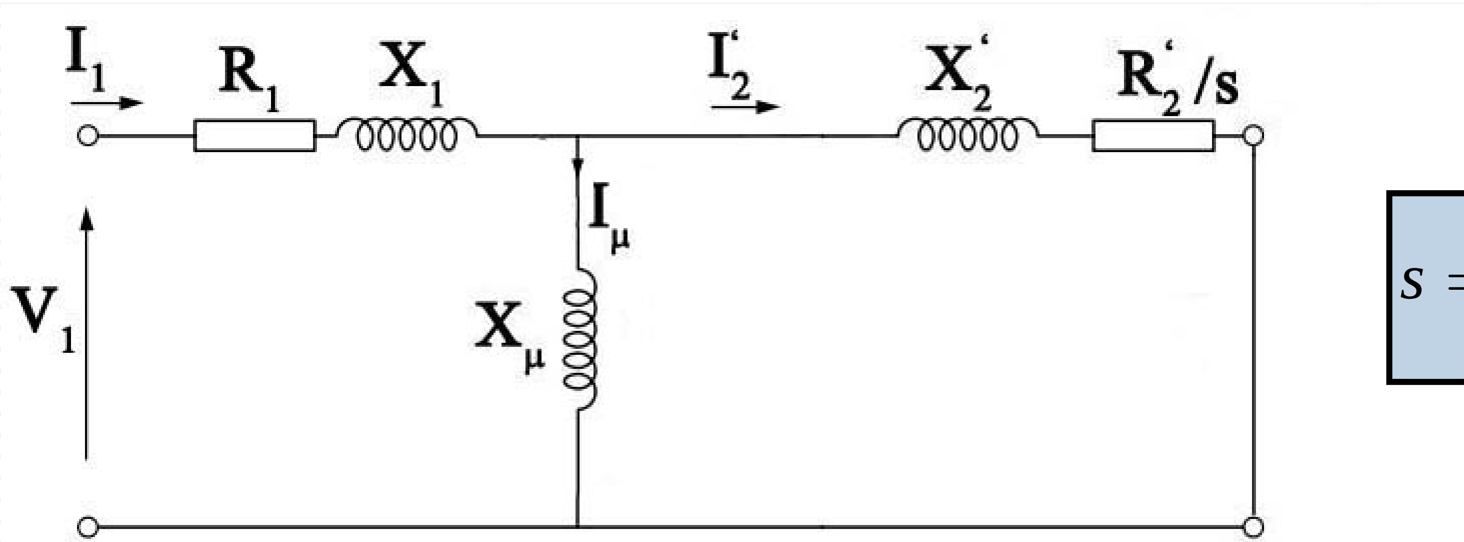
$$M_t = \frac{4}{\pi}\frac{N}{2p} I_m \cos(\omega_e t - p\theta - \phi)$$

Rotating Field



Induction Machines: 2-phase Induction Motors

Equivalent Circuit



$$s = \frac{n_s - n}{n_s}$$

$$P_{in} = 2VI\cos\phi$$

$$P_g = 2I_2^{'2}\frac{R_2^{'}}{s} = P_{in} - P_{cu1}$$

$$P_d = 2I_2^{'2}\frac{(1-s)}{s}R_2^{'} = P_g - P_{cu2}$$

$$P_{cu1} = 2I_1^2 R_1$$

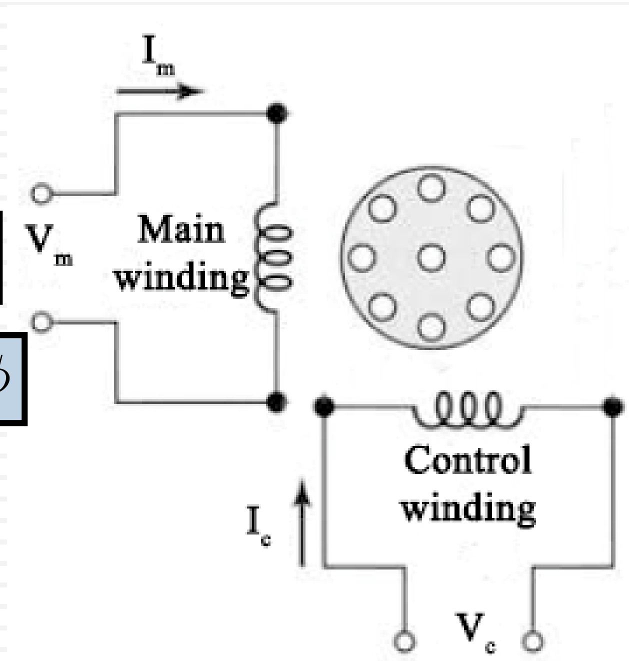
$$P_{cu2} = 2I_2^{'2} R_2^{'}$$

Induction Machines: 2-phase Induction Motors

Unbalanced Operation

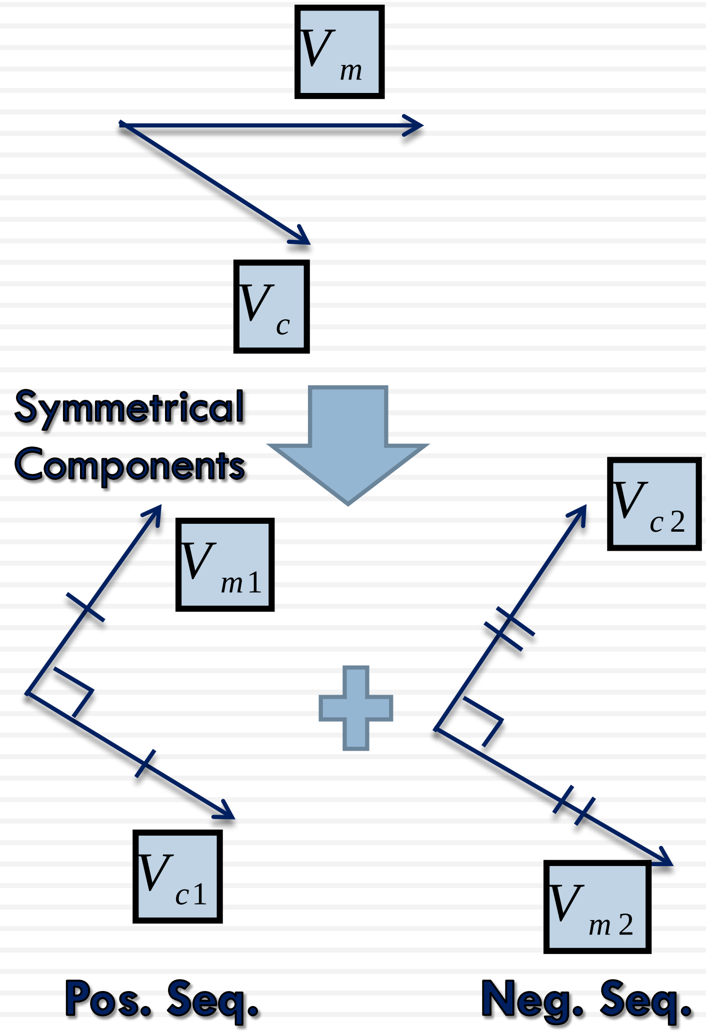
$$V_m = V_m \angle 0$$

$$I_m = I_m \angle -\phi$$



$$V_c = V_c \angle -\theta_c$$

$$I_c = I_c \angle -\phi - \theta_c$$



Induction Machines: 2-phase Induction Motors

Unbalanced Operation

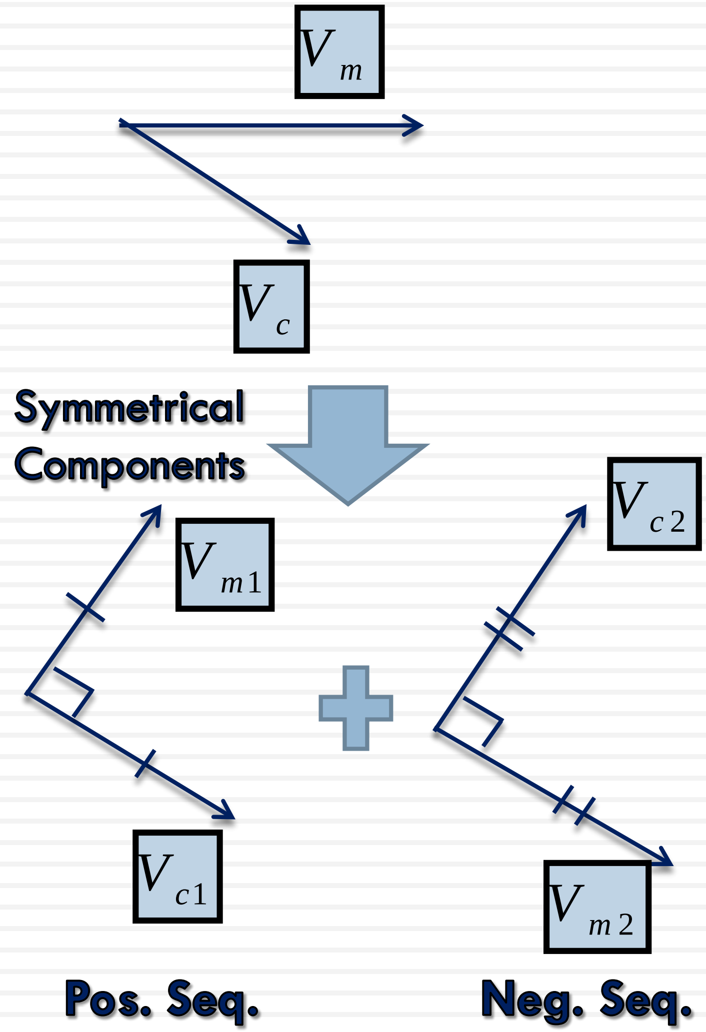
$$V_m = V_{m1} + V_{m2}$$

$$V_c = V_{c1} + V_{c2}$$

$$V_{c1} = -jV_{m1} \qquad V_{c2} = jV_{m2}$$

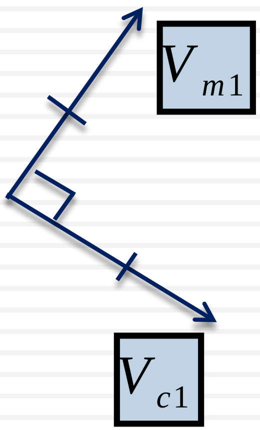
$$\begin{bmatrix} V_m \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \begin{bmatrix} V_{m1} \\ V_{m2} \end{bmatrix}$$

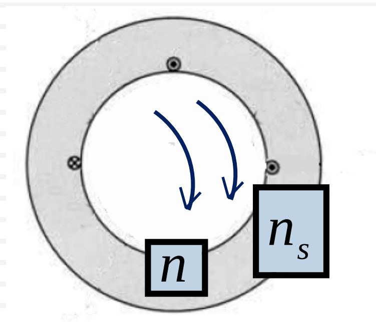
$$\begin{bmatrix} V_{m1} \\ V_{m2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} V_m \\ V_c \end{bmatrix}$$



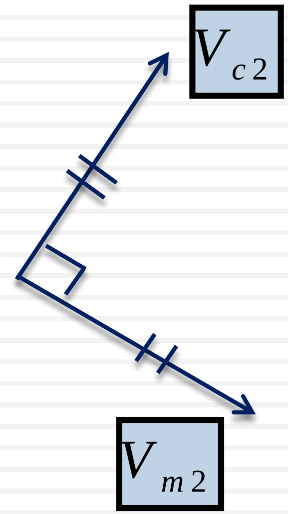
Induction Machines: 2-phase Induction Motors

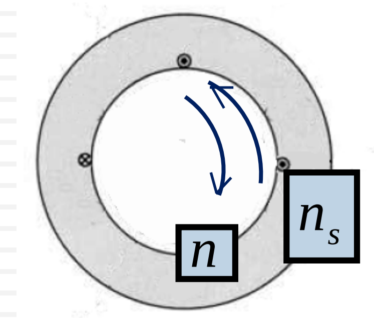
Unbalanced Operation



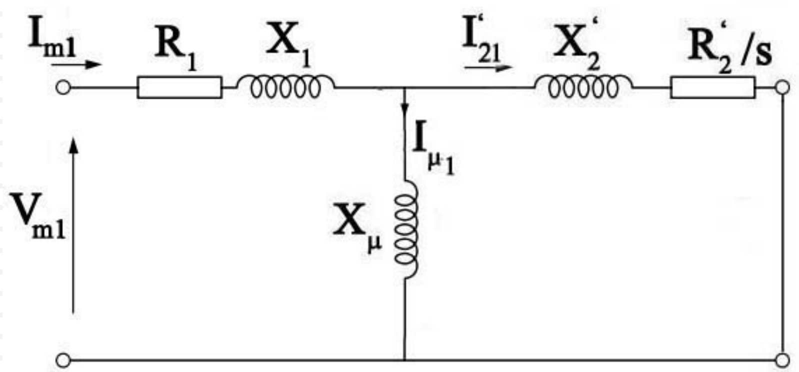


$$s_1 = \frac{n_s - n}{n_s} = s$$

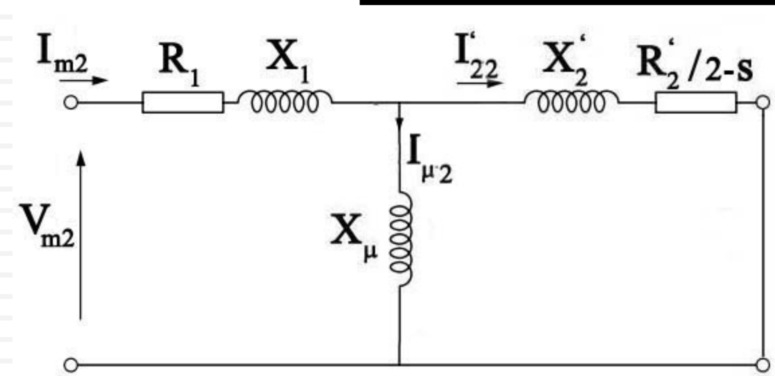




$$s_2 = \frac{n_s + n}{n_s} = 2 - s$$



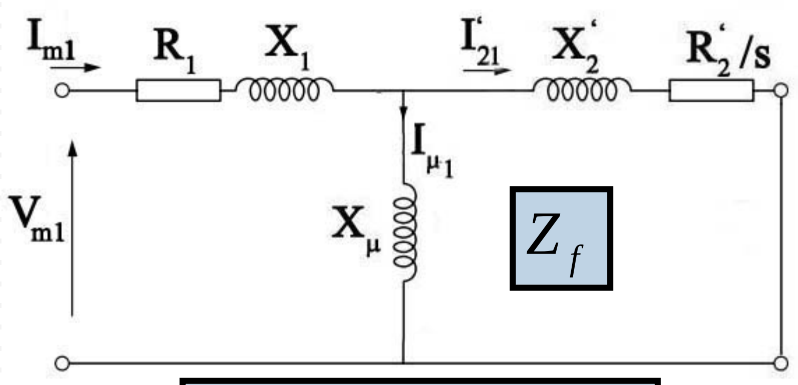
Forward

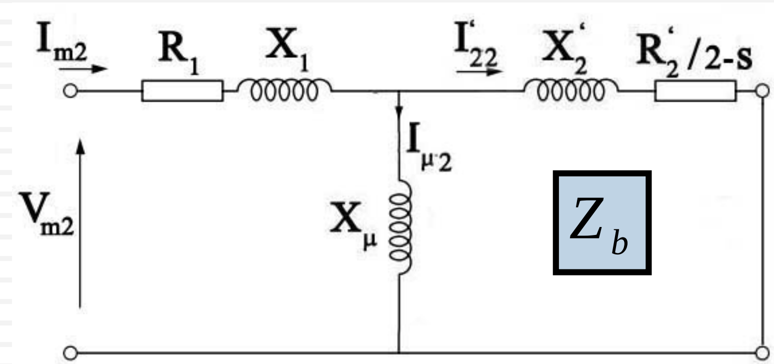


Backward

Induction Machines: 2-phase Induction Motors

Unbalanced Operation





$$I_{m1} = \frac{V_{m1}}{R_1 + jX_1 + Z_f}$$

$$I_{m2} = \frac{V_{m2}}{R_1 + jX_1 + Z_b}$$

$$P_{g1} = 2I_{m1}^2 R_f$$

$$P_{g2} = 2I_{m2}^2 R_b$$

$$\begin{array}{ccc} P_{g1} : & P_{cu21} : & P_{d1} \\ 1 \; : & s & :(1-s) \end{array}$$

$$T_1 = \frac{P_{g1}}{\omega_s}$$

$$\begin{array}{ccc} P_{g2} : & P_{cu22} : & P_{d2} \\ 1 \; : & 2-s & :(1-(2-s)) \end{array}$$

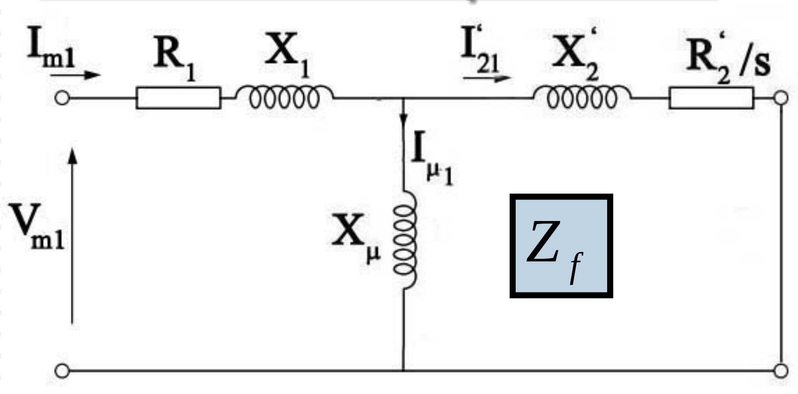
$$T_2 = \frac{P_{g2}}{\omega_s}$$

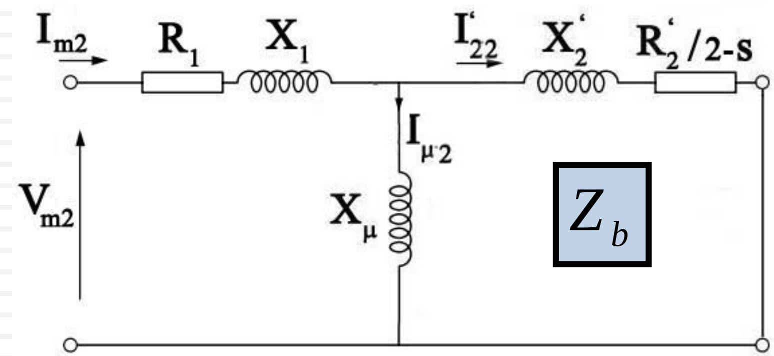
$$P_d = P_{d1} + P_{d2} = (1-s)(P_{g1} - P_{g2})$$

$$T_{net} = \frac{P_g}{\omega_s} = \frac{P_d}{\omega} = T_1 - T_2$$

Induction Machines: 2-phase Induction Motors

Unbalanced Operation





$$\begin{bmatrix} I_m \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \begin{bmatrix} I_{m1} \\ I_{m2} \end{bmatrix}$$

$$P_{cu1} = P_{cu11} + P_{cu12}$$

$$P_{cu11} = 2I_{m1}^2 R_1$$

$$P_{cu12} = 2I_{m2}^2 R_1$$

$$P_{cu2} = P_{cu21} + P_{cu22}$$

$$P_{in} = P_m + P_c$$

$$P_m = V_m I_m \cos(\theta_{V_m} - \theta_{I_m})$$

$$P_c = V_c I_c \cos(\theta_{V_c} - \theta_{I_c})$$

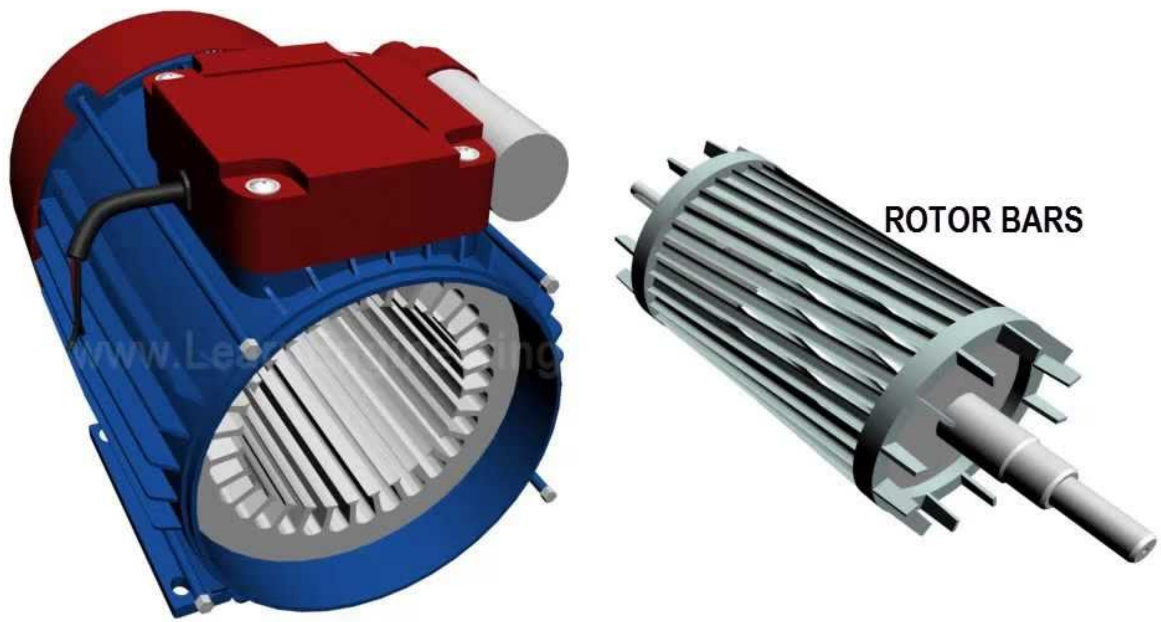
Induction Machines: 1-phase Induction Motors

- Construction
- Principle of Operation
- Equivalent Circuit
- Power and Torque
- Motor Types
- Starting Winding Design
- Testing

Induction Machines: 1-phase Induction Motors

Induction Machines: 1-phase Induction Motors

Construction



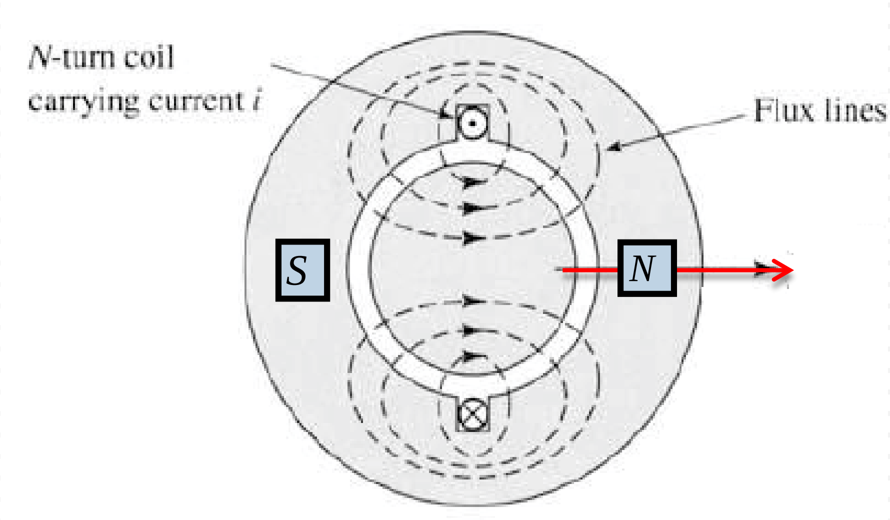
Induction Machines: 1-phase Induction Motors

Magnetic Field Production & Distribution

$$\oint_c \underline{H}.\underline{dl} = I_{en}$$

$$\oint_c \underline{H}.\underline{dl} = Ni$$

$$H \times g + H \times g \simeq Ni$$

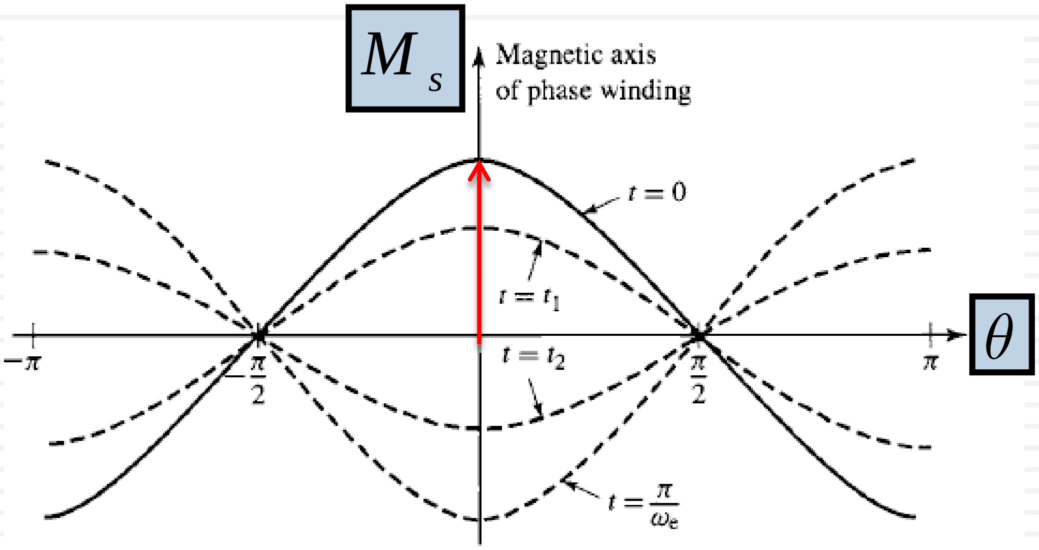


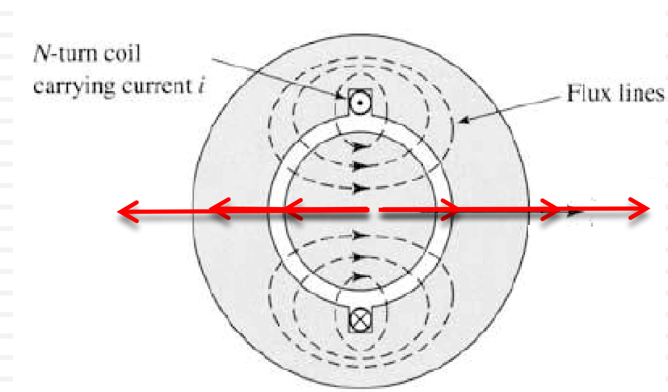
$$H_g\, g = \frac{Ni}{2} = M$$

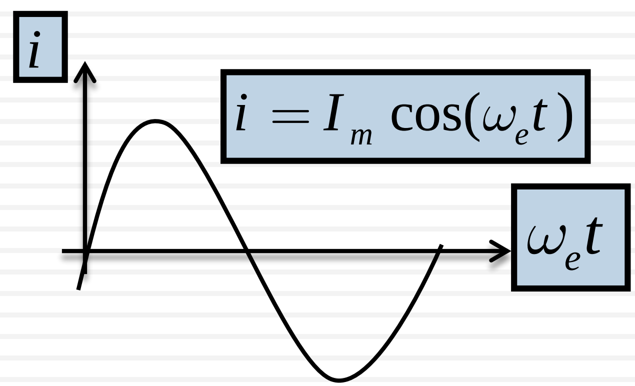
$$M_f = \frac{B_g}{\mu_o} g$$

Induction Machines: 1-phase Induction Motors

Magnetic Field Production & Distribution







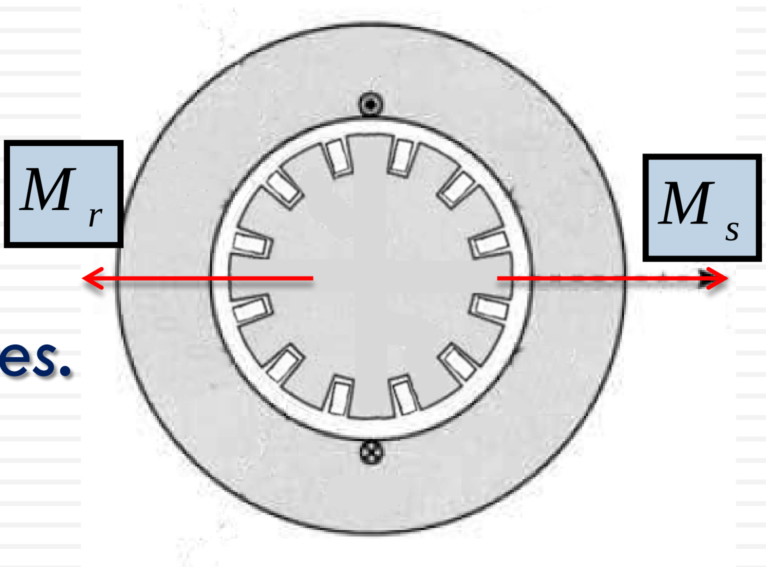
MMF Space Distribution

AC Current → **Pulsating Field**

Current Time Distribution

<u>*Rotor Reaction*</u>

At standstill, the rotor field will always be shifted by 180° from the stator field at all instances.



$$T = kB_s B_r \sin\gamma$$

$$T = kB_s B_r \sin 180^o$$

$$T = zero$$

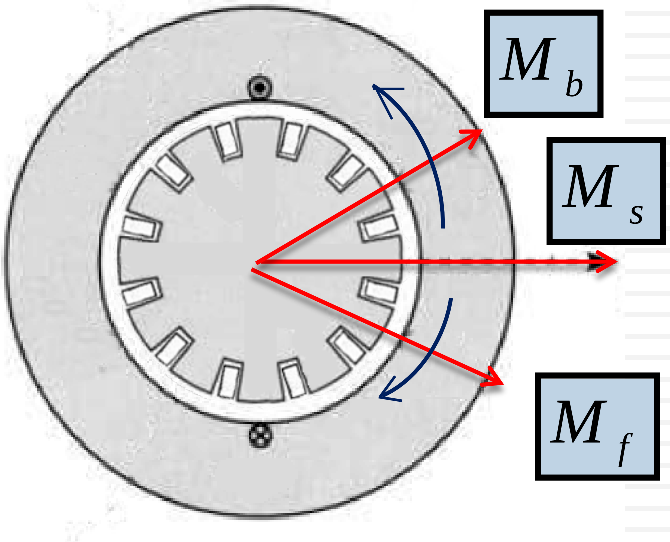
Induction Machines: 1-phase Induction Motors

Double Rotating Field Theory

$$I = I_m \cos(\omega_e t)$$

$$M = \frac{4}{\pi}\frac{N}{2p} I \cos\theta$$

$$M = \frac{4}{\pi}\frac{N}{2p} I_m \cos(\omega_e t)\cos p\theta$$



$$M = \frac{4}{\pi}\frac{N}{2p}\frac{I_m}{2}[\cos(\omega_e t - p\theta) + \cos(\omega_e t + p\theta)]$$

$$M_f = \frac{4}{\pi}\frac{N}{2p}\frac{I_m}{2}\cos(\omega_e t - p\theta)$$

Speed of Rotation

$$n_s = \frac{60f}{p}$$

Forward Field

$$M_b = \frac{4}{\pi}\frac{N}{2p}\frac{I_m}{2}\cos(\omega_e t + p\theta)$$

Speed of Rotation

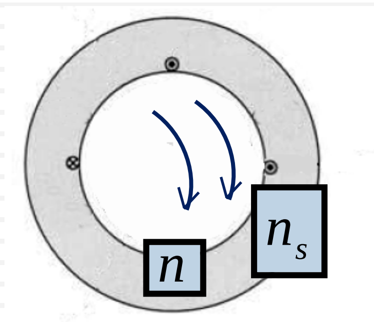
$$-n_s = -\frac{60f}{p}$$

Backward Field

Induction Machines: 2-phase Induction Motors

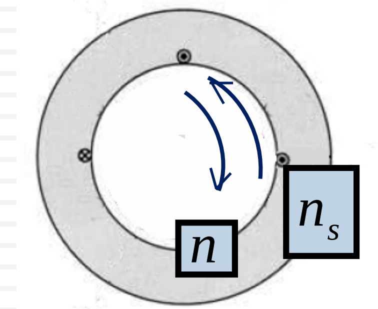
Double Rotating Field Theory

Forward Field

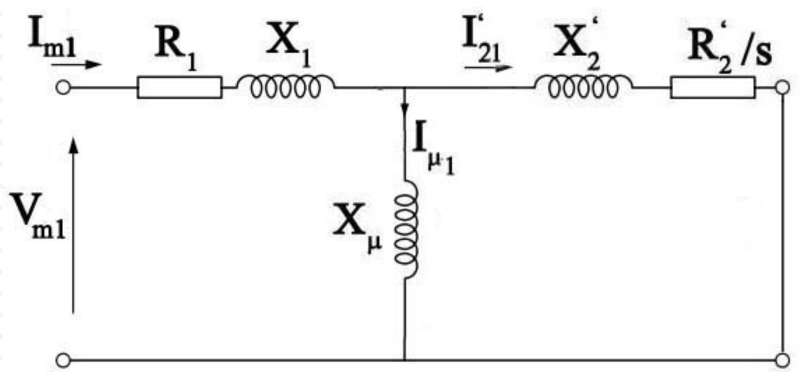


$$s_1 = \frac{n_s - n}{n_s} = s$$

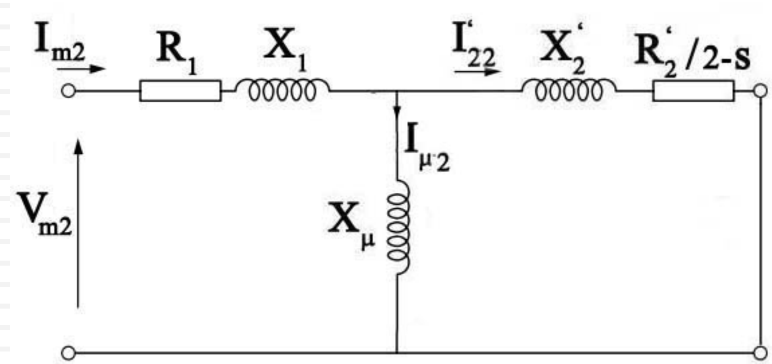
Backward Field



$$s_2 = \frac{n_s + n}{n_s} = 2 - s$$



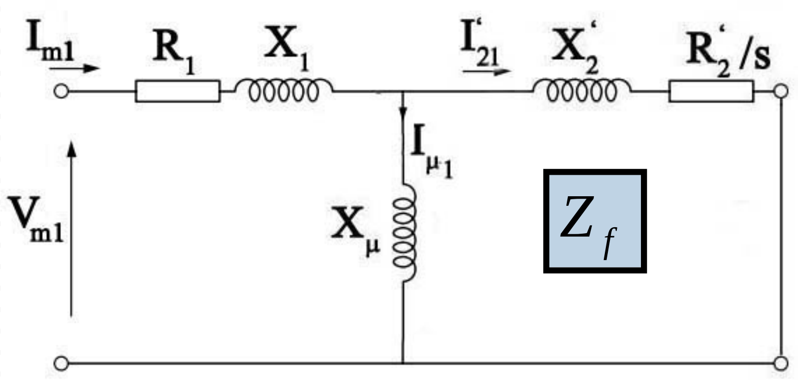
Forward



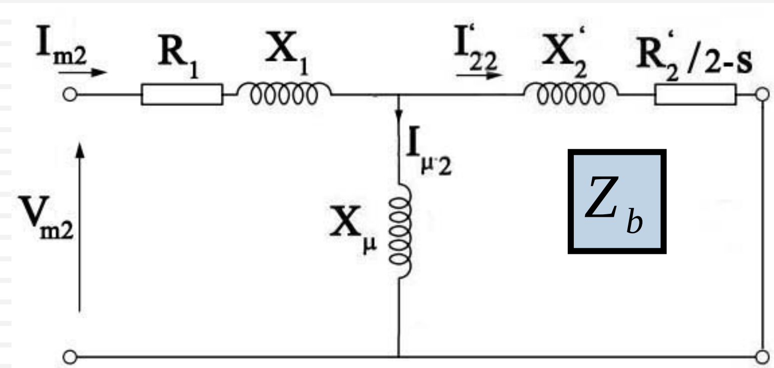
Backward

Induction Machines: 2-phase Induction Motors

Double Rotating Field Theory



Forward



Backward

$$\begin{bmatrix} V_{m1} \\ V_{m2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} V_m \\ V_c \end{bmatrix}$$

$$V_{m1} = V_{m2} = \frac{1}{2} V_m$$

At s = 1

$$Z_f = Z_b = Z$$

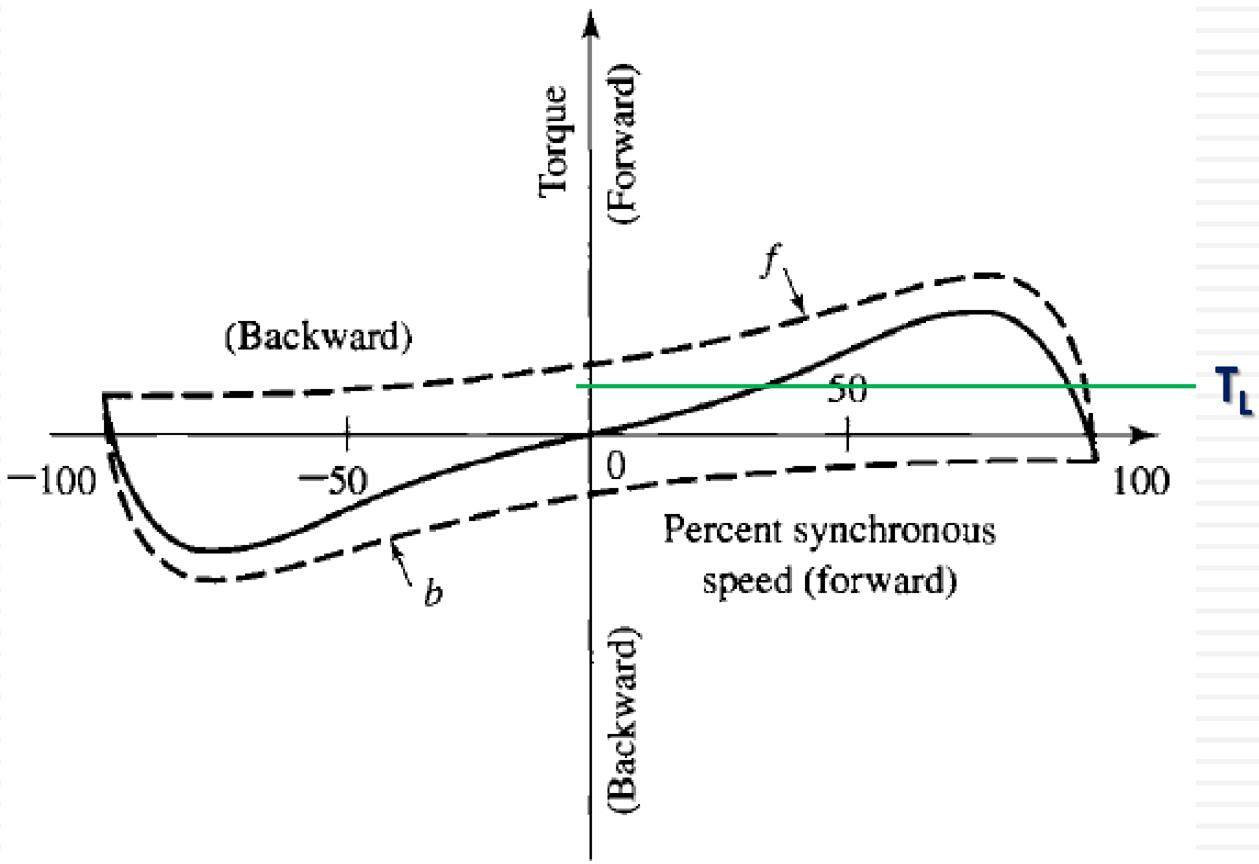
$$I_{m1} = I_{m2}$$

$$P_{g1} = P_{g2}$$

$$T_1 = T_2$$

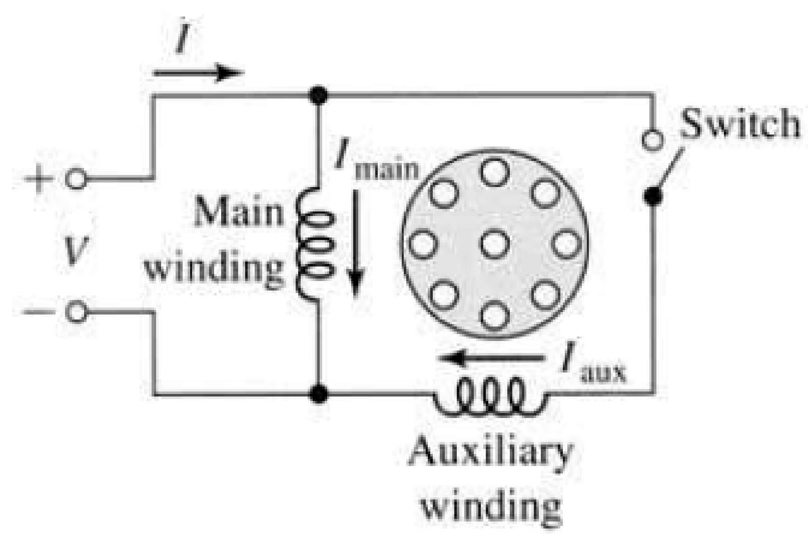
$$T_{net} = T_1 - T_2 = zero$$

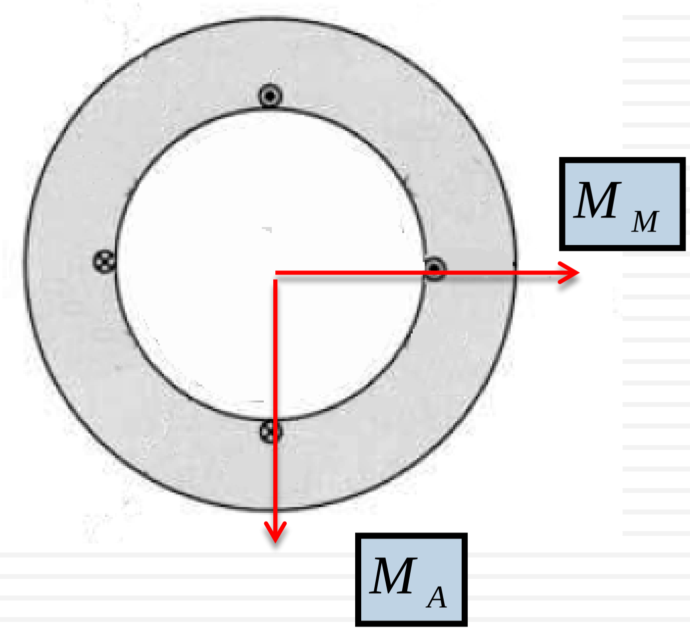
Torque-Speed Characteristics



No starting torque

Motor Starting



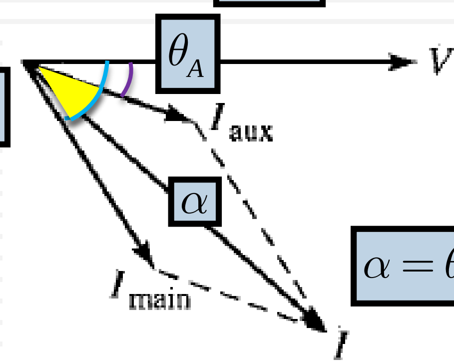


$N_M > N_A$ ⇒ $X_M > X_A$

$A_{cM} > A_{cA}$ ⇒ $R_M < R_A$

⇒ $\dfrac{R_A}{X_A} > \dfrac{R_M}{X_M}$

$T_{st} \propto I_M I_A \sin\alpha$



$\alpha = \theta_M - \theta_A$

Motor Starting

$$I_M = I_{Mm} \cos(\omega_e t)$$

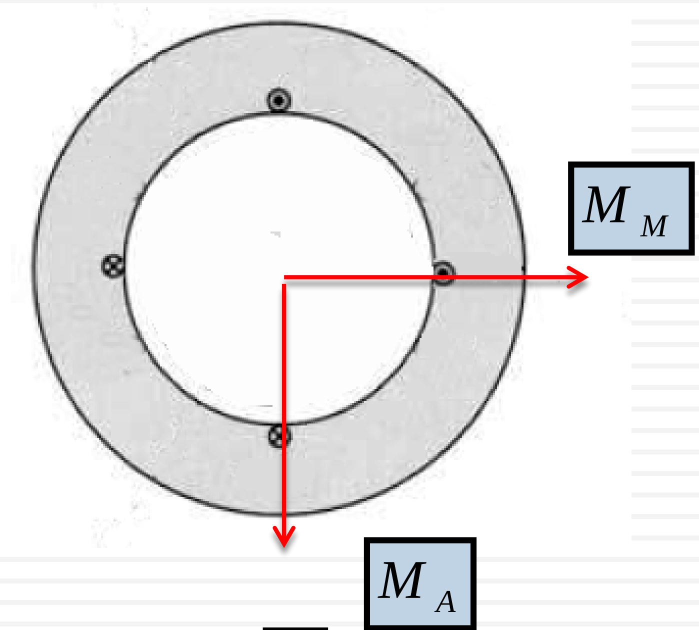
$$M_M = \frac{4}{\pi}\frac{N_M}{2p} I_M \cos\theta$$

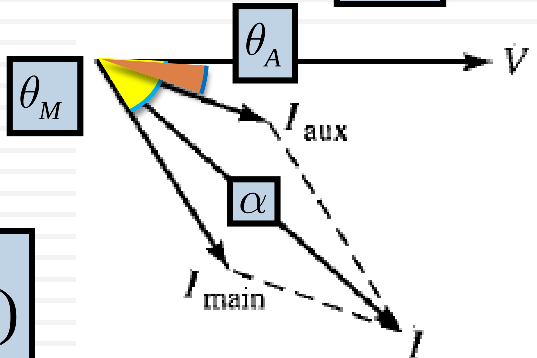
$$M_M = \frac{4}{\pi}\frac{N_M}{2p} I_{Mm} \cos(\omega_e t)\cos p\theta$$

$$I_A = I_{Am} \cos(\omega_e t + \alpha)$$

$$M_A = \frac{4}{\pi}\frac{N_A}{2p} I_A \cos(\theta + 90^o)$$

$$M_A = \frac{4}{\pi}\frac{N_A}{2p} I_{Am} \cos(\omega_e t + \alpha)\cos(p\theta + 90^o)$$





Induction Machines: 1-phase Induction Motors

Motor Starting

$$M_t = kN_M I_{Mm}\cos(\omega_e t)\cos p\theta - kN_A I_{Am}\cos(\omega_e t + \alpha)\sin p\theta$$

$$k = \frac{4}{\pi(2p)}$$

$$M_t = k[N_M I_{Mm}\cos(\omega_e t)\cos p\theta - N_A I_{Am}\sin p\theta[\cos(\omega_e t)\cos(\alpha) - \sin(\omega_e t)\sin(\alpha)]]$$

$$M_t = \frac{kN_M I_{Mm}}{2}[\cos(\omega_e t - p\theta) + \cos(\omega_e t + p\theta)] - \frac{kN_A I_{Am}}{2}\cos(\alpha)[\sin(\omega_e t + p\theta) - \sin(\omega_e t - p\theta)]$$
$$+\frac{kN_A I_{Am}}{2}\sin(\alpha)[\cos(\omega_e t - p\theta) - \cos(\omega_e t + p\theta)]$$

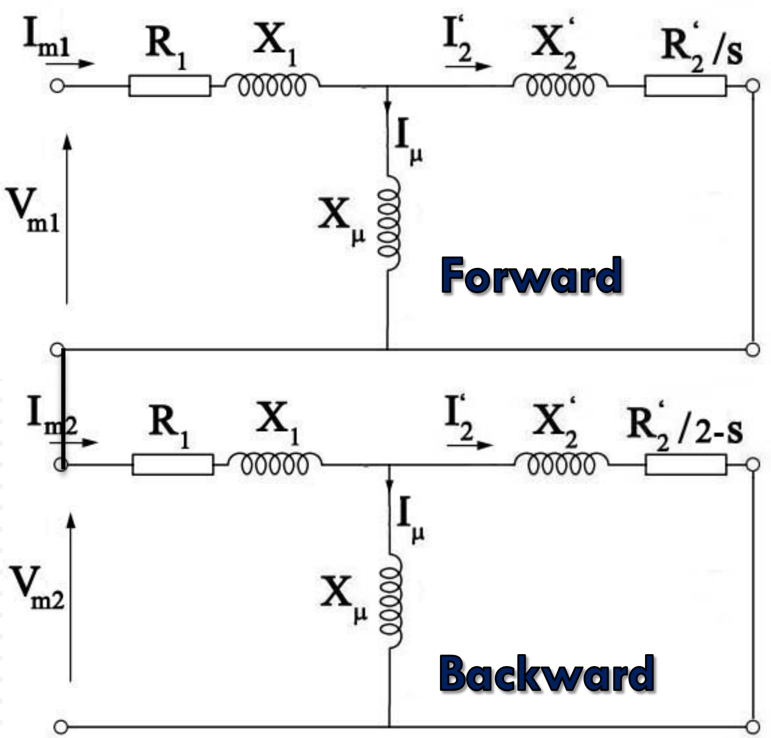
$$M_t = \frac{k}{2}[N_M I_{Mm} + N_A I_{Am}\sin(\alpha)][\cos(\omega_e t - p\theta)] + \frac{kN_A I_{Am}}{2}\cos(\alpha)\sin(\omega_e t - p\theta)$$
$$+\frac{k}{2}[N_M I_{Mm} - N_A I_{Am}\sin(\alpha)][\cos(\omega_e t + p\theta)] - \frac{kN_A I_{Am}}{2}\cos(\alpha)\sin(\omega_e t + p\theta)$$

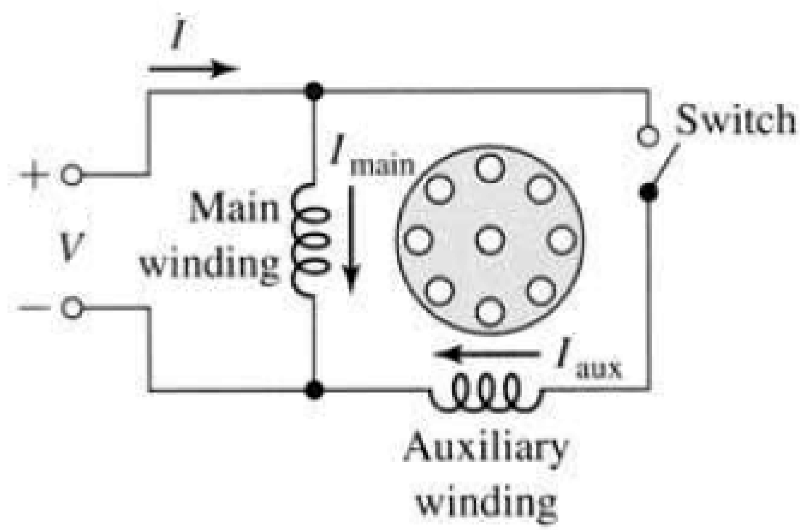
Induction Machines: 1-phase Induction Motors

Equivalent Circuit at running conditions

$$\begin{bmatrix} I_{m1} \\ I_{m2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix}\begin{bmatrix} I_m \\ \not{I}_a \end{bmatrix}$$

$$I_{m1} = I_{m2} = \frac{1}{2} I_m$$

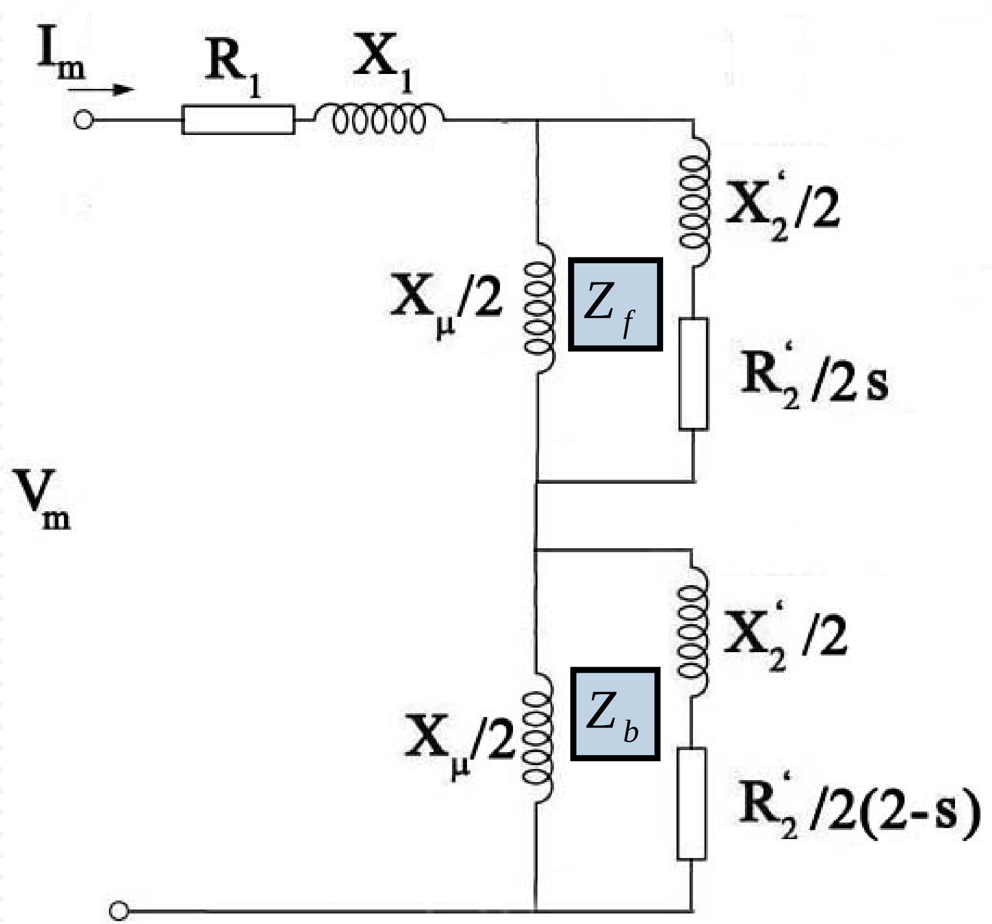


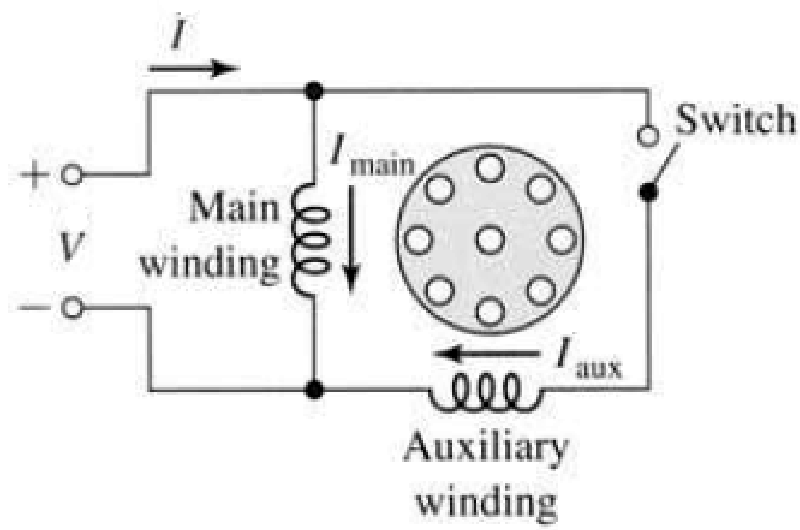


$$I_a = zero$$

$$V_{m1} = V_{m2} = \frac{1}{2} V_m$$

Equivalent Circuit at running conditions



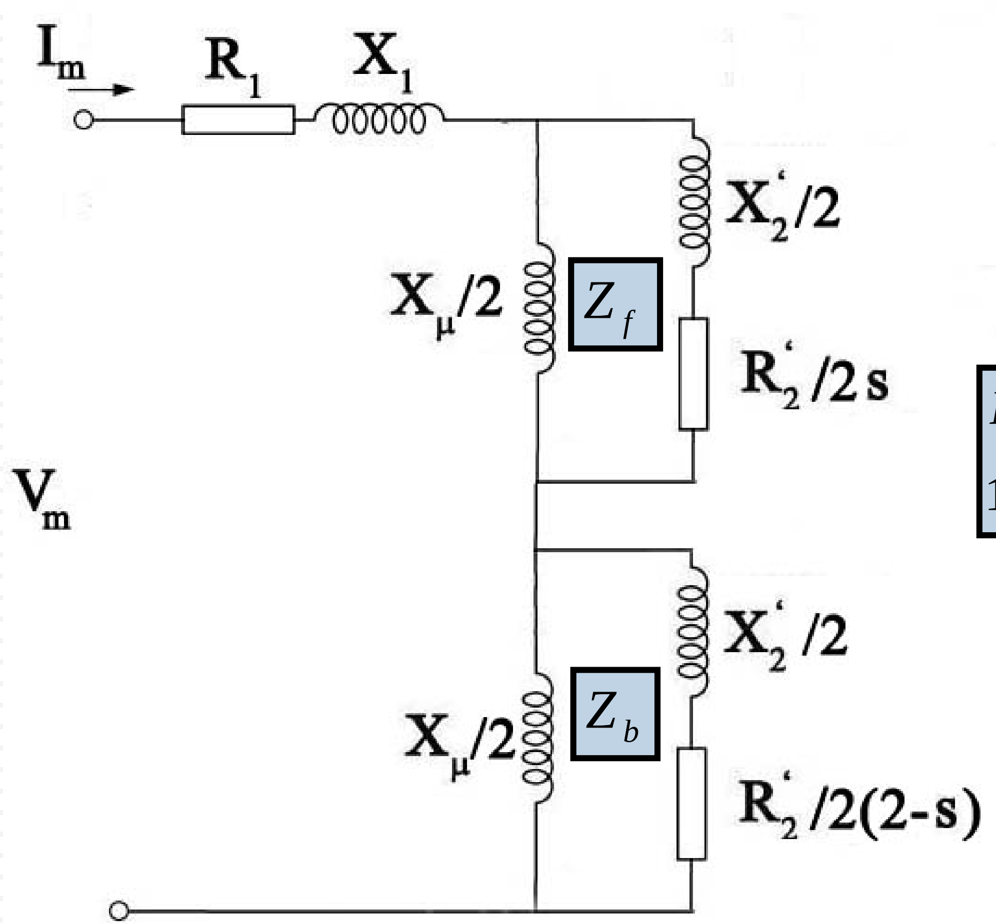


$$I_a = zero$$

$$I_m = \frac{V_m}{R_1 + jX_1 + Z_f + Z_b}$$

Induction Machines: 1-phase Induction Motors

Power Flow



$$P_{in} = V_m I_m \cos\phi_m$$

$$P_{cu1} = I_m^2 R_1$$

$$P_{gf} = I_m^2 R_f$$

$$P_{gb} = I_m^2 R_b$$

$$P_{gf} : P_{cuf\,2} : P_{df}$$
$$1 : s : (1-s)$$

$$P_{gb} : P_{cub\,2} : P_{db}$$
$$1 : 2-s : (1-(2-s))$$

$$P_{cu\,2} = P_{cu\,2f} + P_{cu\,2b}$$

$$P_d = P_{df} + P_{db} = (1-s)(P_{gf} - P_{gb})$$

$$P_{out} = P_d - P_{mech}$$

$$T_{out} = \frac{P_{out}}{\omega}$$