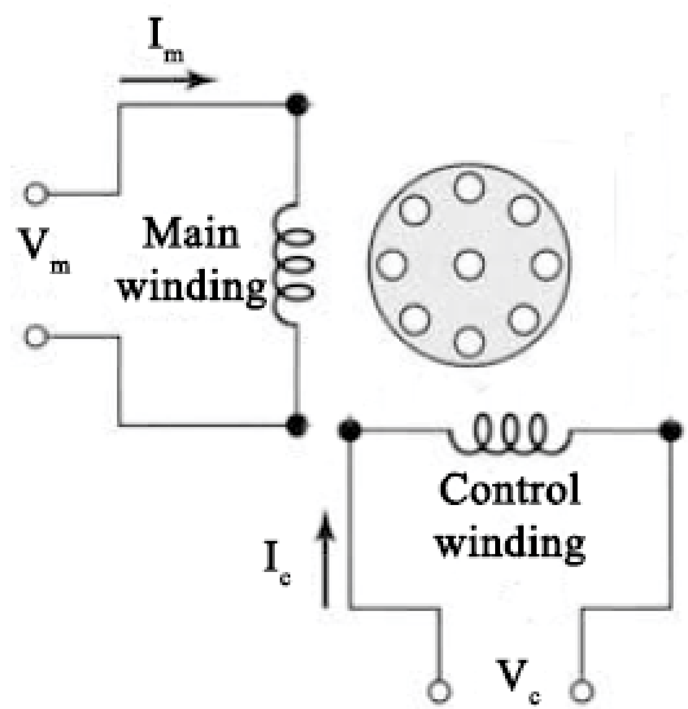
Induction Machines: 2-phase Induction Motors

- Principle of Operation
- Equivalent Circuit
- Design Formula and Torque speed characteristics
- Transfer Function

Induction Machines: 2-phase Induction Motors



Construction

Similar to 3-phase induction motor, have two identical stator windings (main & control) which are shifted in space 90 electrical degrees. The rotor is a squirrel cage rotor.

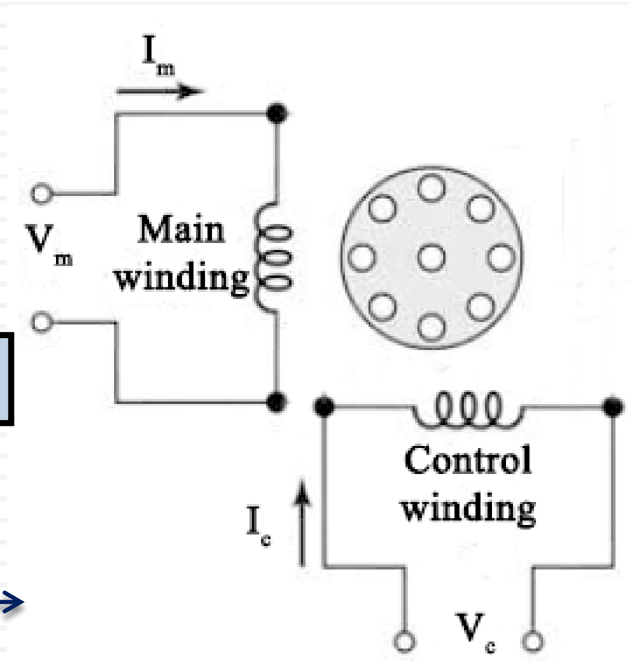
The aim of this motor is to obtain speed and position proportional to the voltage applied on the control winding

Induction Machines: 2-phase Induction Motors

Balanced Operation

$$V_m = V\angle 0$$

$$I_m = I\angle -\phi$$

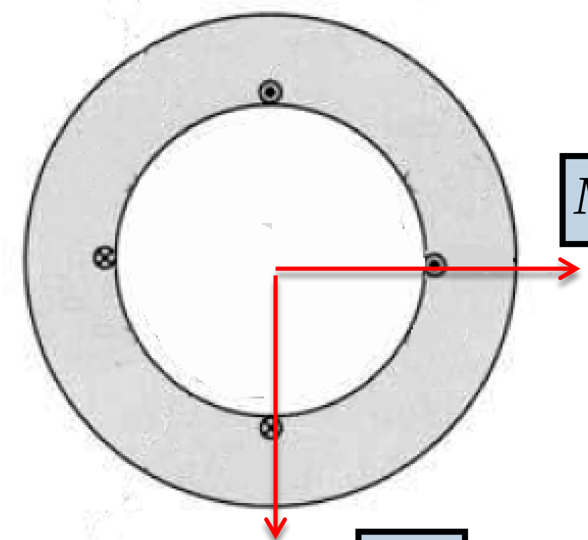


V_m

V_c

$$V_c = V\angle -90^o$$

$$I_c = I\angle -\phi - 90^o$$



M_M

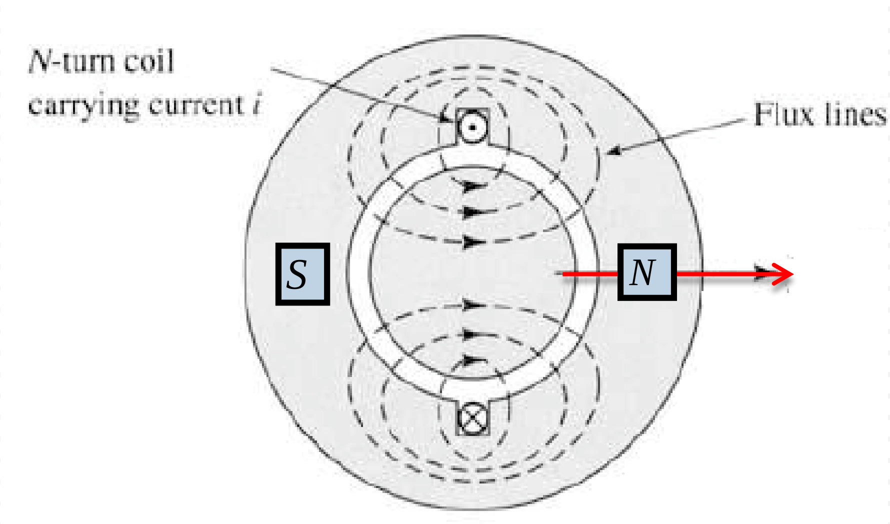
M_C

Induction Machines: 2-phase Induction Motors

Magnetic Field Production & Distribution

$$\oint_c \underline{H}.\underline{dl} = I_{en}$$

$$\oint_c \underline{H}.\underline{dl} = Ni$$



$$H \times g + H \times g \simeq Ni$$

$$H_g\, g = \frac{Ni}{2} = M$$

$$M = \frac{Ni}{2p}$$

Induction Machines: 2-phase Induction Motors

Magnetic Field Production & Distribution

$$i_m = I_m \cos(\omega_e t - \phi)$$

$$i_c = I_m \cos(\omega_e t - \phi - 90^o) \qquad i_c = I_m \sin(\omega_e t - \phi)$$

$$M = \frac{4}{\pi}\frac{N}{2p} I \cos\theta$$

Mech. degrees

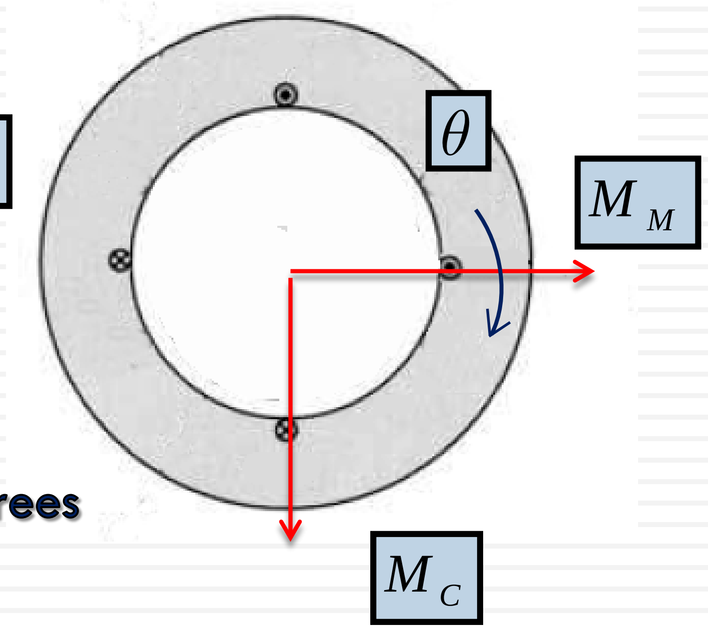
$$M_m = \frac{4}{\pi}\frac{N}{2p} I_m \cos(\omega_e t - \phi)\cos p\theta$$

Electrical degrees

$$M_c = \frac{4}{\pi}\frac{N}{2p} I_m \sin(\omega_e t - \phi)\sin p\theta$$

$$M_t = \frac{4}{\pi}\frac{N}{2p} I_m \cos(\omega_e t - p\theta - \phi)$$

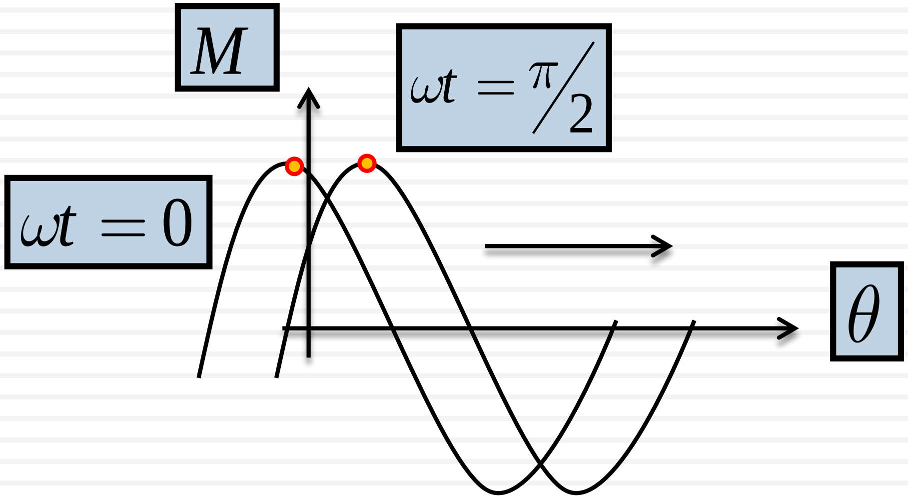
Rotating Field



Induction Machines: 2-phase Induction Motors

Rotating Field and its speed

$$M_t = \frac{4}{\pi}\frac{N}{2p} I_m \cos(\omega_e t - p\theta - \phi)$$



For a particular point:

$$\omega_e t - p\theta = const.$$

$$\omega_e - p\frac{d\theta}{dt} = zero$$

p = number of per poles

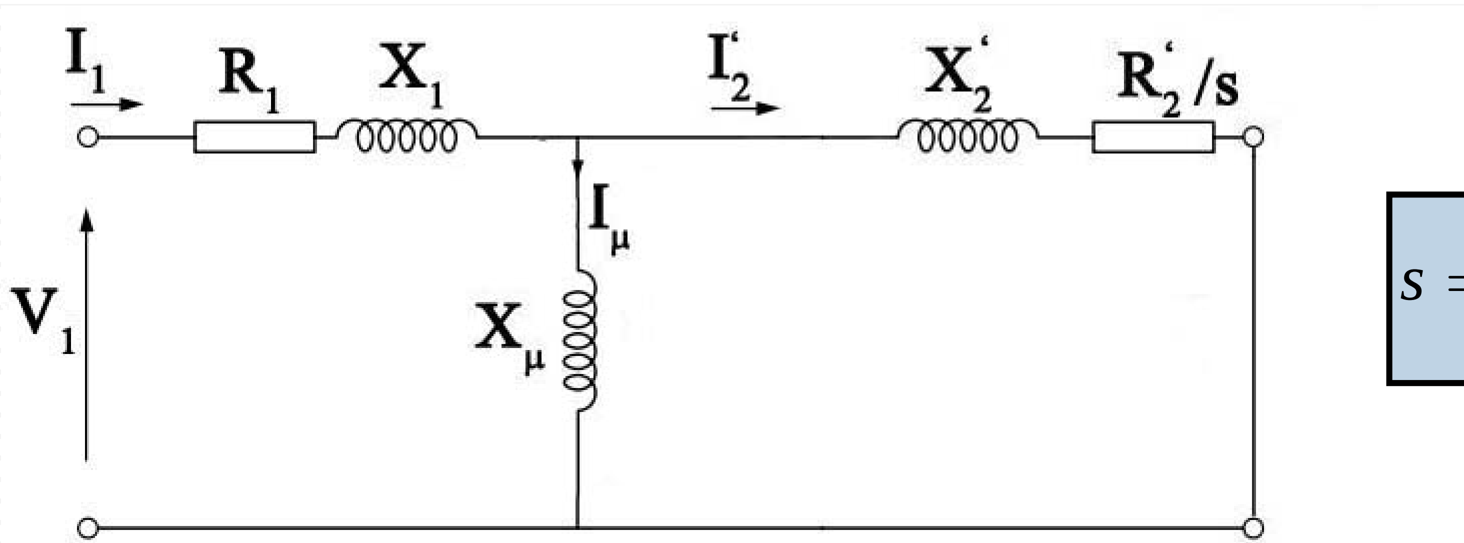
$$\frac{d\theta}{dt} = \frac{\omega_e}{p}$$

$$Speed = \frac{\omega_e}{p} = \frac{2\pi f}{p}$$ **Rad/sec**

$$n_s = \frac{60f}{p}$$ **rpm**

Induction Machines: 2-phase Induction Motors

Equivalent Circuit



$$s = \frac{n_s - n}{n_s}$$

$$P_{in} = 2VI\cos\phi$$

$$P_g = 2I_2'^2 \frac{R_2'}{s} = P_{in} - P_{cu1}$$

$$P_d = 2I_2'^2 \frac{(1-s)}{s} R_2' = P_g - P_{cu2}$$

$$P_{cu1} = 2I_1^2 R_1$$

$$P_{cu2} = 2I_2'^2 R_2'$$

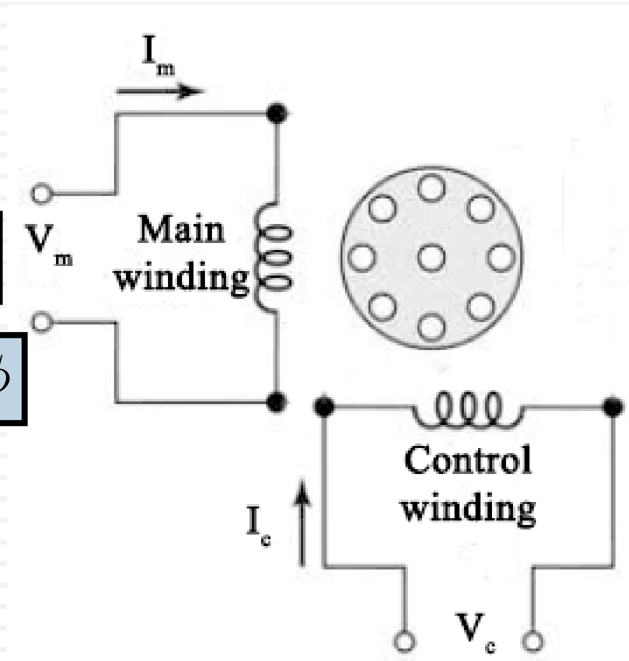
$$P_g : P_{cu2} : P_d$$
$$1 : s : (1-s)$$

Induction Machines: 2-phase Induction Motors

Unbalanced Operation

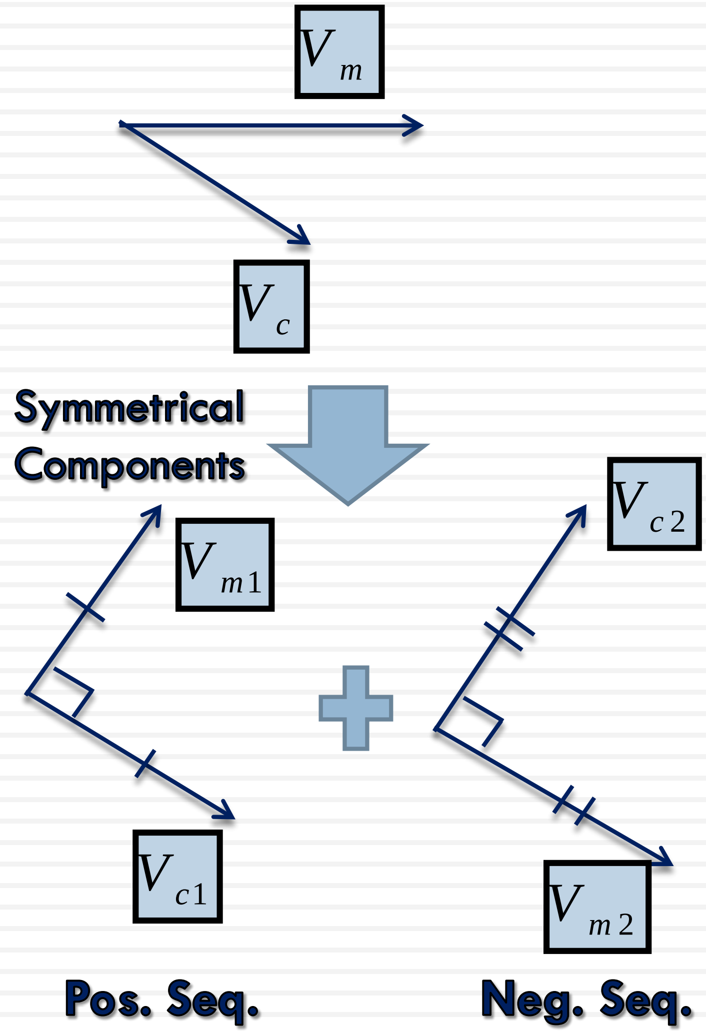
$$V_m = V_m \angle 0$$

$$I_m = I_m \angle -\phi$$



$$V_c = V_c \angle -\theta_c$$

$$I_c = I_c \angle -\phi - \theta_c$$



Induction Machines: 2-phase Induction Motors

Unbalanced Operation

$$V_m = V_{m1} + V_{m2}$$

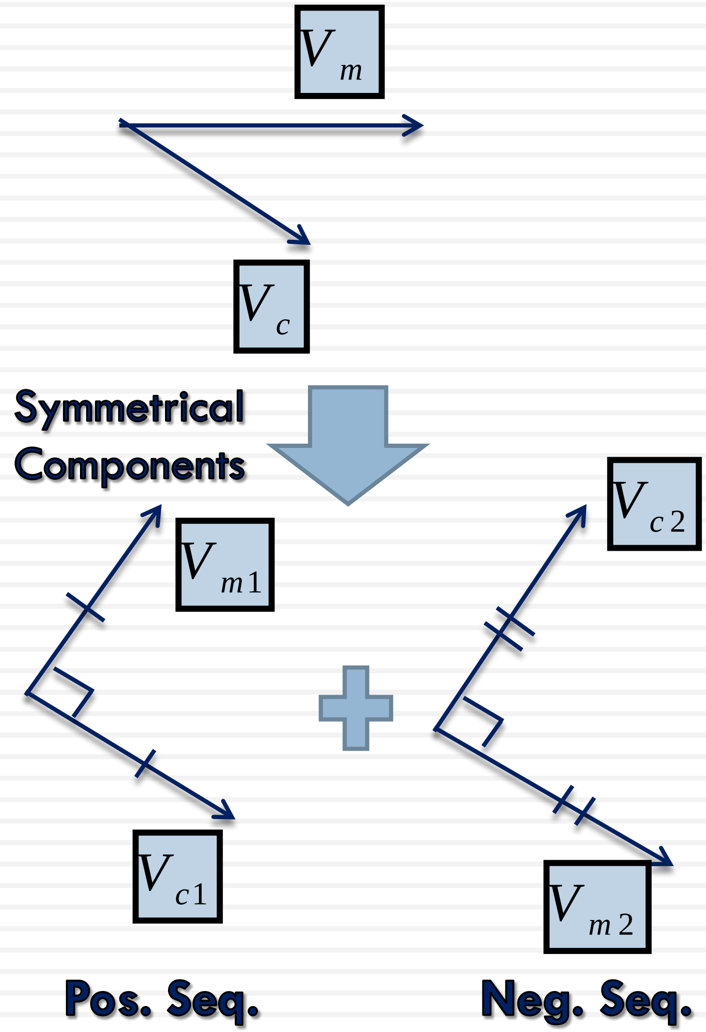
$$V_c = V_{c1} + V_{c2}$$

$$V_{c1} = -jV_{m1}$$

$$V_{c2} = jV_{m2}$$

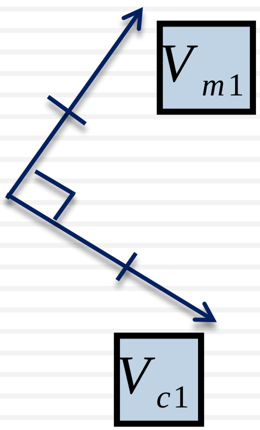
$$\begin{bmatrix} V_m \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \begin{bmatrix} V_{m1} \\ V_{m2} \end{bmatrix}$$

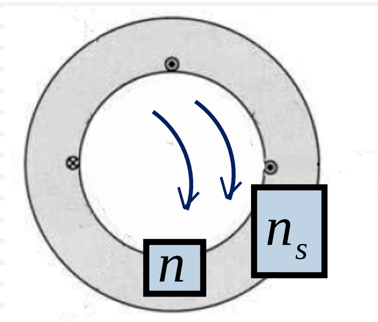
$$\begin{bmatrix} V_{m1} \\ V_{m2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} V_m \\ V_c \end{bmatrix}$$



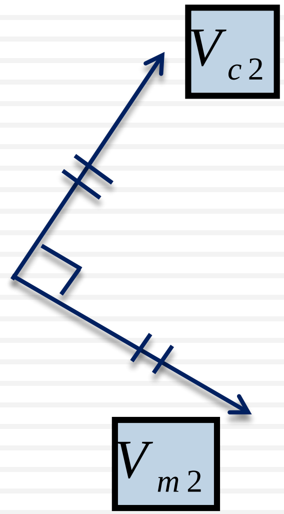
Induction Machines: 2-phase Induction Motors

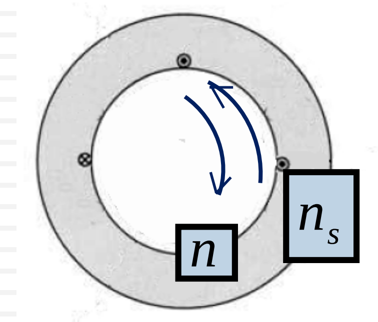
Unbalanced Operation



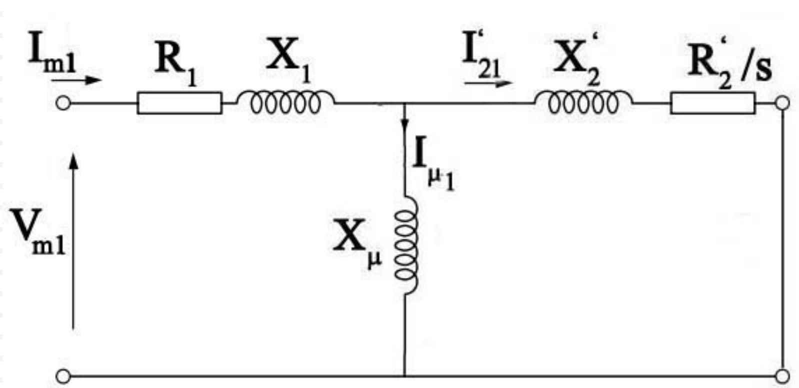


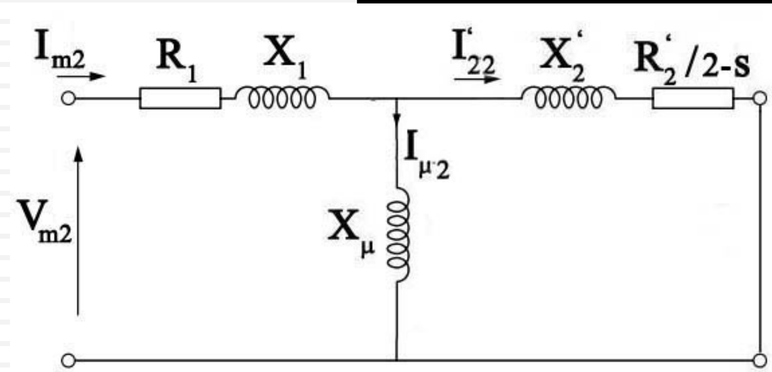
$$s_1 = \frac{n_s - n}{n_s} = s$$





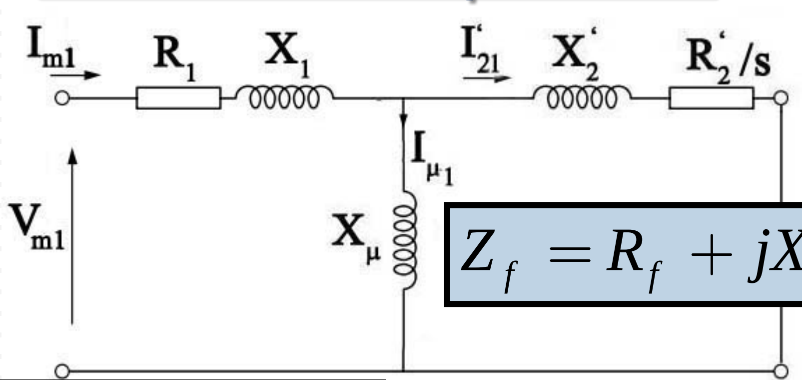
$$s_2 = \frac{n_s + n}{n_s} = 2 - s$$



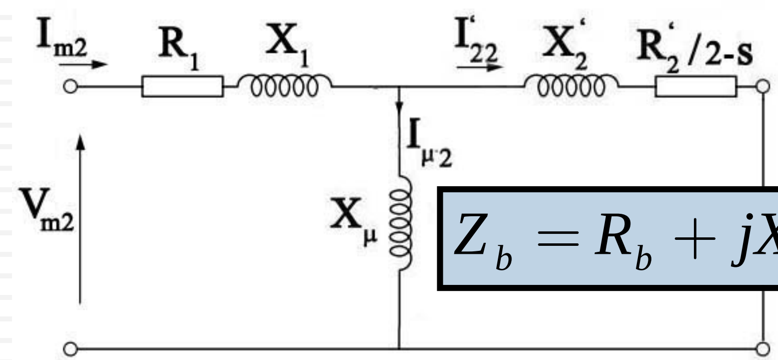


Induction Machines: 2-phase Induction Motors

Unbalanced Operation



$$Z_f = R_f + jX_f$$



$$Z_b = R_b + jX_b$$

$$I_{m1} = \frac{V_{m1}}{R_1 + jX_1 + Z_f}$$

$$Z_f = \frac{jX_\mu (R_2' / s + jX_2')}{R_2' / s + jX_2' + jX_\mu}$$

$$I_{m2} = \frac{V_{m2}}{R_1 + jX_1 + Z_b}$$

$$Z_b = \frac{jX_\mu (R_2' / (2-s) + jX_2')}{R_2' / (2-s) + jX_2' + jX_\mu}$$

$$P_{g1} = 2I_{m1}^2 R_f$$

$$P_{g2} = 2I_{m2}^2 R_b$$

$$\begin{array}{lll} P_{g1} : & P_{cu21} : & P_{d1} \\ 1 \ : & s & : (1-s) \end{array}$$

$$T_1 = \frac{P_{g1}}{\omega_s}$$

$$\begin{array}{lll} P_{g2} : & P_{cu22} : & P_{d2} \\ 1 \ : & 2-s & : (1-(2-s)) \end{array}$$

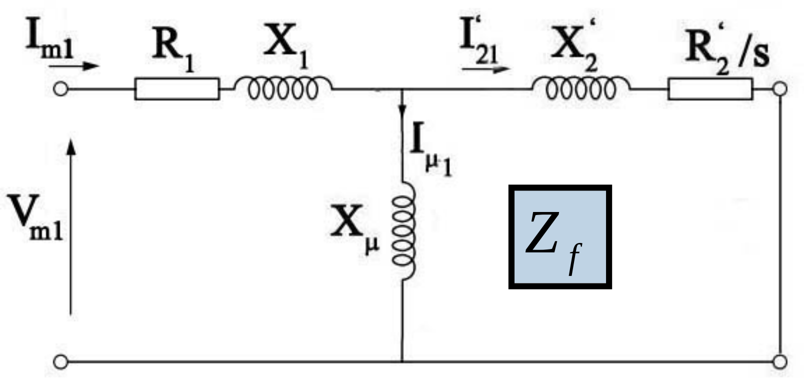
$$T_2 = \frac{P_{g2}}{\omega_s}$$

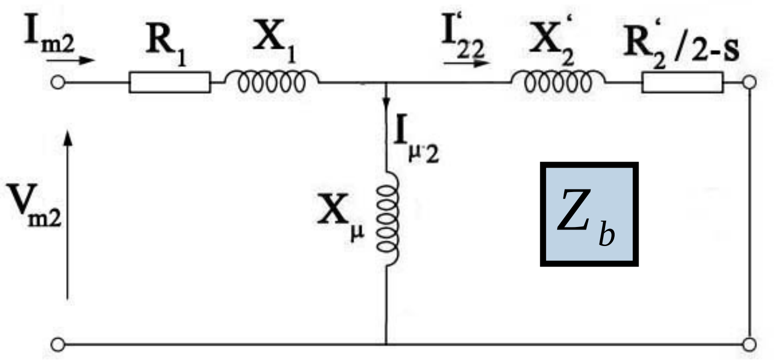
$$P_d = P_{d1} + P_{d2} = (1-s)(P_{g1} - P_{g2})$$

$$T_{net} = \frac{P_g}{\omega_s} = \frac{P_d}{\omega} = T_1 - T_2$$

Induction Machines: 2-phase Induction Motors

Unbalanced Operation





$$\begin{bmatrix} I_m \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \begin{bmatrix} I_{m1} \\ I_{m2} \end{bmatrix}$$

$$P_{cu1} = P_{cu11} + P_{cu12}$$

$$P_{cu11} = 2I_{m1}^2 R_1$$

$$P_{cu12} = 2I_{m2}^2 R_1$$

$$P_{cu2} = P_{cu21} + P_{cu22}$$

$$P_{in} = P_m + P_c$$

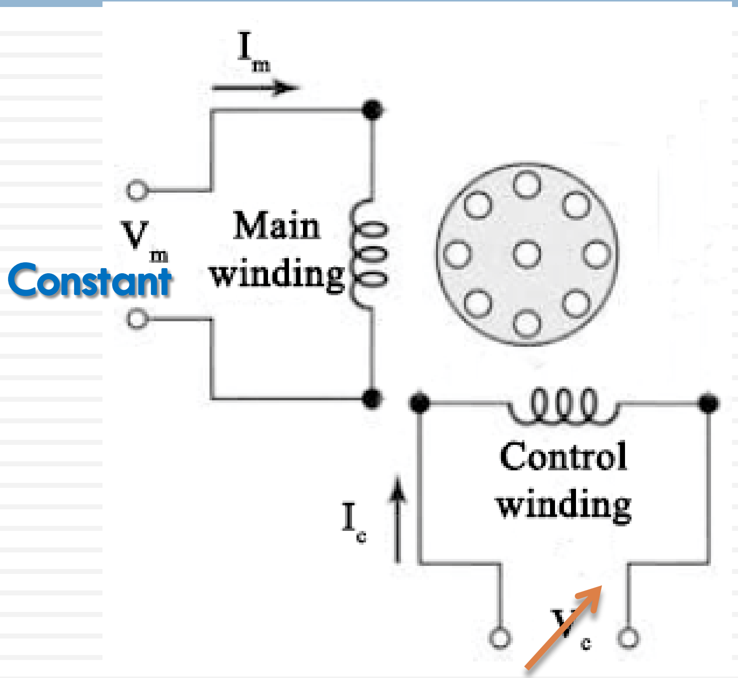
$$P_m = V_m I_m \cos(\theta_{V_m} - \theta_{I_m})$$

$$P_c = V_c I_c \cos(\theta_{V_c} - \theta_{I_c})$$

Induction Machines: 2-phase Induction Motors

Application: AC servo motor

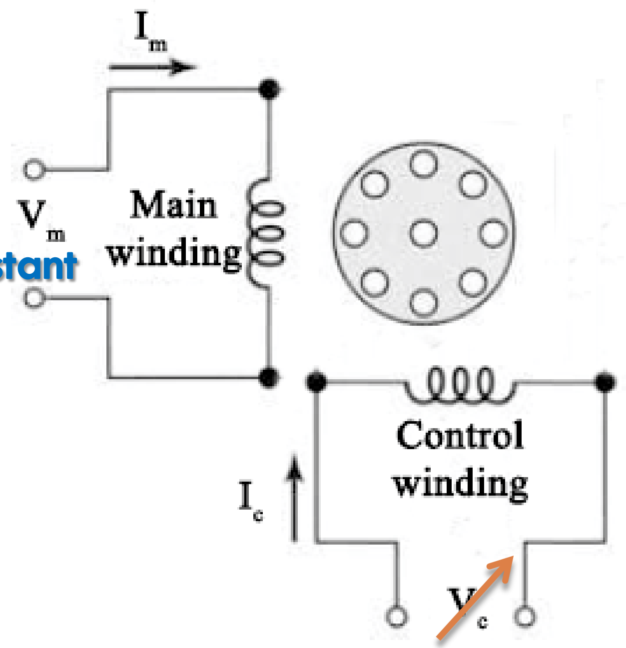
A *servo motor* is typically part of a feedback loop containing electronic, mechanical, and electrical components. The servo loop is a means of controlling the motion of an object via the motor.



A requirement of many such systems is fast response. One phase is connected to the single phase line; the other winding is driven by a variable amplitude sine wave to control motor speed.

Induction Machines: 2-phase Induction Motors

AC servo motor



$$T_{net} = T_1 - T_2$$

$$T_1 = \frac{P_{g1}}{\omega_s}$$

$$T_2 = \frac{P_{g2}}{\omega_s}$$

$$I_{m1} = \frac{V_{m1}}{Z_1}$$

$$P_{g1} = 2I_{m1}^2 R_f$$

$$I_{m2} = \frac{V_{m2}}{Z_2}$$

$$P_{g2} = 2I_{m2}^2 R_b$$

For Balanced Operation:

$$V_{m1} = V_{m2} = V_m = V_{rated}$$

$$I_{m1B} = \frac{V_m}{Z_1}$$

$$I_{m2B} = \frac{V_m}{Z_2}$$

$$T_{1B} = \frac{P_{g1B}}{\omega_s}$$

$$T_{2B} = \frac{P_{g2B}}{\omega_s}$$

$$\frac{T_1}{T_{1B}} = \left(\frac{V_{m1}}{V_m}\right)^2$$

$$\frac{T_2}{T_{2B}} = \left(\frac{V_{m2}}{V_m}\right)^2$$

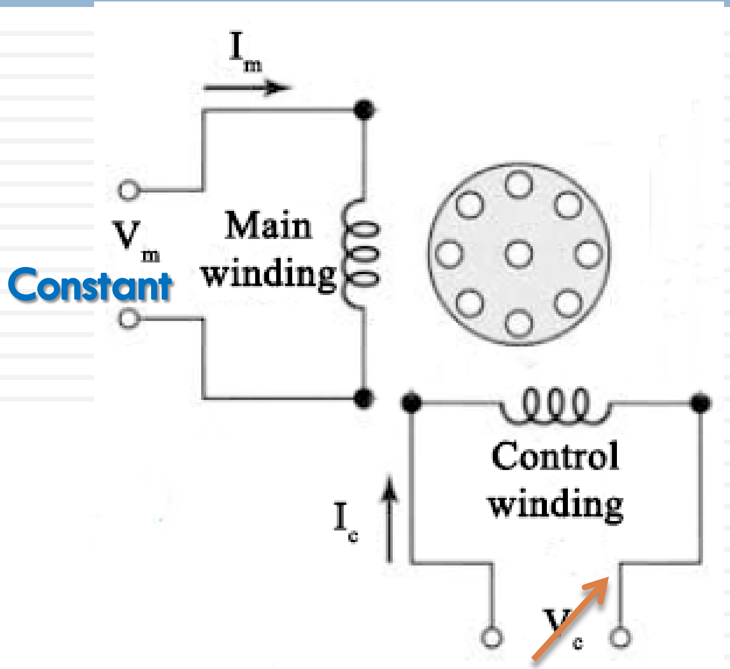
Design Formula

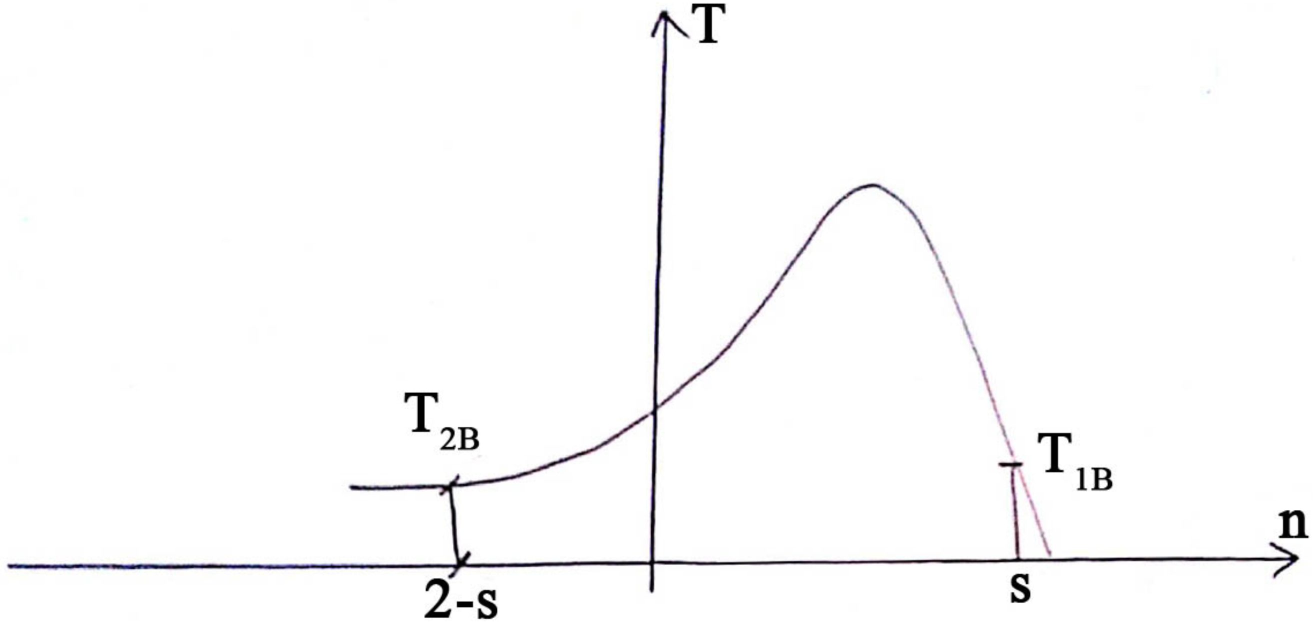
$$T_{net} = T_{1B}\left(\frac{V_{m1}}{V_m}\right)^2 - T_{2B}\left(\frac{V_{m2}}{V_m}\right)^2$$

Induction Machines: 2-phase Induction Motors

AC servo motor

$$T_{net} = T_{1B}\left(\frac{V_{m1}}{V_m}\right)^2 - T_{2B}\left(\frac{V_{m2}}{V_m}\right)^2$$

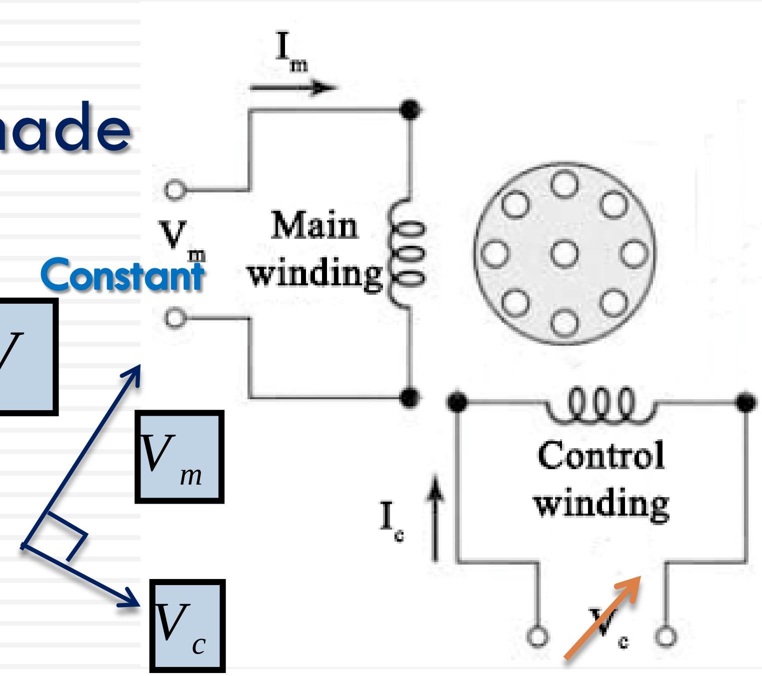




Induction Machines: 2-phase Induction Motors

AC servo motor

Typically, the two voltages are made 90° shifted.



$$V_m = V\angle 0$$

$$V_c = \rho V\angle -90^o = -j\rho V$$

$$\begin{bmatrix} V_{m1} \\ V_{m2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix}\begin{bmatrix} V \\ -j\rho V \end{bmatrix}$$

$$V_{m1} = \left(\frac{1+\rho}{2}\right)V$$

$$V_{m2} = \left(\frac{1-\rho}{2}\right)V$$

$$T_{net} = T_{1B}\left(\frac{1+\rho}{2}\right)^2 - T_{2B}\left(\frac{1-\rho}{2}\right)^2$$

Induction Machines: 2-phase Induction Motors

AC servo motor: Single Phasing Problem

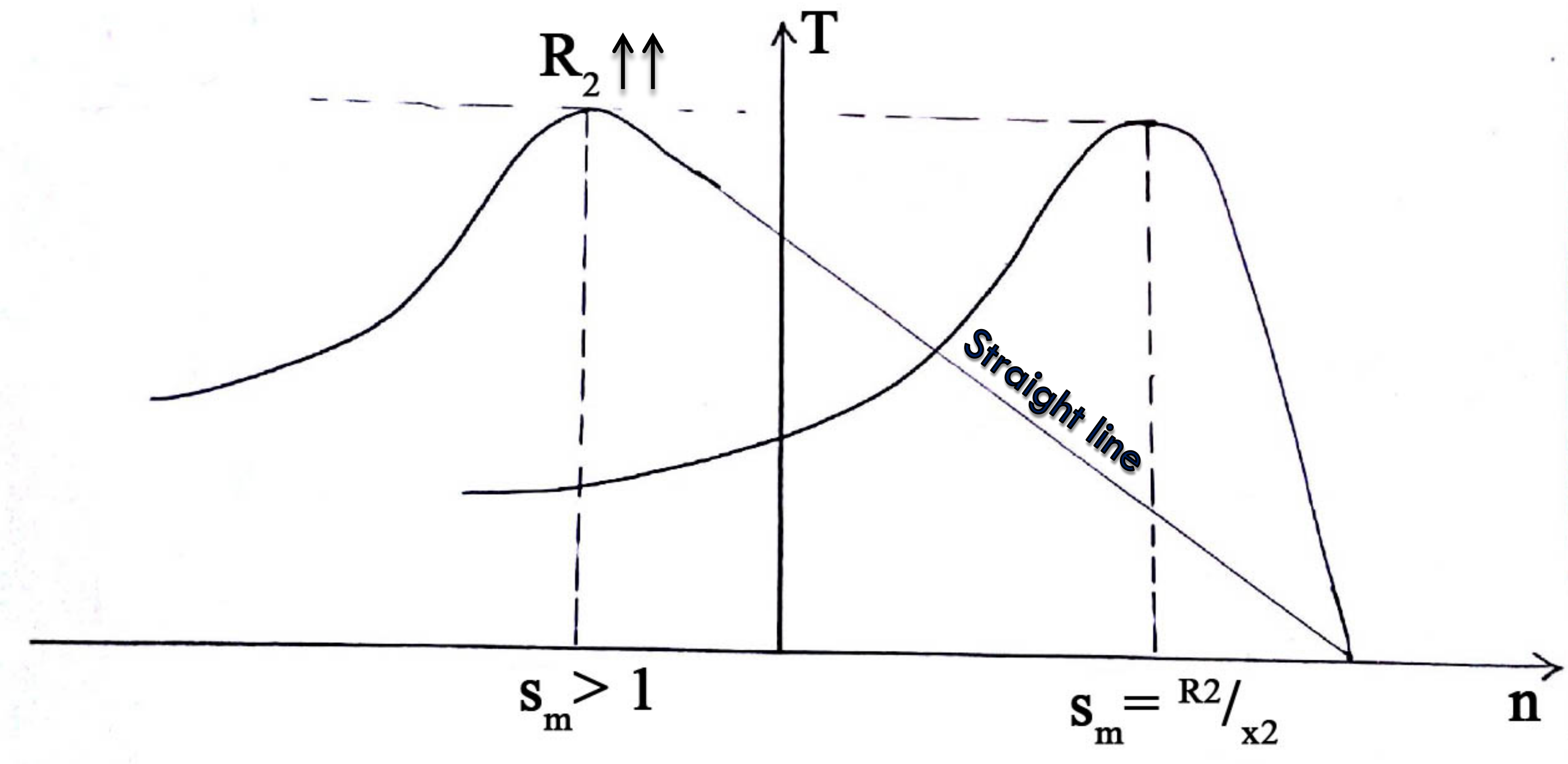
$$T_{net} = T_{1B}\left(\frac{1+\rho}{2}\right)^2 - T_{2B}\left(\frac{1-\rho}{2}\right)^2$$

Supposedly, when V_c = zero (ρ = zero) the motor should stop (i.e. T_{net} = zero or negative). But this will only happen if $T_{2B} \geq T_{1B}$.

To avoid this problem, the design is adjusted to achieve $R_2' \geq X_2' + X_m$ ⇨ $s_m \geq 1$

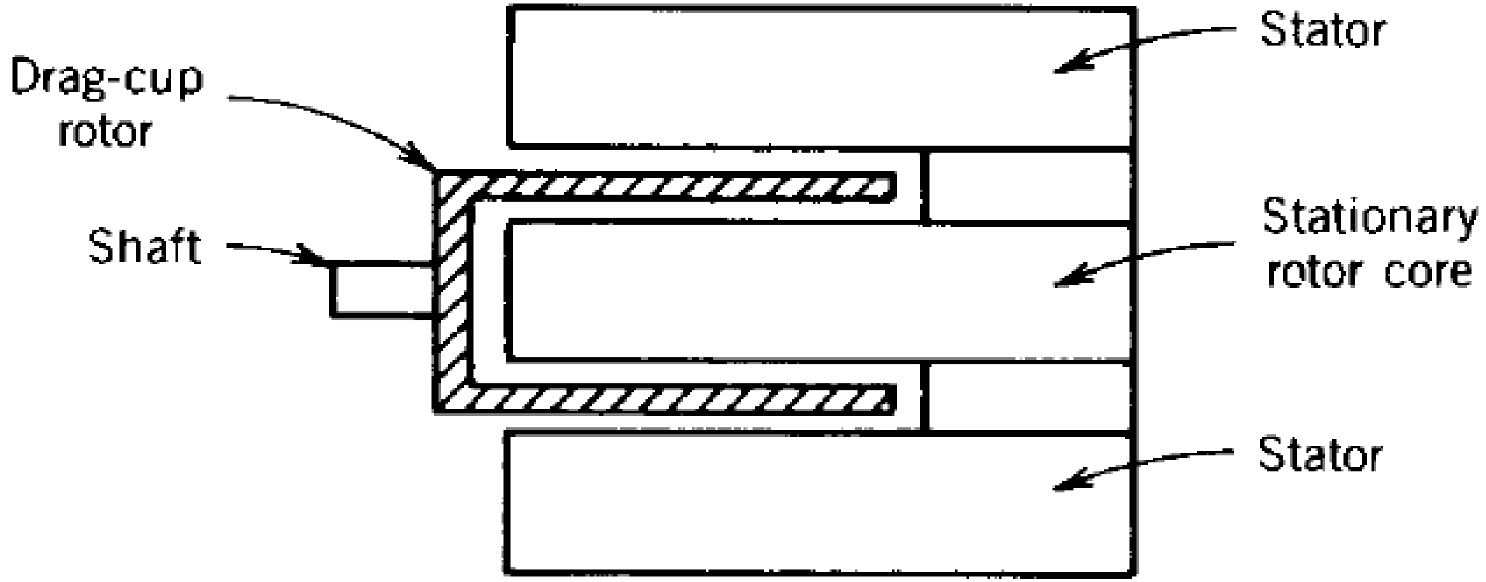
Induction Machines: 2-phase Induction Motors

AC servo motor: Single Phasing Problem



Induction Machines: 2-phase Induction Motors

AC servo motor: Drag-cup Rotor Design



AC servo motor: Torque-speed characteristics

At starting (s=1)

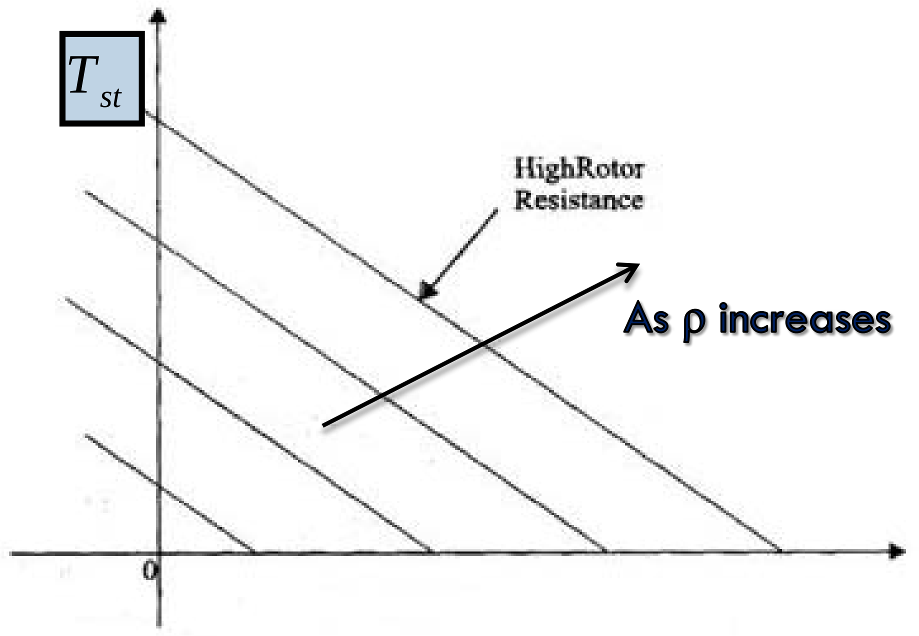
$$T_{1B} = T_{2B} = T_{stB}$$

$$T_{st} = T_{stB}\left(\frac{1+\rho}{2}\right)^2 - T_{stB}\left(\frac{1-\rho}{2}\right)^2 = \rho T_{stB}$$

$$T_m = T_{st} - B_m \omega$$

$$T_m = \frac{V_c}{V_m} T_{stB} - B_m \omega$$

$$T_m = k_m V_c - B_m \omega$$



Motor Torque is function of the control voltage

AC servo motor: Transfer Function

$$T_m - T_L = J\frac{d\omega}{dt}$$

$$J = J_L + J_m$$

$$T_m = k_m V_c - B_m \omega$$

$$T_L = B_L \omega$$

Viscous Friction Load

$$J\frac{d\omega}{dt} = k_m V_c - B_m \omega - B_L \omega$$

$$J\frac{d\omega}{dt} + B_T \omega = k_m V_c$$

La Place Transform

$$Js\omega(s) + B_T \omega(s) = k_m V_c(s)$$

$$\frac{\omega(s)}{V_c(s)} = \frac{k_m}{Js + B_T} = \frac{k_m / B_T}{J/B_T \, s + 1}$$

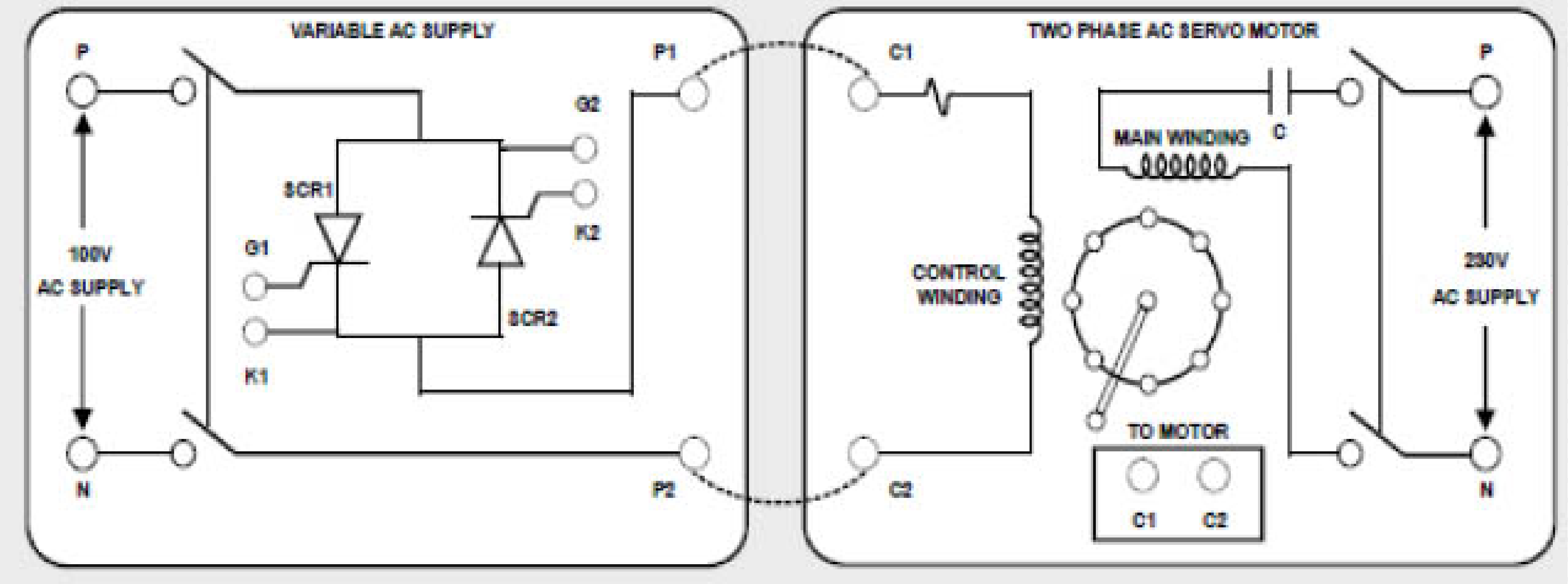
$$\frac{\omega(s)}{V_c(s)} = \frac{k_m}{Js + B_T} = \frac{k_m / B_T}{\tau_m s + 1}$$

Mech. Time const.

$$\tau_m = J / B_T$$

Induction Machines: 2-phase Induction Motors

AC servo motor



Induction Machines: 2-phase Induction Motors

AC servo motor