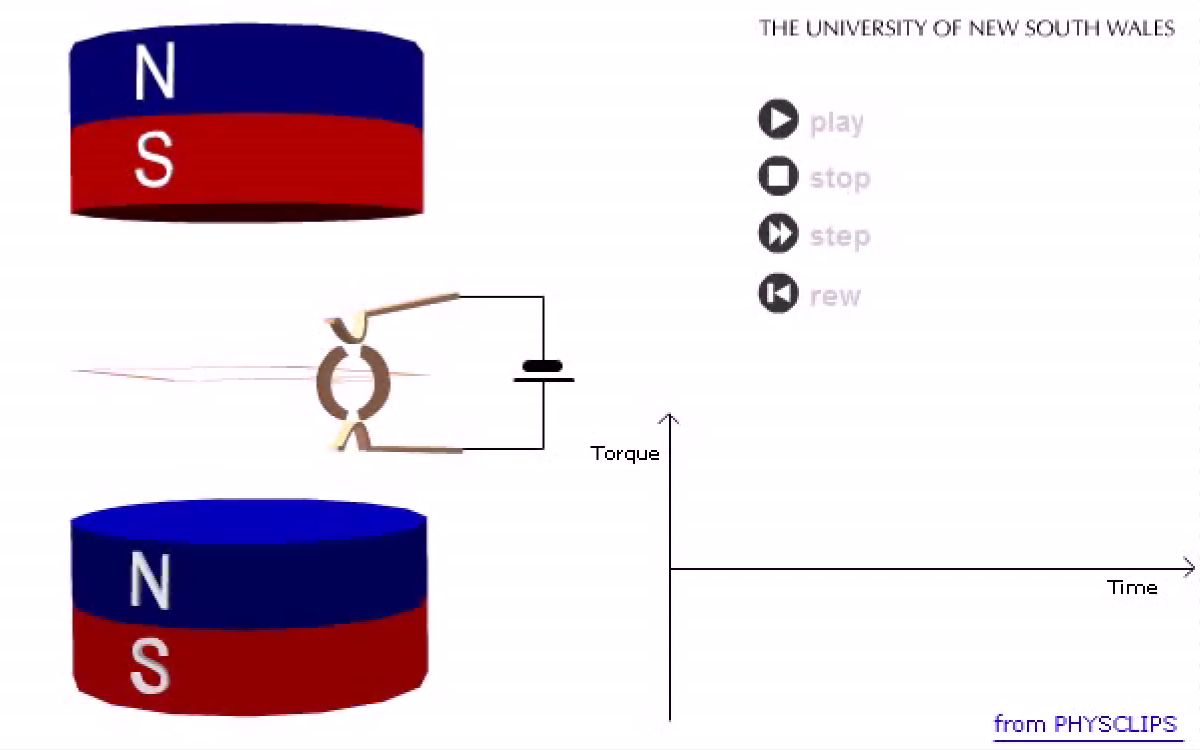
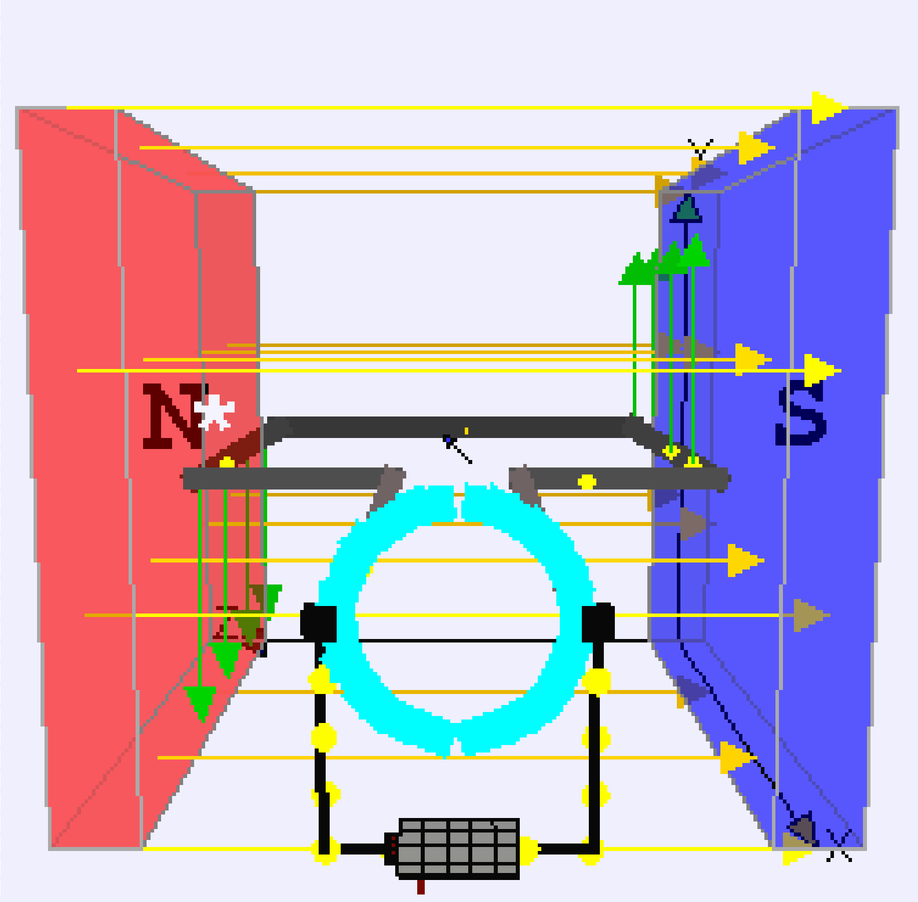
Faraday’s Law Applications

1. Inductors.
2. Transformers.
3. Electrical Generators.
4. Electrical Motors.
5. Transmission Lines.

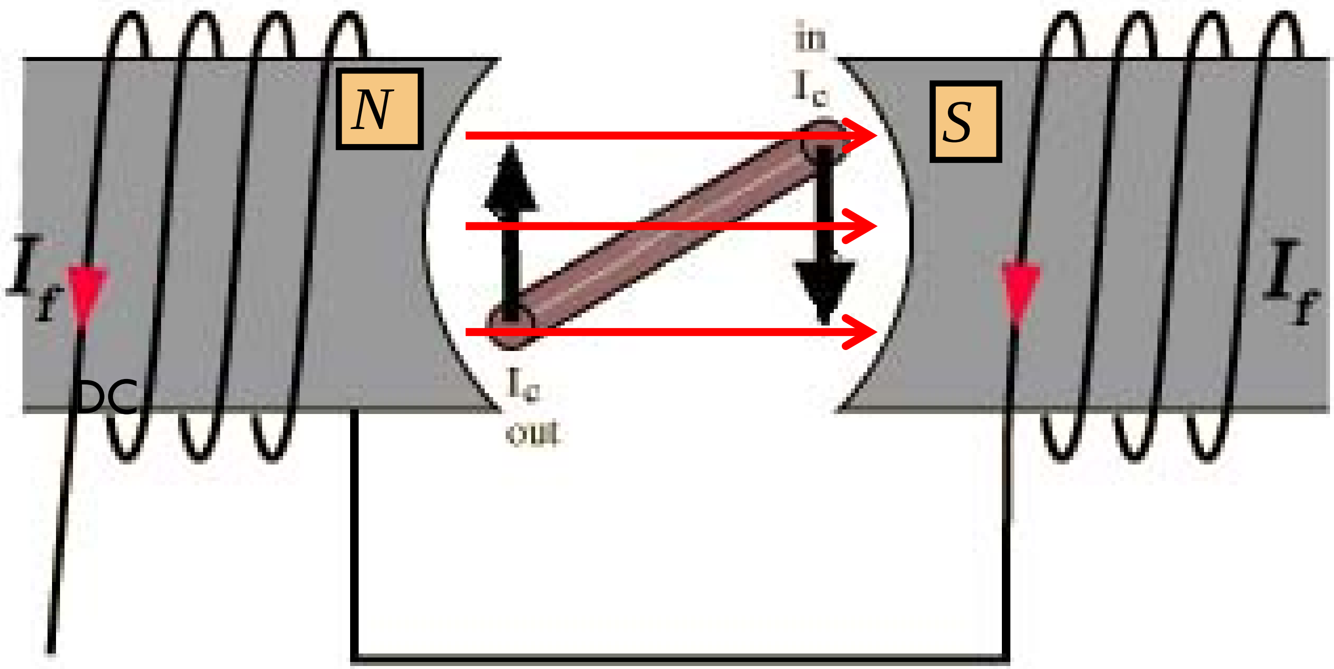
Motors: *DC Motor*



Motors: *DC Motor*

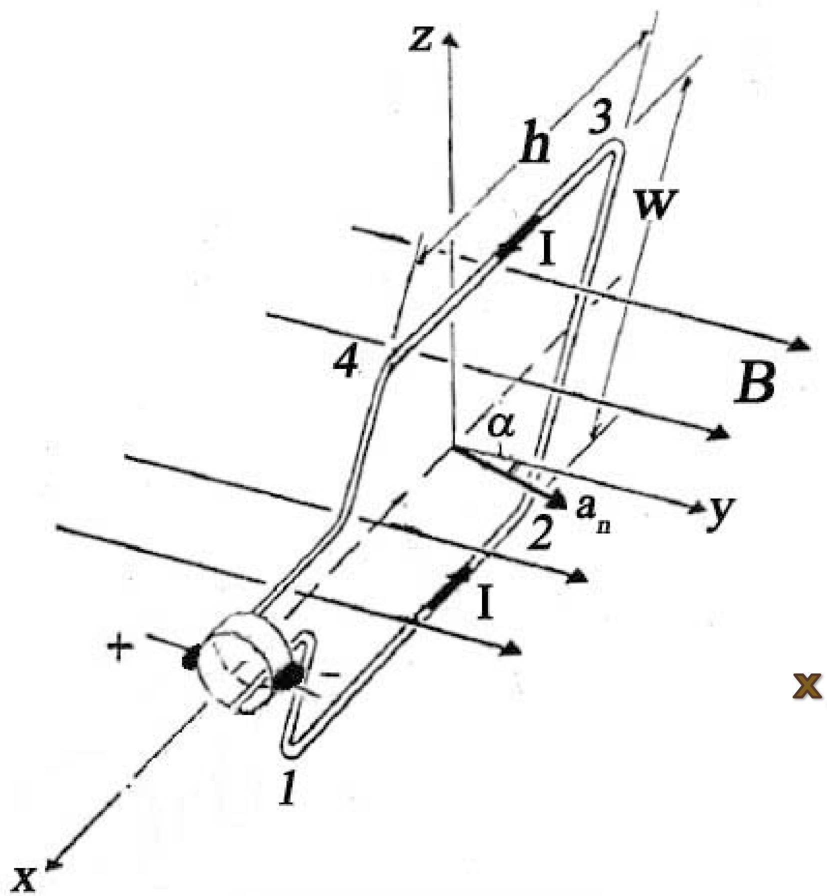


Motors: ***DC Motor***

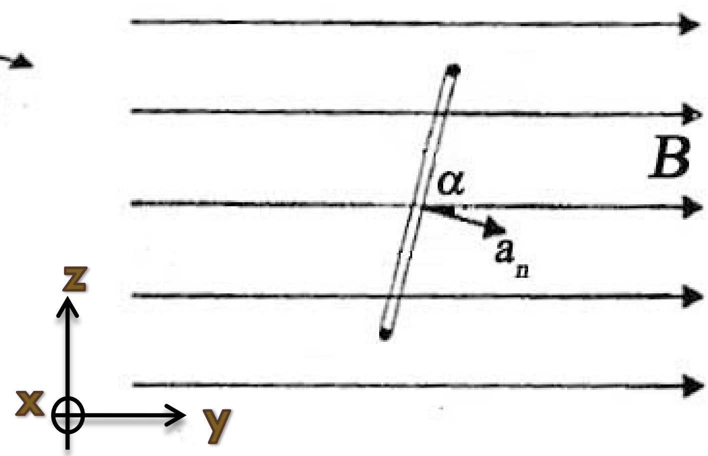


$$\boxed{B = B_o}$$

Motors: *DC Motor*



(a) Prespective View



(b) View from +x direction

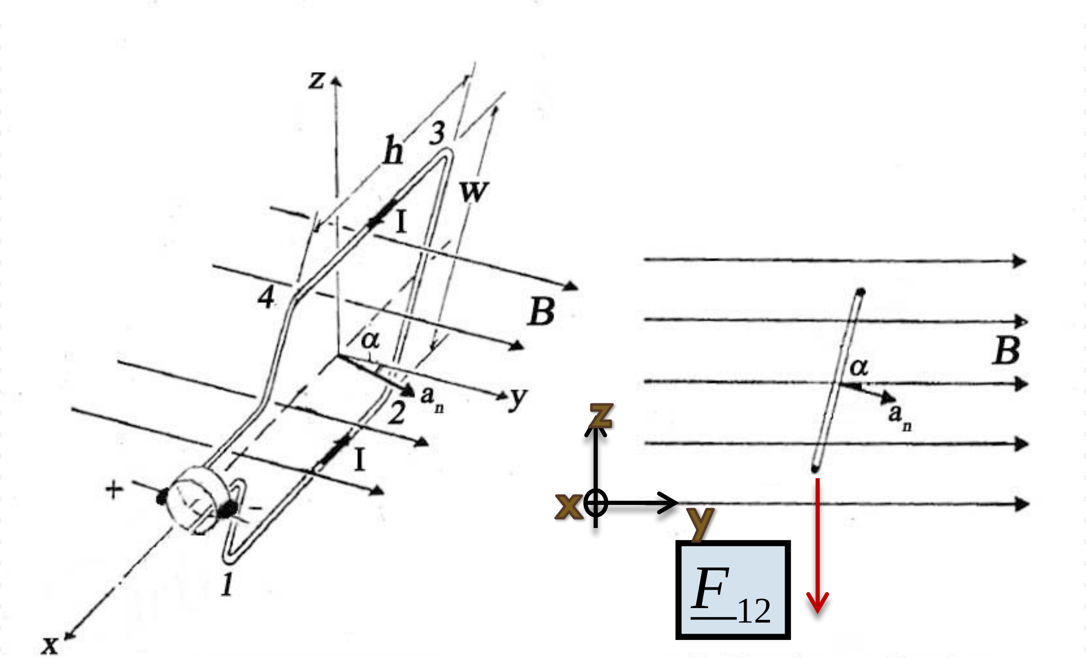
Motors: ***DC Motor***

$$\underline{F}_m = I\int \underline{dl} \times \underline{B}$$

$$\underline{F}_{12} = I\int_1^2 \underline{dl} \times \underline{B}$$

$$\underline{B} = B_o \underline{u}_y$$

$$\Rightarrow \quad \underline{F}_{12} = I\int_{1=0}^{2=h} dx\,(-\underline{u}_x) \times B_o \underline{u}_y \quad \Rightarrow \quad \underline{F}_{12} = IB_o h(-\underline{u}_z)$$

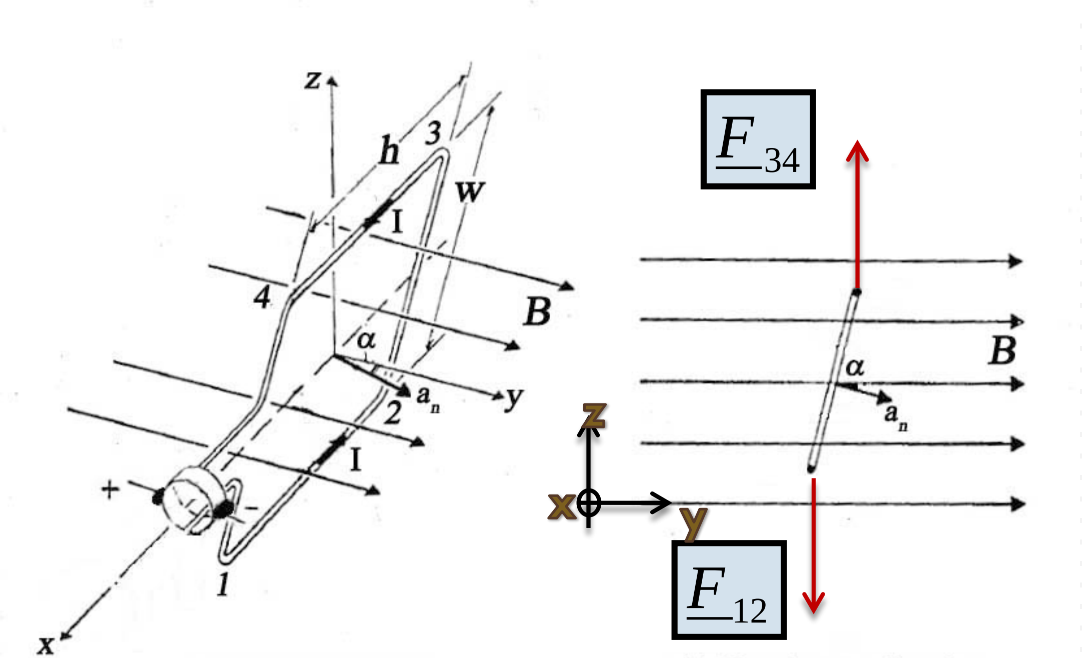


(a) Prespective View

(b) View from +x direction

Motors: ***DC Motor***

$$\underline{F}_{34} = I\int_{3}^{4} \underline{dl} \times \underline{B}$$



(a) Prespective View (b) View from +x direction

$$\underline{F}_{34} = I\int_{3=0}^{4=h} dx\,(\underline{u}_x)\times B_o\underline{u}_y$$

$$\underline{F}_{12} = IB_o h(\underline{u}_z)$$

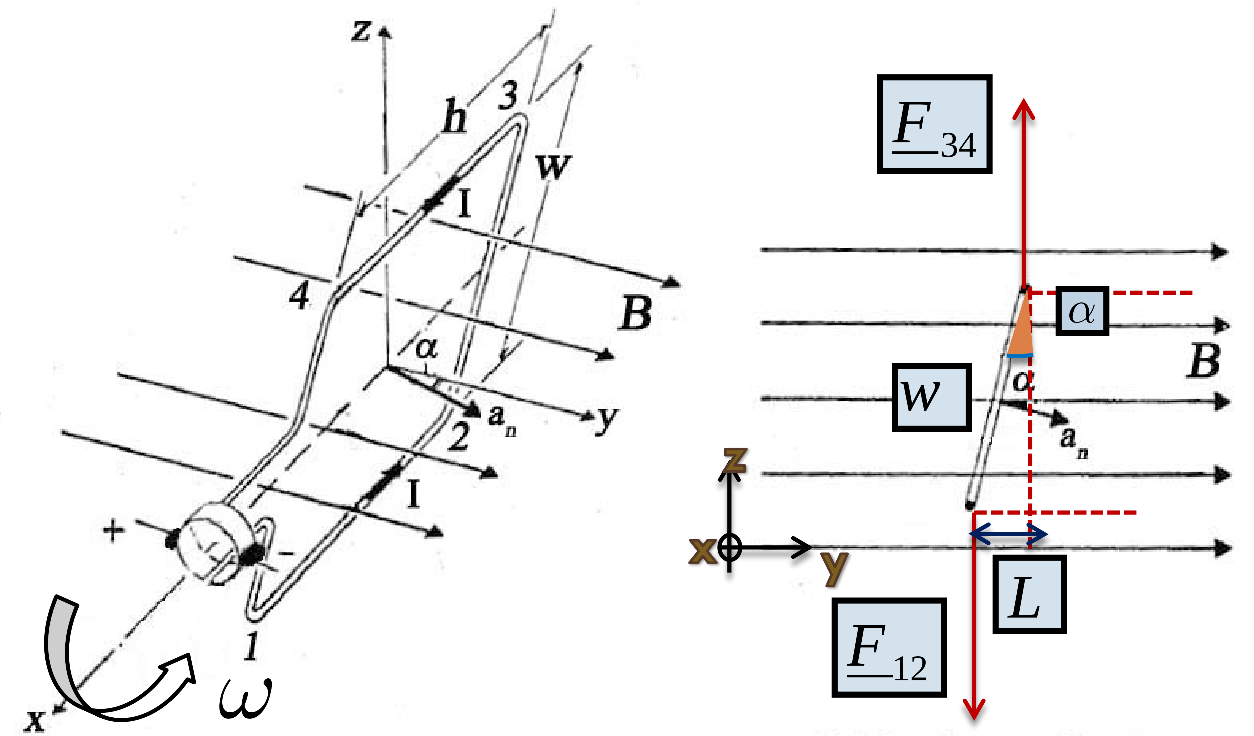
Motors: ***DC Motor***

$$T = \underline{r} \times \underline{F}$$

$$T = FL$$

$$L = w \sin\alpha$$

$$F = IB_o h$$

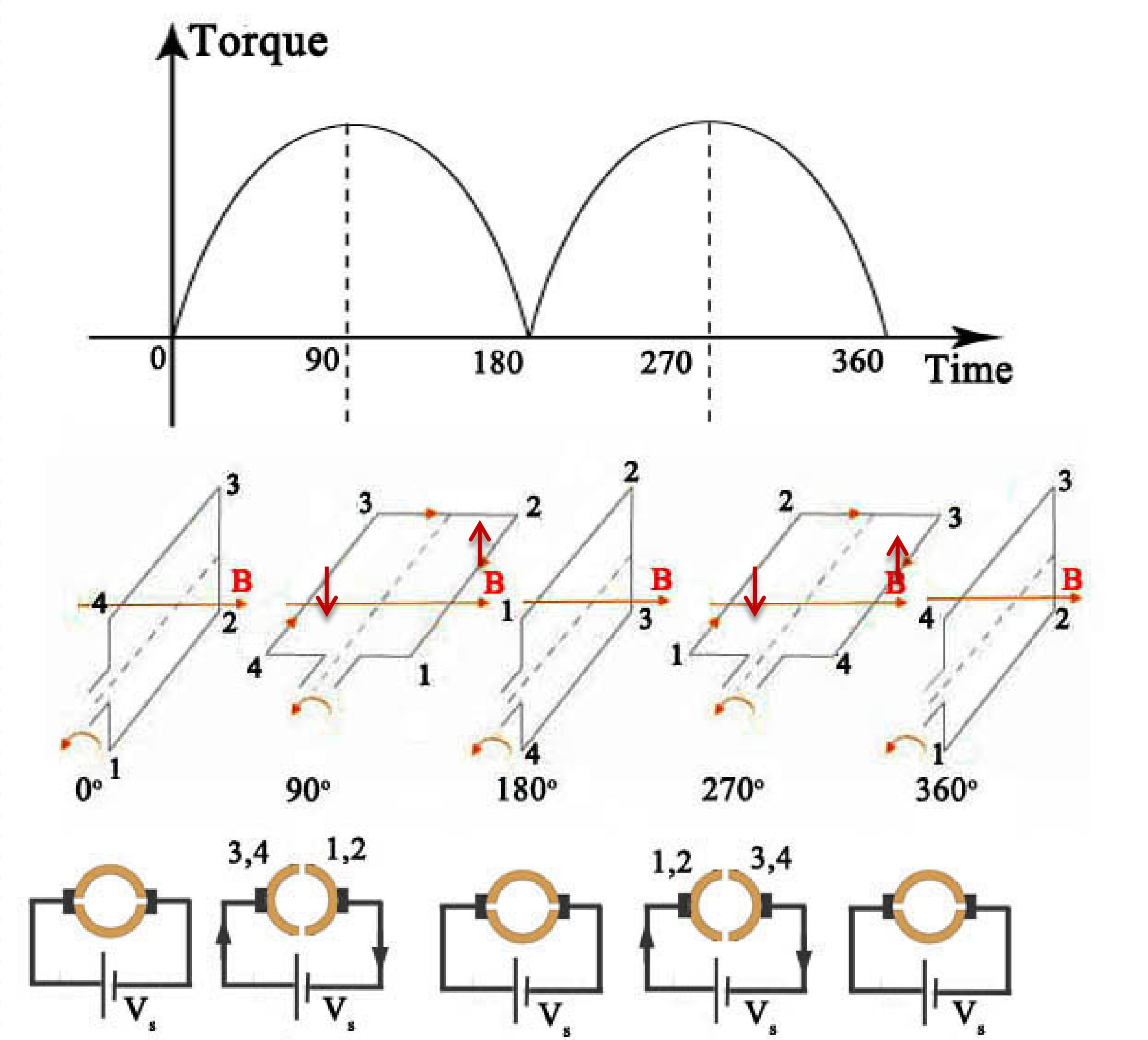


(a) Prespective View (b) View from +x direction

$$T = IB_o(hw)\sin\alpha$$

$$T = IB_o(hw)\sin(\omega t)$$

Motors: *DC Motor*



Motors: *DC Motor*

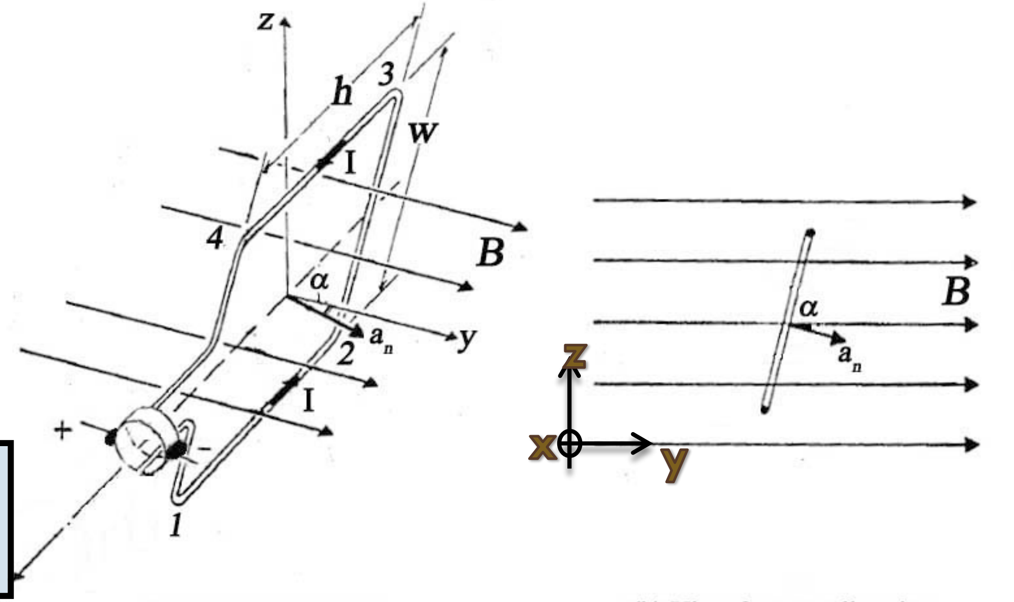
$$\underline{a}_n \times \underline{u}_y = (1)(1)\sin\alpha(\underline{u}_x)$$

$$\boxed{\underline{T} = \underline{m} \times \underline{B}}$$

$$\boxed{\underline{m} = I\,\underline{S}}$$

$$\boxed{\underline{m} = I\,(hw)\underline{a}_n}$$

$$\boxed{\underline{T} = I\,(hw)\underline{a}_n \times B_o\underline{u}_y}$$



(a) Prespective View

(b) View from +x direction

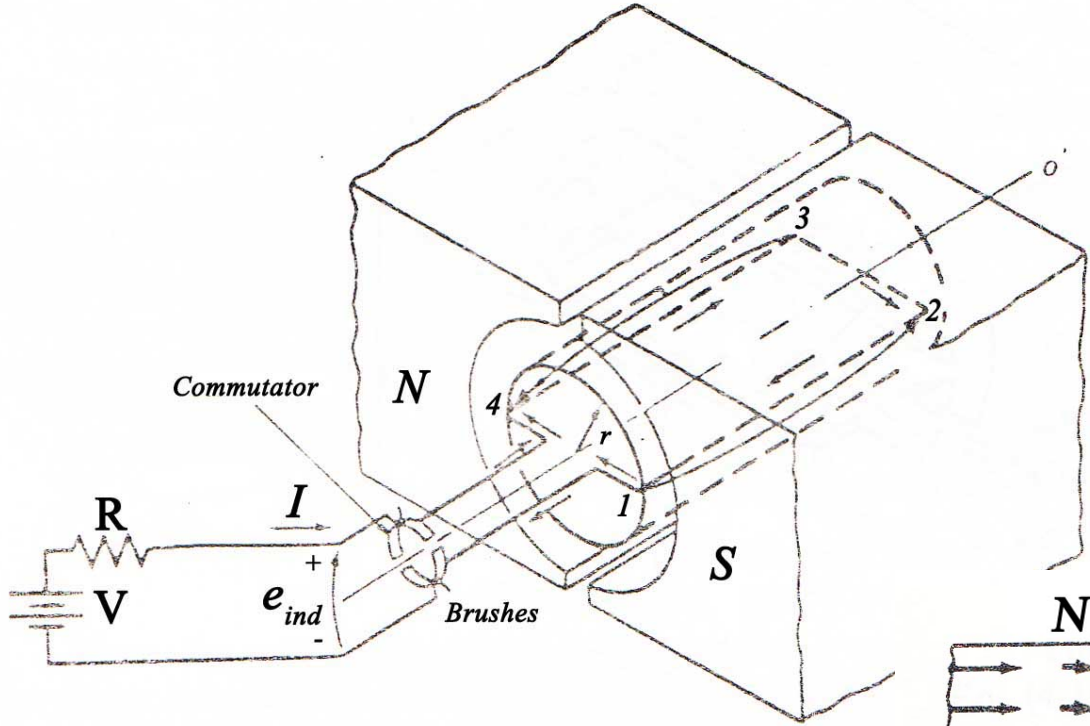
$$\boxed{\underline{T} = IB_oA\sin\alpha(\underline{u}_x)}$$

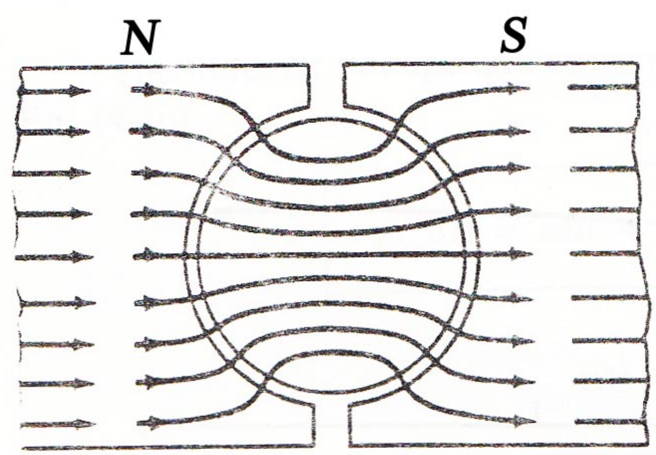
$$\boxed{A = hw}$$

$$\boxed{P_m = T\,\omega}$$

$$\boxed{P_m = \omega IB_oA\sin\alpha}$$

Motors: ***DC Motor***



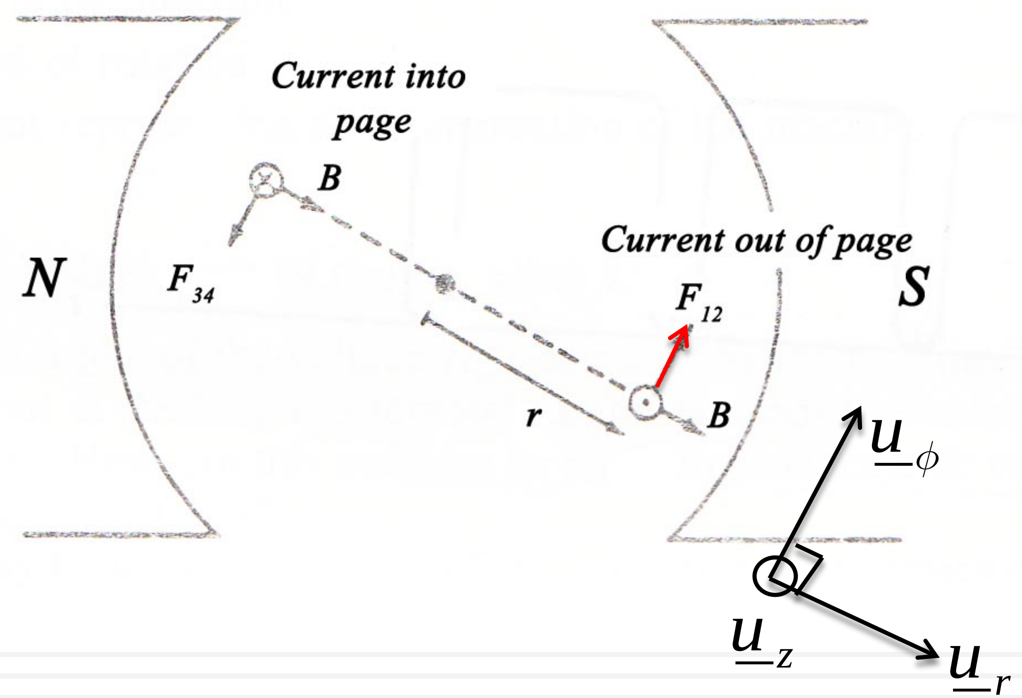


Motors: ***DC Motor***

$$\underline{F}_m = I \int \underline{dl} \times \underline{B}$$

$$\underline{F}_{12} = I \int_1^2 \underline{dl} \times \underline{B}$$

$$\underline{B} = B_o \underline{u}_r$$



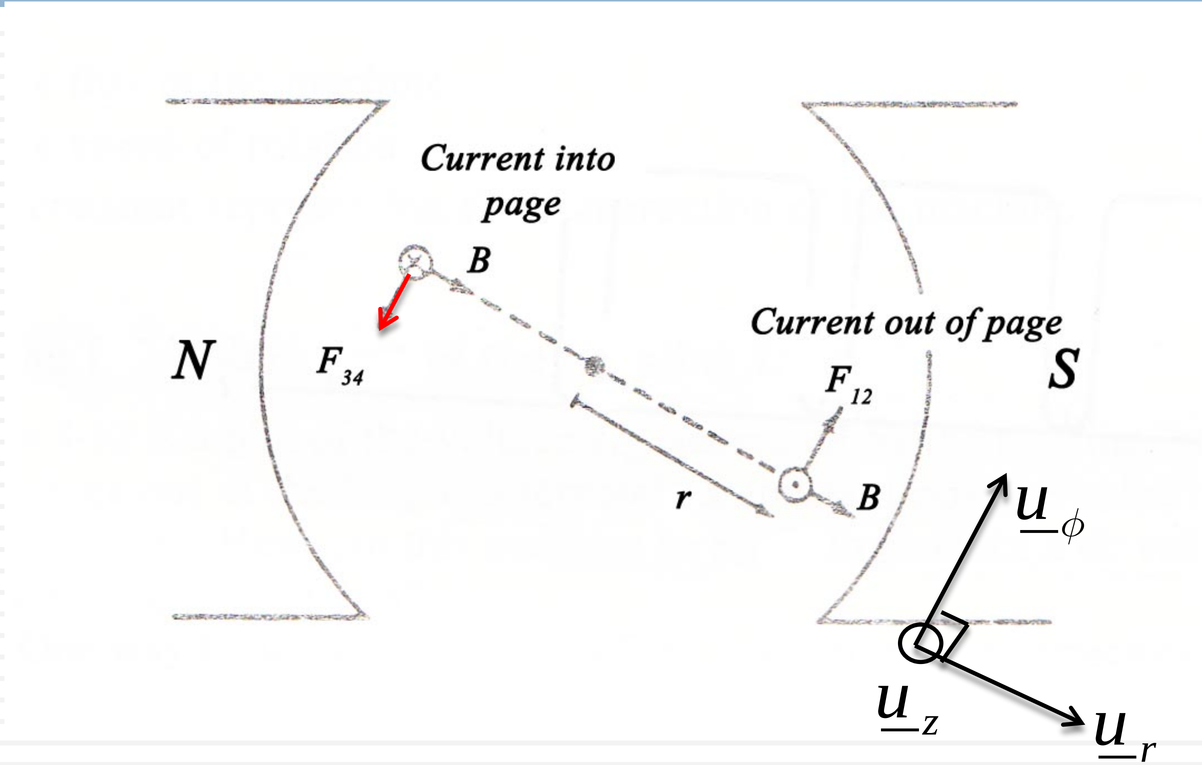
$$\underline{F}_{12} = I \int_{1=0}^{2=h} dz\,(\underline{u}_z) \times B_o \underline{u}_r$$

$$\underline{F}_{12} = I B_o h (\underline{u}_\phi)$$

Motors: *DC Motor*

$$\underline{F}_{34} = I\int_{3}^{4} \underline{dl} \times \underline{B}$$

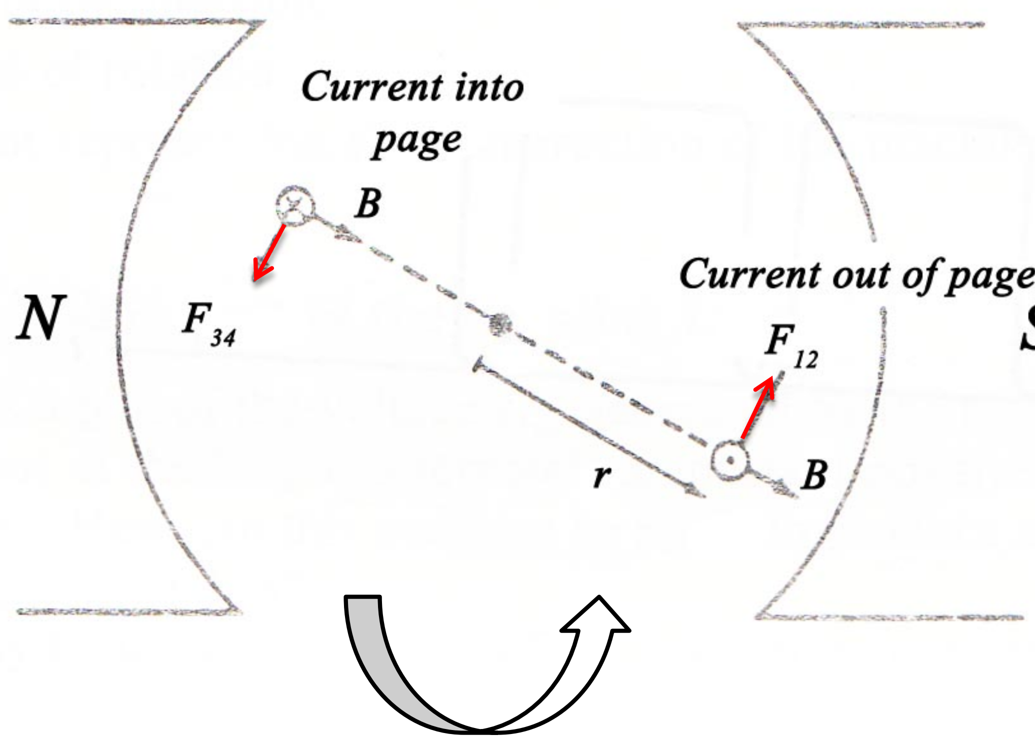
$$\underline{B} = B_o(-\underline{u}_r)$$



$$\underline{F}_{34} = I\int_{3=0}^{4=h} dz\,(-\underline{u}_z) \times B_o(-\underline{u}_r)$$

$$\underline{F}_{34} = IB_o h(\underline{u}_\phi)$$

Motors: ***DC Motor***



Motors: ***DC Motor***

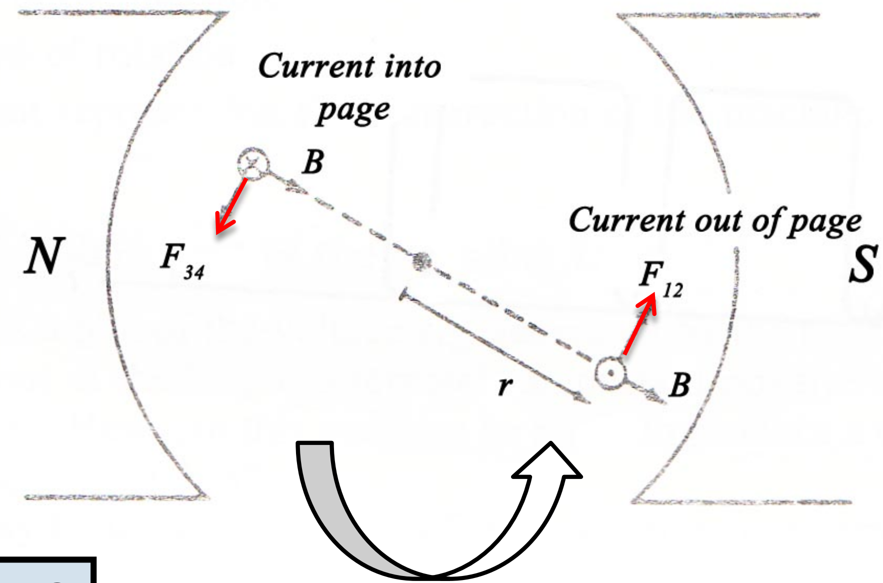
$$\boxed{T = \underline{r} \times \underline{F}}$$

$$\boxed{T = FL}$$

$$\boxed{T = (2r)IB_o h}$$

$$\boxed{T = 2IB_o(rh)}$$

Under the pole faces



$$\boxed{T = 0}$$

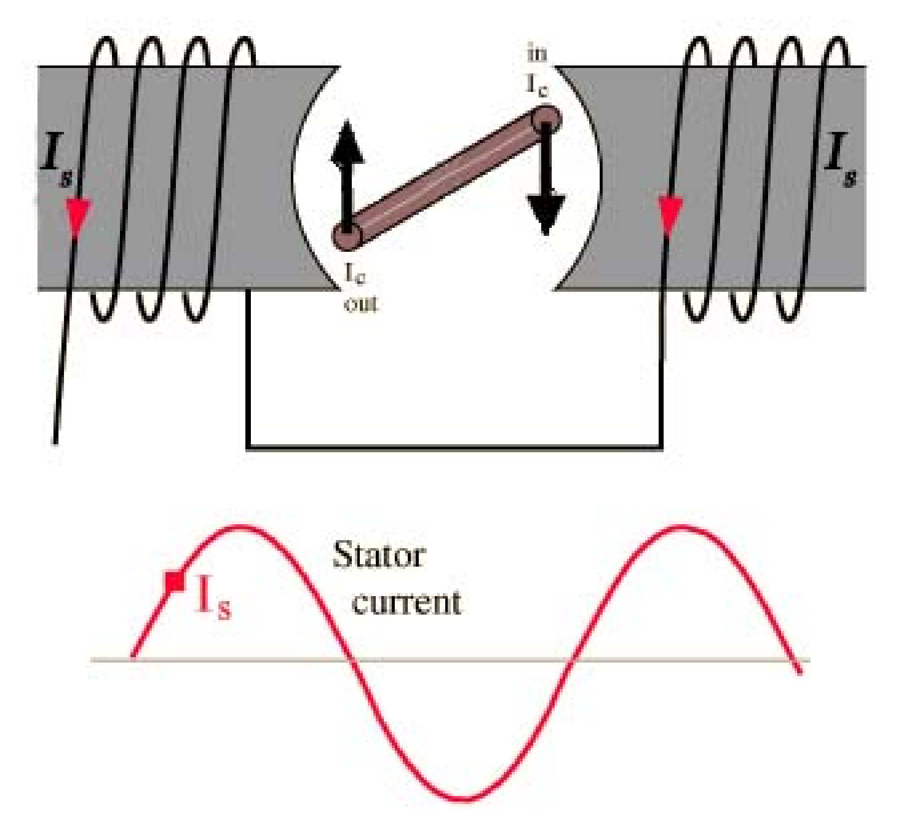
Beyond the pole edges

$$\boxed{T = \frac{2}{\pi} A_p B_o I}$$

$$\boxed{T = \frac{2}{\pi} \phi I}$$

Φ = Total flux under one pole

Motors: *AC Motor*

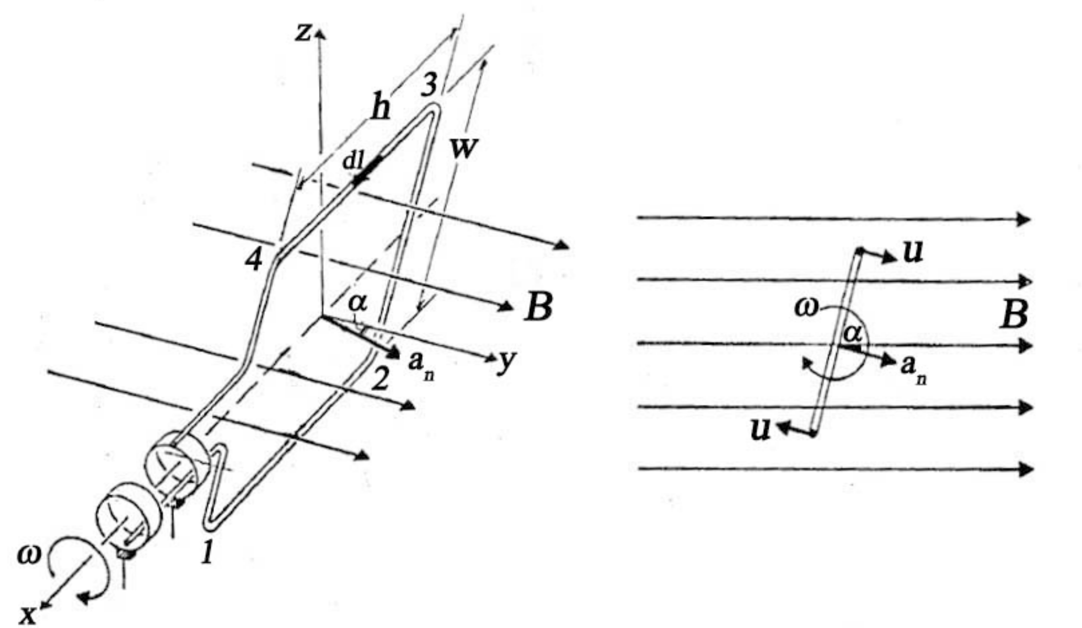


$$B = B_o \sin \omega_1 t$$

$$\omega_1 = 2\pi f$$

Motors: ***AC Motor***

$$e = -\int_S \frac{\partial \underline{B}}{\partial t} . \underline{dS} + \oint_c \underline{v} \times \underline{B} . \underline{dl}$$



(a) Prespective View

(b) View from +x direction

$$\underline{B} = B_o \sin \omega_1 t \, \underline{u}_y$$

$$e = e_1 + e_2$$

$$e_1 = -\int_S \frac{\partial \underline{B}}{\partial t} . \underline{dS}$$

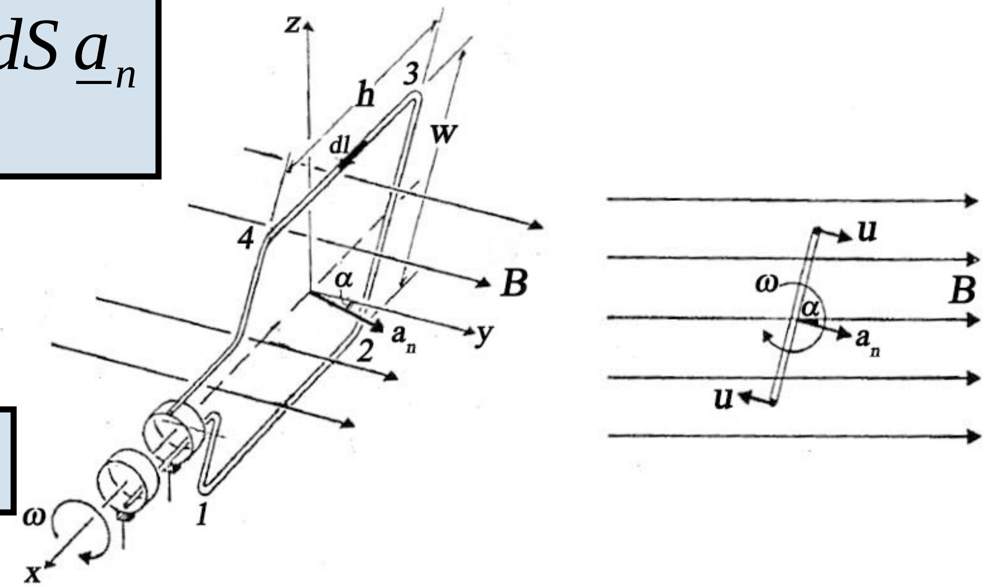
$$e_1 = -\int_{S=hw} \omega_1 B_o \cos(\omega_1 t) \underline{u}_y . dS \, \underline{a}_n$$

Motors: ***AC Motor***

$$e_1 = -\int_{S=wh} \omega_1 B_o \cos(\omega_1 t)\underline{u}_y . dS\, \underline{a}_n$$

$$\underline{a}_n \cdot \underline{u}_y = (1)(1)\cos\alpha$$

$$e_1 = -\omega_1 B_o (wh)\cos(\omega_1 t)\cos(\alpha)$$



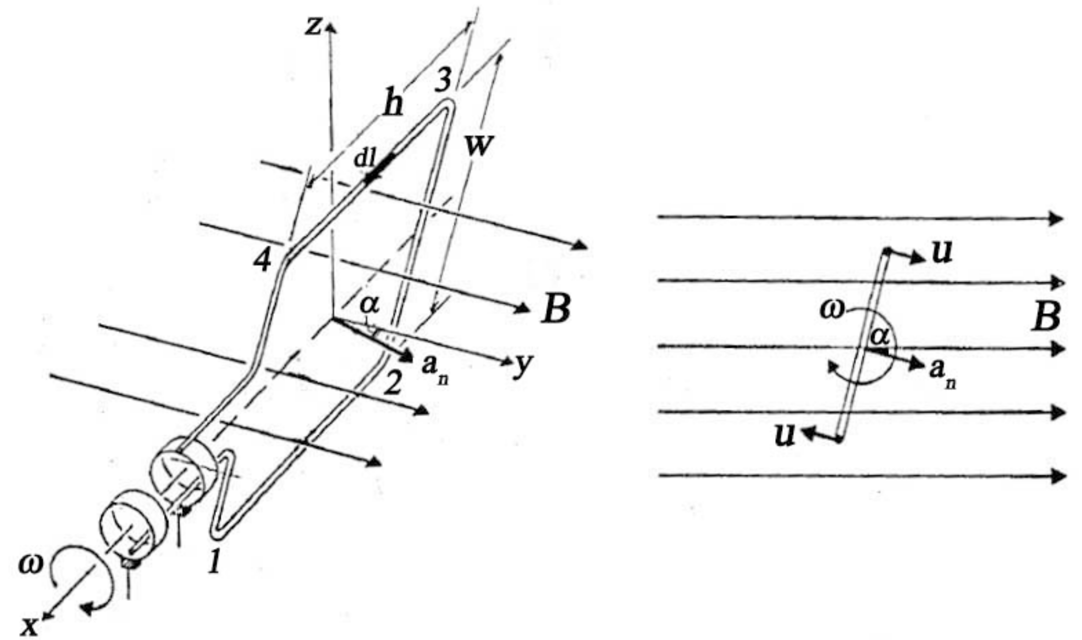
(a) Prespective View (b) View from +x direction

$$e_1 = -\omega_1 B_o (wh)\cos(\omega_1 t)\cos(\omega t + \theta_o)$$

$$e_1 = -\omega_1 B_o \frac{A}{2}[\cos((\omega_1 - \omega)t - \theta_o) + \cos((\omega_1 + \omega)t + \theta_o)]$$

Motors: ***AC Motor***

$$e_2 = \oint_c \underline{v} \times \underline{B}.\underline{dl}$$



(a) Prespective View (b) View from +x direction

$$e_2 = \omega B_o (wh) \sin(\omega_1 t) \sin(\omega t + \theta_o)$$

$$e_2 = \omega B_o \frac{A}{2}[\cos((\omega_1 - \omega)t - \theta_o) - \cos((\omega_1 + \omega)t + \theta_o)]$$

Motors: ***AC Motor***

$$e = e_1 + e_2$$

$$e_1 = -\omega_1 B_o \frac{A}{2}[\cos((\omega_1 - \omega)t - \theta_o) + \cos((\omega_1 + \omega)t + \theta_o)]$$

$$e_2 = \omega B_o \frac{A}{2}[\cos((\omega_1 - \omega)t - \theta_o) - \cos((\omega_1 + \omega)t + \theta_o)]$$

$$e = -B_o \frac{A}{2}[(\omega_1 - \omega)\cos((\omega_1 - \omega)t - \theta_o) + (\omega_1 + \omega)\cos((\omega_1 + \omega)t + \theta_o)]$$

$$e = e_f + e_b$$

Motors: *AC Motor*

$e_f \Rightarrow i_f$

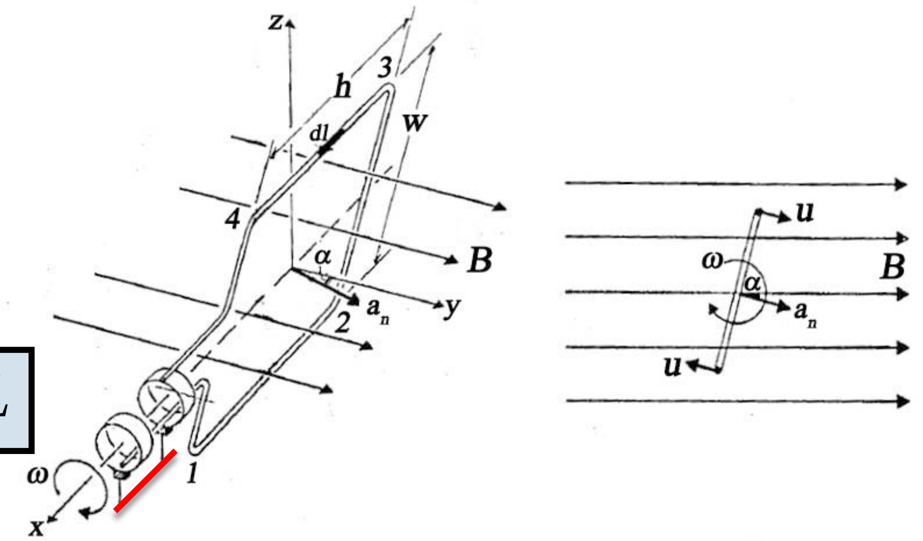
$e_b \Rightarrow i_b$

$i_f \Rightarrow \dfrac{e_f}{Z_f}$, $\quad Z_f = R + j(\omega_1 - \omega)L$

$i_b \Rightarrow \dfrac{e_b}{Z_b}$, $\quad Z_b = R + j(\omega_1 + \omega)L$

$$i_f = \frac{B_o A}{2\left|Z_f\right|}[(\omega_1 - \omega)\cos((\omega_1 - \omega)t - \theta_o - \phi_f)]$$

$$i_b = \frac{B_o A}{2\left|Z_b\right|}[(\omega_1 + \omega)\cos((\omega_1 + \omega)t + \theta_o - \phi_b)]$$



(a) Prespective View

(b) View from +x direction

Motors: ***AC Motor***

$$i_f = I_{fm}\cos((\omega_1-\omega)t-\theta_o-\phi_f)$$

$$i_b = I_{bm}\cos((\omega_1+\omega)t+\theta_o-\phi_b)$$

Torque

$$T_f = \underline{m}\times\underline{B}$$

$$\underline{m} = i_f\,\underline{S}$$

$$\underline{B} = B_o\sin\omega_1 t\,\underline{u}_y$$

$$T_f = I_{fm}\cos((\omega_1-\omega)t-\theta_o-\phi_f)(A\underline{a}_n)\times B_o\sin\omega_1 t\,\underline{u}_y$$

$$T_f = I_{fm}B_oA\cos((\omega_1-\omega)t-\theta_o-\phi_f)\sin\omega_1 t\sin(\omega t+\theta_o)$$

$$\sin\omega_1 t\sin(\omega t+\theta_o) = \frac{1}{2}[\cos((\omega_1-\omega)t-\theta_o)-\cos((\omega_1+\omega)t+\theta_o)]$$

Motors: *AC Motor*

Torque

$$T_f = \frac{I_{fm} B_o A}{2} \cos((\omega_1 - \omega)t - \theta_o - \phi_f)$$
$$*[\cos((\omega_1 - \omega)t - \theta_o) - \cos((\omega_1 + \omega)t + \theta_o)]$$

$$\cos((\omega_1 - \omega)t - \theta_o - \phi_f) * \cos((\omega_1 - \omega)t - \theta_o) =$$
$$\frac{1}{2}\left[\cos\phi_f + \cos[2((\omega_1 - \omega)t - \theta_o) - \phi_f)]\right]$$

$$\cos((\omega_1 - \omega)t - \theta_o - \phi_f) * \cos((\omega_1 + \omega)t + \theta_o) =$$
$$\frac{1}{2}\left[\cos[2(\omega t + \theta_o) + \phi_f)] + \cos(2\omega_1 t - \phi_f)\right]$$

Motors: ***AC Motor***

Torque

$$T_f = \frac{I_{fm} B_o A}{4} * \begin{bmatrix} \cos\phi_f + \cos[2((\omega_1 - \omega)t - \theta_o) - \phi_f)] \\ -\cos(2\omega_1 t - \phi_f) - \cos[2(\omega t + \theta_o) + \phi_f)] \end{bmatrix}$$

$$T_{favg} = \frac{I_{fm} B_o A}{4} \cos\phi_f$$

Similarly

$$T_{bavg} = -\frac{I_{bm} B_o A}{4} \cos\phi_b$$

$$T_{net} = \frac{B_o A}{4} \left[I_{fm} \cos\phi_f - I_{bm} \cos\phi_b \right]$$

Motors: ***AC Motor***

$$\boxed{T_{net} = \frac{B_o A}{4}\left[I_{fm}\cos\phi_f - I_{bm}\cos\phi_b\right]}$$

For ω = 0

$$\boxed{I_{fm} = I_{bm}} \qquad \boxed{\phi_f = \phi_b}$$

$$\boxed{T_{net} = zero}$$

This motor **<u>cannot</u>** start from rest by its own developed torque

For any speed ω

$$\boxed{I_{fm} \neq I_{bm}} \qquad \boxed{\phi_f \neq \phi_b}$$

$$\boxed{T_{net} \neq zero}$$

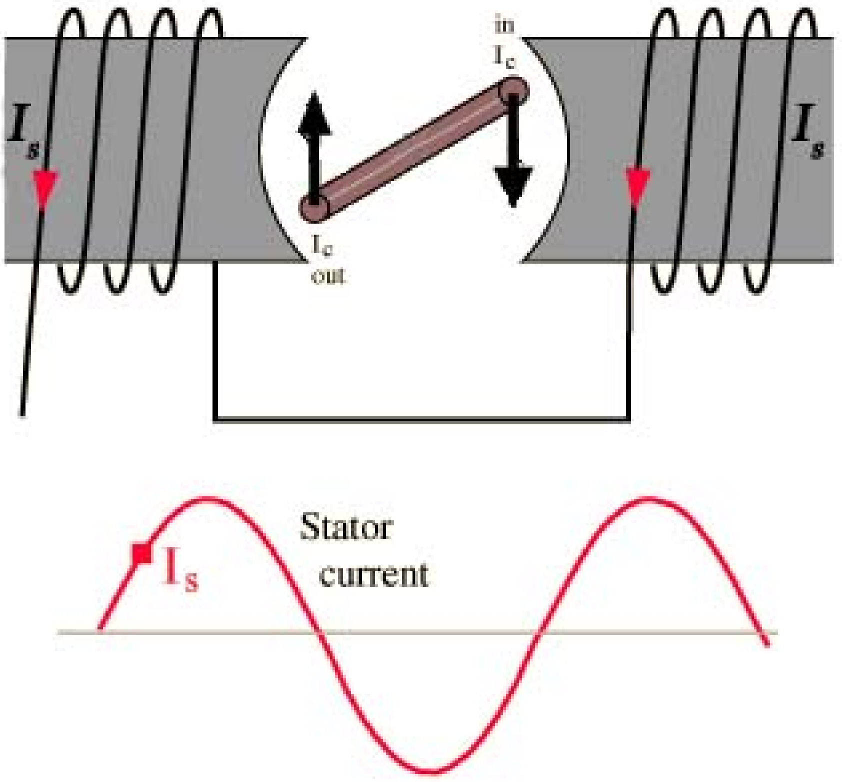
Motors: ***AC Motor***

The motor needs an external method to start-up

It is called <u>Induction Motor</u>

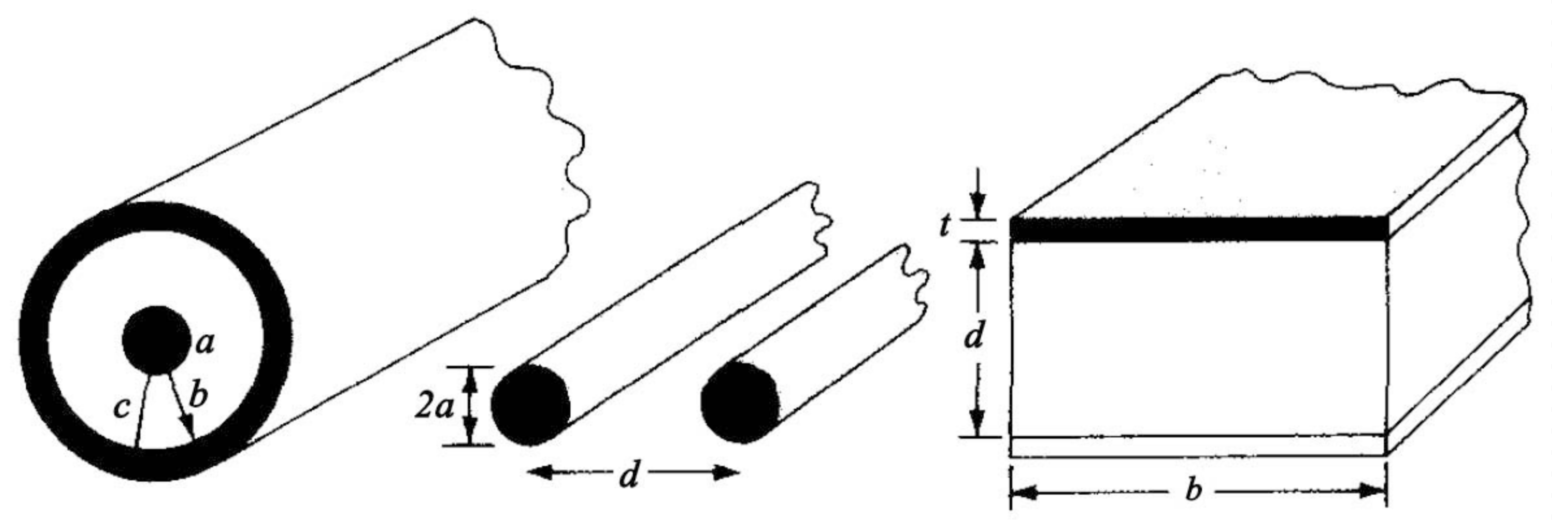
<u>Single Phase</u>

<u>Induction Motor</u>



Transmission Line Parameters

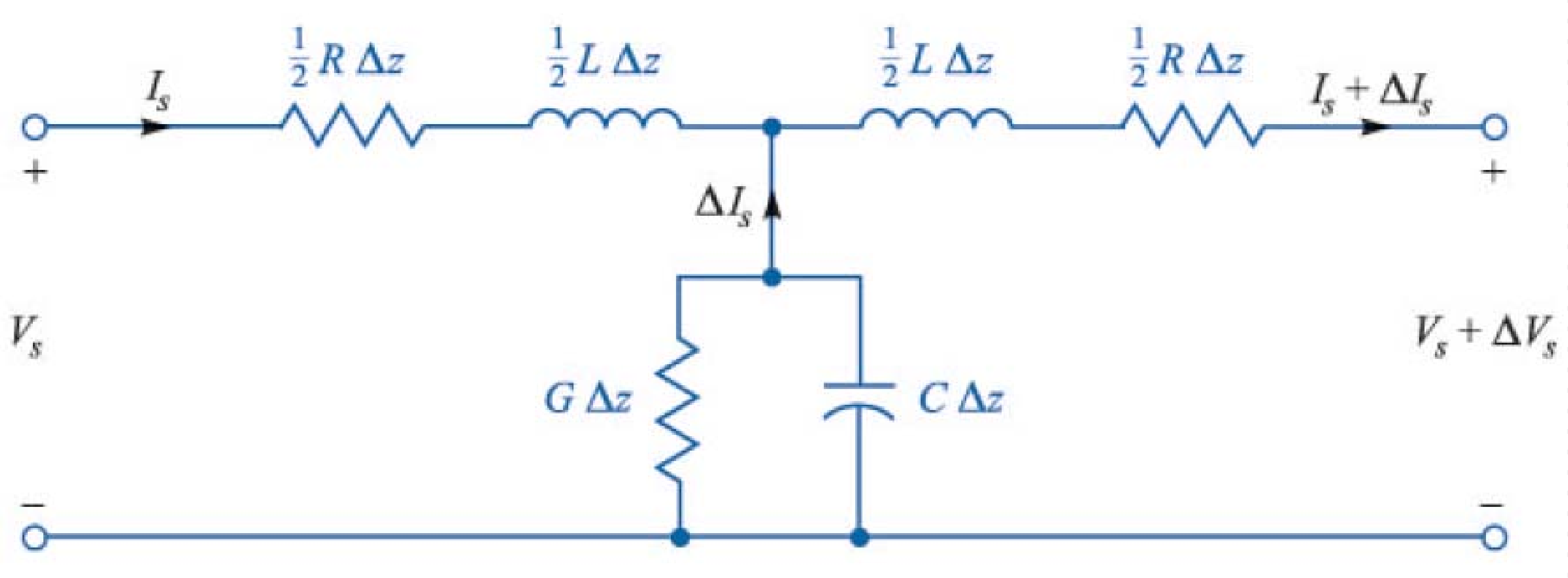
Transmission lines are used to transmit electric energy (power distribution at low frequency) and signals (communication at high frequency) from one point to another, specially from a source to a load.



Coaxial Line ***Two Wire Line*** ***Planar Line***

Transmission Line Parameters

Transmission line circuit model

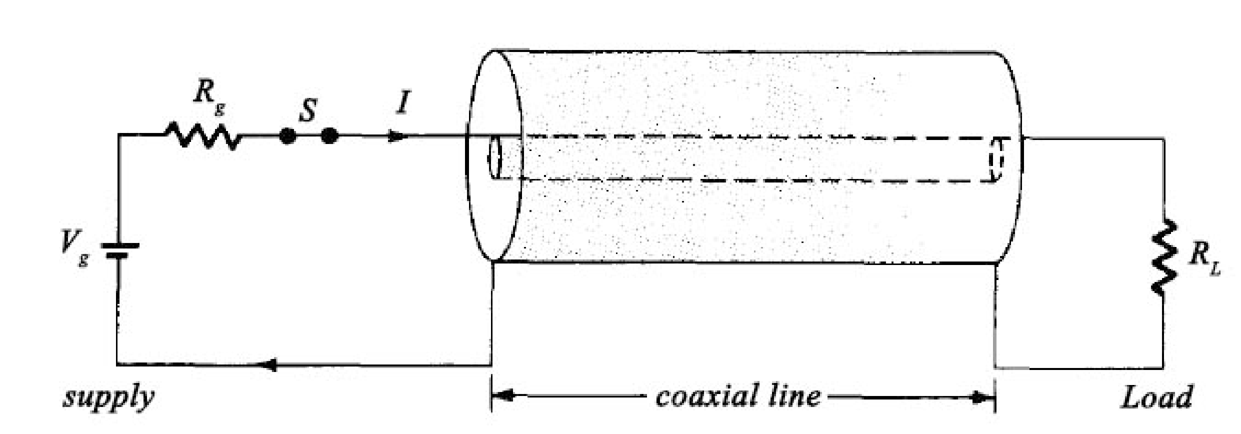


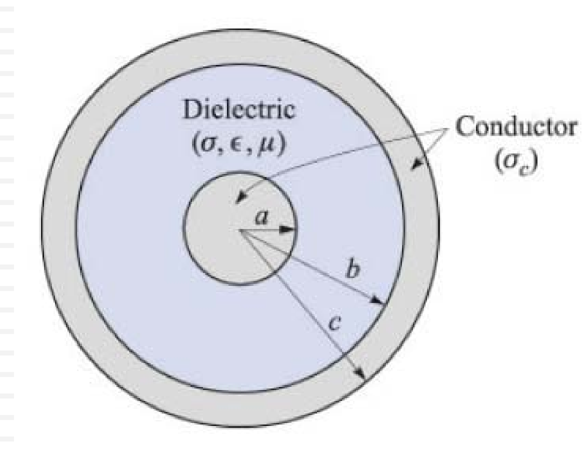
R, L, C & G are functions of the transmission line configuration and materials

Transmission Line Parameters

1. Coaxial Line

At Low frequencies:





Resistance

$$R_{in} = \frac{1}{\sigma_c \pi a^2} \quad \Omega / m$$

$$R_{out} = \frac{1}{\sigma_c \pi (c^2 - b^2)} \quad \Omega / m$$

Transmission Line Parameters

1. Coaxial Line

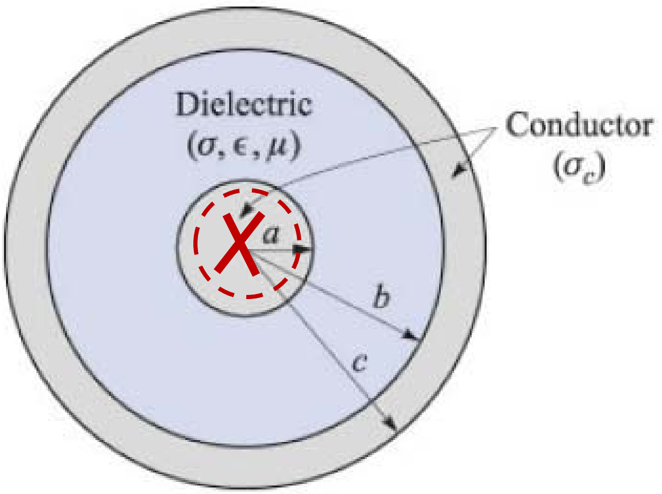
Internal Inductance

$$\underline{H} = \frac{Ir}{2\pi a^2}\underline{u}_\phi$$

$$w_e = \int \frac{1}{2}\mu H^2 dv = \frac{1}{2} L I^2$$

$$w_e = \int_{z=0}^{1}\int_{r=0}^{a}\int_{\phi=0}^{2\pi} \frac{\mu r^2 I^2}{8\pi^2 a^4} r dr d\phi dz = \frac{\mu I^2}{16\pi}$$

$$L_{in} = \frac{\mu}{8\pi} \qquad H/m$$



Transmission Line Parameters

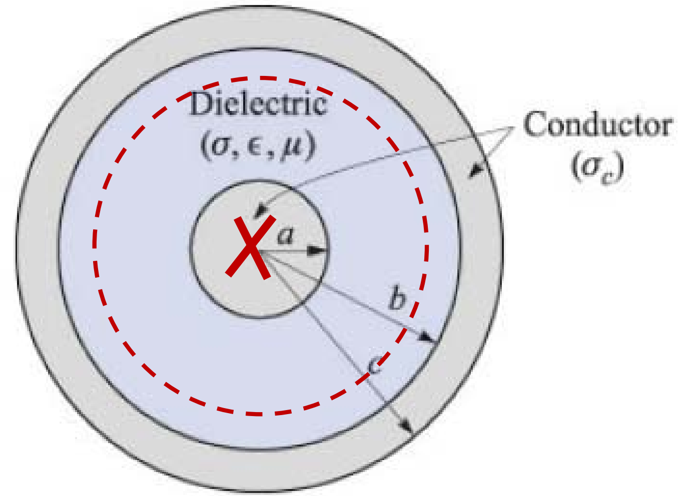
1. Coaxial Line

Inductance

$$\underline{H}=\frac{I}{2\pi r}\underline{u}_{\phi}$$

$$\underline{B}=\frac{\mu I}{2\pi r}\underline{u}_{\phi}$$

$$\lambda=N\int \underline{B}.\underline{ds}$$

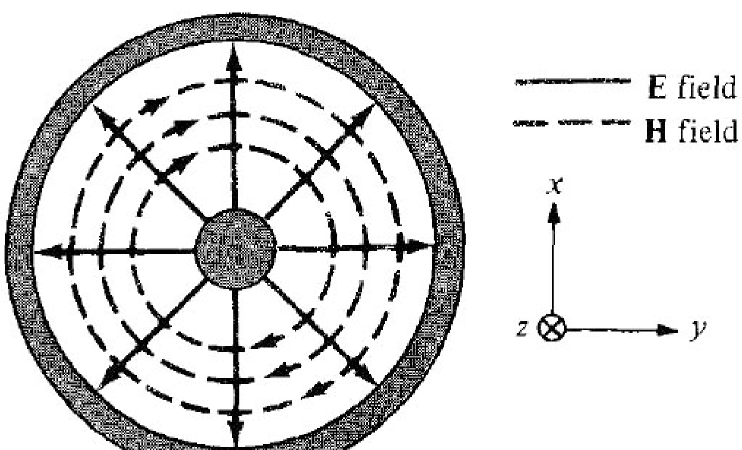


$$\lambda=\int_{z=0}^{1}\int_{r=a}^{b}\frac{\mu I}{2\pi r}drdz=\frac{\mu I}{2\pi}ln\frac{b}{a}$$

$$L=\frac{\lambda}{I}$$

$$L=\frac{\mu}{2\pi}\ln\frac{b}{a}$$

$$H/m$$



Transmission Line Parameters

1. Coaxial Line

Capacitance

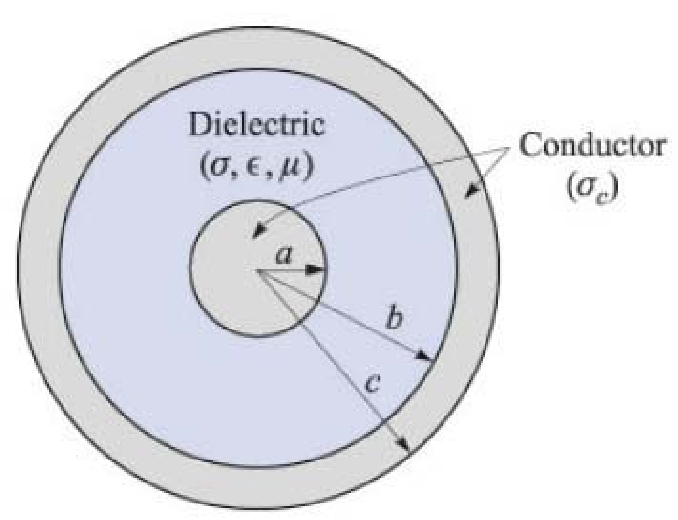
$$\underline{E} = \frac{Q}{2\pi\varepsilon r L}\underline{u}_r$$

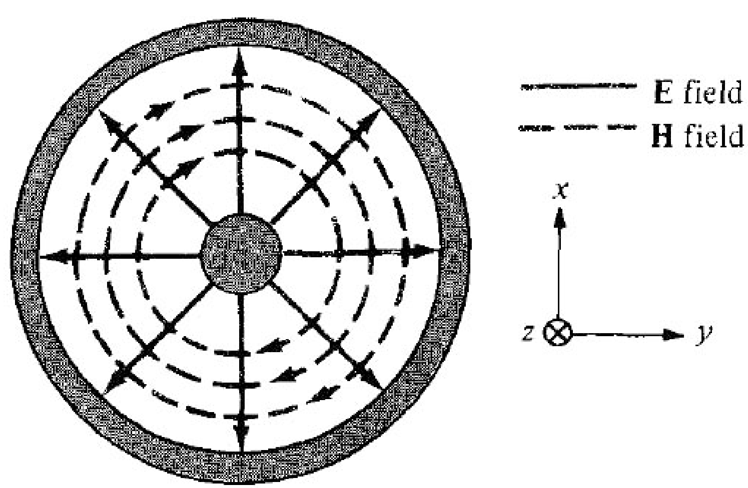
$$V_{ab} = \int_{r=a}^{b} \frac{Q}{2\pi\varepsilon r L} dr = \frac{Q}{2\pi\varepsilon L} ln\frac{b}{a}$$

$$C = \frac{Q}{V}$$

$$C = \frac{2\pi\varepsilon}{ln\, b/a} \quad F/m$$

$$LC = \mu\varepsilon$$





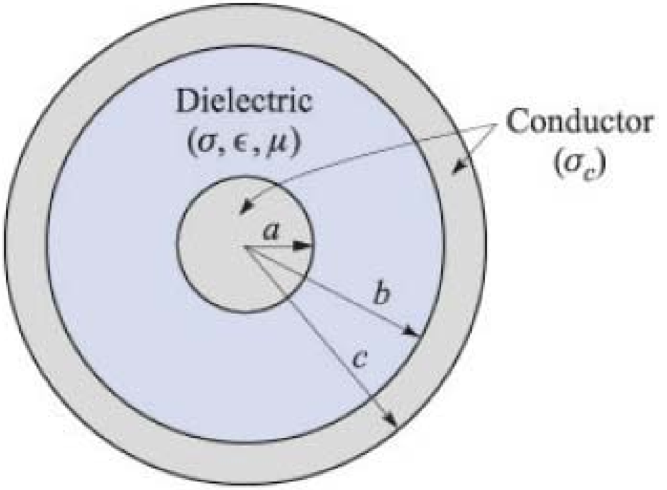
Transmission Line Parameters

1. Coaxial Line

Conductance

$$\underline{E} = \frac{Q}{2\pi\varepsilon rL}\underline{u}_r$$

$$R = \frac{\int \underline{E}.\underline{dl}}{\sigma\int \underline{E}.\underline{dS}} = \frac{V_{ab}}{I_L} = \frac{1}{G}$$



$$V_{ab} = \frac{Q}{2\pi\varepsilon L}\ln\frac{b}{a}$$

$$I_L = \sigma\int_{\phi=0}^{2\pi}\int_{z=0}^{L}\frac{Q}{2\pi\varepsilon rL}r\,d\phi\,dz = \frac{\sigma Q}{\varepsilon}$$

$$G = \frac{2\pi\sigma}{\ln b/a}$$

$$\mho / m$$

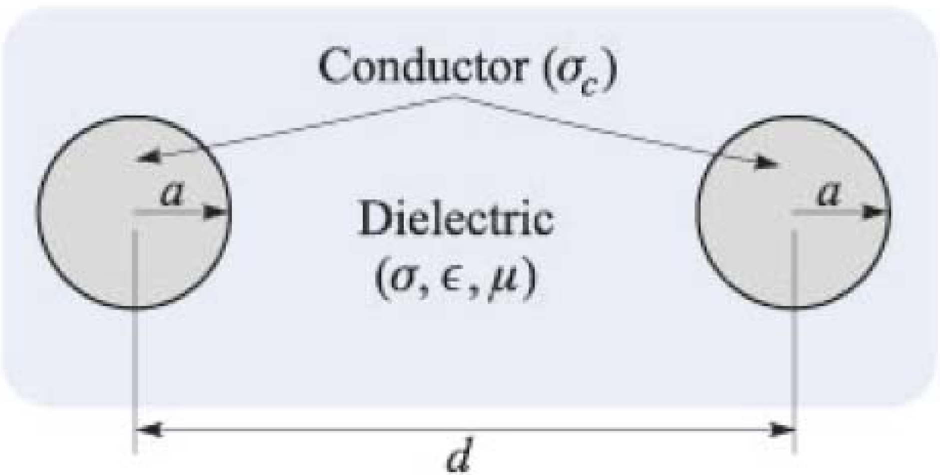
$$\frac{C}{G} = \frac{\varepsilon}{\sigma}$$

Transmission Line Parameters

2. Two Wire Line

At Low Frequencies:

Resistance



$$\boxed{R = \frac{2}{\sigma_c \pi a^2}} \quad \boxed{\Omega / m}$$

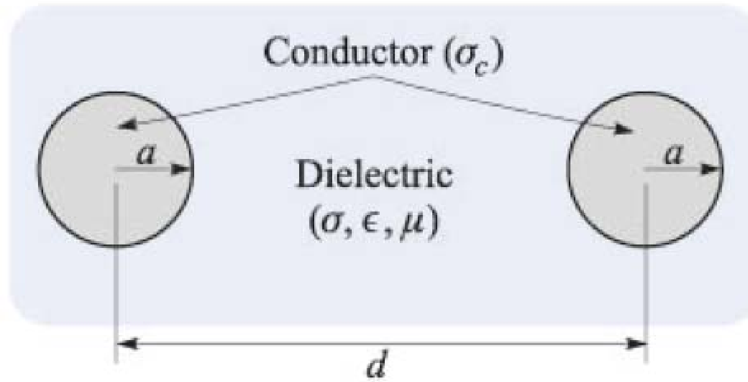
Transmission Line Parameters

2. Two Wire Line

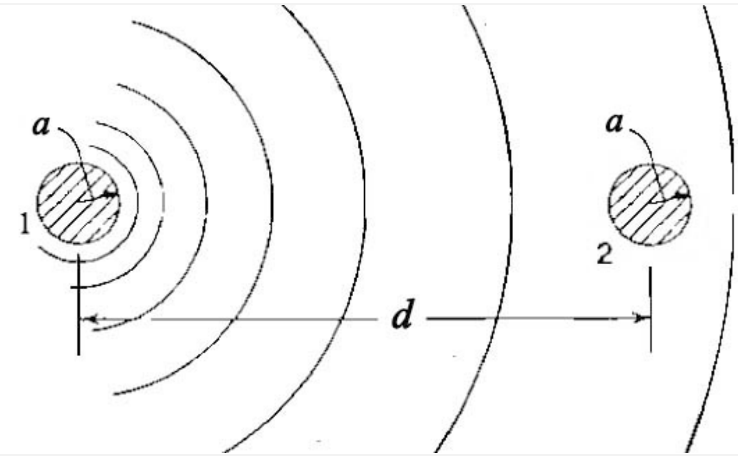
Inductance

$$\boxed{\underline{H} = \frac{I}{2\pi r}\underline{u}_\phi}$$

$$\boxed{\underline{B} = \frac{\mu I}{2\pi r}\underline{u}_\phi}$$



$$\boxed{\lambda_{12} = \int_{z=0}^{1}\int_{r=a}^{d}\frac{\mu I}{2\pi r}drdz = \frac{\mu I}{2\pi}ln\frac{d}{a}}$$



$$\boxed{L_{12} = \frac{\mu}{2\pi}\ln\frac{d}{a}}$$ $$\boxed{H / m}$$

Transmission Line Parameters

2. Two Wire Line

Inductance

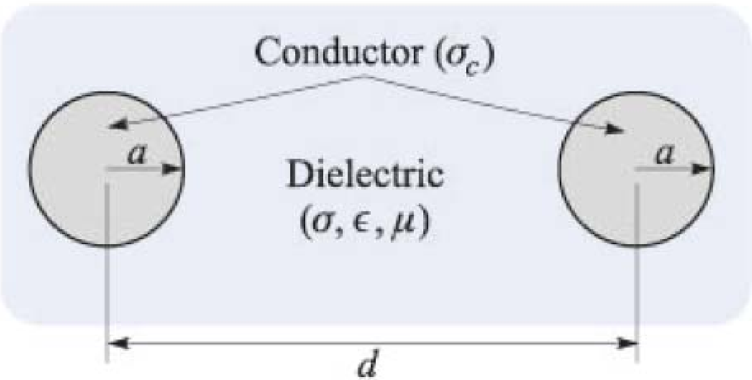
$$L_1 = L_{12} + L_{1in}$$

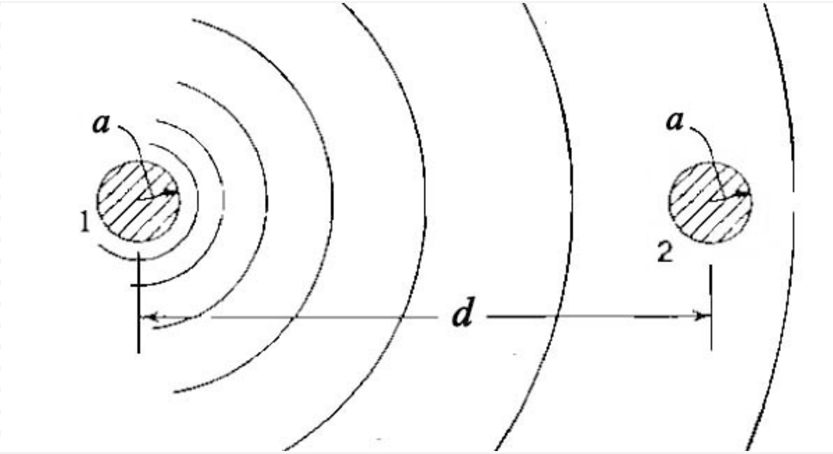
$$L_{in} = \frac{\mu}{8\pi}$$

$$L_1 = (\frac{1}{2} + 2\ln\frac{d}{a}) \times 10^{-7}$$

$$L_1 = (\frac{1}{4} + \ln\frac{d}{a}) 2 \times 10^{-7}$$

$$L_1 = 2 \times 10^{-7} \ln\frac{d}{ae^{-1/4}}$$

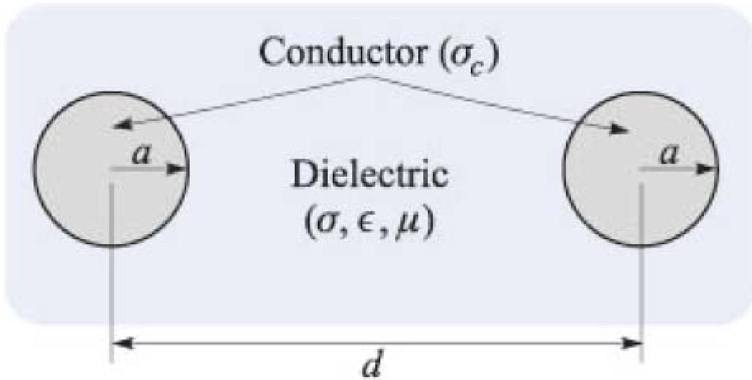




2. Two Wire Line

Inductance

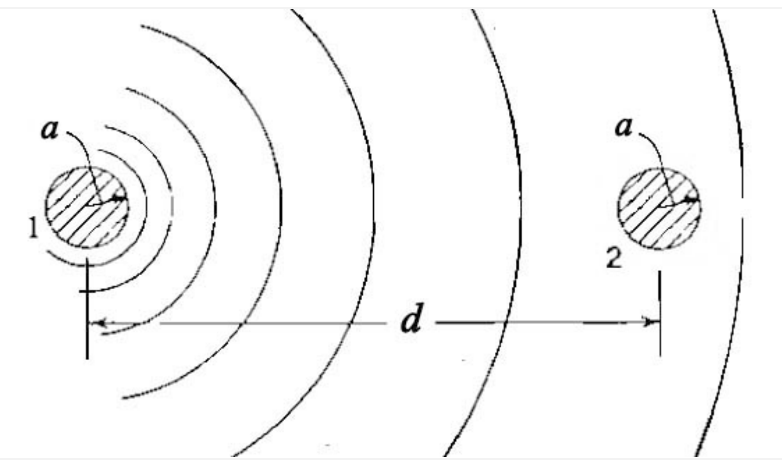
$$L_2 = 2\times10^{-7}\ln\frac{d}{ae^{-1/4}}$$



$$L_{circuit} = L_1 + L_2 = 4\times10^{-7}\ln\frac{d}{ae^{-1/4}}$$

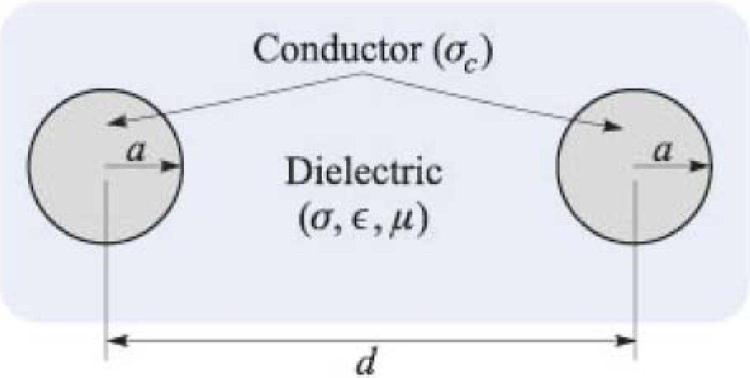
$$L_{circuit} = 4\times10^{-7}\ln\frac{d}{a'} \qquad H/m$$

$$a' = ae^{-0.25}$$



Transmission Line Parameters

2. Two Wire Line



Capacitance

$$L_{12} = \frac{\mu}{2\pi} \ln \frac{d}{a}$$

$$LC = \mu\varepsilon$$

$$C_{12} = \frac{2\pi\varepsilon}{\ln {d}/{a}}$$

$$F / m$$

Conductance

$$\frac{C}{G} = \frac{\varepsilon}{\sigma}$$

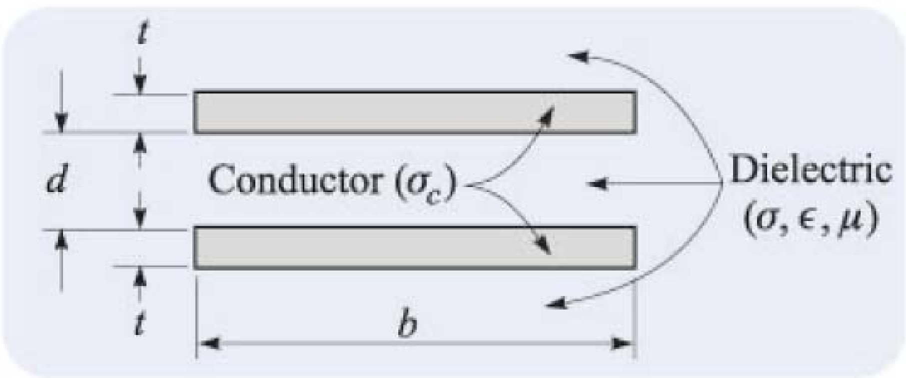
$$G_{12} = \frac{2\pi\sigma}{\ln {d}/{a}}$$

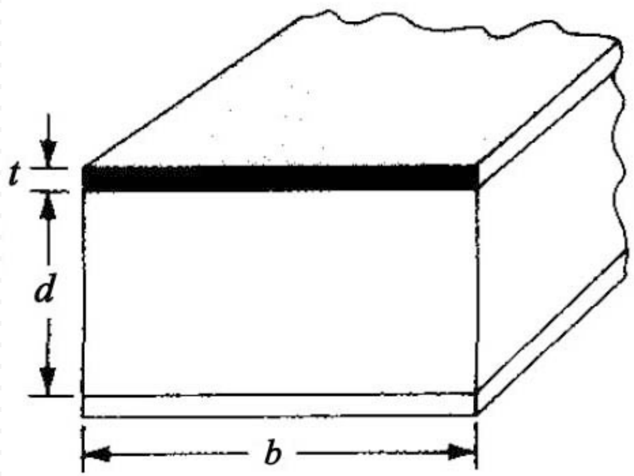
$$\mho / m$$

Transmission Line Parameters

3. Planar Line

At High Frequency:





$$\boxed{R = \frac{2}{\sigma \delta b}} \quad \boxed{\Omega / m} \quad \boxed{C = \frac{\varepsilon b}{d}} \quad \boxed{F / m} \quad \boxed{L = \frac{\mu d}{b}} \quad \boxed{H / m} \quad \boxed{G = \frac{\sigma b}{d}} \quad \boxed{\mho / m}$$

$$\boxed{\delta = skin\ depth}$$

Transmission Line Parameters

4. Power Lines (single-phase and three-phase)