

# Induction Machines

- Magnetic Field Production and Distribution
- Generated Voltage and Torque Production
- Principle of Operation
- Equivalent Circuit
- Power Flow
- Torque-Speed Characteristics
- Loading & Stability
- Induction Machine Modes of Operation
- Testing

# Generated Voltage (EMF)

2

## Induction Machines

### 1. *Wound Rotor (Slip-Ring) Machine*



# Generated Voltage (EMF)

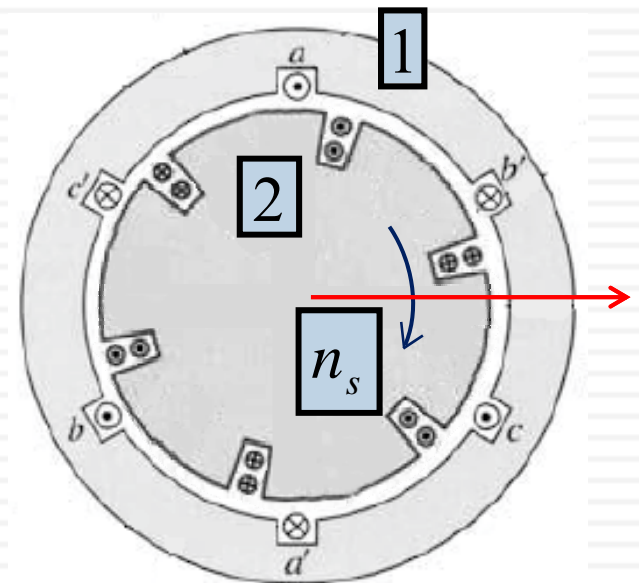
3

## Induction Machines

### 1. Wound Rotor (Slip-Ring) Machine

$$E_{1 \text{ phase}} = 4.44 f \phi N_1 k_{w1}$$

$$E_{2 \text{ phase}} = 4.44 f \phi N_2 k_{w2}$$

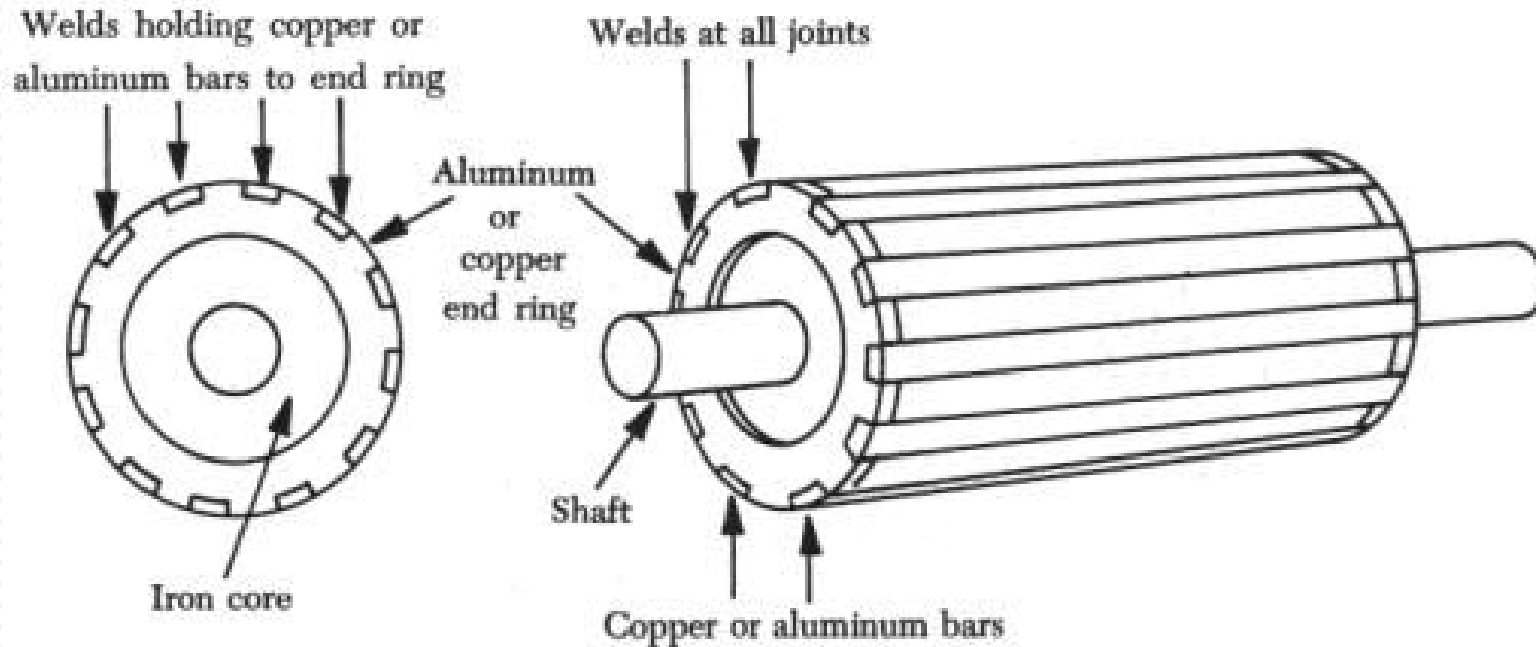


# Generated Voltage (EMF)

4

## Induction Machines

### 2. Squirrel Cage Rotor Machine



# Generated Voltage (EMF)

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## Induction Machines

### 2. Squirrel Cage Rotor Machine



# Generated Voltage (EMF)

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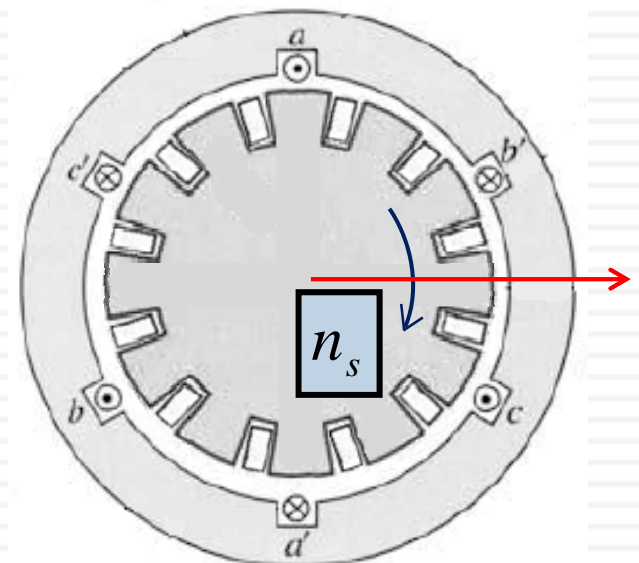
## Induction Machines

### 2. Squirrel Cage Rotor Machine

$$E_{1 \text{ phase}} = 4.44 f \phi N_1 k_{w1}$$

For simplicity, assume:

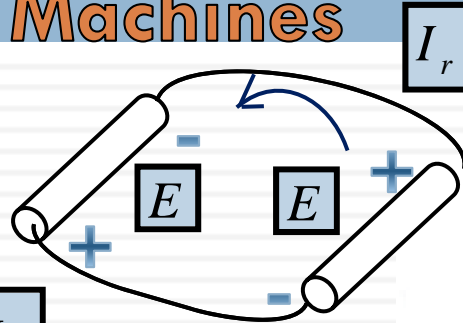
$$E_{2 \text{ phase}} = 4.44 f \phi N_2 k_{w2}$$



# Torque Production

7

## Induction Machines



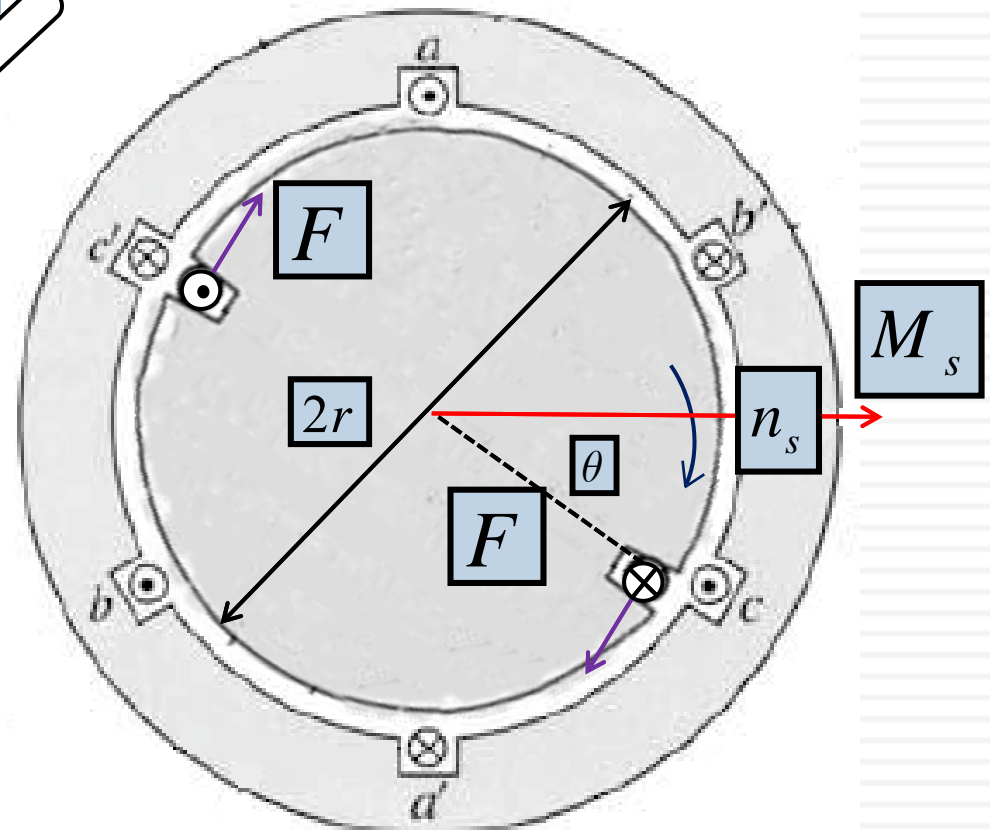
$$F = I_r BL$$

$$F = I_r LB_s \cos \theta$$

$$\underline{T} = \underline{r} \times \underline{F}$$

$$T = F(2r)$$

$$T = 2rI_r LB_s \cos \theta$$



$$B = B_s \cos \theta$$

# Torque Production

8

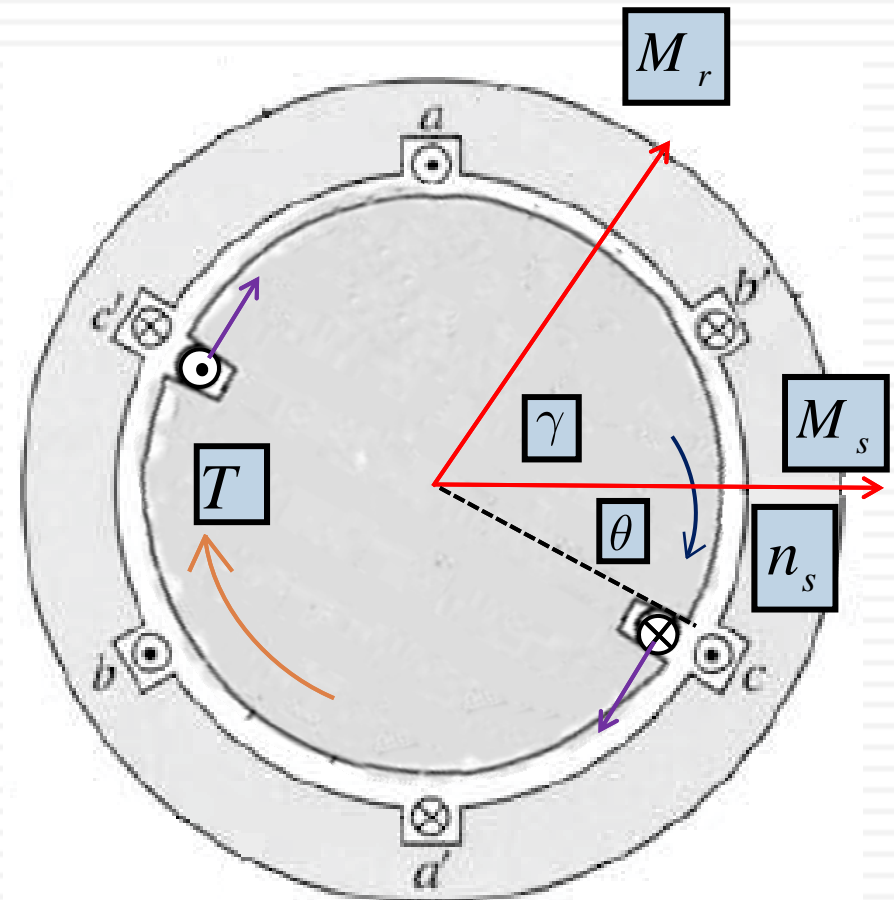
## Induction Machines

$$B_r \propto I_r$$

$$T = k B_s B_r \cos \theta$$

$$\gamma = 90 - \theta$$

$$T = k B_s B_r \sin \gamma$$



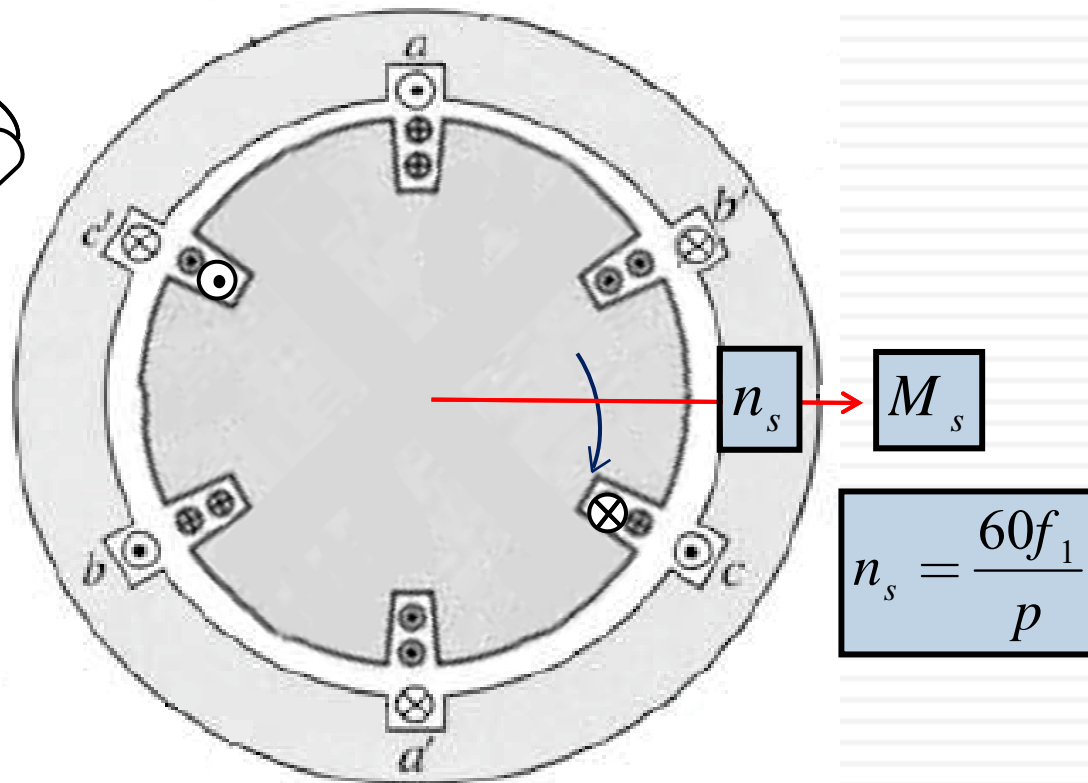
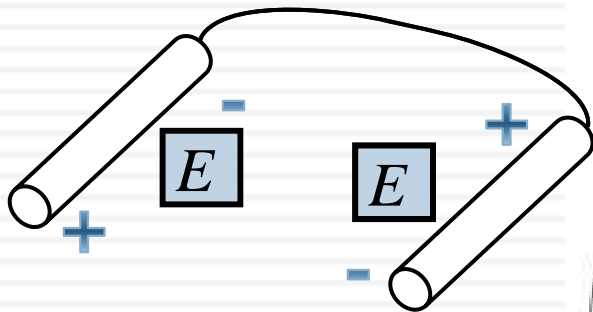
$$B = B_s \cos \theta$$

# Induction Machines: Principle of Operation

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## A. Motor at standstill with rotor open circuited

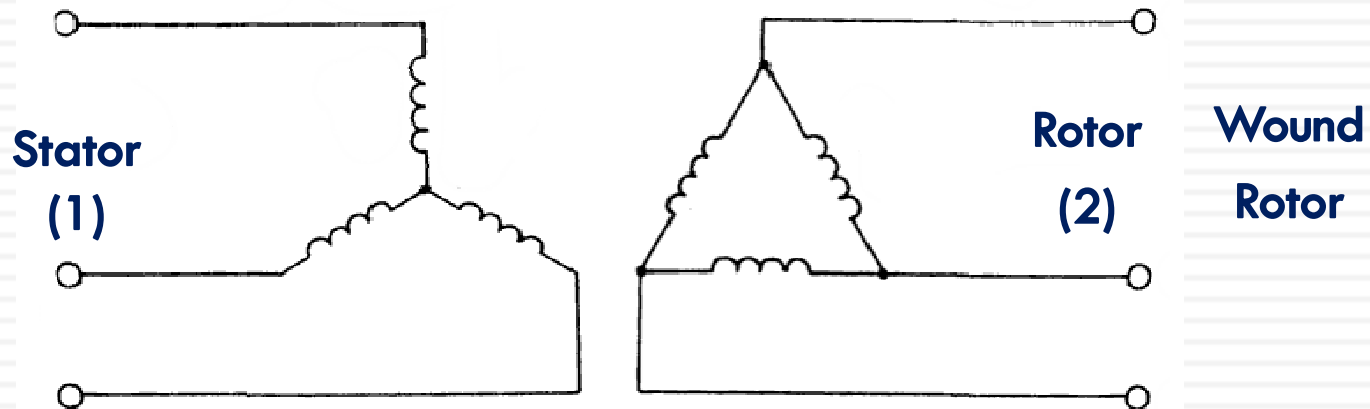
### Induced Voltage



# Induction Machines: Principle of Operation

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## A. Motor at standstill with rotor open circuited



$$E_{1 \text{ phase}} = 4.44f_1\phi N_1 \frac{k_{w1}}{m}$$

$$E_{2 \text{ phase}} = 4.44f_1\phi N_2 \frac{k_{w2}}{m}$$

$$\frac{E_{1 \text{ phase}}}{E_{2 \text{ phase}}} = \frac{N_1 k_{w1}}{N_2 k_{w2}} = a_{\text{eff}}$$

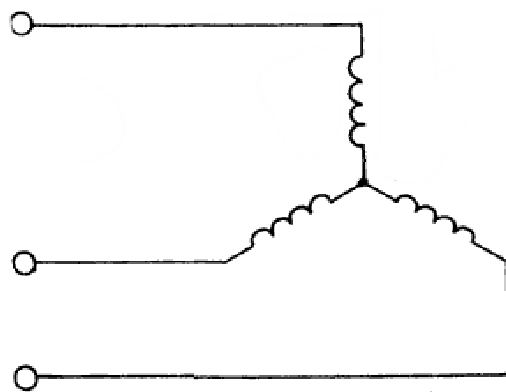
**Effective  
turns ratio**

$$E_{2 \text{ oc}} = 4.44f_1\phi N_2 \frac{k_{w2}}{m}$$

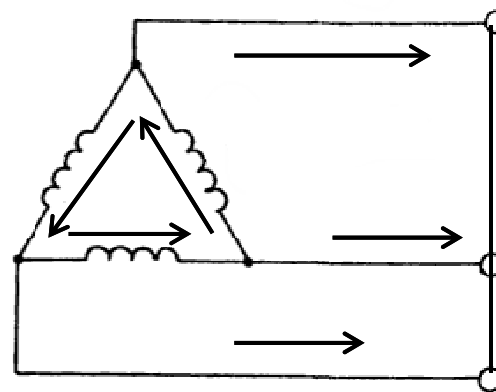
# Induction Machines: Principle of Operation

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## B. Motor at standstill with rotor short circuited

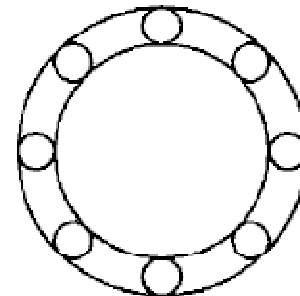
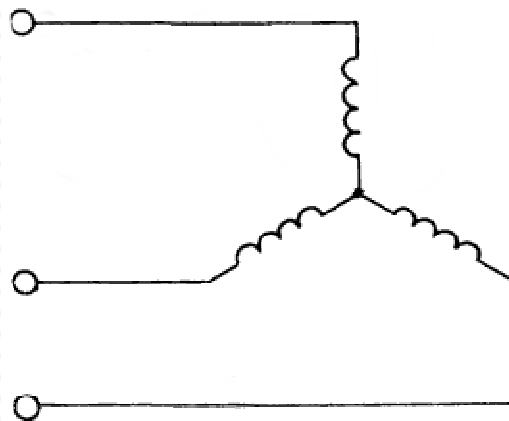


**Stator**



**Rotor**

**Wound  
Rotor**



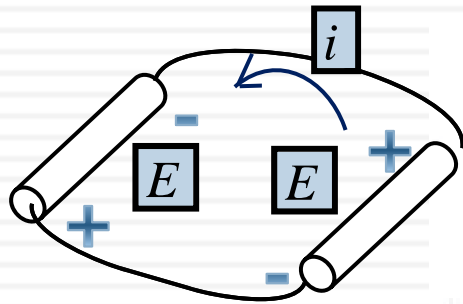
**Squirrel  
Cage  
Rotor**

# Induction Machines: Principle of Operation

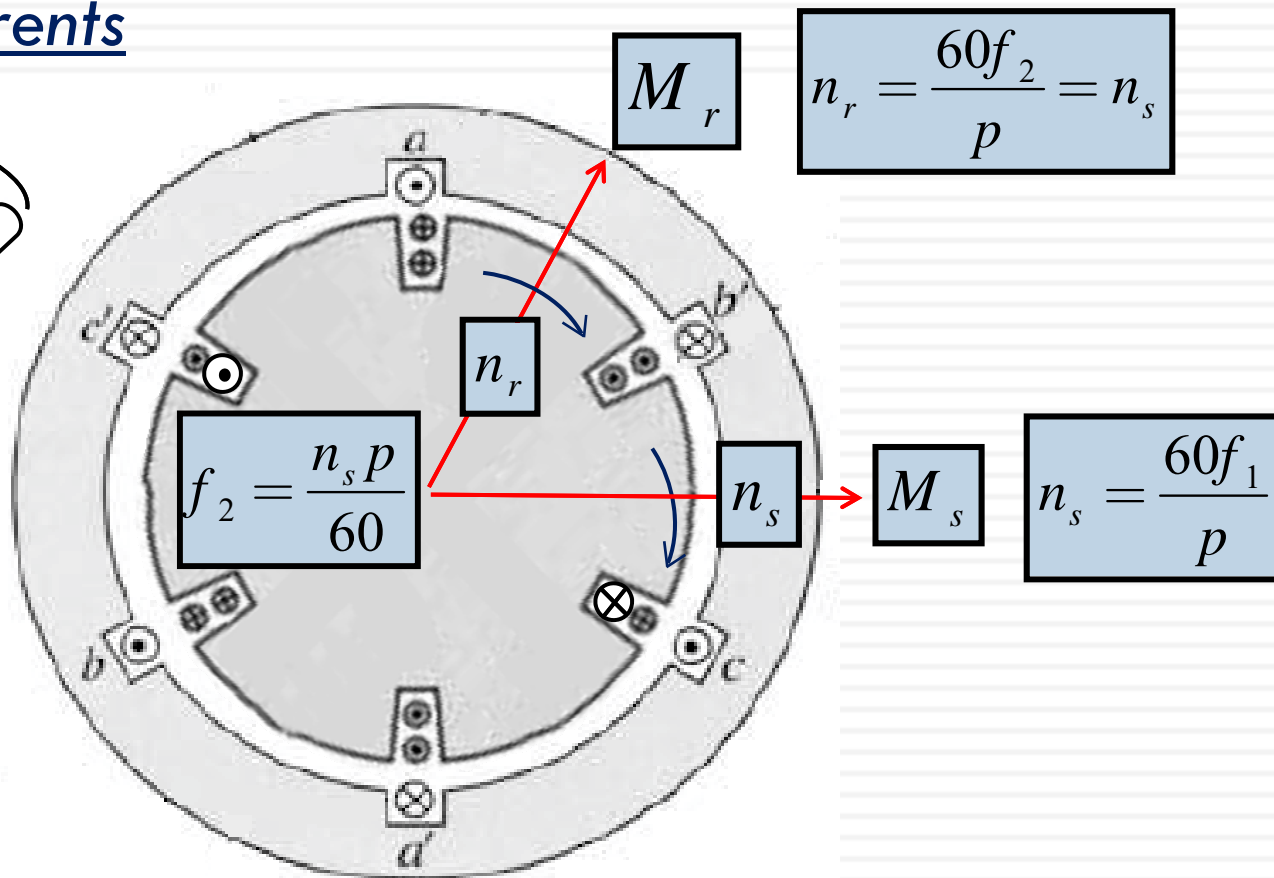
12

## B. Motor at standstill with rotor short circuited

### Induced Currents



$$f_2 = f_1$$



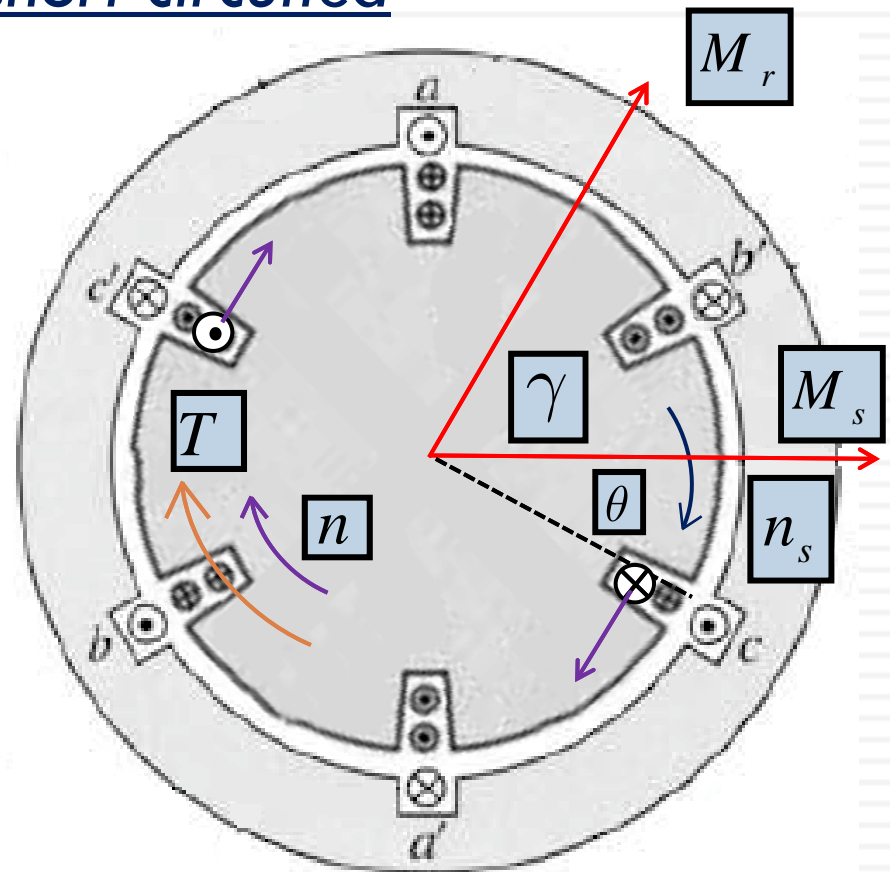
# Induction Machines: Principle of Operation

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## C. Motor running with rotor short circuited

### Torque Production

$$T = k B_s B_r \sin \gamma$$



# Induction Machines: Principle of Operation

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## C. Motor running with rotor short circuited

### Rotor Rotating Field Speed

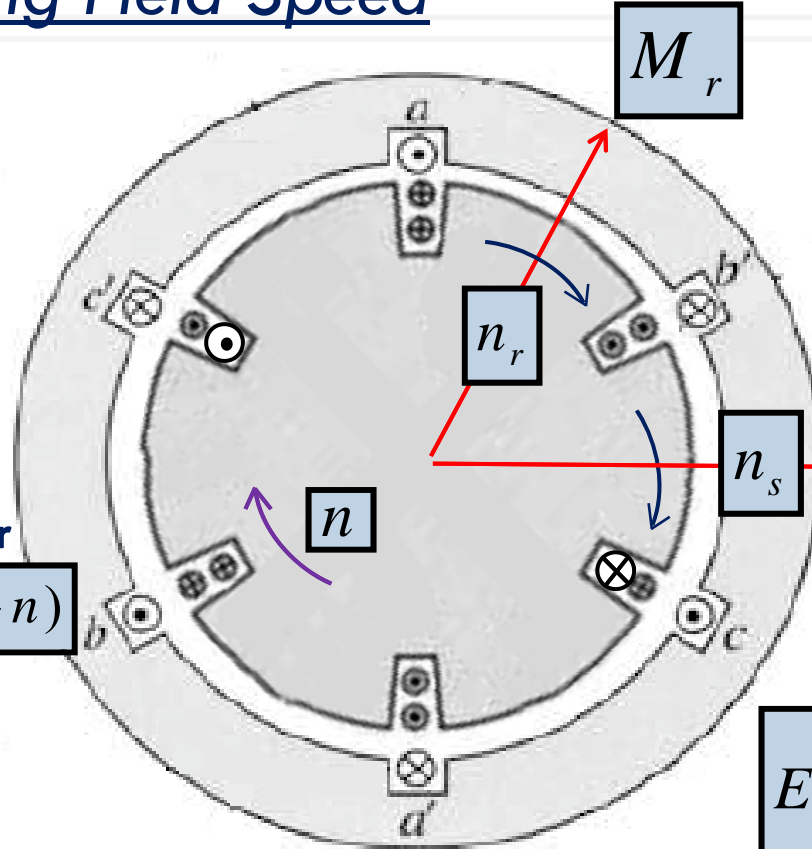
With Respect to Rotor

$$n_r = \frac{60f_2}{p}$$

With Respect to Stator

$$n + n_r = n + (n_s - n)$$

$$n_s$$



Slip

$$f_2 = \frac{(n_s - n)}{60} p$$

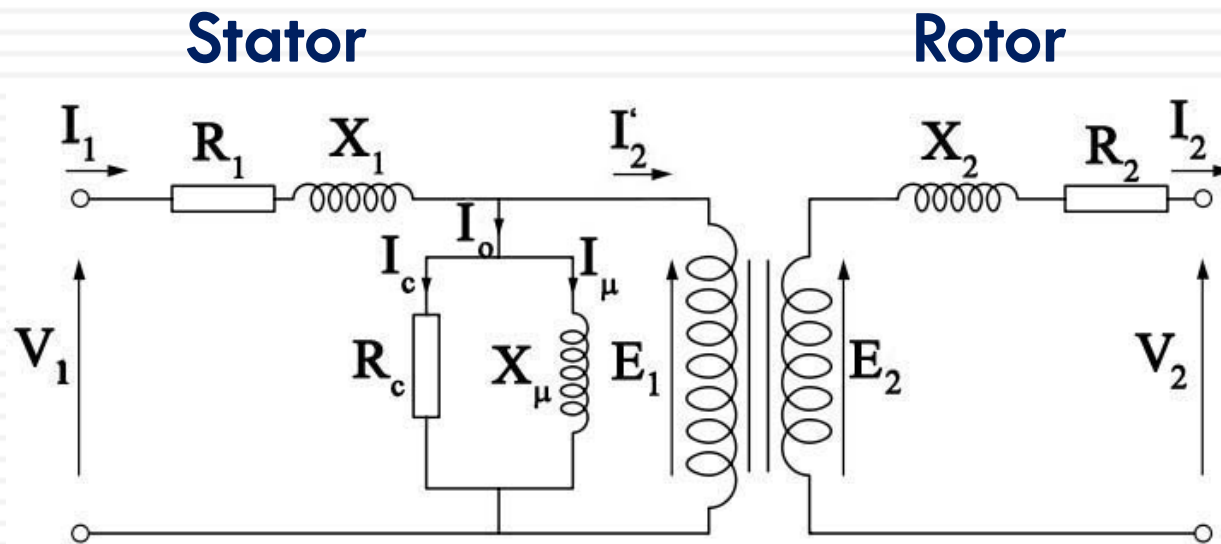
$$n_s = \frac{60f_1}{p}$$

$$E_{2 \text{ run}} = 4.44f_2\phi N_2 \frac{k_w 2}{m}$$

# Induction Machines: Equivalent Circuit

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## A. Rotor open circuited (standstill)



$$E_1 = 4.44 f_1 \phi N_1 k_{w1}$$

$$E_2 = 4.44 f_1 \phi N_2 k_{w2}$$

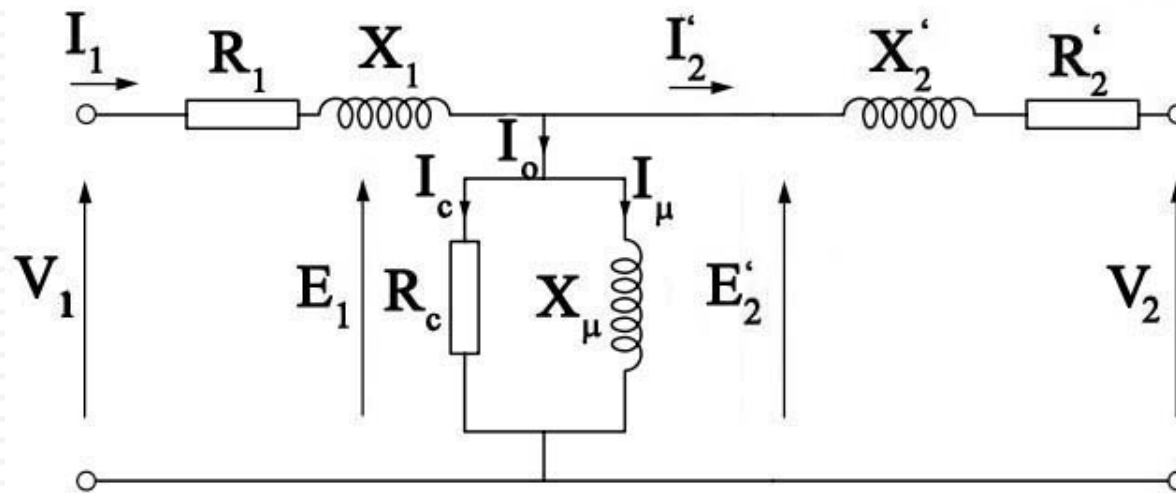
$$\frac{E_1}{E_2} = \frac{N_1 k_{w1}}{N_2 k_{w2}} = a_{eff}$$

$$E_{2\ oc} = 4.44 f_1 \phi N_2 k_{w2}$$

# Induction Machines: Equivalent Circuit

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## A. Rotor open circuited



$$E'_2 = E_1$$

$$\frac{E'_{2\ oc}}{E_{2\ oc}} = \frac{N_1 k_{w1}}{N_2 k_{w2}}$$

$$R'_2 = R_2 \left( \frac{N_1 k_{w1}}{N_2 k_{w2}} \right)^2$$

$$X'_2 = X_2 \left( \frac{N_1 k_{w1}}{N_2 k_{w2}} \right)^2$$

# Induction Machines: Equivalent Circuit

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## B. Rotor running at speed $n$

When Rotor is stationary:

$$E_{2\ oc} = 4.44 f_1 \phi N_2 k_{w2}$$

$$\frac{E_{2\ run}}{E_{2\ oc}} = \frac{f_2}{f_1} = \frac{n_s - n}{n_s} = s$$

$X_2$  = rotor leakage reactance at  $f_1$

$$X_2 = 2\pi f_1 L_2$$

At  $f_2$

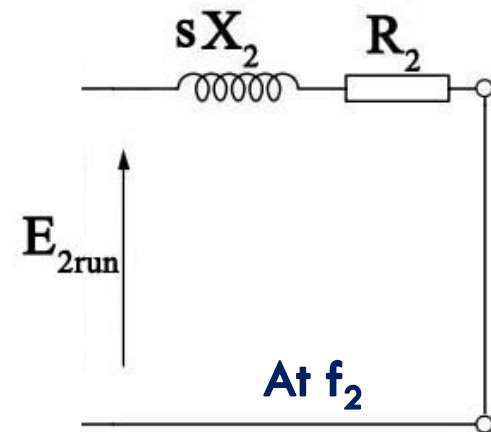
$$X_{2f_2} = 2\pi f_2 L_2 = X_2 \frac{f_2}{f_1} = sX_2$$

When Rotor is running:

$$E_{2\ run} = 4.44 f_2 \phi N_2 k_{w2}$$

$$s = \frac{n_s - n}{n_s}$$

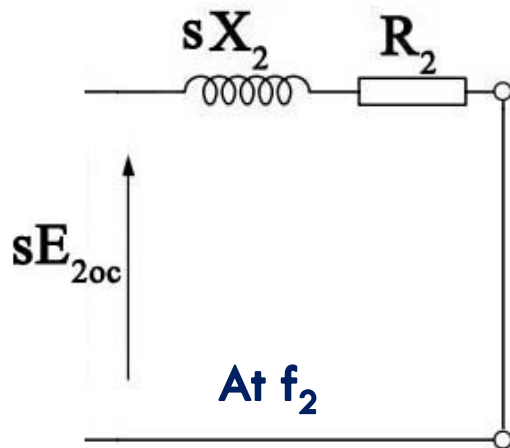
Per-unit Slip



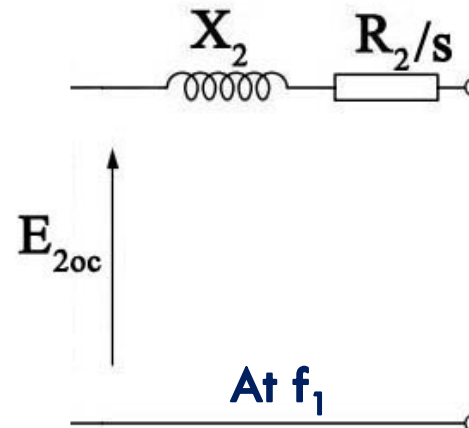
# Induction Machines: Equivalent Circuit

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## B. Rotor running at speed $n$



$$E_{2\ run} = sE_{2\ oc}$$



$$sE_{2\ oc} = I_2(R_2 + jsX_2)$$

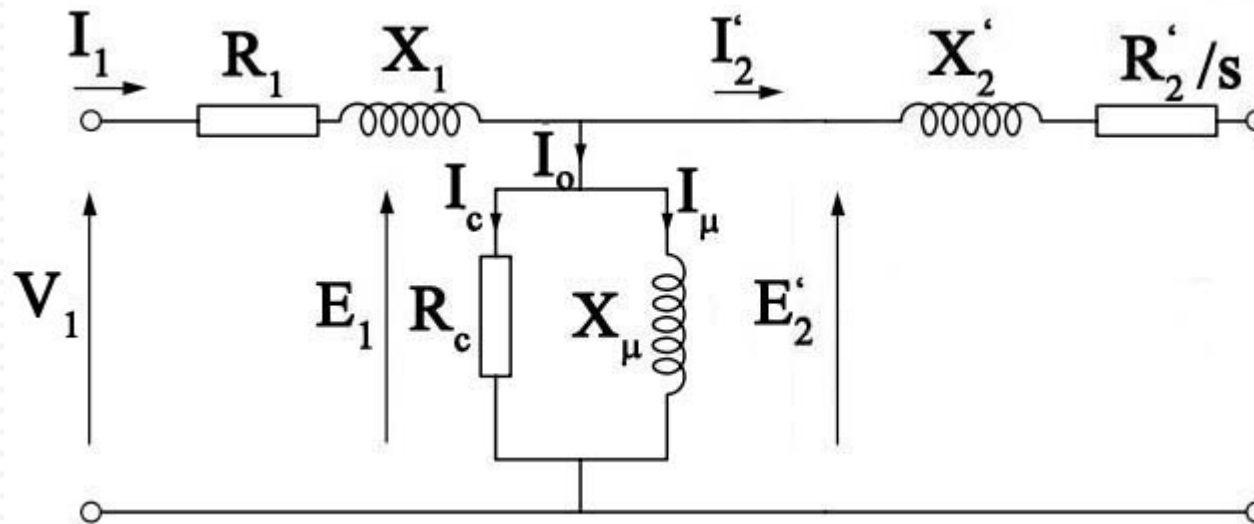
$$E_{2\ oc} = I_2\left(\frac{R_2}{s} + jX_2\right)$$

$$E'_{2\ oc} = I'_2\left(\frac{R'_2}{s} + jX'_2\right)$$

# Induction Machines: Equivalent Circuit

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## B. Rotor running at speed $n$



$$E_1 = E_2' = E$$

$$s = \frac{n_s - n}{n_s} \times \frac{2\pi}{60}$$

$$s = \frac{\omega_s - \omega}{\omega_s}$$

$$n = (1 - s)n_s$$

$$\omega = (1 - s)\omega_s$$

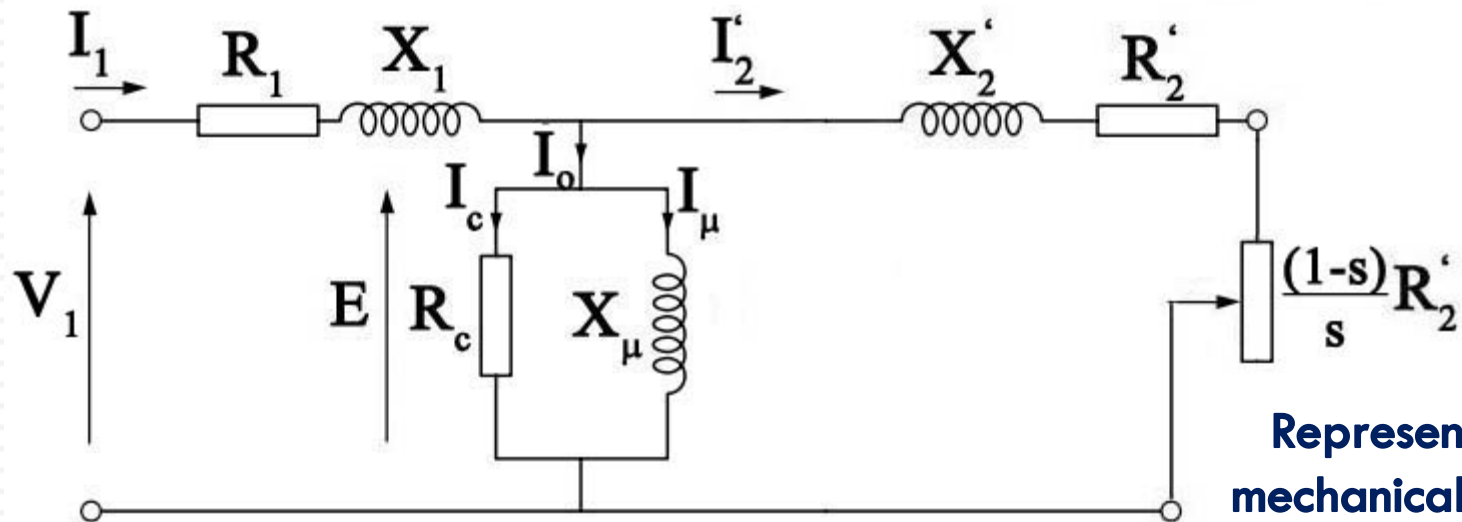
$$n_s = \frac{60f_1}{p} \times \frac{2\pi}{60}$$

$$\omega_s = \frac{2\pi f_1}{p}$$

# Induction Machines: Equivalent Circuit

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## B. Rotor running at speed $n$



**Represents the  
mechanical power  
produced by the  
motor**

$$\frac{R_2'}{s} = R_2' + \frac{R_2'}{s} - R_2'$$

$$\frac{R_2'}{s} = R_2' + \left(\frac{1-s}{s}\right)R_2'$$

# Induction Machines: Power Flow

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$$P_{in} = \sqrt{3} V_L I_L \cos \phi$$

$$P_{cu1} = 3 I_1^2 R_1$$

$$P_{iron} = 3 I_c^2 R_c$$

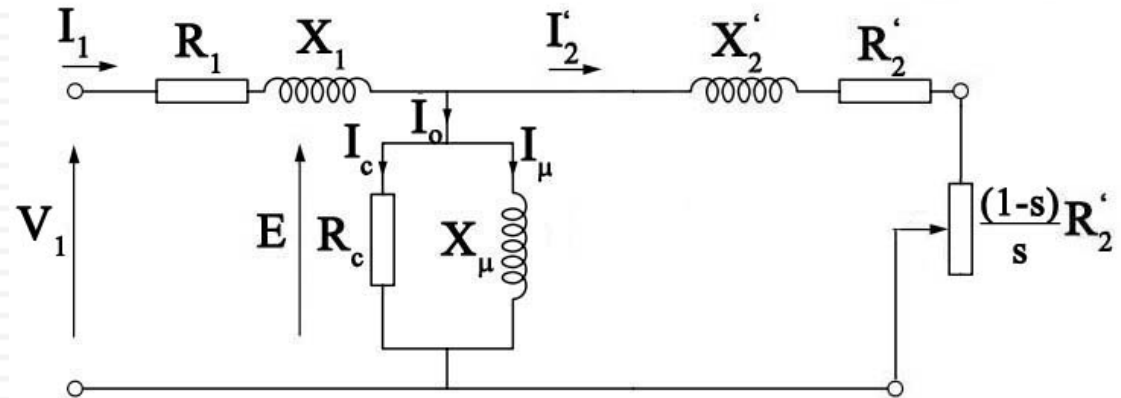
$$P_g$$

$$P_{cu2} = 3 I_2'^2 R_2'$$

$$P_d$$

$$P_{mech\ losses}$$

$$P_{out} = T_{out} \omega$$



$P_g =$  air-gap power

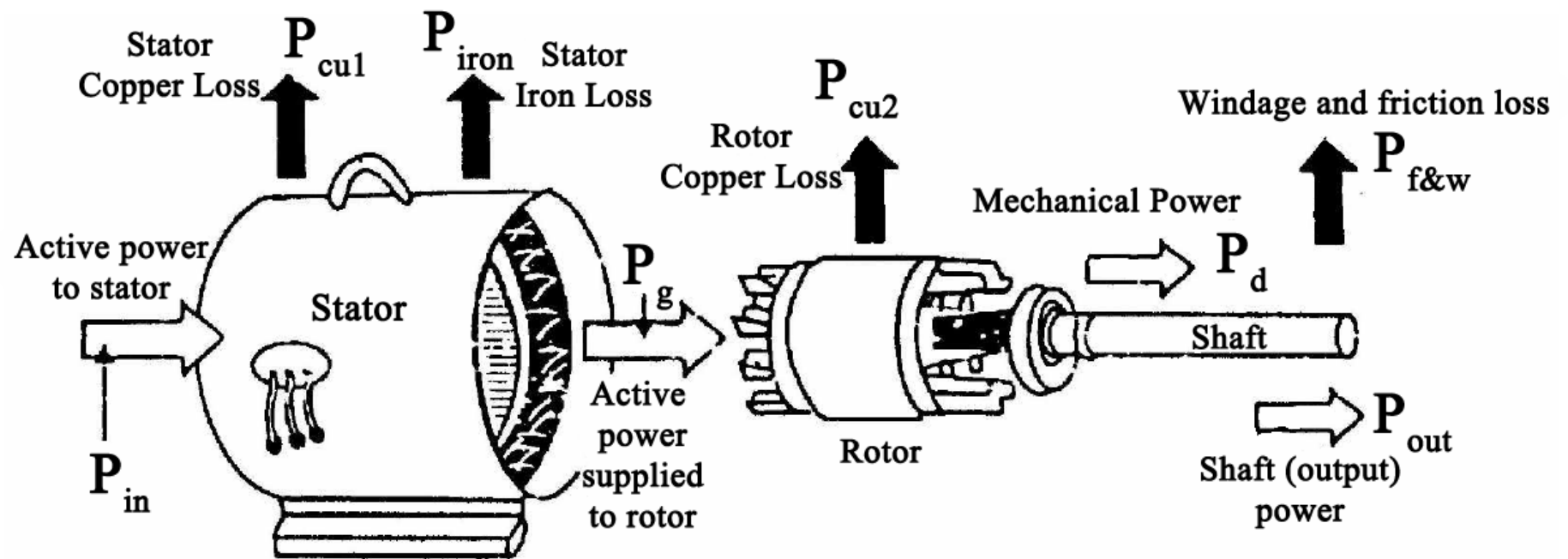
$P_d =$  developed mech. power

$$P_g = 3 I_2'^2 \frac{R_2'}{s} = P_{in} - P_{cu1} - P_{iron}$$

$$P_d = 3 I_2'^2 \frac{(1-s)}{s} R_2' = P_g - P_{cu2}$$

# Induction Machines: Efficiency & Torque

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# Induction Machines: Efficiency & Torque

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$$P_{in} = \sqrt{3} V_L I_L \cos \phi$$

$$\eta = \frac{P_{out}}{P_{in}}$$

$$T_{out} = \frac{P_{out}}{\omega}$$

$$T_d = \frac{P_d}{\omega}$$

$$P_{cu1} = 3I_1^2 R_1$$

$$P_{iron} = 3I_c^2 R_c$$

$$P_g$$

$$P_g = 3I_2'^2 \frac{R_2'}{s}$$

$$P_g : P_{cu2} : P_d$$

$$1 : s : (1-s)$$

$$P_{cu2} = 3I_2'^2 R_2'$$

$$P_d$$

$$P_d = 3I_2'^2 \frac{(1-s)}{s} R_2'$$

$$P_{mech losses}$$

$$T_d = \frac{P_d}{(1-s)\omega_s} = \frac{P_g}{\omega_s}$$

$$P_{out} = T_{out} \omega$$

