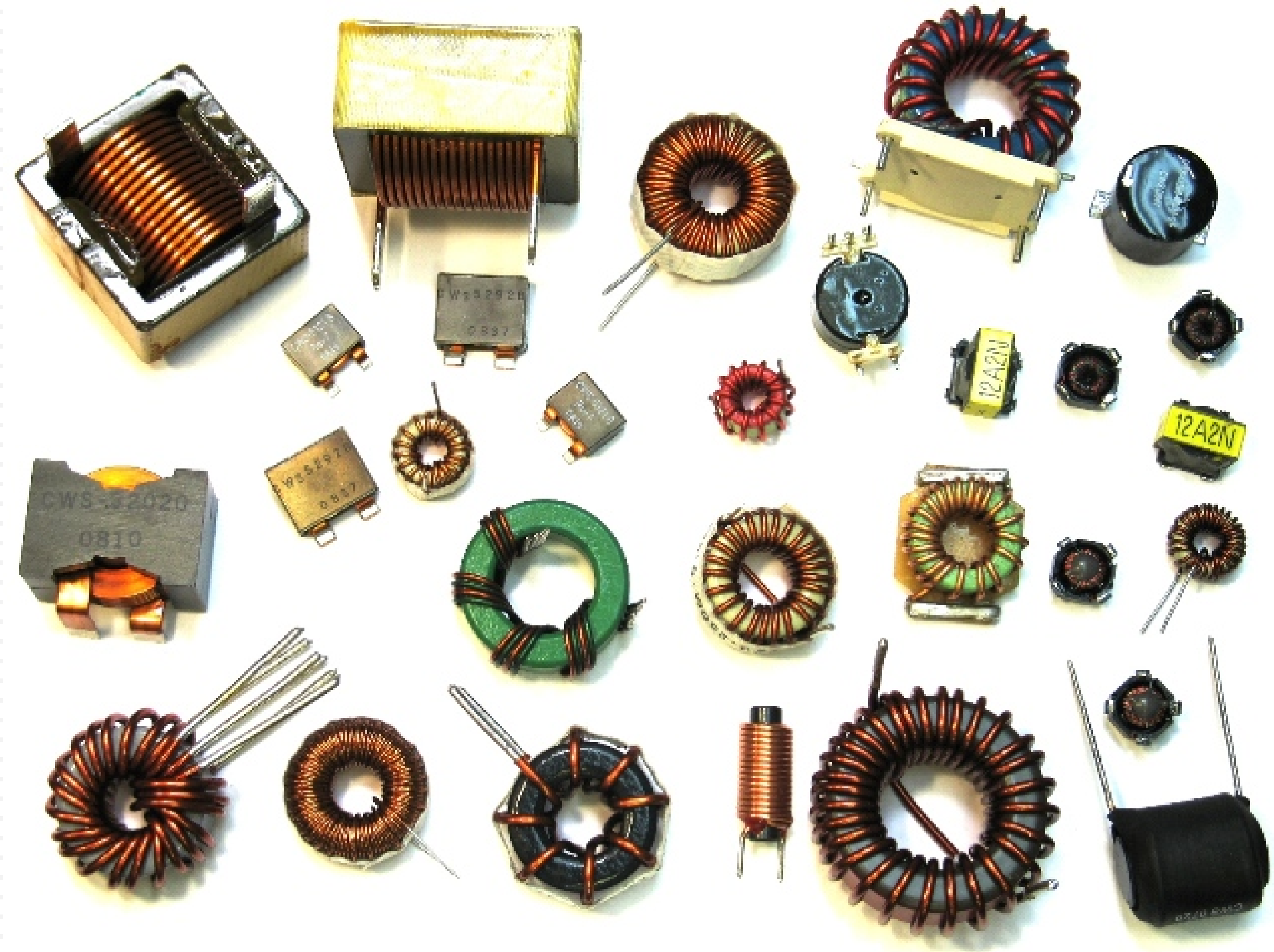
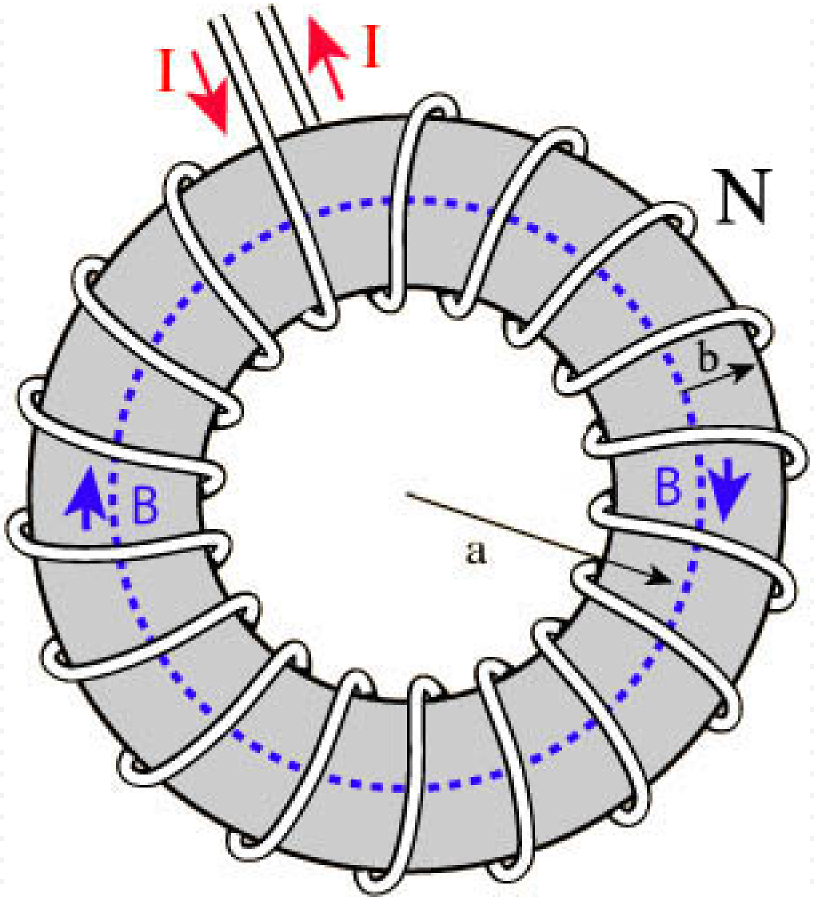
Faraday's Law Applications

1. Inductors.
2. Transformers.
3. Electrical Generators.
4. Electrical Motors.
5. Transmission Lines.

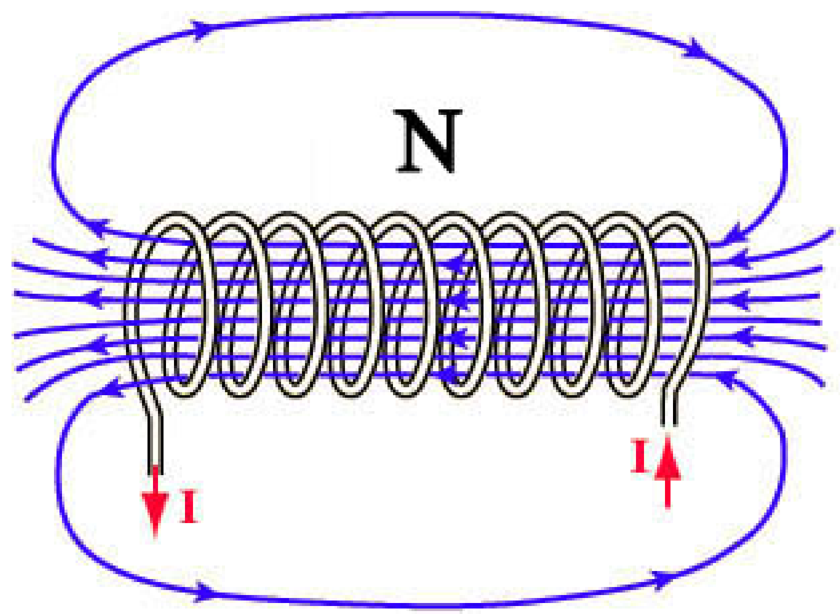
Inductors



Inductors



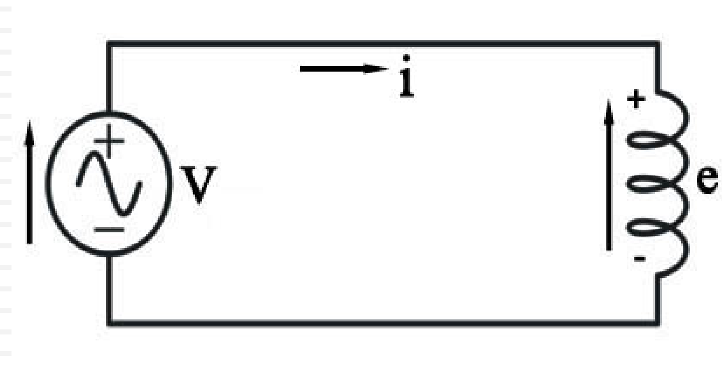
Toroid



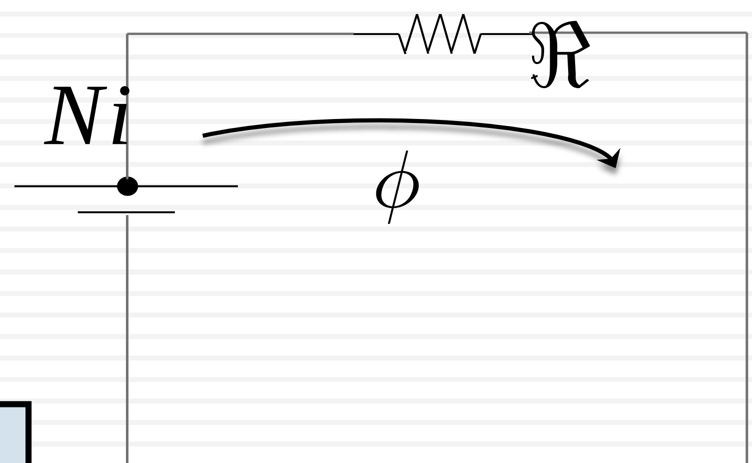
Solenoid

Inductors

$$v = N\frac{d\phi}{dt}$$



$$Ni = \phi\Re \Rightarrow \phi = \frac{Ni}{\Re}$$



$$\Rightarrow v = \frac{N^2}{\Re}\frac{di}{dt}$$

$$\Rightarrow v = L\frac{di}{dt} \Rightarrow L = \frac{N^2}{\Re}$$

Inductors

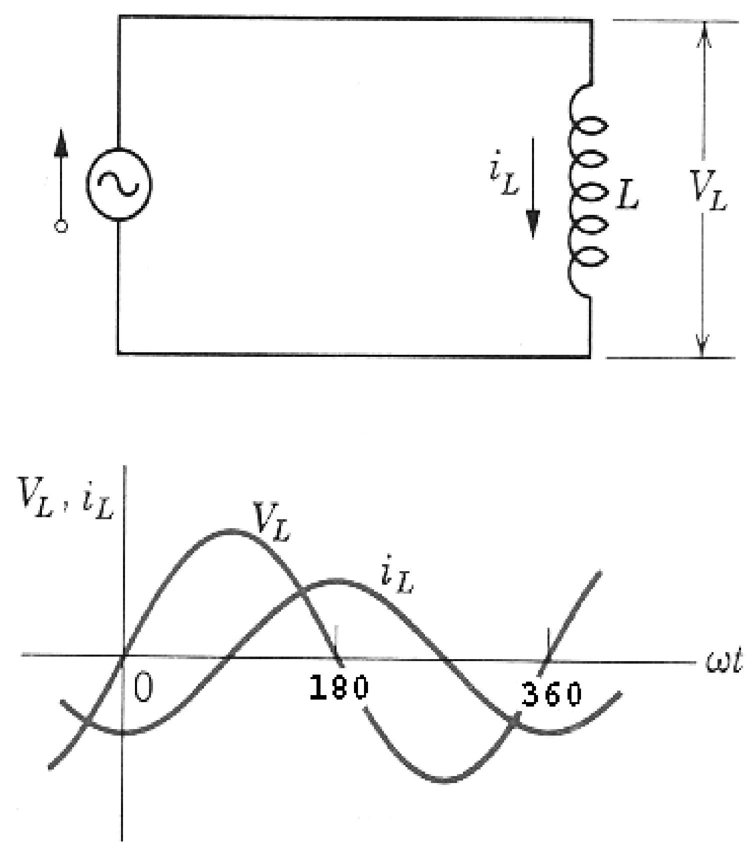
By Integration

$$i = \frac{V_m}{\omega L} \sin(\omega t - \frac{\pi}{2})$$

$$I = \frac{V}{j\omega L}$$

$$V = j(\omega L)I$$

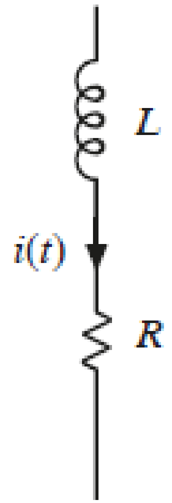
$$V = jX_L I$$



Inductors: Introduction to Inductor Design

Objective:

Design an inductor having a given inductance L, which carries worst-case current I_{max} without saturating the core (B_{max}), and which has a given winding resistance R, or, equivalently, exhibits a worst-case copper loss of I_m^2R

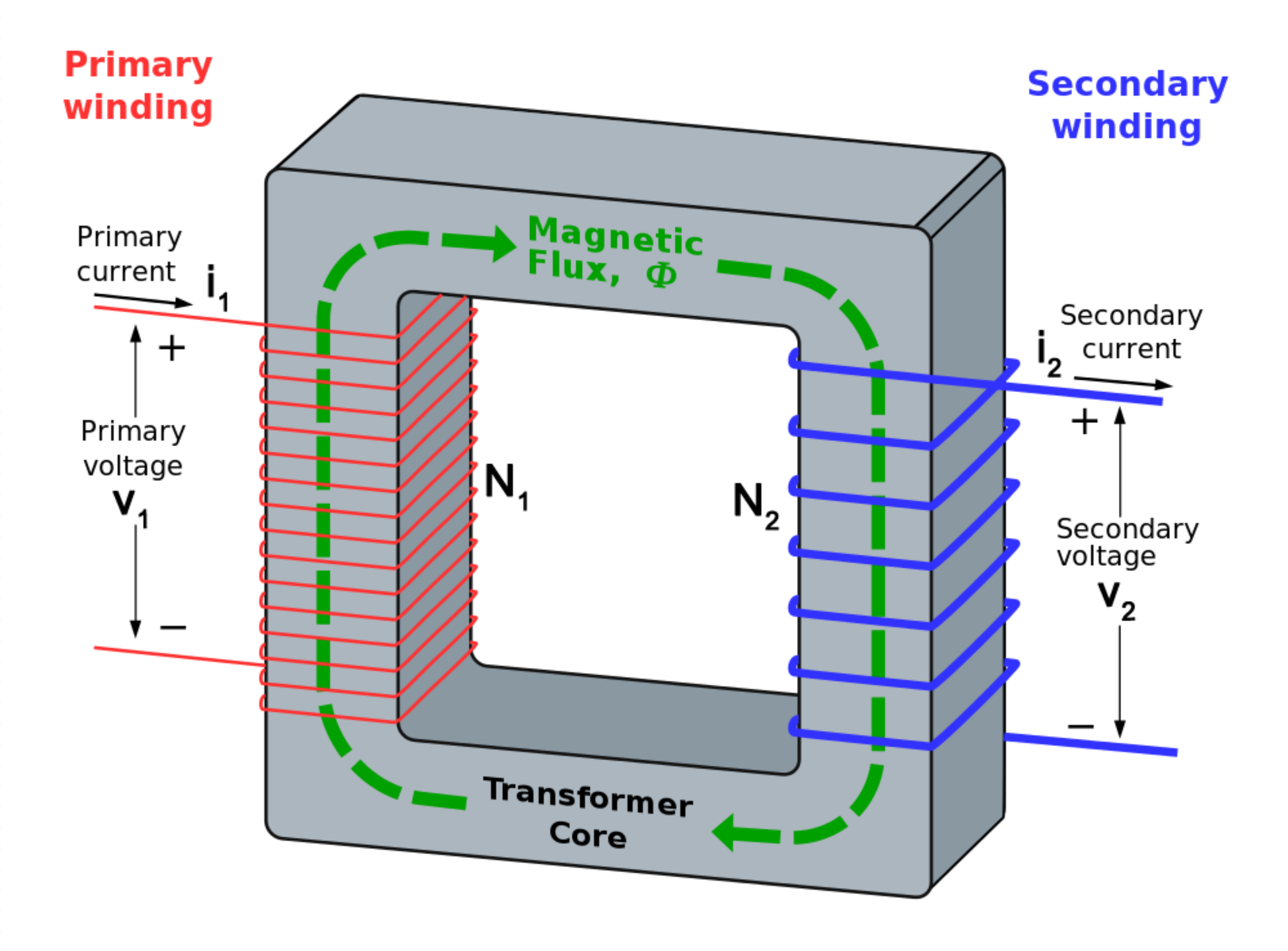


Transformer

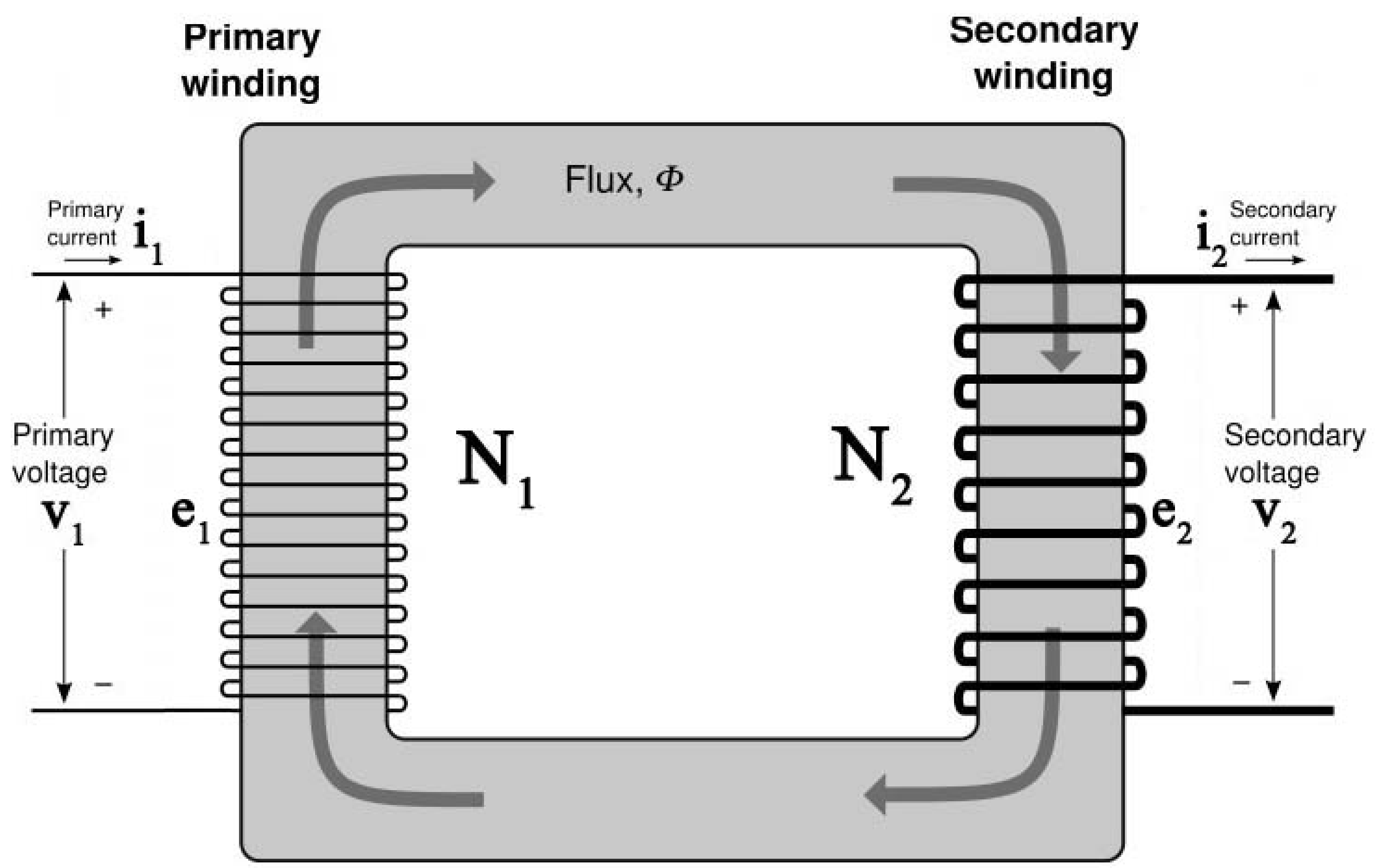
Single-phase transformer

Transformer

Single-phase transformer



Transformer



Transformer

Ideal Transformer

- R_{coil} = zero
- μ → ∞
- No leakage flux
- No core losses

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_2}{dt}$$

Since no leakage

$$\phi_1 = \phi_2$$

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

$$\frac{e_1}{e_2} = \frac{N_1}{N_2}$$

Transformer

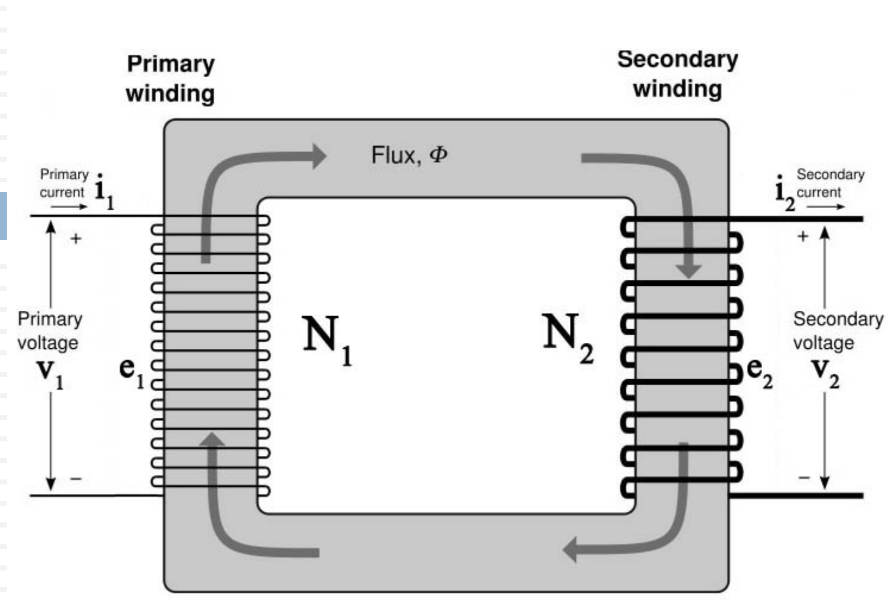
Ideal Transformer

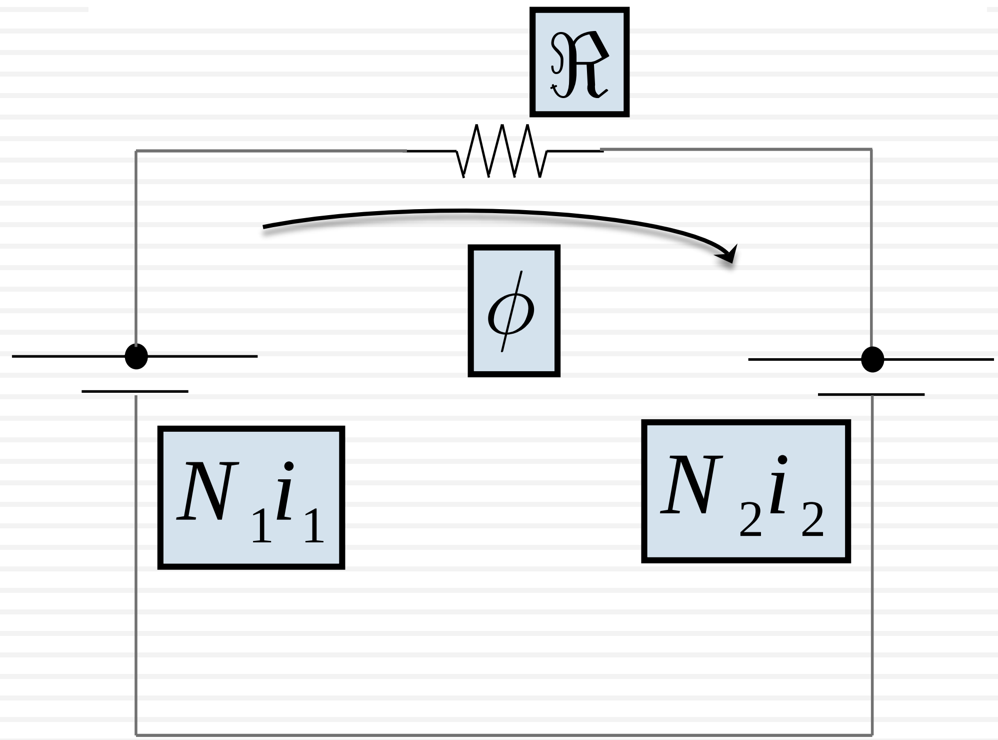
$$N_1 i_1 - N_2 i_2 = \phi \Re$$

Since μ → ∞

$$\Re = zero$$

$$\Rightarrow \quad \frac{N_1}{N_2} = \frac{i_2}{i_1}$$





Transformer

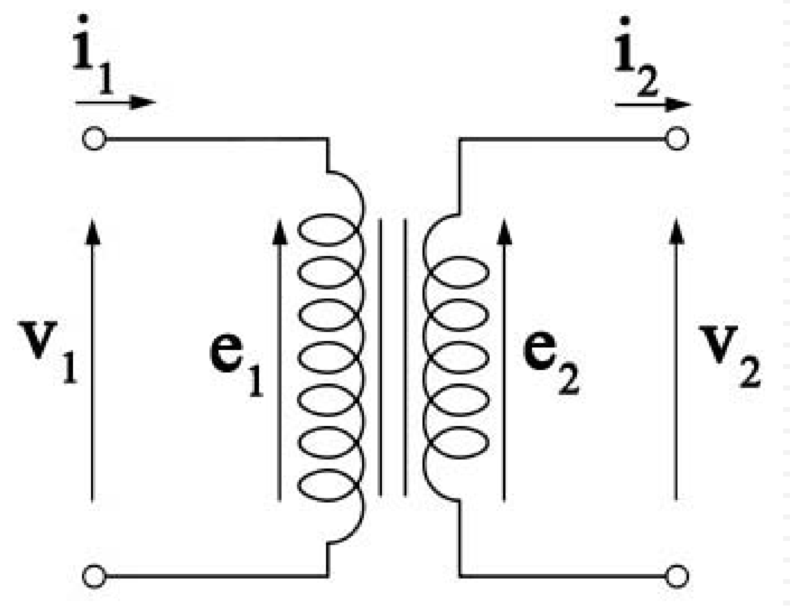
Ideal Transformer

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

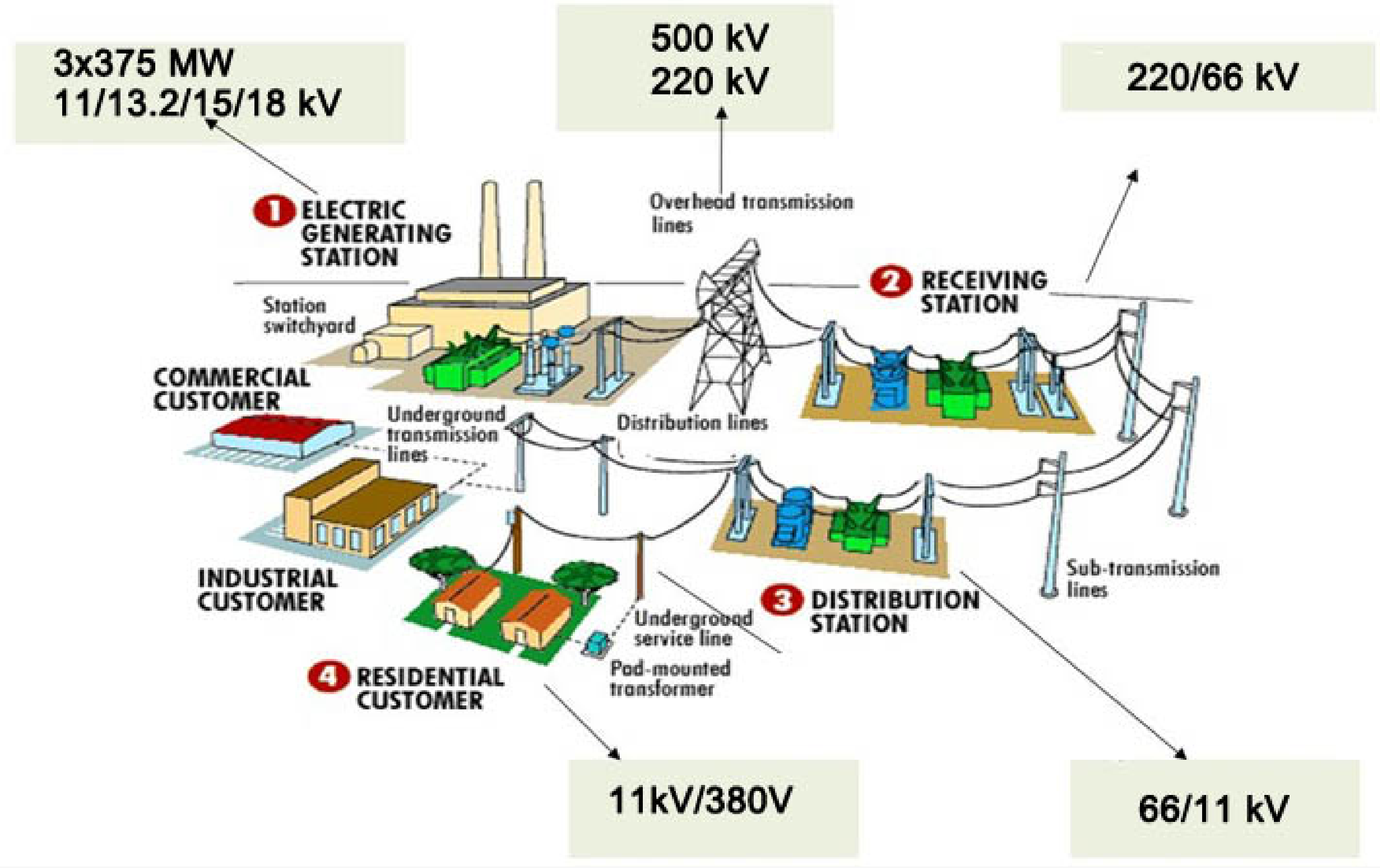
$$\frac{e_1}{e_2} = \frac{N_1}{N_2}$$

$$\frac{i_2}{i_1} = \frac{N_1}{N_2}$$

$$v_1 i_1 = v_2 i_2$$



Transformer: Applications



Transformer: Equivalent Circuit

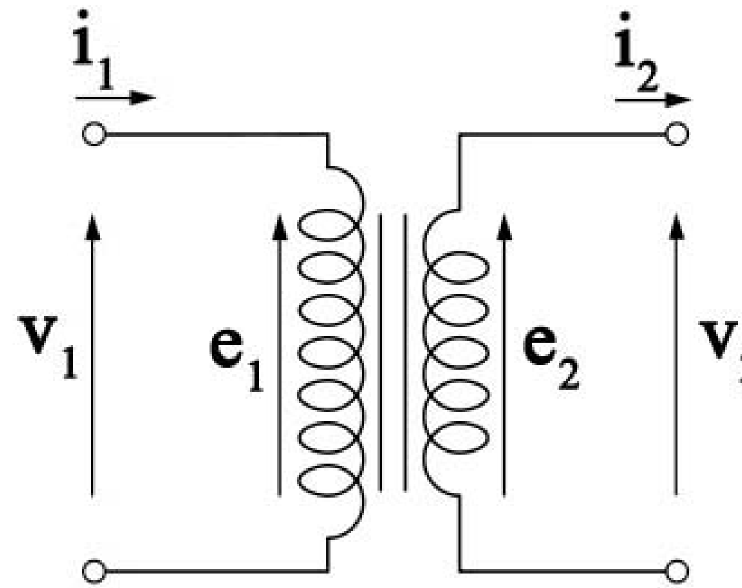
Case A: Ideal Transformer

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

$$\frac{e_1}{e_2} = \frac{N_1}{N_2}$$

$$\frac{i_2}{i_1} = \frac{N_1}{N_2}$$

$$v_1 i_1 = v_2 i_2$$



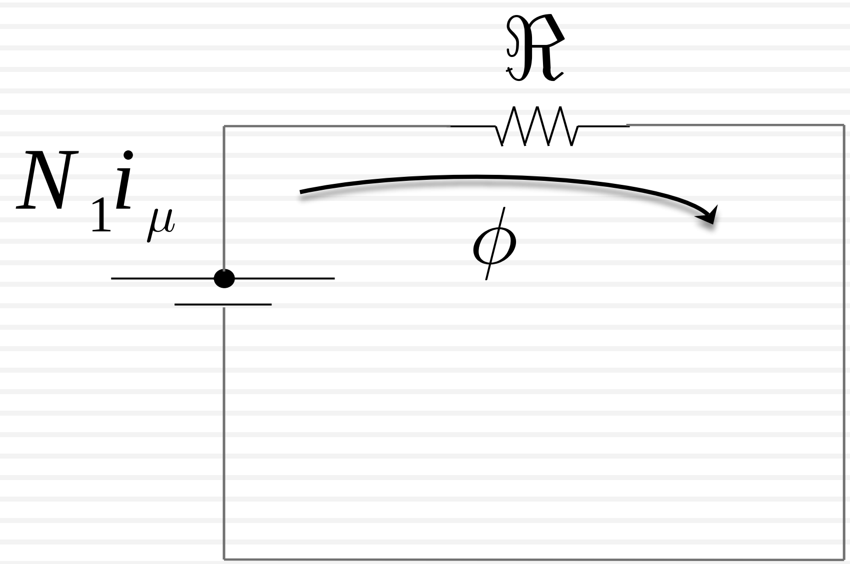
Transformer: Equivalent Circuit

Case B: - R_{coil} = zero - $\mu \neq \infty$

- No leakage flux - No core losses

At No-load: (i_2 = zero)

$$N_1 i_\mu = \phi \Re$$



From Faraday's Law

$$\phi = \frac{V_{1m}}{\omega N_1} \sin(\omega t - \frac{\pi}{2})$$

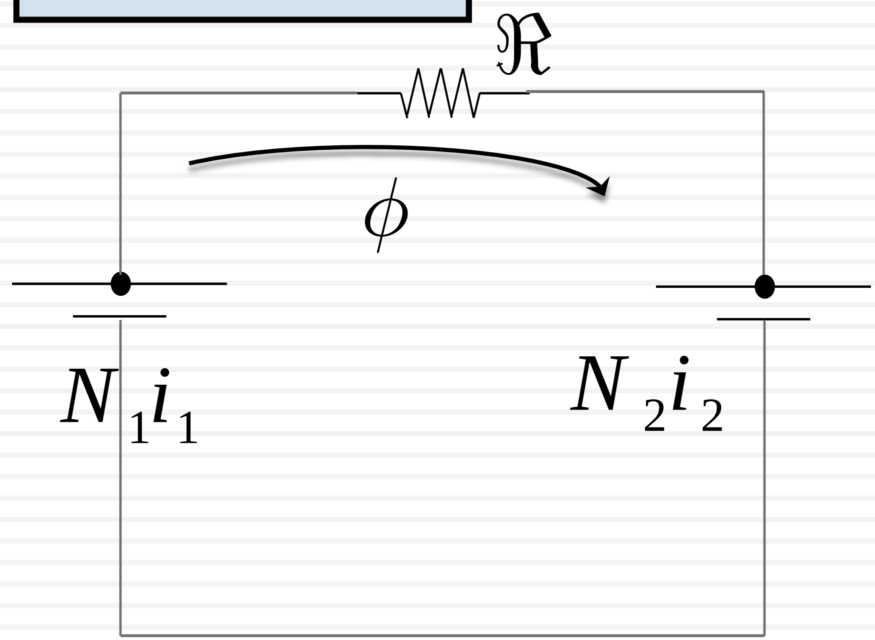
$$i_\mu = \frac{V_{1m}}{\omega N_1^2} \Re \sin(\omega t - \frac{\pi}{2})$$

Transformer: Equivalent Circuit

$$i_\mu = \frac{V_{1m}}{\omega L_{core}} \sin(\omega t - \frac{\pi}{2})$$

$$I_\mu = \frac{V_1}{j\omega L_{core}}$$

$$X_\mu = \omega L_{core}$$



If the transformer is loaded:

$$N_1 i_1 - N_2 i_2 = \phi \Re$$

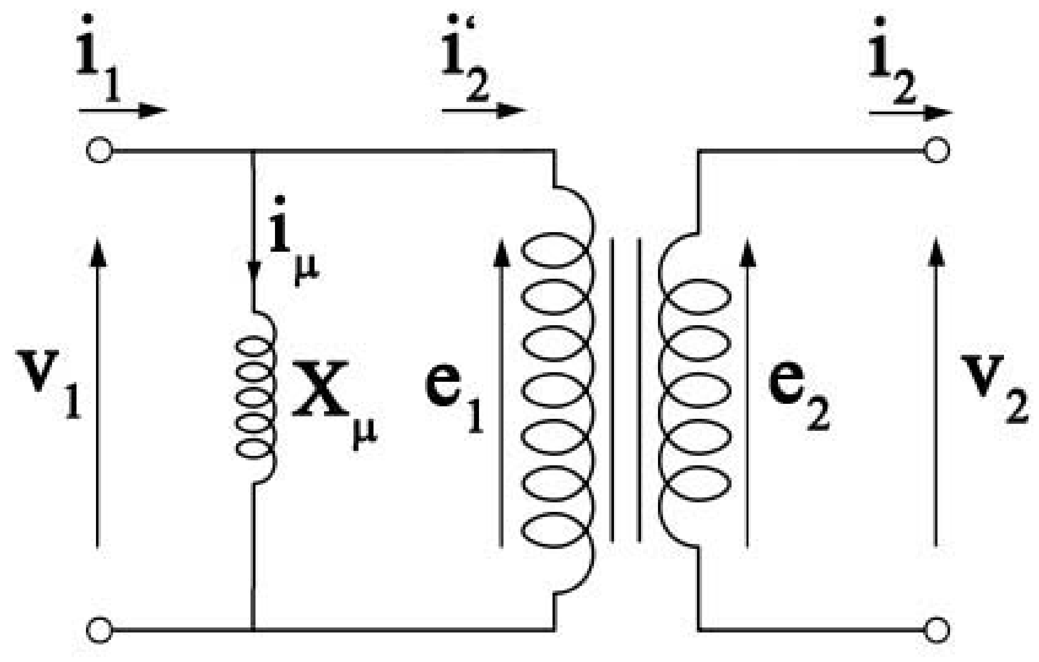
$$N_1 i_1 - N_2 i_2 = N_1 i_\mu$$

Transformer: Equivalent Circuit

$$i_\mu = i_1 - \frac{N_2}{N_1} i_2$$

$$i_1 = i_\mu + \frac{N_2}{N_1} i_2$$

$$i_2^{'} = \frac{N_2}{N_1} i_2$$



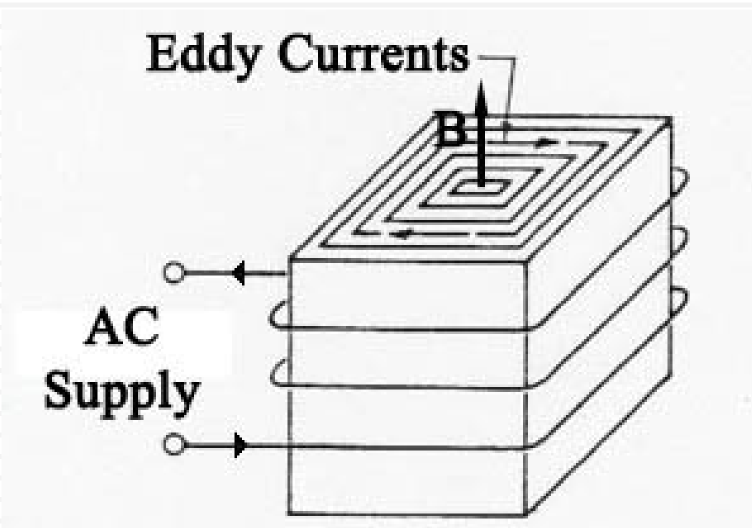
Transformer: Equivalent Circuit

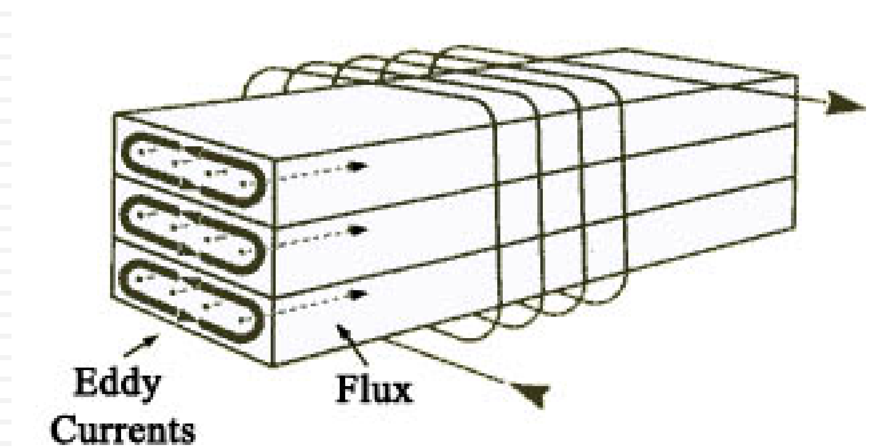
Case C: - R_{coil} = zero - $\mu \neq \infty$

- No leakage flux - core losses exist.

Core (iron) losses

1. Eddy Current Losses





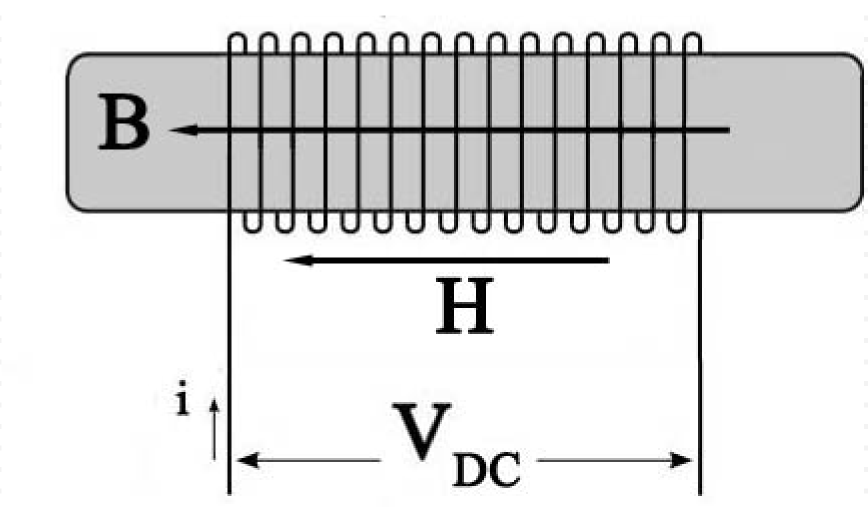
$$P_e \propto B_m{}^2 f^{\,2} (thickness)^2$$

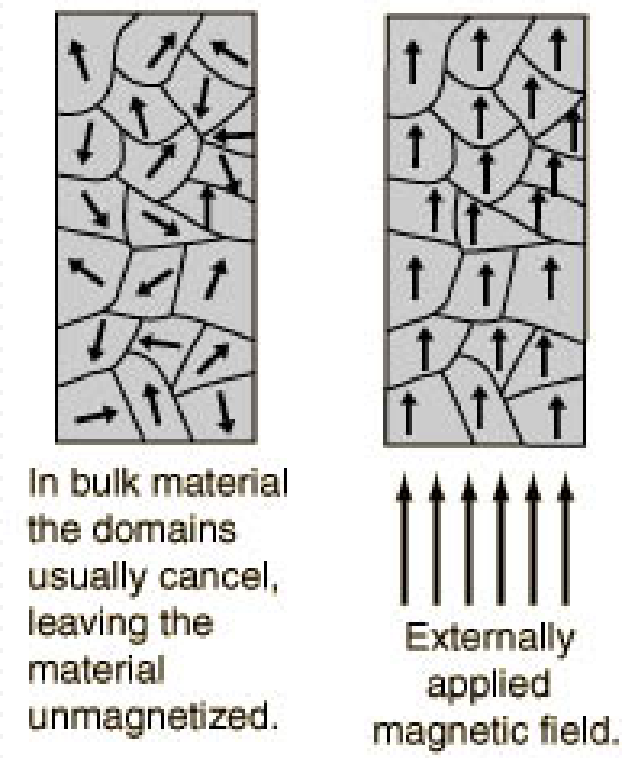
$$P_e = k_e B_m{}^2 f^{\,2} (thickness)^2 \quad W/m^3$$

Transformer: Equivalent Circuit

Case C:

2. Hysteresis Losses

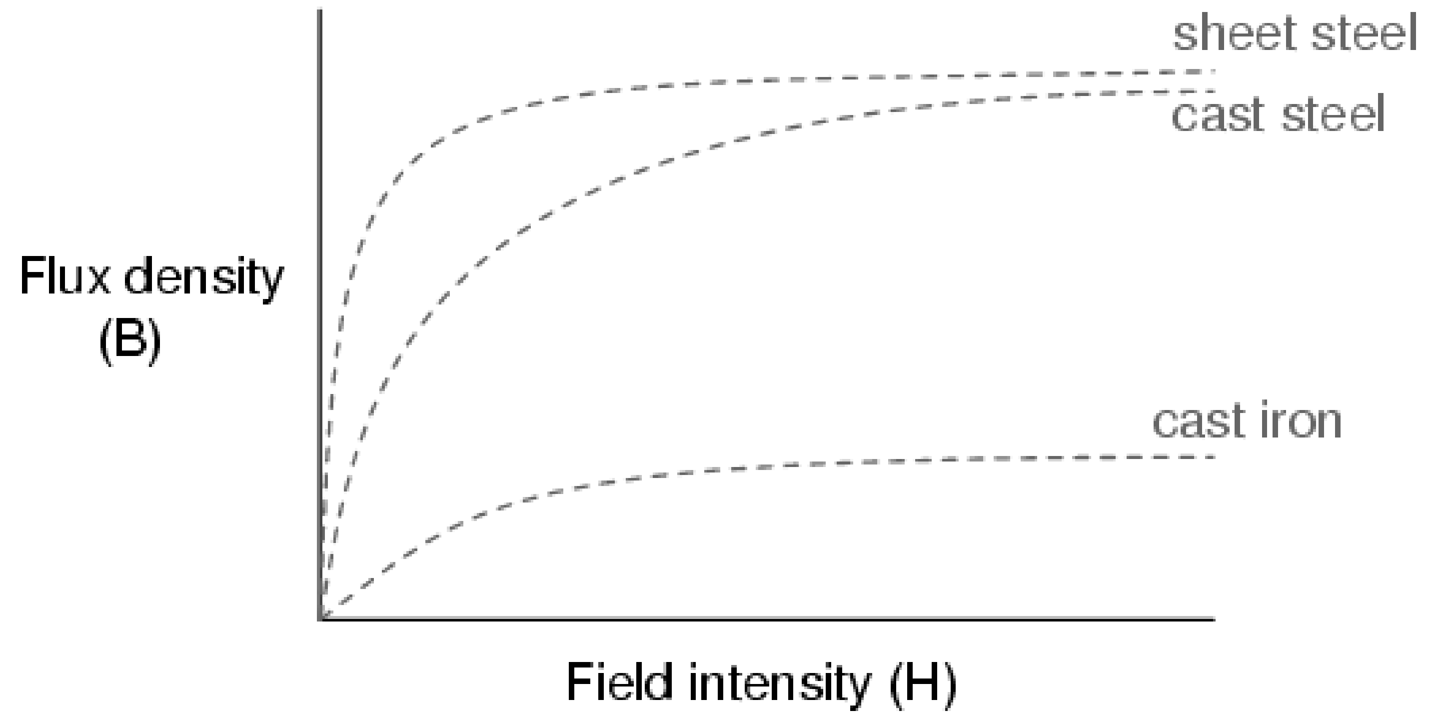




Transformer: Equivalent Circuit

Case C:

2. Hysteresis Losses

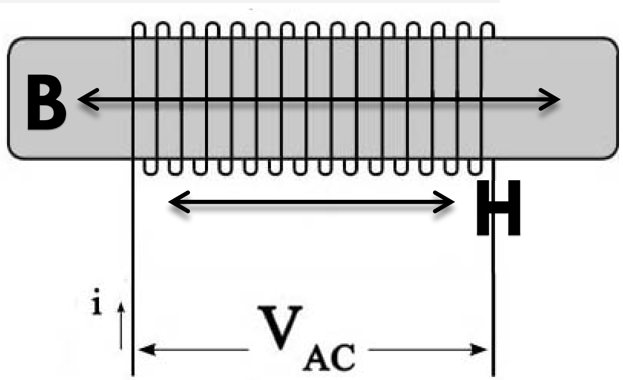


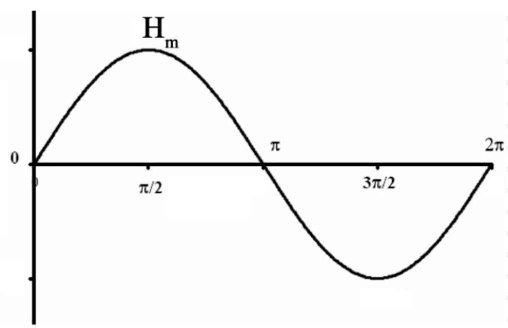
BH Curve (Magnetization Curve)

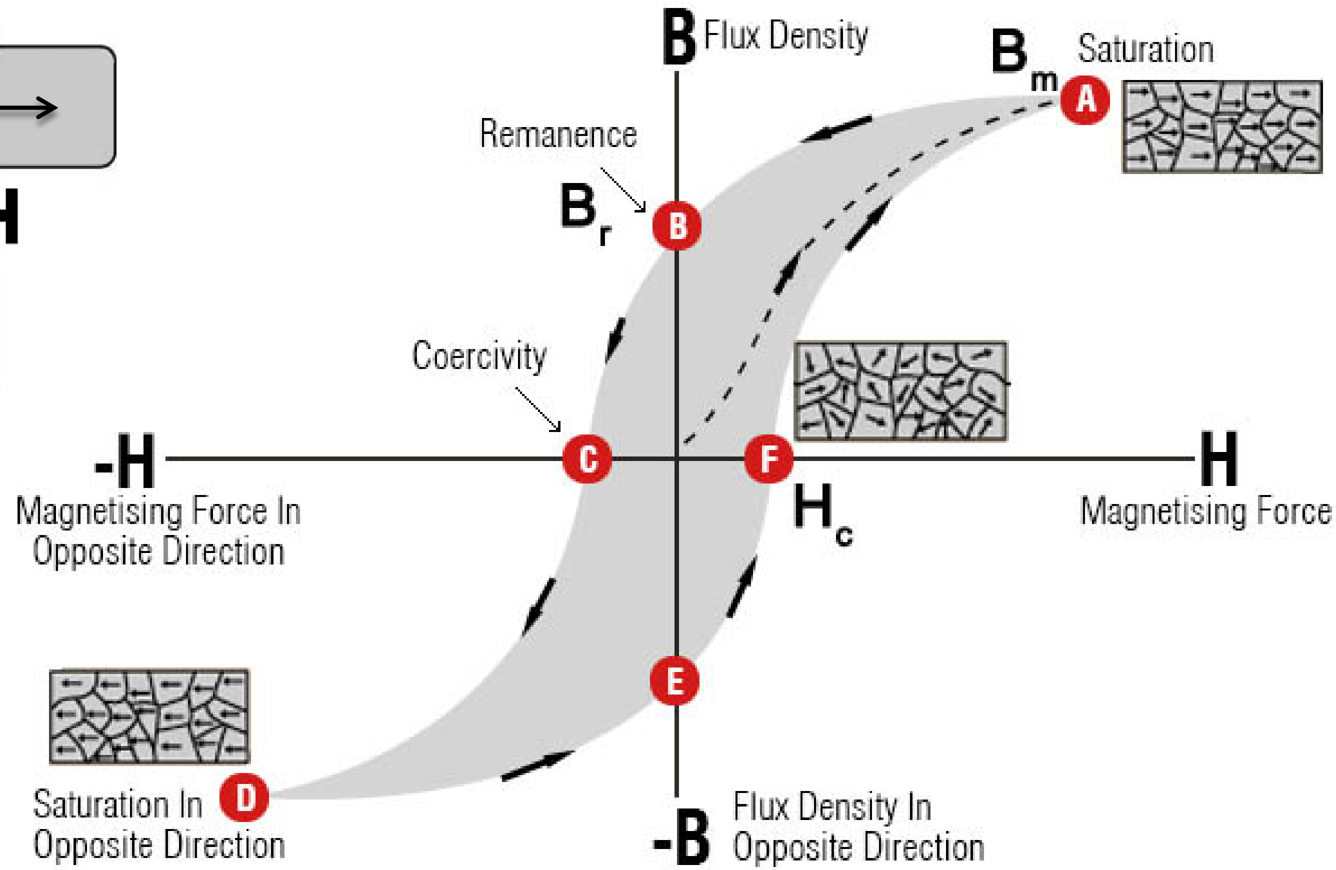
Transformer: Equivalent Circuit

Case C:

2. Hysteresis Losses







Transformer: Equivalent Circuit

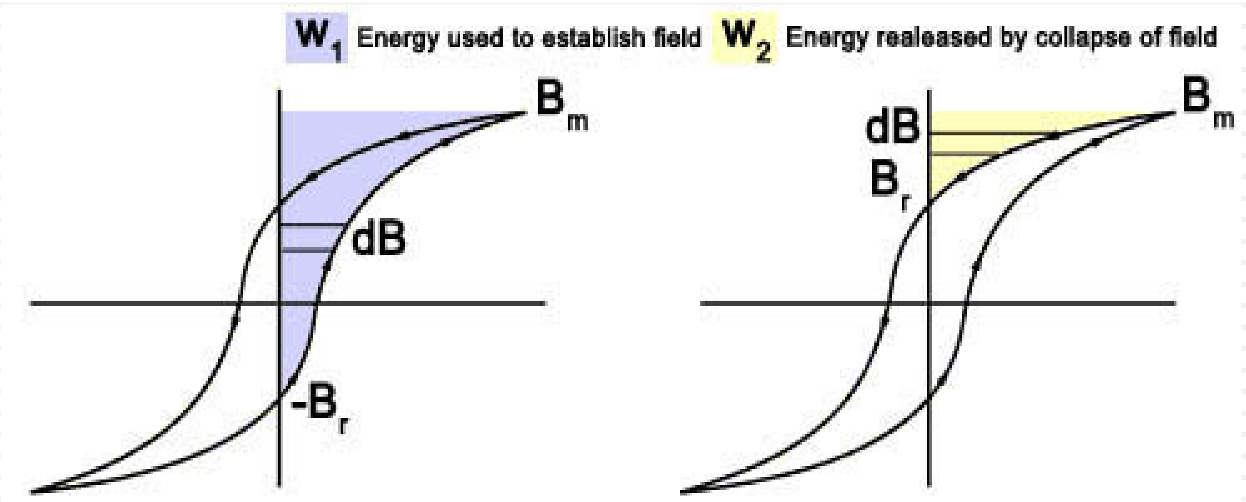
Case C:

2. Hysteresis Losses

$$H\,(\mathrm{A.m^{-1}}) \times B\,(T = J.A^{-1}.m^{-2}) = J.m^{-3}$$

$$w_1 = \int_{-B_r}^{B_m} \underline{H}.\underline{dB}$$

$$w_2 = \int_{B_m}^{B_r} \underline{H}.\underline{dB}$$



Energy lost /unit volume/cycle = 2($w_1 - w_2$)= area of the loop

Transformer: Equivalent Circuit

Case C:

2. Hysteresis Losses

$$P_h \propto B^n f\ (volume)$$

$$P_h = k_h B^n f$$ W/m^3

***n* depends on the material (1.6 – 2.4)**

***Total core losses*:**

$$P_c = k_e B^2 f^2 (thickness)^2 + k_h B^n f$$ W/m^3

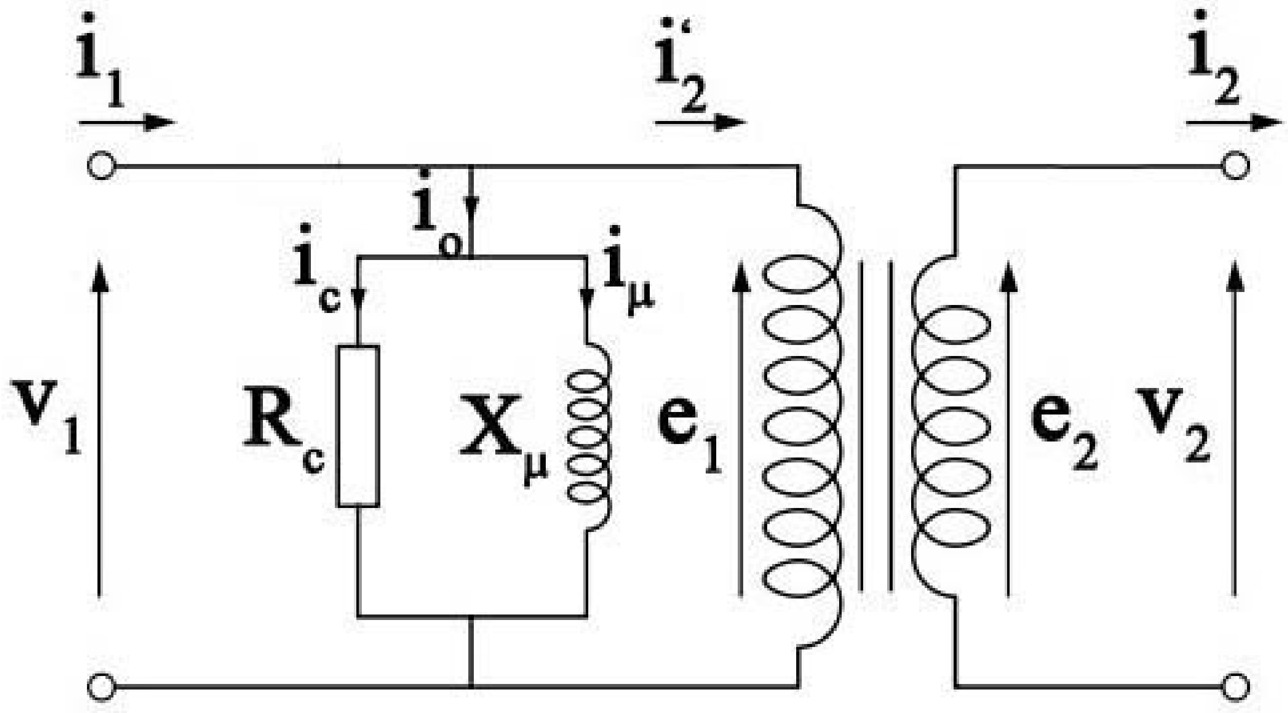
Since f is usually constant

$$\Rightarrow P_c \propto B^2 \Rightarrow P_c \propto \phi^2 \Rightarrow P_c \propto V^2$$

Transformer: Equivalent Circuit

<u>Case C:</u>

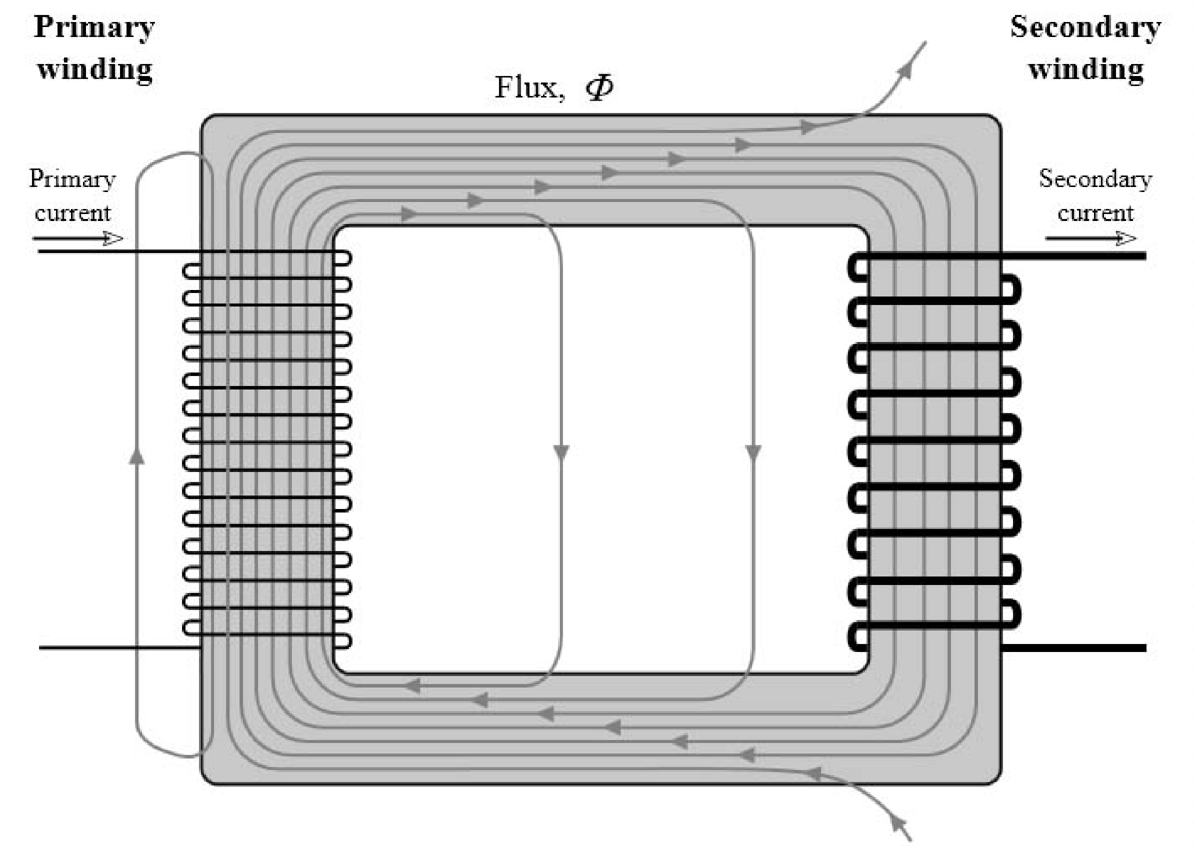
$$P_c = \frac{V_1^{\,2}}{R_c}$$



Transformer: Equivalent Circuit

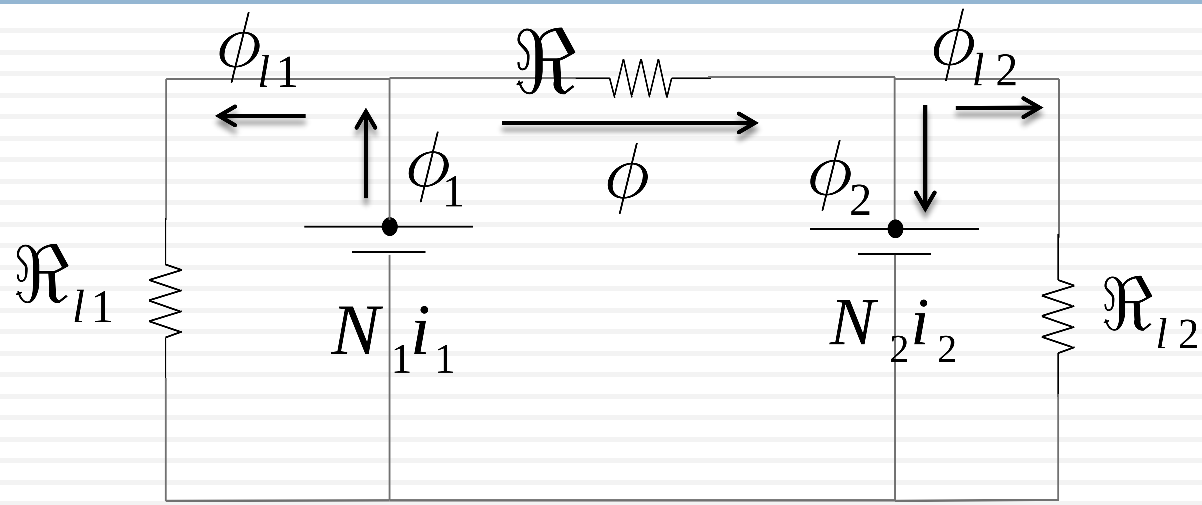
Case D: - R_{coil} = zero - $\mu \neq \infty$

- Leakage flux exists. - core losses exist.



Transformer: Equivalent Circuit

Case D:



$$\phi_1 = \phi + \phi_{l1}$$ (1)

$$\phi_2 = \phi - \phi_{l2}$$ (2)

$$N_1 i_1 = \phi_{l1} \Re_{l1}$$ (3)

$$N_2 i_2 = \phi_{l2} \Re_{l2}$$ (4)

From (1) & (3)

$$N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi}{dt} + N_1 \frac{d\phi_{l1}}{dt}$$

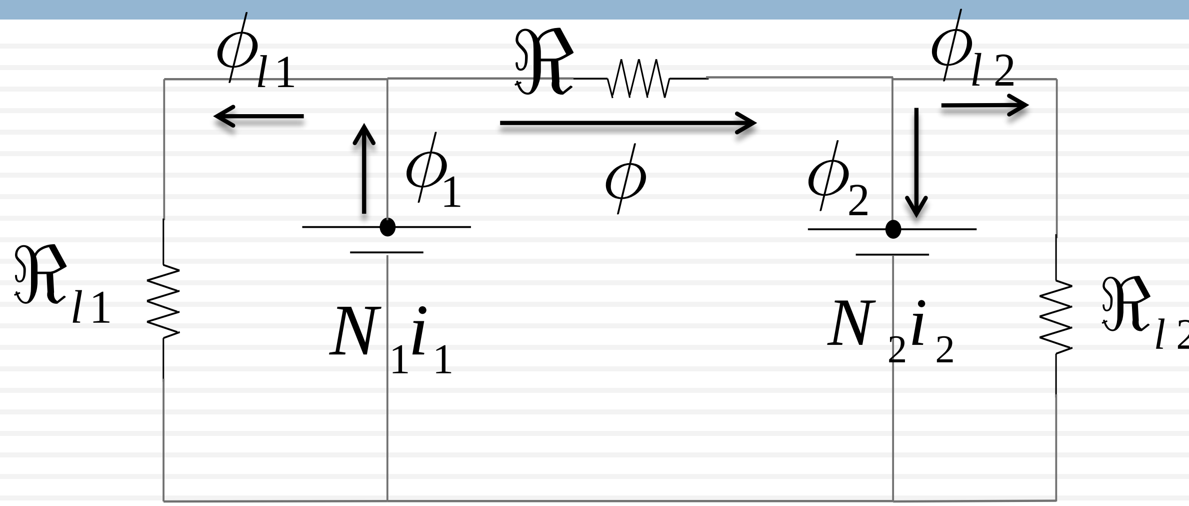
$$v_{l1} = N_1 \frac{d\phi_{l1}}{dt} = \frac{N_1^2}{R_{l1}} \frac{di_1}{dt}$$

Transformer: Equivalent Circuit

Case D:

$$\phi_2 = \phi - \phi_{l2} \quad (2)$$

$$N_2 i_2 = \phi_{l2} \Re_{l2} \quad (4)$$



From (2) & (4)

$$N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi}{dt} - N_2 \frac{d\phi_{l2}}{dt}$$

$$v_{l2} = N_2 \frac{d\phi_{l2}}{dt} = \frac{N_2^2}{R_{l2}} \frac{di_2}{dt}$$

Transformer: Equivalent Circuit

Case D:

let

$$N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi}{dt} + N_1 \frac{d\phi_{l1}}{dt}$$

$$N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi}{dt} - N_2 \frac{d\phi_{l2}}{dt}$$

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

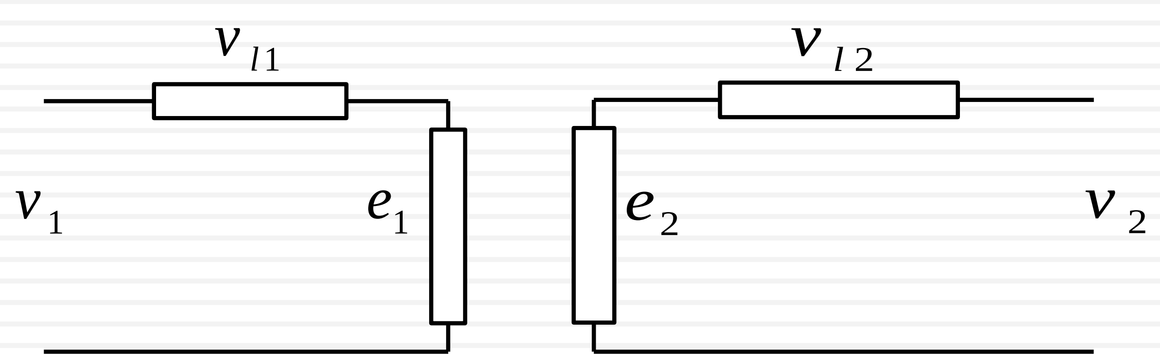
$$e_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = N_2 \frac{d\phi_2}{dt}$$

$$e_2 = N_2 \frac{d\phi}{dt}$$

$$v_1 = e_1 + v_{l1}$$

$$v_2 = e_2 - v_{l2}$$



Transformer: Equivalent Circuit

Case D:

$$v_{l1} = \frac{N_1^2}{R_{l1}} \frac{di_1}{dt}$$

$$v_{l1} = L_{l1} \frac{di_1}{dt}$$

$$v_{l2} = \frac{N_2^2}{R_{l2}} \frac{di_2}{dt}$$

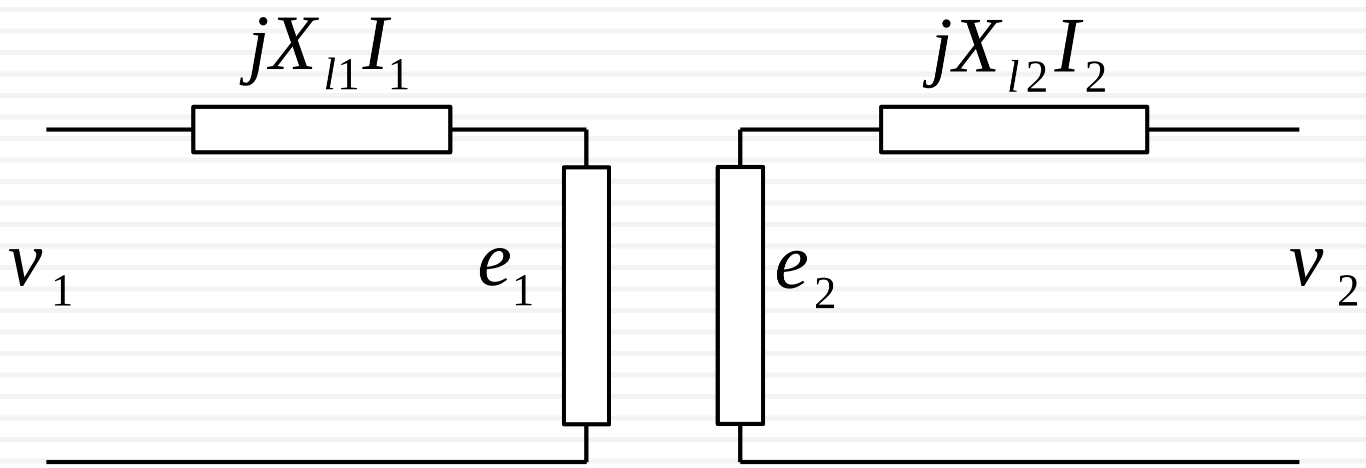
$$v_{l2} = L_{l2} \frac{di_2}{dt}$$

$$V_{l1} = j\omega L_{l1} I_1$$

$$V_{l1} = jX_{l1} I_1$$

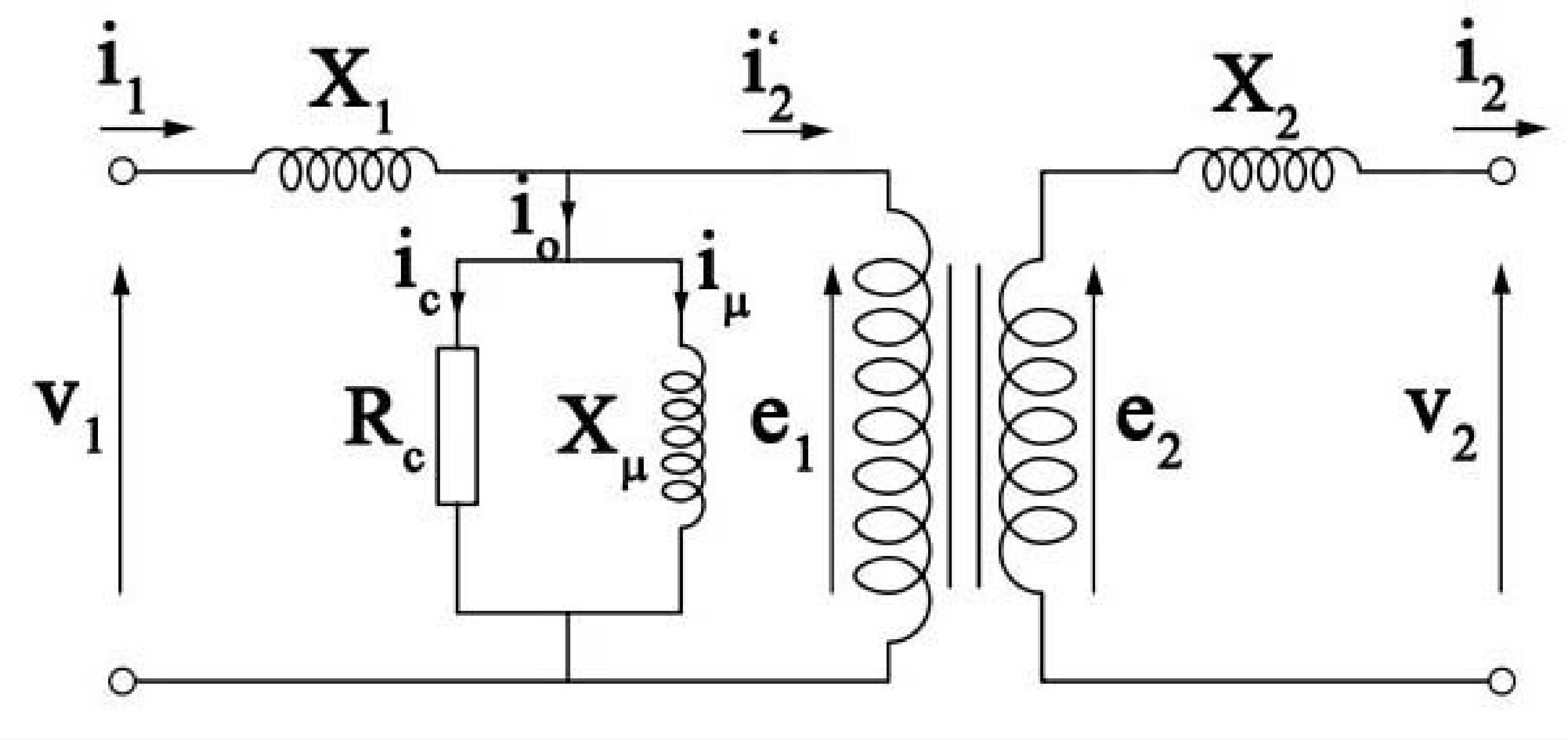
$$V_{l2} = j\omega L_{l2} I_2$$

$$V_{l2} = jX_{l2} I_2$$



Transformer: Equivalent Circuit

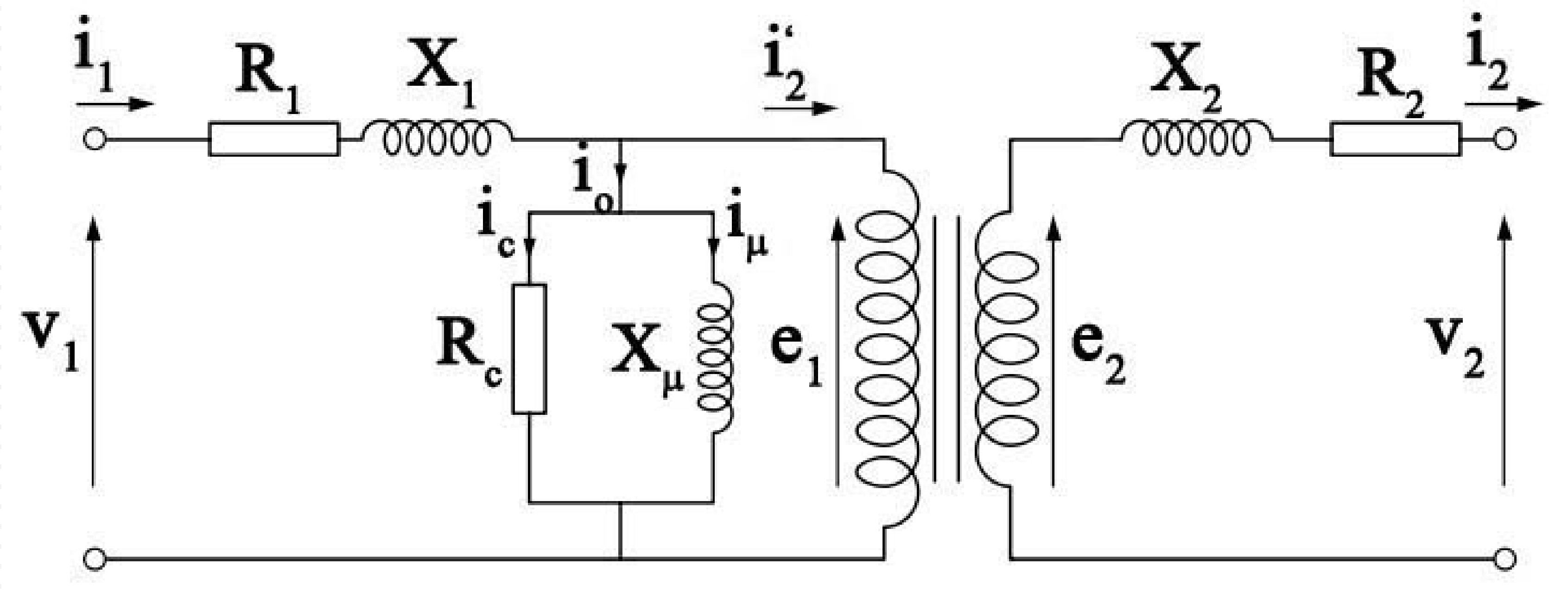
<u>*Case D:*</u>



Transformer: Equivalent Circuit

<u>*Case E*:</u> - $R_{coil} \neq$ zero - $\mu \neq \infty$

- Leakage flux exists. - core losses exist.



Exact Equivalent Circuit of the Transformer