Transformer

- Construction
- Theory of Operation
- Equivalent Circuit
- Transformer Rating
- Per-Unit System
- Efficiency
- Voltage Regulation
- Tests
- Applications

Transformer: Ratings

- ☐ *Full Load Apparent Power (S_{FL}) (KVA-MVA)*
- ☐ *Rated Voltages (High Voltage V_{RHV} / Low Voltage V_{RLV}) (V)*
- ☐ *Full Load Current (High Voltage I_{FLHV}/ Low Voltage I_{FLLV}) (A)*
- ☐ *Frequency (Hz)*

$$\boxed{I_{FLHV} = \frac{S_{FL}}{V_{HV}}}$$

$$\boxed{I_{FLLV} = \frac{S_{FL}}{V_{LV}}}$$

$$\boxed{\frac{V_{RHV}}{V_{LHV}} = \frac{N_{HV}}{N_{LV}}}$$

Transformation Ratio

TRANSFORMER

TO STANDARD DDS-84-2007

TRANSFORMER: IDB-100-11
TYPE OF COOLING: ONAN
KVA: 100
FREQUENCY HZ: 50
VOLTS (NO LOAD) H.V.: 11000
IMPEDANCE %: 4
L.V.: 415
OIL SPECS: IEC-60296
AMPERES H.V.: 5.25
WEIGHT OF OIL KGS: 112
L.V.: 139.1
LIFTABLE ASSY. KGS: 301.5
PHASES H.V.: 3
TOTAL WEIGHT KGS: 600
L.V.: 3
YEAR OF MANUFACTURE: 2010
DIAGRAM DRG NO.: S-1760
MAKER'S S.NO.: 291144
VECTOR SYMBOL: Dyn-11
P.O.NO.: 15024-28

LOW VOLTAGE
n a2 b2 c2
A8 B8 C8
HIGH VOLTAGE

H.T. VOLTS	SWITCH POSITION
11275	1
11000	2
10725	3
10450	4
10175	5

Transformer: Per-Unit System

- Per-unit value of a quantity = $\boxed{\dfrac{Actual\ Value}{Base\ Value}}$

- Choose base values as follows:

Apparent Power (S) S_b= rated volt-ampere = S_{FL}

$$S_b = P_b = Q_b$$

Voltages (V_1/V_2) $V_{1b} = V_{1rated}$ $V_{2b} = V_{2rated}$

- Calculate the other base values:

$$\boxed{I_{1b} = \frac{S_b}{V_{1b}} = I_{1FL}} \qquad \boxed{I_{2b} = \frac{S_b}{V_{2b}} = I_{2FL}}$$

$$\boxed{Z_{1b} = R_{1b} = X_{1b} = \frac{V_{1b}^2}{S_b} = \frac{V_{1b}}{I_{1b}}} \qquad \boxed{Z_{2b} = R_{2b} = X_{2b} = \frac{V_{2b}^2}{S_b} = \frac{V_{2b}}{I_{2b}}}$$

Transformer: Per-Unit System

Advantages of Per-unit System:

1. Per-unit (p.u.) values of circuit parameters are the same on either side of the a transformer (i.e. weather referred to primary or secondary).

$$\boxed{R_{2pu} = \frac{R_{2actual}}{R_{2b}\,(\text{or } Z_{2b})} = R_{2actual}\left(\frac{S_b}{V_{2b}^2}\right)}$$

$$\boxed{R'_{2pu} = \frac{R'_{2actual}}{R_{1b}\,(\text{or } Z_{1b})} = R'_{2actual}\left(\frac{S_b}{V_{1b}^2}\right)}$$

$$\boxed{R_{2actual}\left(\frac{N_1}{N_2}\right)^2\left(\frac{S_b}{V_{1b}^2}\right) = R_{2actual}\left(\frac{V_{1b}}{V_{2b}}\right)^2\left(\frac{S_b}{V_{1b}^2}\right) = R_{2actual}\left(\frac{S_b}{V_{2b}^2}\right)}$$

⇨ $\boxed{R_{2pu} = R'_{2pu}}$

⇨ $\boxed{R_{1pu} = R'_{1pu}}$ $\boxed{X_{1pu} = X'_{1pu}}$ $\boxed{X_{2pu} = X'_{2pu}}$

Transformer: Per-Unit System

Advantages of Per-unit System:

2. The per-unit value of the resistance represents the percentage of power loss at full load.

$$\boxed{R_{1pu} = \frac{R_{1actual}}{R_{1b}\,(\text{or } Z_{1b})} = \frac{R_{1actual}}{S_b} * I_{1b}^{2} = \frac{Power\ Losses\ at\ FL}{Rated\ Power}}$$

$$\boxed{I_{1b} = I_{1FL}}$$

$\boxed{R_{1pu}}$ = Primary power losses at FL as a ratio of rated power

Transformer: Per-Unit System

Advantages of Per-unit System:

3. The per-unit value of the impedance represents the percentage voltage drop at full load.

$$\boxed{Z_{1pu} = \frac{Z_{1actual}}{Z_{1b}} = \frac{Z_{1actual}}{V_{1b}} * I_{1b} = \frac{Voltage\ Drop\ at\ FL}{Rated\ Voltage}}$$

$\boxed{Z_{1pu}}$ = Primary voltage drop at FL as a ratio of rated voltage

4. Transformer parameters lie within a narrow range of values when expressed in p.u.

Transformer: Per-Unit System

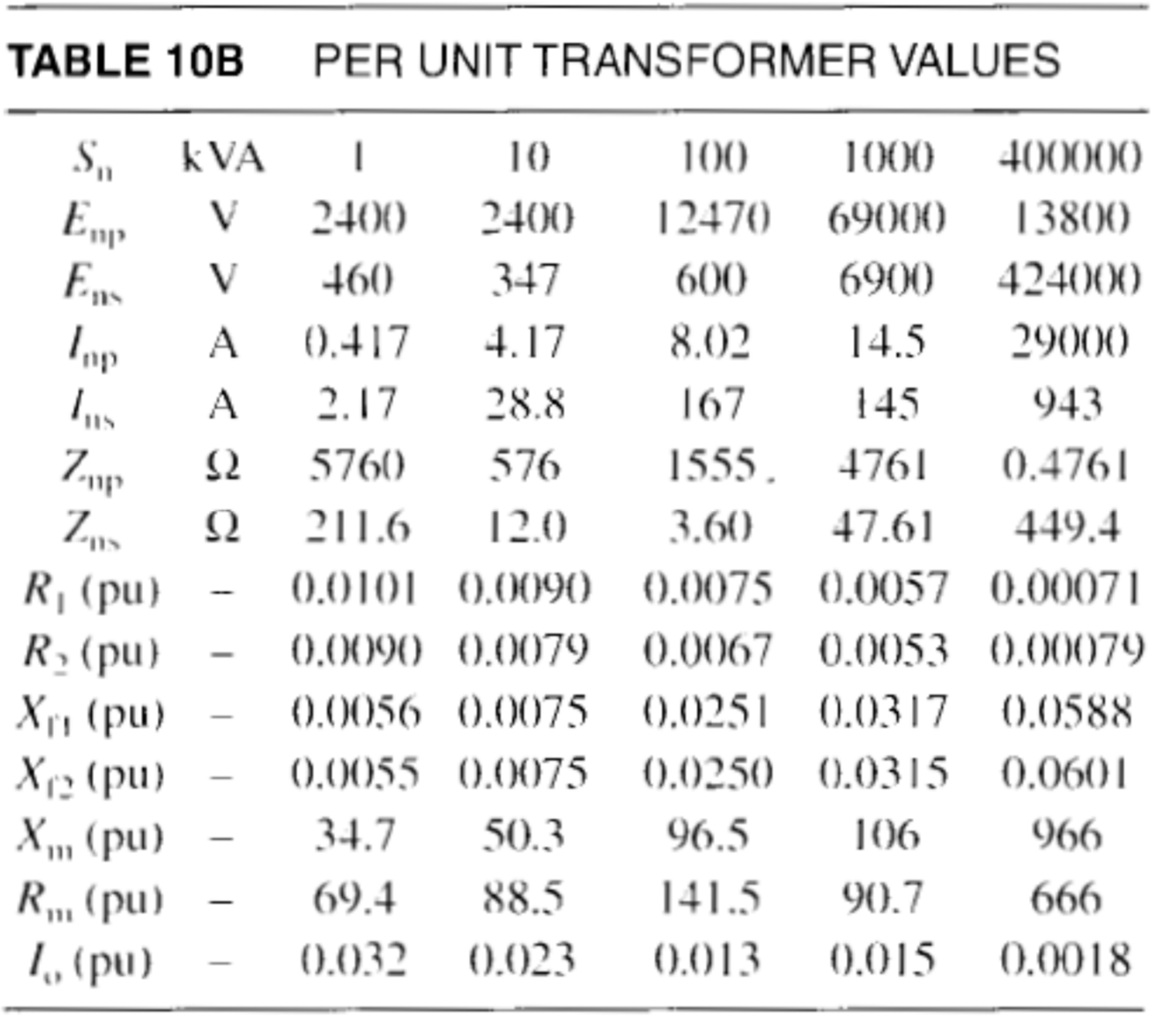


TABLE 10B PER UNIT TRANSFORMER VALUES

S_n	kVA	1	10	100	1000	400000
E_{np}	V	2400	2400	12470	69000	13800
E_{ns}	V	460	347	600	6900	424000
I_{np}	A	0.417	4.17	8.02	14.5	29000
I_{ns}	A	2.17	28.8	167	145	943
Z_{np}	Ω	5760	576	1555	4761	0.4761
Z_{ns}	Ω	211.6	12.0	3.60	47.61	449.4
R_1 (pu)	–	0.0101	0.0090	0.0075	0.0057	0.00071
R_2 (pu)	–	0.0090	0.0079	0.0067	0.0053	0.00079
X_{l1} (pu)	–	0.0056	0.0075	0.0251	0.0317	0.0588
X_{l2} (pu)	–	0.0055	0.0075	0.0250	0.0315	0.0601
X_m (pu)	–	34.7	50.3	96.5	106	966
R_m (pu)	–	69.4	88.5	141.5	90.7	666
I_o (pu)	–	0.032	0.023	0.013	0.015	0.0018

Transformer: Per-Unit System

☐ Example:

A 50 kVA, 2400/240V, 50 Hz transformer has a resistance and leakage inductance of 0.72+j0.92 for the primary side and 0.007+j0.009 for the secondary side. At rated voltage and frequency, the admittance for the shunt branch of the excitation current is $0.324+j2.24 \times 10^{-2}$ mho when viewed from the low voltage side.

Draw the exact equivalent circuit:

a. Referred to the primary.
b. Referred to the secondary.
c. In per-unit values.

Transformer: Per-Unit System

Solution:

Rated VA = S_{rated} = 50 kVA = S_b

Rated primary voltage = V_{1r} = 2400 V = V_{1b}

Rated secondary voltage = V_{2r} = 240 V = V_{2b}

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = 10$$

$$I_{1FL} = \frac{S}{V_1} = I_{2FL}\left(\frac{N_2}{N_1}\right)$$

$$I_{2FL} = \frac{S}{V_2} = I_{1FL}\left(\frac{N_1}{N_2}\right)$$

$$I_{1FL} = 20.8A$$

$$I_{2FL} = 208A$$

$$R_1 + jX_1 = 0.72 + j0.92$$

$$R_2 + jX_2 = 0.007 + j0.009$$

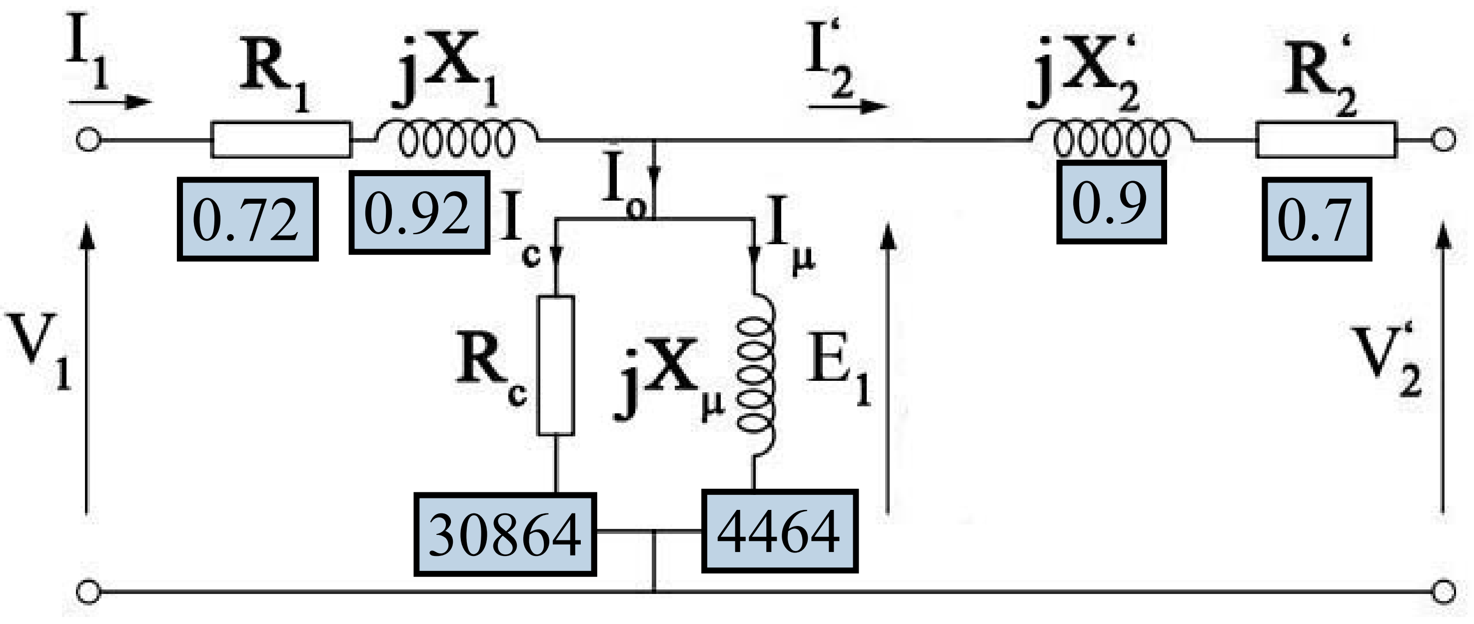
$$R_c^{'} = \frac{1}{0.324 \times 10^{-2}} = 308.64$$

$$X_\mu^{'} = \frac{1}{2.24 \times 10^{-2}} = 44.64$$

Transformer: Per-Unit System

Solution:

$$R_2' + jX_2' = (R_2 + jX_2)\left(\frac{N_1}{N_2}\right)^2 = 0.7 + j0.9$$

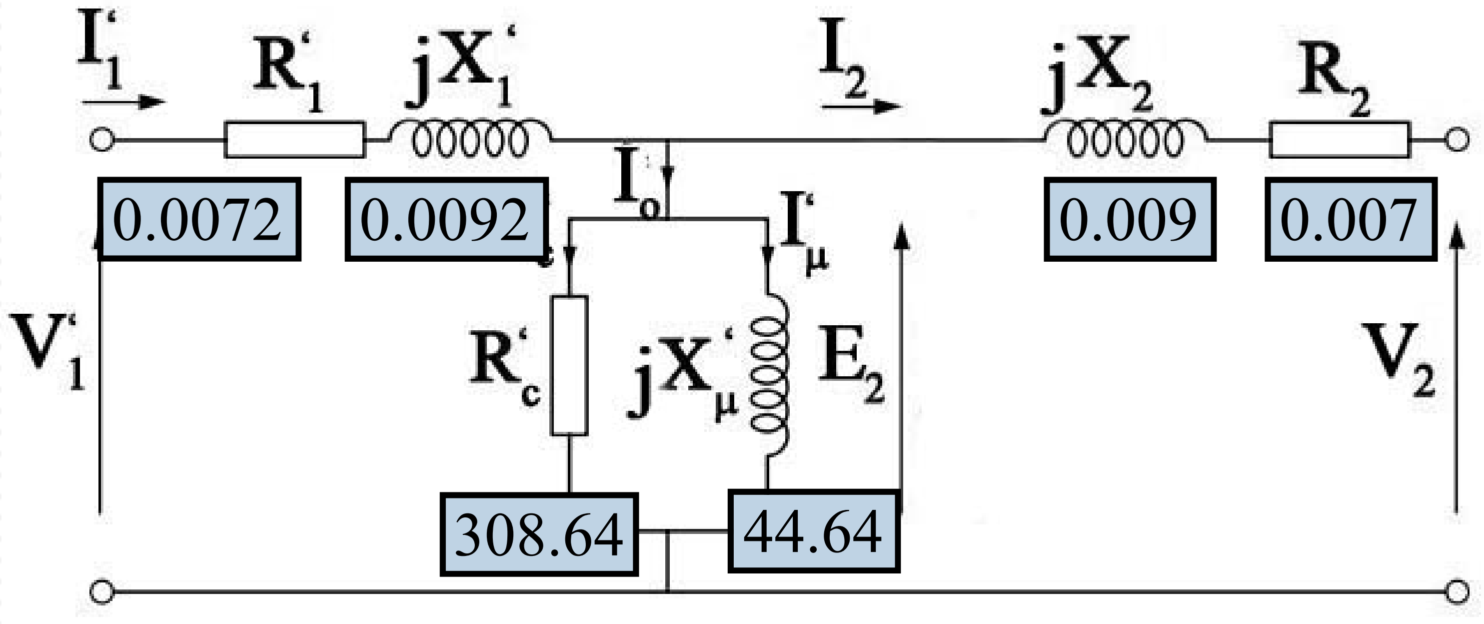


Exact Equivalent Circuit referred to Primary

Transformer: Per-Unit System

Solution:

$$R_1' + jX_1' = (R_1 + jX_1)\left(\frac{N_2}{N_1}\right)^2 = 0.0072 + j\,0.0092$$



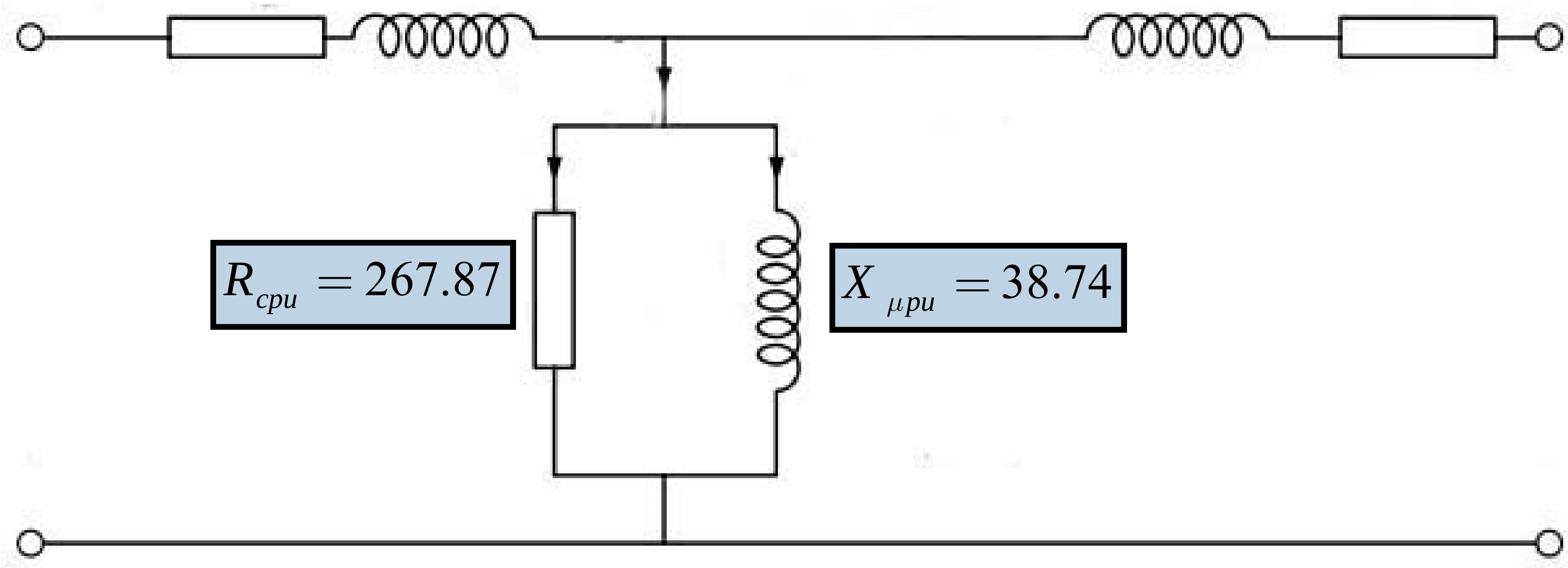
Exact Equivalent Circuit referred to Secondary

Transformer: Per-Unit System

Solution:

$$Z_{1b} = \frac{V_{1b}^2}{S_b} = 115.2\Omega$$

$$Z_{2b} = \frac{V_{2b}^2}{S_b} = 1.152\Omega$$



$$R_{1pu} = 6.25\times10^{-3}$$

$$X_{1pu} = 7.98\times10^{-3}$$

$$X_{2pu} = 7.8\times10^{-3}$$

$$R_{2pu} = 6.08\times10^{-3}$$

$$R_{cpu} = 267.87$$

$$X_{\mu pu} = 38.74$$

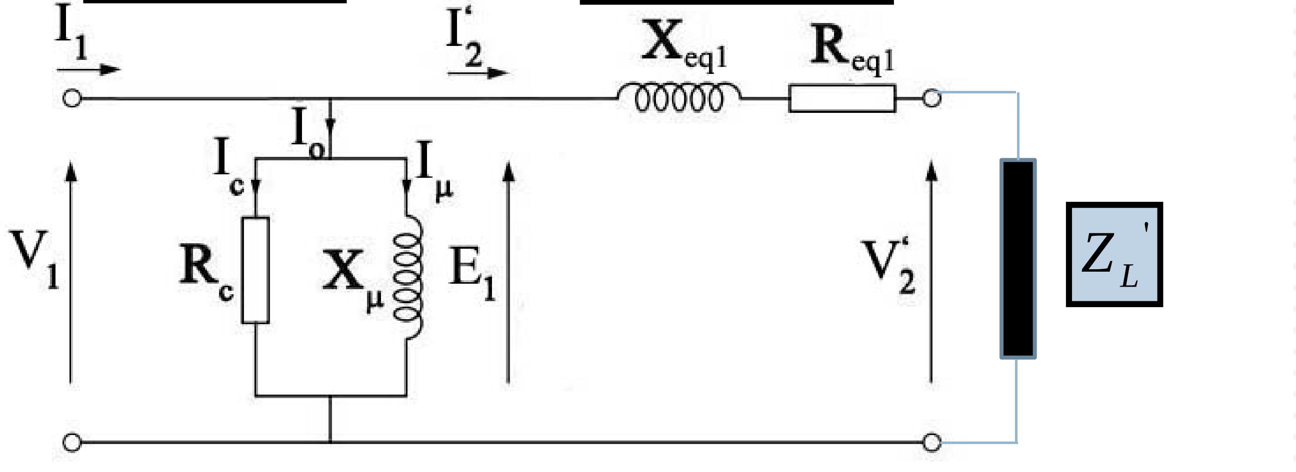
Exact Equivalent Circuit in per-unit

Transformer: Load Factor

Load factor

It's the ratio between the apparent power (current) drawn by the load at a certain condition and the full load power (current) of the transformer.

$$\boxed{k = \frac{S_L}{S_{FL}}} \Rightarrow \boxed{k = \frac{V_2 I_2}{V_2 I_{FL}}} \Rightarrow \boxed{k = \frac{I_2}{I_{2FL}}}$$

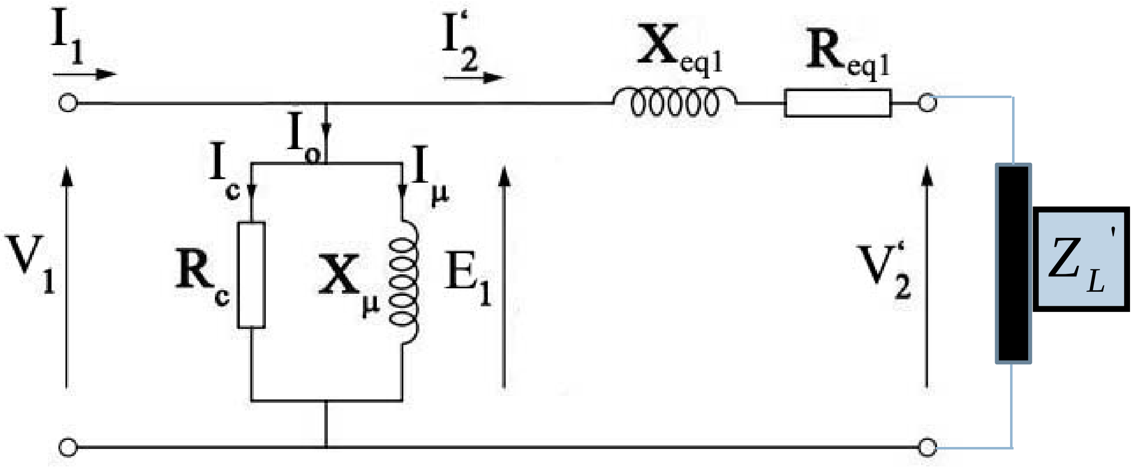


Transformer: Voltage Regulation

Voltage Regulation

It’s the difference between the load voltage (secondary voltage) at no-load and the load voltage at a certain loading condition as a percentage of the no-load voltage. (i.e. the percentage voltage drop)

$$VR = \varepsilon = \frac{|V_{2NL}| - |V_2|}{|V_{2NL}|} \times 100$$



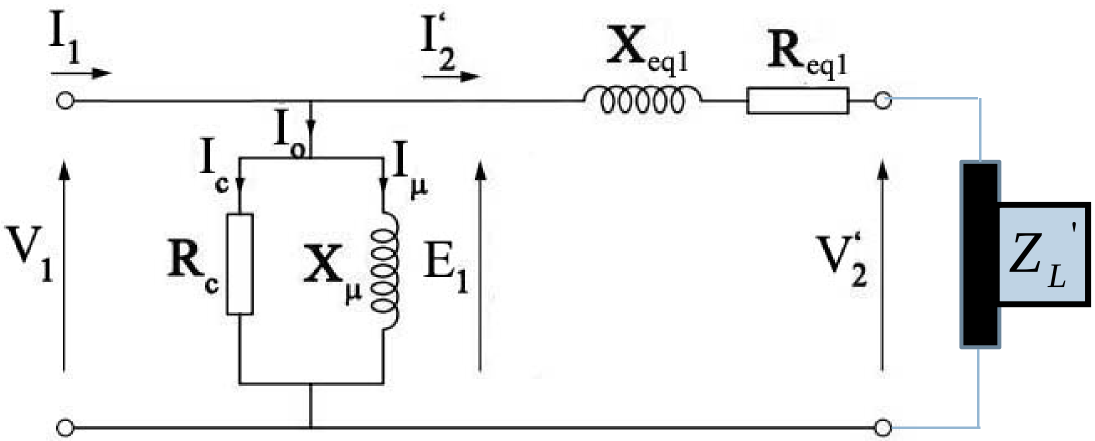
Transformer: Voltage Regulation

$$VR = \varepsilon = \frac{\left|V_{2NL}\right| - \left|V_2\right|}{\left|V_{2NL}\right|} \times 100$$

$$\varepsilon = \frac{\left|V_{2NL}{'}\right| - \left|V_2{'}\right|}{\left|V_{2NL}{'}\right|} \times 100$$

$$\left|V_{2NL}{'}\right| = \left|V_{1NL}\right|$$

$$\varepsilon = \frac{\left|V_1\right| - \left|V_2{'}\right|}{\left|V_1\right|} \times 100$$

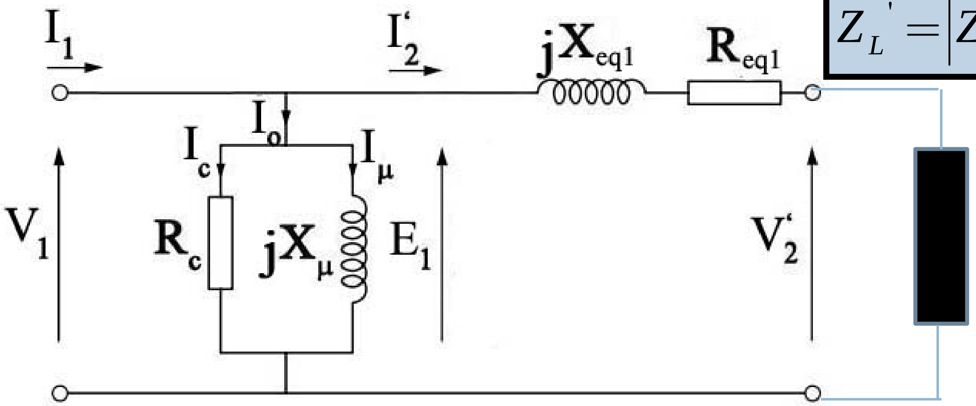


Transformer: Voltage Regulation

$$V_1 = I_2'(R_{eq1} + jX_{eq1}) + V_2'$$

$$\phi_2 = \tan^{-1}\frac{X_L}{R_L}$$

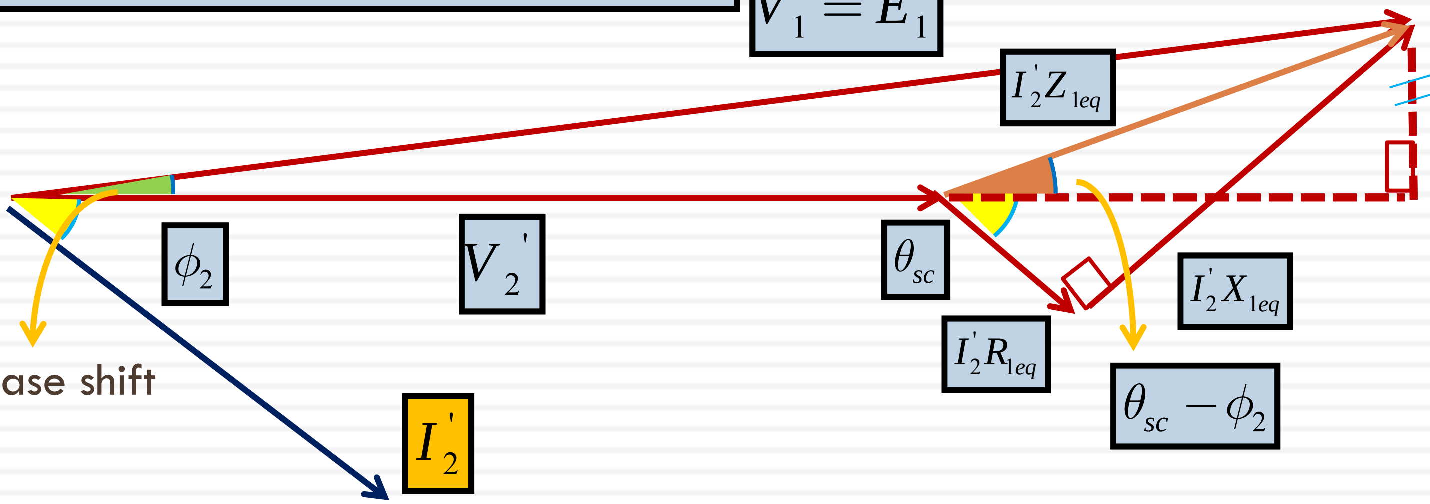
$$\theta_{sc} = \tan^{-1}\frac{X_{eq1}}{R_{eq1}}$$



$$Z_L' = |Z_L'|\angle\phi_2$$

$$|V_1| - |V_2'| \cong I_2' Z_{eq1}\cos(\theta_{sc} - \phi_2)$$

$$V_1 = E_1$$



$I_2' Z_{1eq}$

negligible

ϕ_2

V_2'

θ_{sc}

$I_2' X_{1eq}$

$I_2' R_{1eq}$

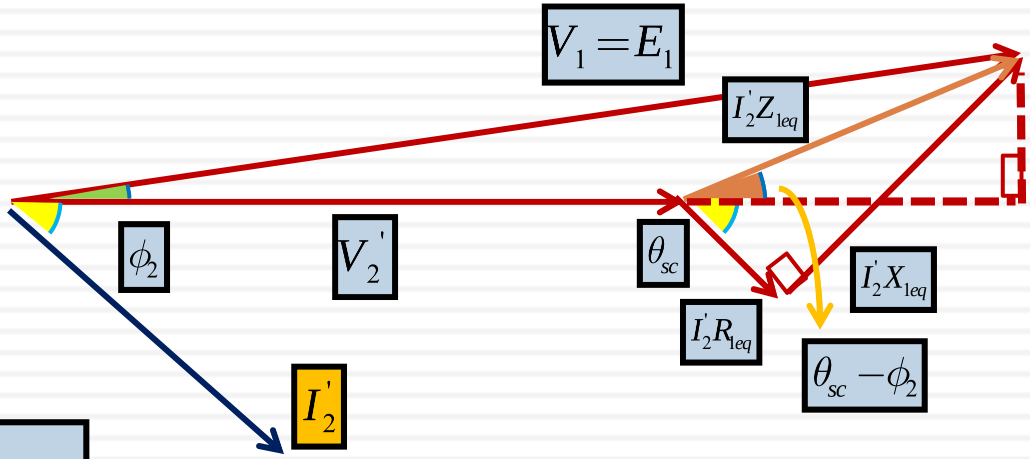
Very small phase shift

$\theta_{sc} - \phi_2$

I_2'

Transformer: Voltage Regulation

$$\varepsilon = \frac{|V_1| - |V_2^{'}|}{|V_1|} \times 100$$



$$\varepsilon = \frac{I_2^{'} Z_{eq1} \cos(\theta_{sc} - \phi_2)}{|V_1|} \times 100$$

$$\varepsilon = \frac{I_2 Z_{eq2} \cos(\theta_{sc} - \phi_2)}{|V_{2NL}|} \times 100$$

$$\theta_{sc} = \tan^{-1} \frac{X_{eq1}}{R_{eq1}}$$

Transformer: Voltage Regulation

$$\varepsilon = \frac{\boxed{(I_2 / I_{2FL})} Z_{eq\,2} \cos(\theta_{sc} - \phi_2)}{(|V_{2NL}| / I_{2FL})} \times 100$$

Load Factor

$$k = \frac{I_2}{I_{2FL}}$$

$$\frac{Z_{eq\,2}}{(|V_{2NL}| / I_{2FL})} = \frac{Z_{eq\,2}}{(|V_{2b}| / I_{2b})} = \frac{Z_{eq\,2}}{Z_{2b}} = Z_{eq\;pu}$$

$$\varepsilon = k Z_{eq\;pu} \cos(\theta_{sc} - \phi_2) \times 100$$

Transformer: Voltage Regulation

- In general:

$$\boxed{\varepsilon = kZ_{eq\ pu}\cos(\theta_{sc} \mp \phi_2)\times 100}$$

-ve for lagging , +ve for leading

- Maximum regulations happens when:

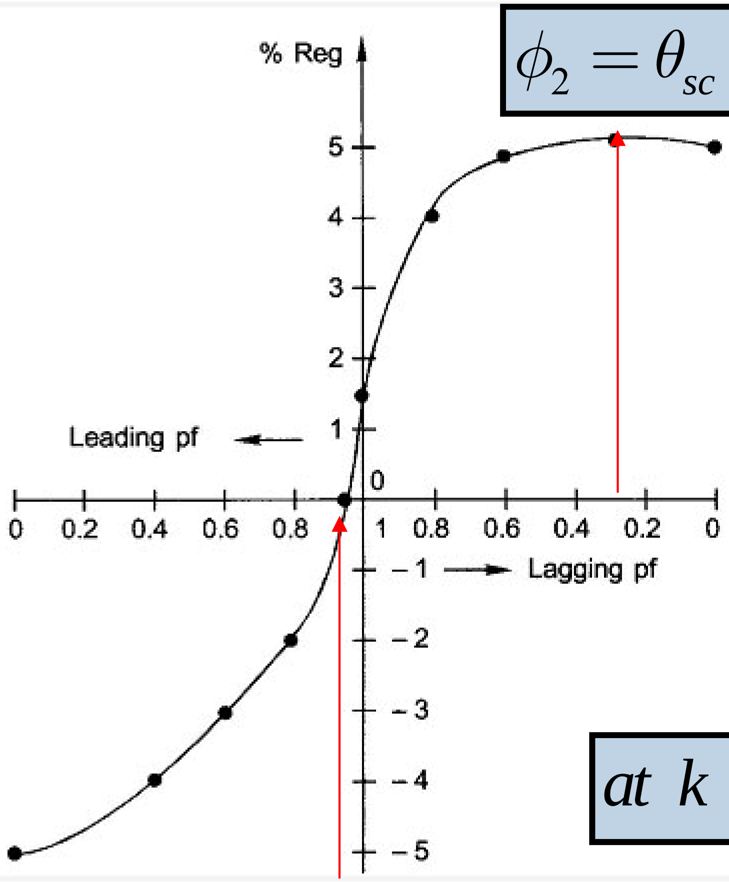
$$\boxed{\cos(\theta_{sc} \mp \phi_2) = 1} \Rightarrow \boxed{\phi_2 = \theta_{sc}} \text{ lagging}$$

- Zero regulation happens when:

$$\boxed{\cos(\theta_{sc} \mp \phi_2) = zero} \Rightarrow \boxed{\theta_{sc} + \phi_2 = 90^o}$$

$$\boxed{\phi_2 = 90^o - \theta_{sc}} \text{ leading}$$

Transformer: Voltage Regulation



$\phi_2 = \theta_{sc}$

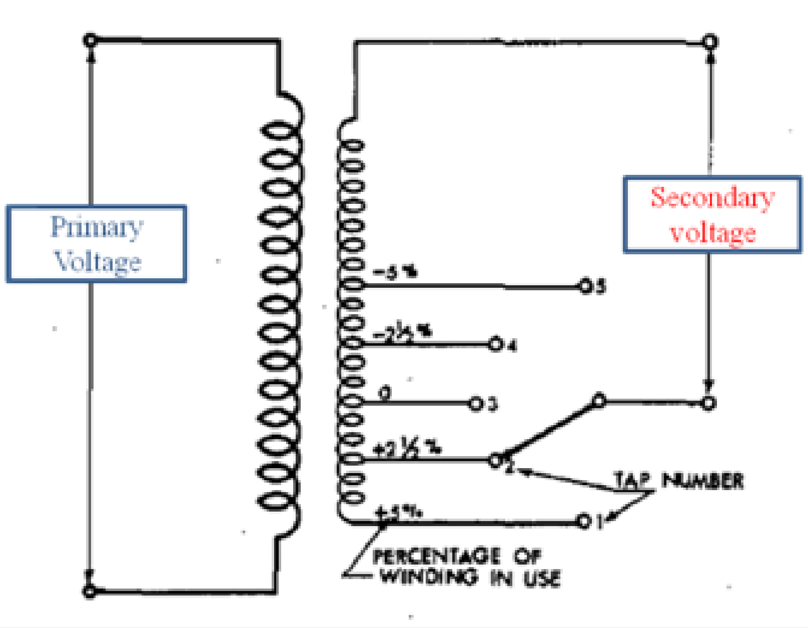
$at \ \ k = k_1$

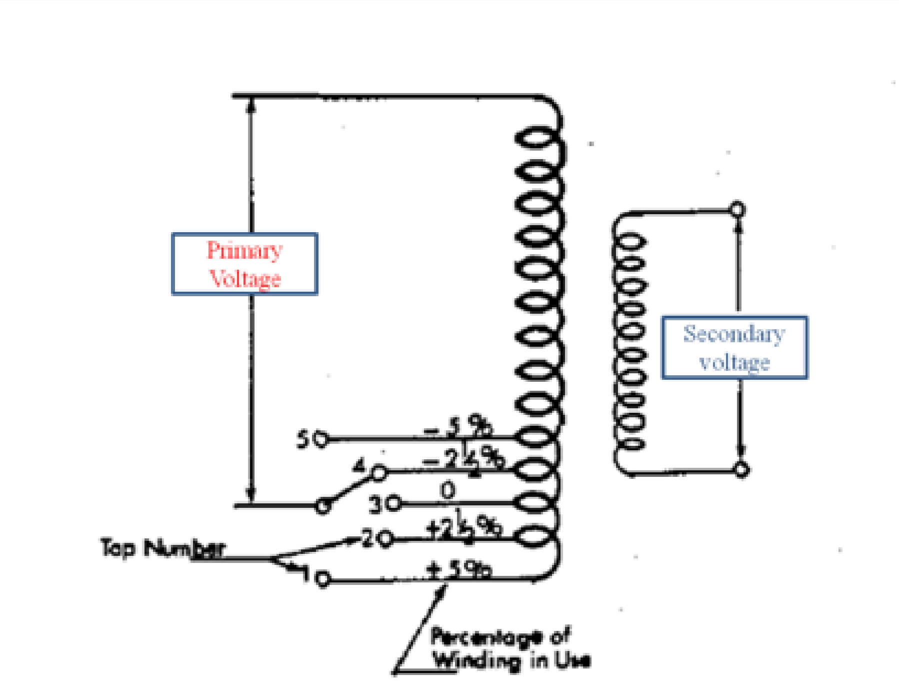
$\phi_2 = 90^o - \theta_{sc}$

Transformer: Tapping

- *The transformer voltage at the load side desired to be constant or as close to the nominal value. But the load voltage may vary according to current drawn by the load or supply voltage.*
- *Taps are provided on a transformer winding for selecting/cutting out a certain number of turns on the transformer winding thus obtaining a variable turns ratio. This allows adjustments of the output voltage.*
- *Typically, transformers are provided with four tapes allowing ±5% adjustments above or below the nominal voltage.*
- *The taps are usually on the high voltage side.*
- *The taps are connected to a tap-changer.*
- *Tap changers are either on-load type or off-load type.*

Transformer: Tapping





Transformer: Efficiency

$$\boxed{\eta = \frac{P_{out}}{P_{in}} \times 100}$$

$$\boxed{\eta = \frac{P_{out}}{P_{out} + P_{losses}} \times 100}$$

$$\boxed{\eta = \frac{P_{in} - P_{losses}}{P_{in}} \times 100}$$

Transformer: Efficiency

Transformer Losses

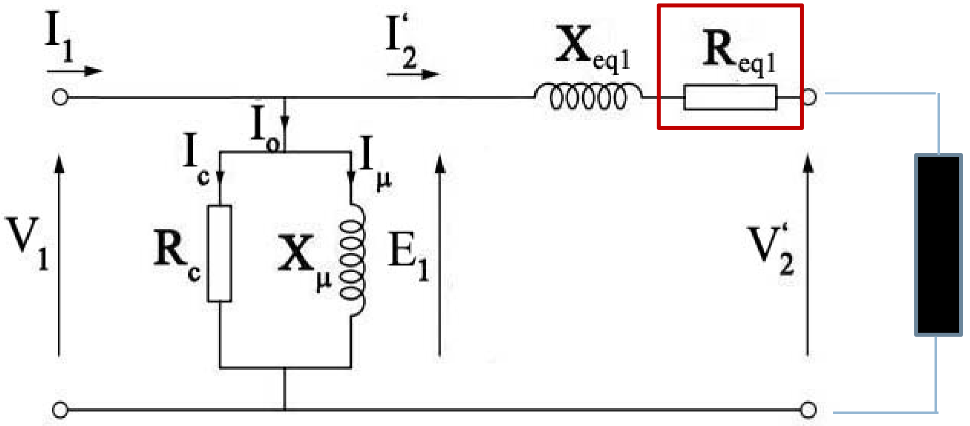
1. Copper losses: *Winding losses due to winding resistances.*

$$P_{cu} = I_1^{\,2} R_1 + I_2^{\,2} R_2$$

$$P_{cu} \simeq I_2'^{\,2} R_{eq1}$$

$$P_{cu} \simeq I_2^{\,2} R_{eq2}$$

Variable losses



$$Z_L' = |Z_L'| \angle \phi_2$$

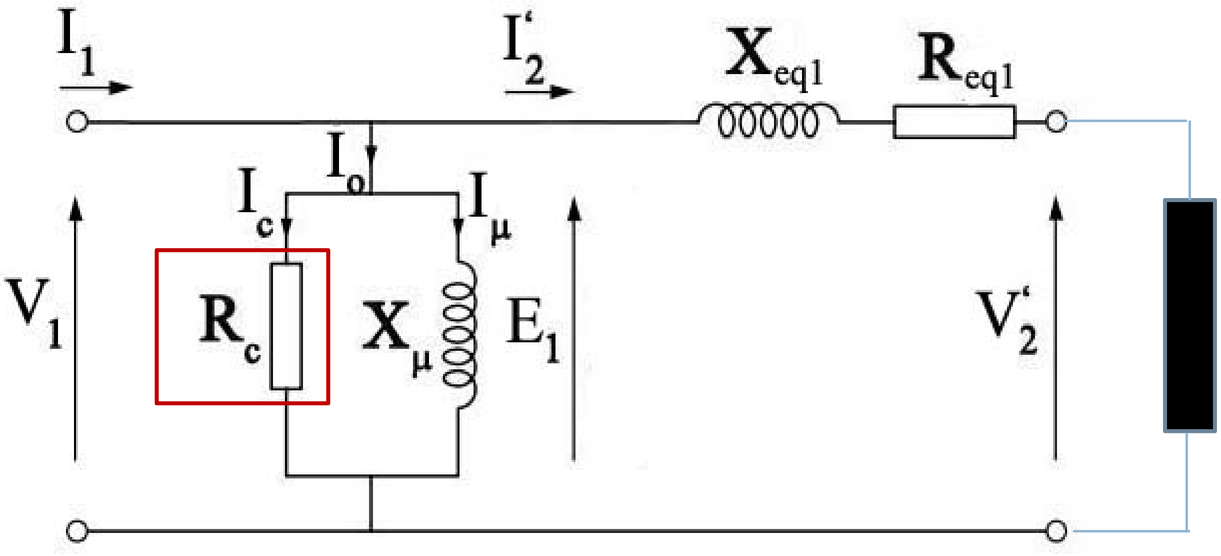
Transformer: Efficiency

Transformer Losses

2. Iron (core) losses: Eddy loss + Hysteresis loss

$$\boxed{P_{iron} = \frac{V_1^{\,2}}{R_c}}$$

Constant losses



$$\boxed{Z_L{}' = \left|Z_L{}'\right| \angle \phi_2}$$

Transformer: Efficiency

Transformer Losses

3. Stray losses: Not all flux flow in the iron core, some leakage flux may induce eddy currents in the metal structures supporting the core or the transformer housing, producing power loss. (5-10% of the transformer losses)

4. Dielectric losses: happens in the insulating materials in the transformer and in the transformer oil. Very small and often neglected.

Transformer: Efficiency

$$\boxed{\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{losses}}}$$

$$\boxed{P_{out} = S_L \cos\phi_2}$$

$$\boxed{\eta = \frac{P_{out}}{P_{in}} = \frac{S_L \cos\phi_2}{S_L \cos\phi_2 + P_{iron} + P_{cu}}}$$

$$\boxed{k = \frac{I_2}{I_{2FL}} = \frac{S_L}{S_{FL}}}$$

$$\boxed{P_{out} = kS_{FL} \cos\phi_2}$$

Transformer: Efficiency

$$\boxed{P_{cu} = I_2^{\,2} R_{eq2}}$$

$$\boxed{k = \frac{I_2}{I_{2FL}} = \frac{S_L}{S_{FL}}}$$

$$\boxed{P_{cu} = k^2 I_{2FL}^{\,2} R_{eq2}}$$

$$\boxed{P_{cu} = k^2 P_{cuFL}}$$

$$\boxed{\eta = \frac{kS_{FL}\cos\phi_2}{kS_{FL}\cos\phi_2 + P_{iron} + k^2 P_{cuFL}}}$$

Transformer: Efficiency

$$\eta = \frac{kS_{FL}\cos\phi_2}{kS_{FL}\cos\phi_2 + P_{iron} + k^2 P_{cuFL}}$$

$$\eta = \frac{k\cos\phi_2}{k\cos\phi_2 + \frac{P_{iron}}{S_{FL}} + k^2\frac{P_{cuFL}}{S_{FL}}}$$

$$\frac{P_{iron}}{S_b} = \frac{V_1^2}{R_c} * \frac{Z_{1b}}{V_{1b}^2} = \frac{1}{R_c / Z_{1b}} = \frac{1}{R_{c\ pu}}$$

$$\frac{P_{CuFL}}{S_b} = \frac{I_{2FL}^2 R_{eq2}}{S_b} = \frac{R_{eq2}}{S_b / I_{2FL}^2} = R_{eqpu}$$

$$\eta = \frac{k\cos\phi_2}{k\cos\phi_2 + \frac{1}{R_{c\ pu}} + k^2 R_{eqpu}}$$

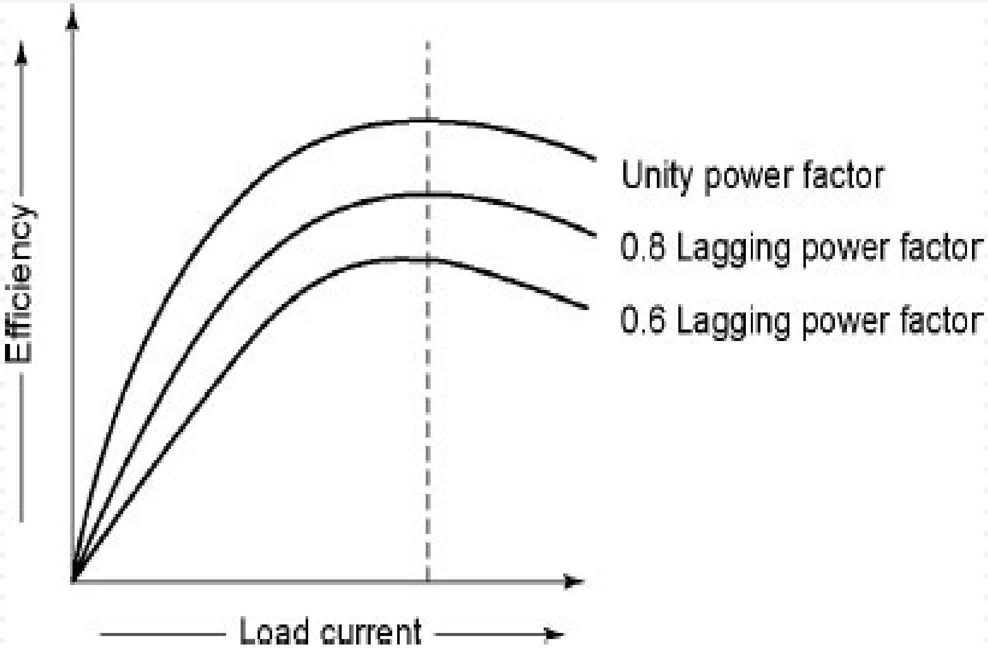
Transformer: Efficiency

Maximum Efficiency

$$\eta = \frac{kS_{FL}\cos\phi}{kS_{FL}\cos\phi + P_{iron} + k^2 P_{cuFL}}$$

$$k_m = \sqrt{\frac{P_{iron}}{P_{cuFL}}}$$

$$\eta_m = \frac{k_m S_{FL}\cos\phi}{k_m S_{FL}\cos\phi + 2P_{iron}}$$



Transformer: Efficiency

All Day Efficiency

$$\boxed{\eta_{AllDay} = \frac{Output\ \ Energy\ \ during\ \ the\ \ day}{Output\ \ Energy\ \ during\ \ the\ \ day + Energy\ \ losses} \times 100}$$

$$\boxed{\eta_{AllDay} = \frac{E_{out}}{E_{out} + E_{losses}} \times 100}$$

$$\boxed{\eta_{AllDay} = \frac{\sum_{i=1}^{n} (k_i S_{FL} \cos\phi) \times T_i}{\sum_{i=1}^{n} (k_i S_{FL} \cos\phi) \times T_i + P_{iron} \times T_{total} + \sum_{i=1}^{n} k_i^{\,2} P_{cuFL} \times T_i}}$$

Transformer: Tests

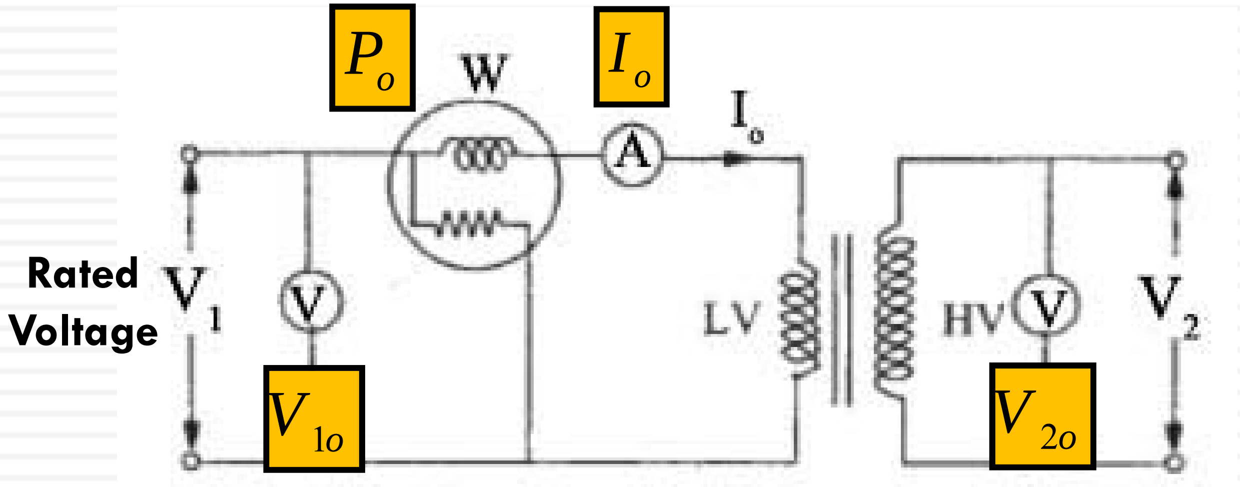
The equivalent circuit parameters are determined using two standard experiments.

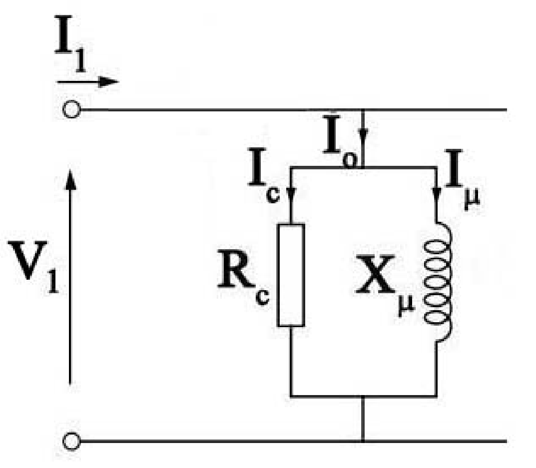
1. Open Circuit Test:

- *Performed by supplying the rated voltage to one side while keeping the other side open circuit.*
- *Preferably done by low voltage side with the rated voltage while keeping the high voltage side open.*
- *The HV and LV sides voltages, the input current and the input power are measured.*

Transformer: Tests

1. Open Circuit test:





$$P_o = P_{iron} = V_{1o} I_o \cos\phi_o$$

$$R_c = \frac{V_{1o}}{I_o \cos\phi_o}$$

$$X_\mu = \frac{V_{1o}}{I_o \sin\phi_o}$$

$$\frac{V_{1o}}{V_{2o}} = \frac{N_1}{N_2}$$

$$R_{cLV}$$

$$X_{\mu LV}$$

$$\frac{N_1}{N_2}$$

Transformer: Tests

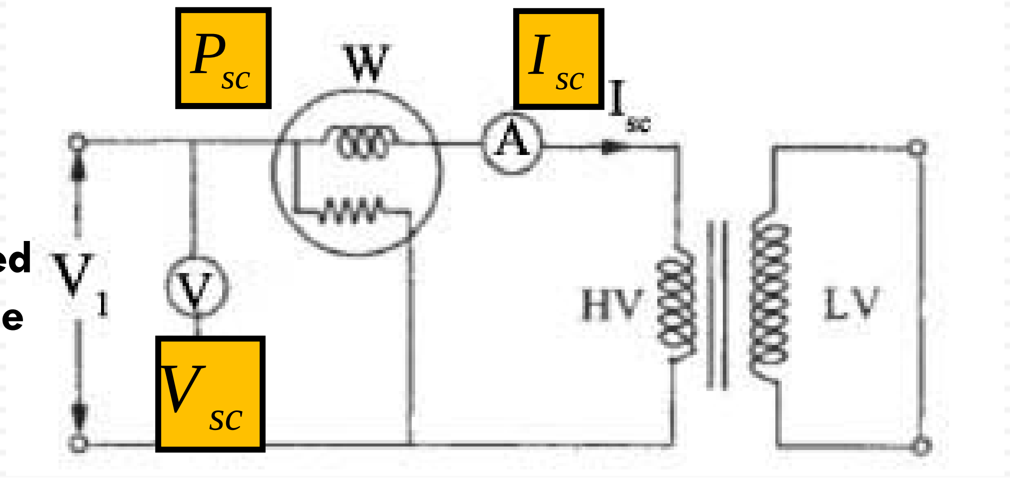
2. Short Circuit Test:

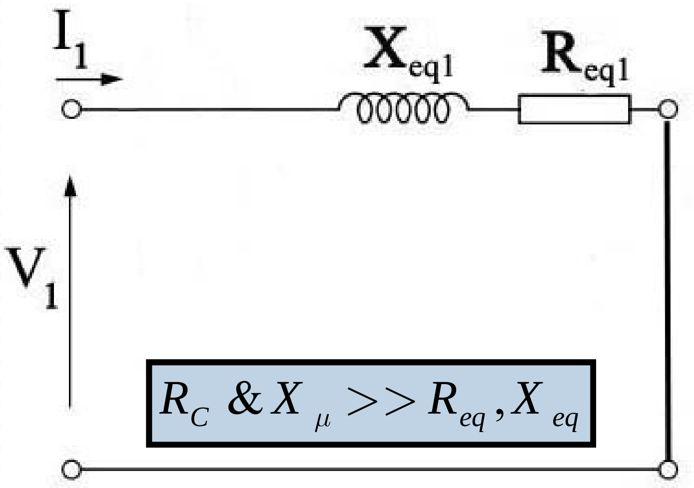
- *Performed by short-circuiting one side while supplying a reduced voltage to the other side.*
- *Preferably done by supplying the reduced voltage to the high voltage side while the low voltage side is short-circuited.*
- *The reduced voltage is usually adjusted so that the current equals the full load current.*
- *The supply voltage, the input current and the input power are measured.*

Transformer: Tests

2. Short Circuit Test:

Reduced Voltage





$R_C \,\&\, X_\mu >> R_{eq}, X_{eq}$

$$P_{sc} = I_{sc}^{\;2} R_{eq(sc)}$$

$$R_{eq(sc)} = \frac{P_{sc}}{I^2}$$

$$Z_{eq(sc)} = \frac{V_{sc}}{I_{sc}} = \sqrt{R^2_{eq(sc)} + X^{\;2}_{eq(sc)}}$$

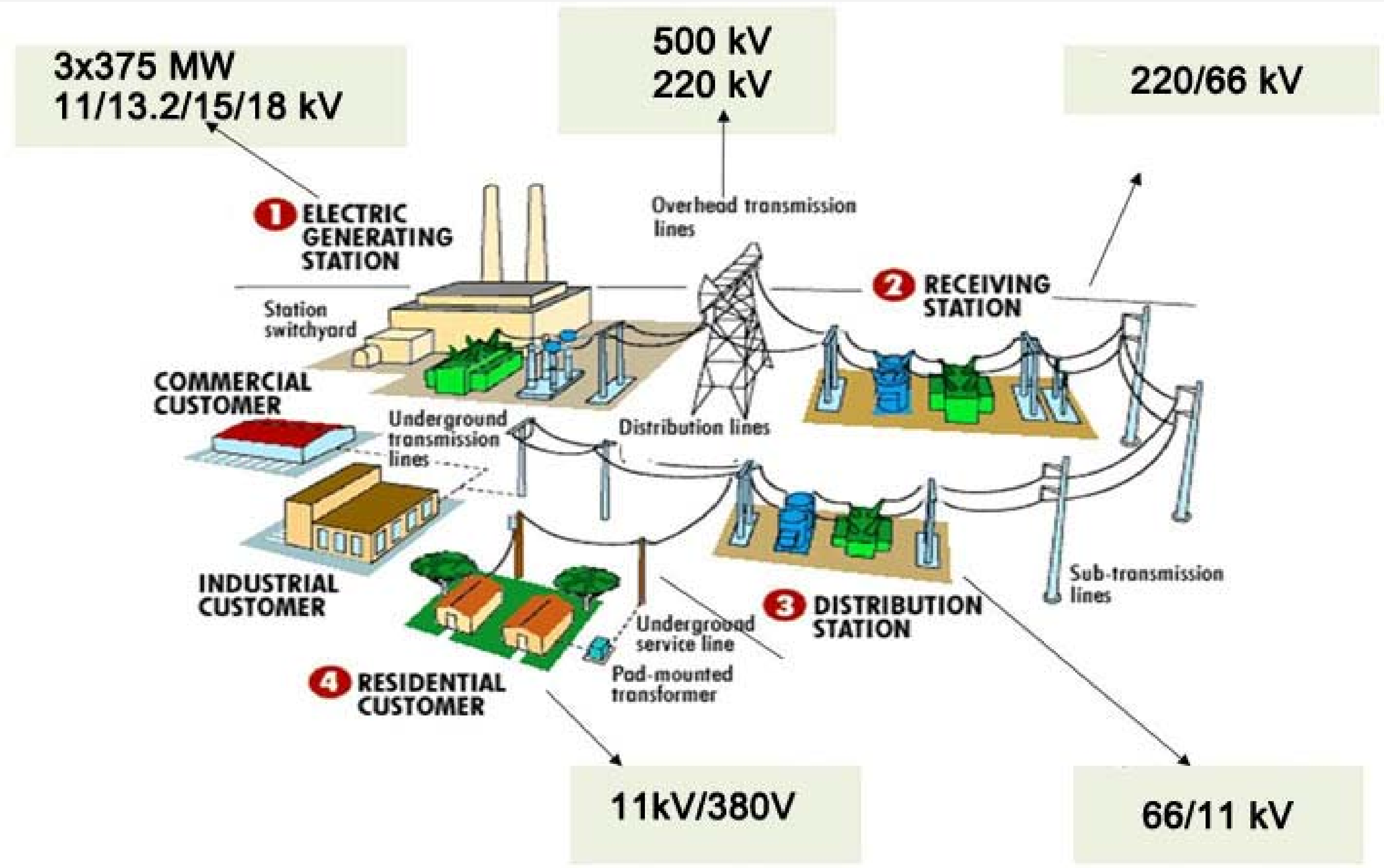
$$X_{\;eq(sc)} = \sqrt{Z^{\;2}_{eq(sc)} - R^{\;2}_{eq(sc)}}$$

R_{eqHV}

$X_{\;eqHV}$

Transformer: Applications

Power Transformers

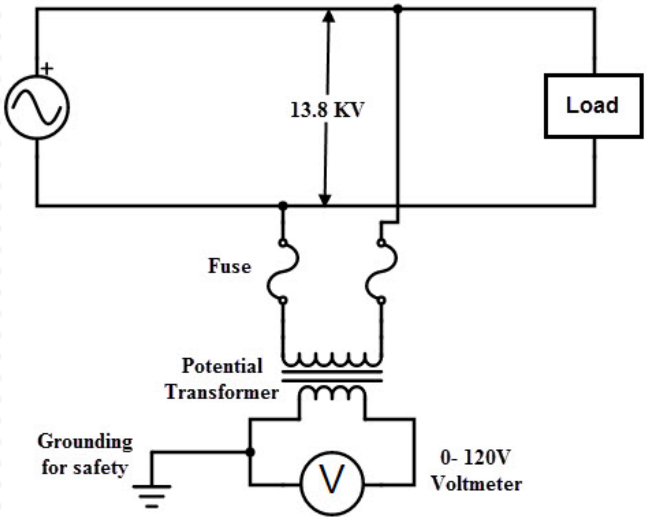


Transformer: Special Transformers

1. Instrumental Transformers

a. Potential (Voltage) Transformers: (PT/VT)

It’s a step-down transformer used with measuring equipment. The secondary rated voltage level is typically 120 V.

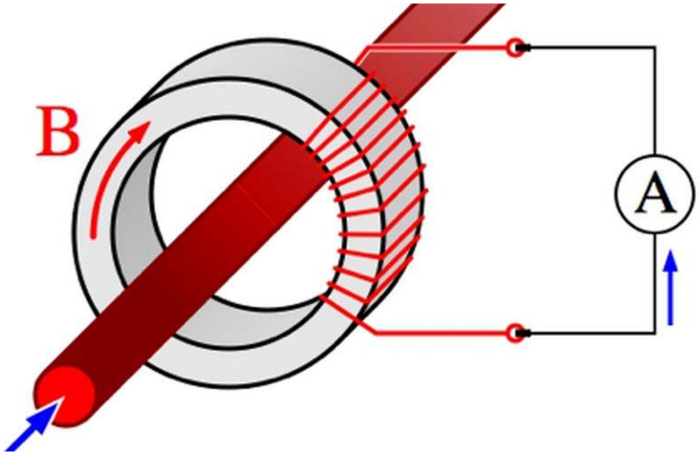


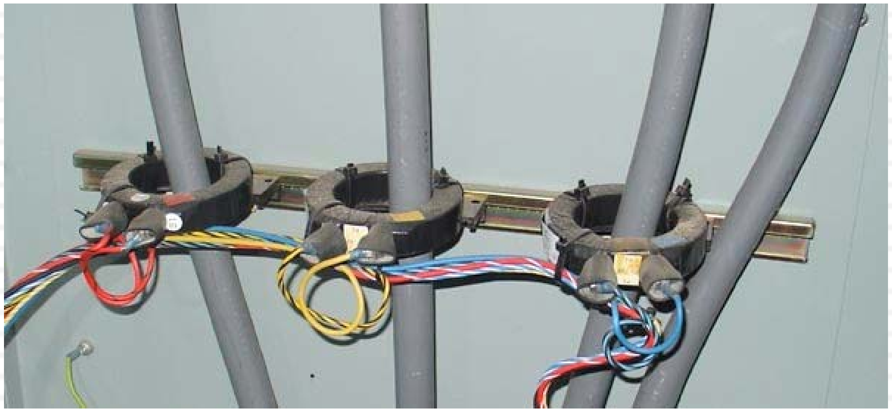
Transformer: Special Transformers

1. Instrumental Transformers

b. Current Transformers: (CT)

The current transformer is a step-up transformer, with the current carrying conductor acting as primary and the secondary winding wound around it. The secondary rated current level is typically 5 A.





Transformer: Special Transformers

2. Communication Transformers

a. RF transformers

RF transformers are widely used in electronic circuits for

- *Impedance matching to achieve maximum power transfer and to suppress undesired signal reflection.*
- *Voltage, current step-up or step-down.*
- *DC isolation between circuits while affording efficient AC transmission.*
- *Interfacing between balanced and unbalanced circuits; example: balanced amplifiers.*

Transformer: Special Transformers

2. Communication Transformers

a. RF transformers

Laminated steel is not suitable for RF.

Types:

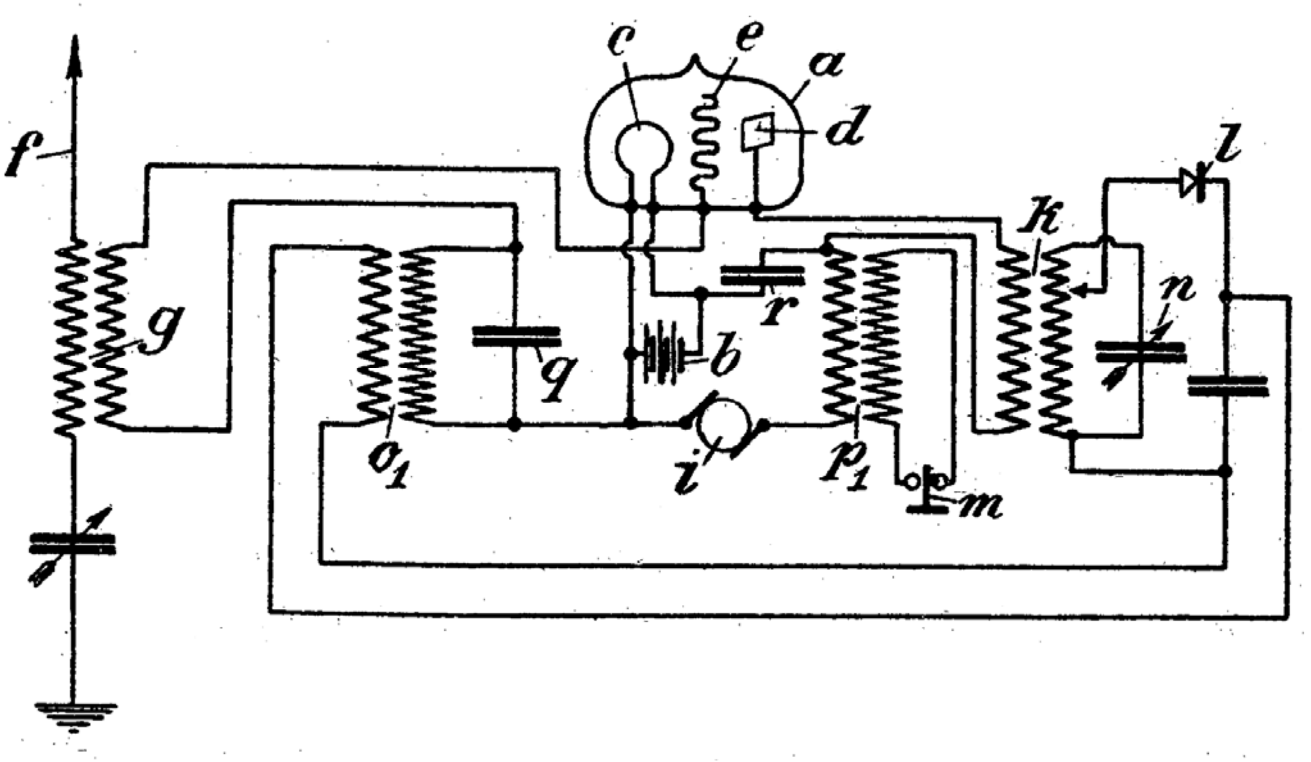
- Air-core transformer: These are used for high frequency work. The lack of a core means very low inductance.

- Ferrite-core transformers: These are widely used in (intermediate frequency) (IF) stages in superheterodyne radio receivers (superhet: uses frequency mixing to convert a received signal to a fixed intermediate frequency (IF) which can be more conveniently processed than the original carrier frequency.)

Transformer: Special Transformers

2. Communication Transformers

a. RF transformers



Transformer: Special Transformers

2. Communication Transformers

b. LF transformers

The power supplies in the receivers and other loads will create small amounts of pulse currents with harmonics reaching into the LF band of interest. The receiver chassis injects these small currents up the antenna cable shield. These currents give rise to voltages between the antenna ground and the local ground. This local voltage field is then injected into the antenna signal.

When this occurs in a LF receiving setup, power line harmonic interference is created. The placement of a 1:1 isolation transformer in the antenna line significantly reduces this source of interference.

Transformer: Special Transformers

3. Pulse Transformers

A pulse transformer is a transformer that is optimized for transmitting rectangular electrical pulses (that is, pulses with fast rise and fall times and a relatively constant amplitude). To minimize distortion of the pulse shape, a pulse transformer needs to have low values of leakage inductance and distributed capacitance, and a high open-circuit inductance. Small versions called signal types are used in digital logic and telecommunications circuits, often for matching logic drivers to transmission lines.

Transformer: Special Transformers

3. Pulse Transformers

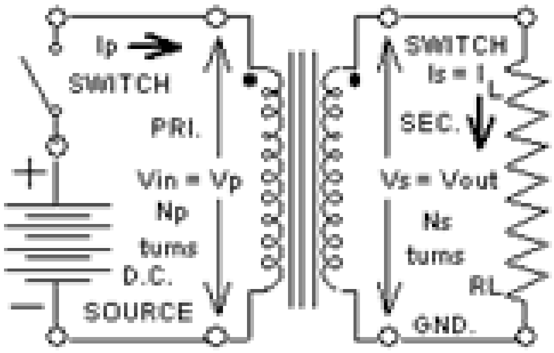


FIG. 3A --- SCHEMATIC; IDEAL PULSE TRANSFORMER APPLICATION

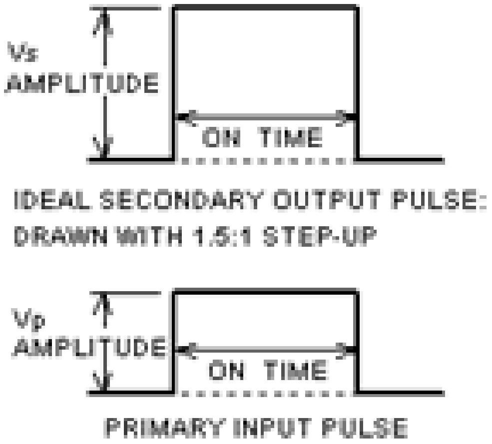


FIG. 3B --IDEAL PULSE, PULSE TRANSFORMER

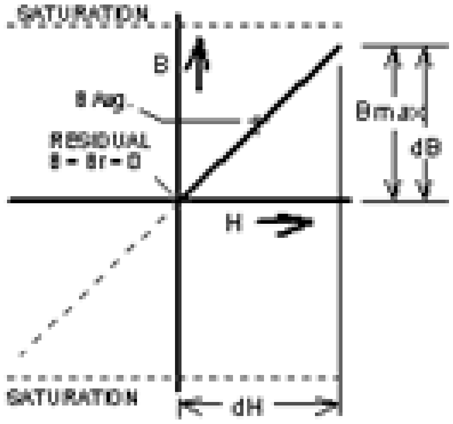


FIGURE 3C --- B-H CURVE; IDEAL UNIPOLAR APPLICATION, AVG. B > 0.

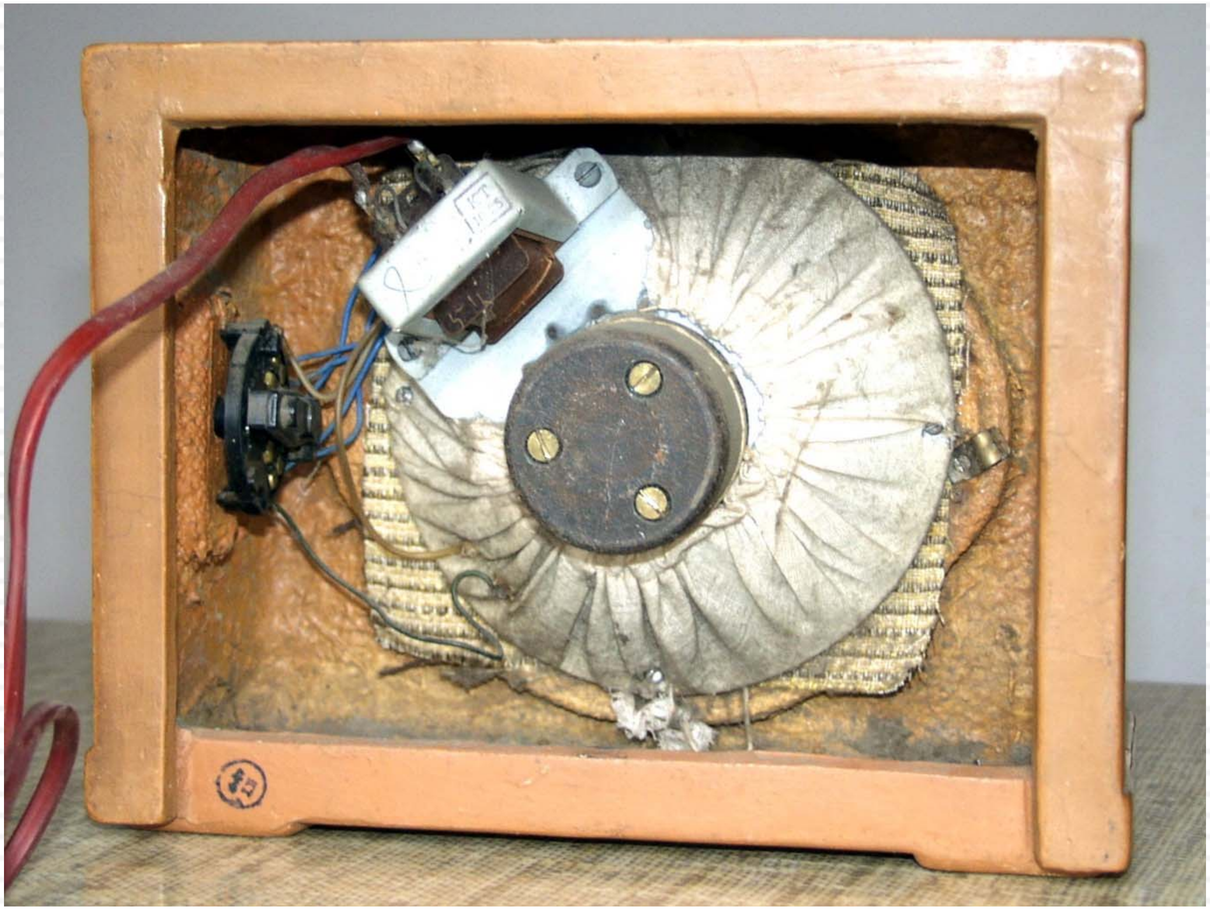
Transformer: Special Transformers

4. Audio Transformers

Audio transformers are those specifically designed for use in audio circuits to carry audio signal. They can be used to block radio frequency interference or the DC component of an audio signal, to split or combine audio signals, or to provide impedance matching between high and low impedance circuits, such as between a high impedance tube (valve) amplifier output and a low impedance loudspeaker, or between a high impedance instrument output and the low impedance input of a mixing console.

Transformer: Special Transformers

4. Audio Transformers



Transformer: Special Transformers

5. Auto Transformers