TRANSFORMERS

Electrical Machines – Elective Course for ELC

Transformer

- Construction
- Theory of Operation
- Equivalent Circuit
- Transformer Rating
- Efficiency
- Voltage Regulation
- Tests
- Applications

Transformer: Introduction

What's a transformer?

It's a stationary electric device used to transfer electric energy from one circuit to another (mostly of different voltage levels but with the same frequency). The two circuits are electrically isolated and the energy transfer occurs through ***magnetic induction*** *between the two circuits.*

It is not an energy conversion device.

AC Supply → Transformer → Load

Transformer: Introduction

What's a transformer?

video

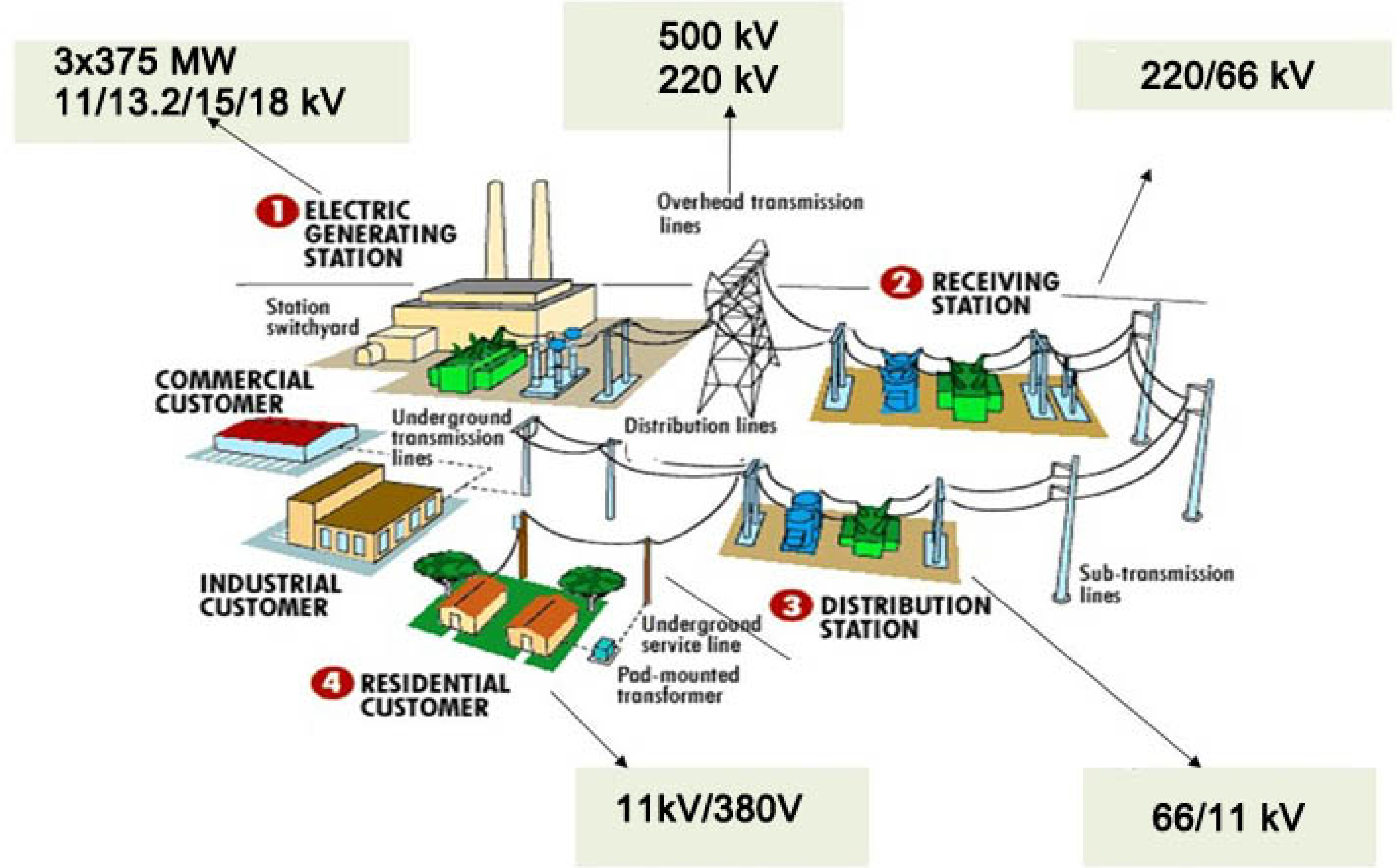
Transformer: Introduction

Transformer Applications

1. *In power transmission and distribution systems.*
2. *When the required voltage is different from the supply voltage.*
3. *To provide electrical isolation for circuits.*
4. *For matching circuits to change the impedance.*
5. *In measuring and control instruments (current and potential transformers)*

Transformer: Introduction

Transformer Applications



3x375 MW
11/13.2/15/18 kV

500 kV
220 kV

220/66 kV

11kV/380V

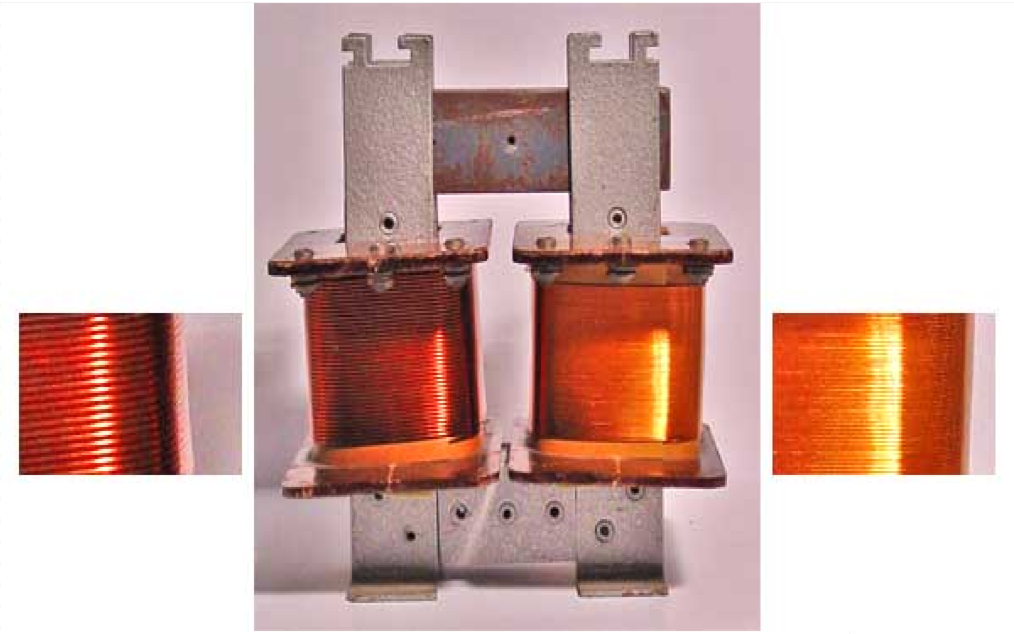
66/11 kV

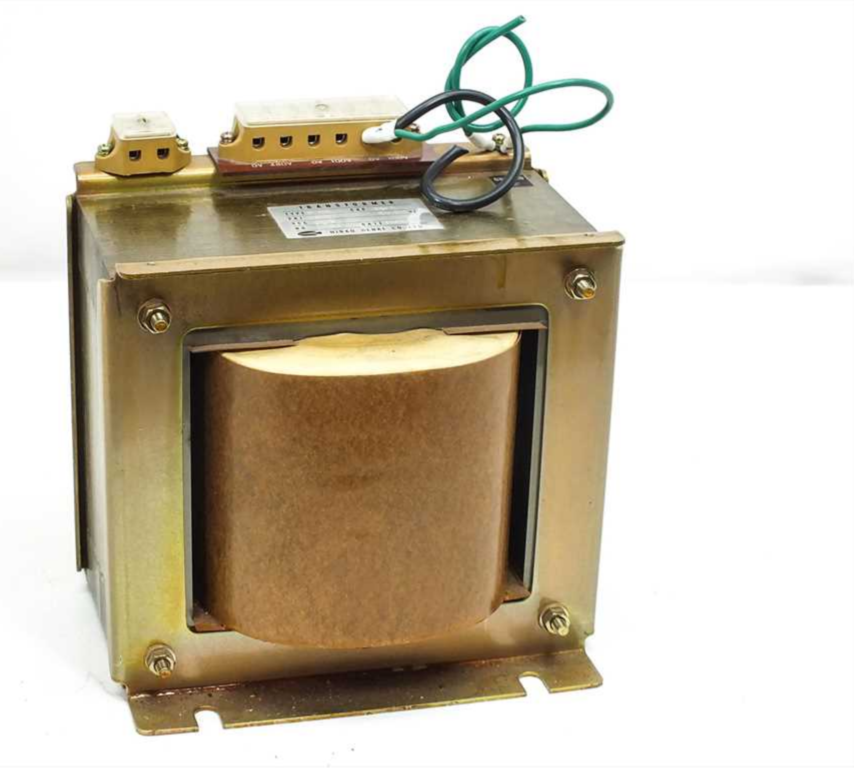
Transformer: Construction

- *It consists of two or more coils of wire (windings) wrapped around a common ferromagnetic core.*
- *The winding connected to the supply is called the Primary Winding, while the winding connected to the load is called Secondary Winding.*
- *The transformer windings wires are made of high conductivity copper.*
- *The transformer core is made of insulated steel laminations.*

Transformer

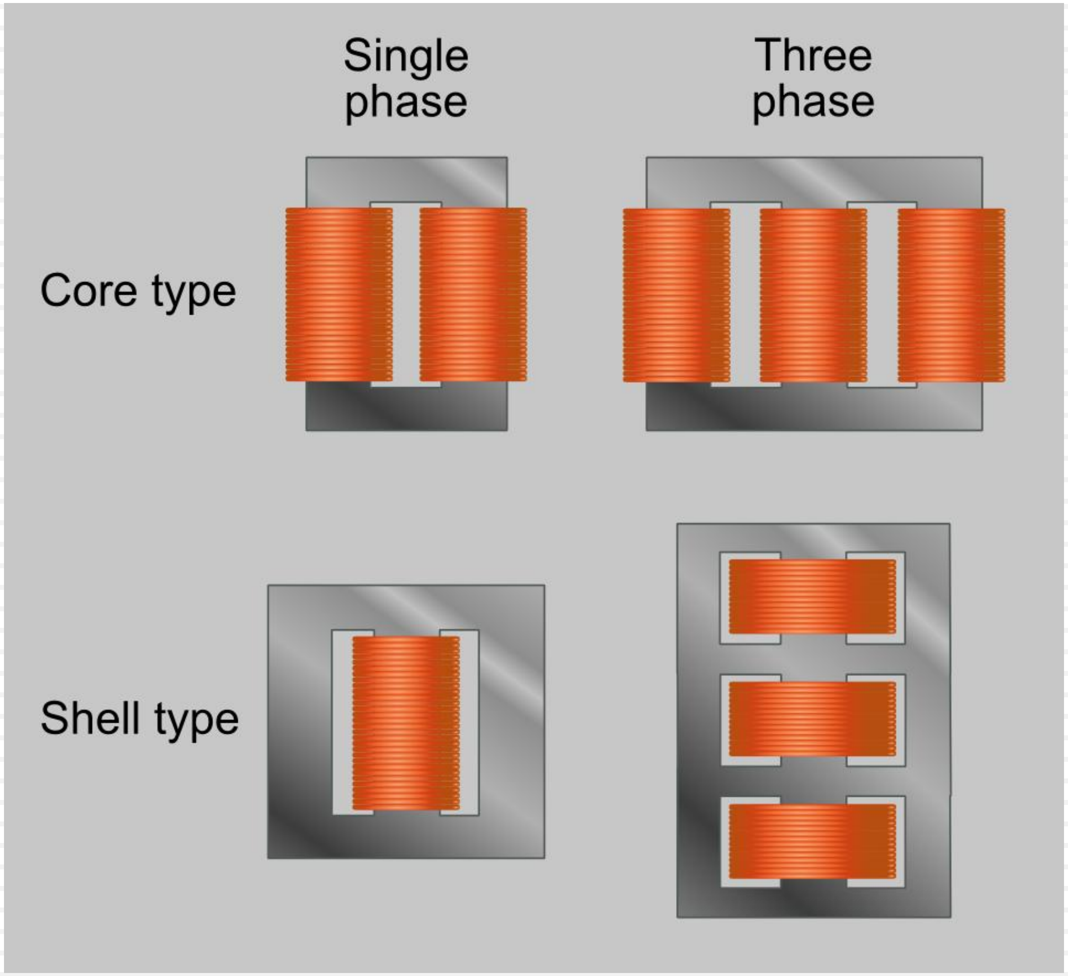
Single-phase transformer





Transformer

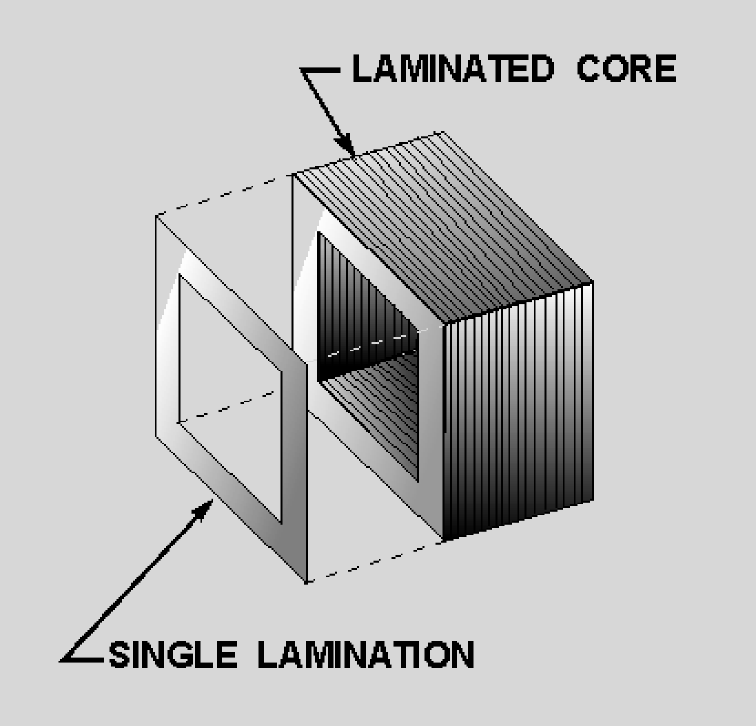
Construction

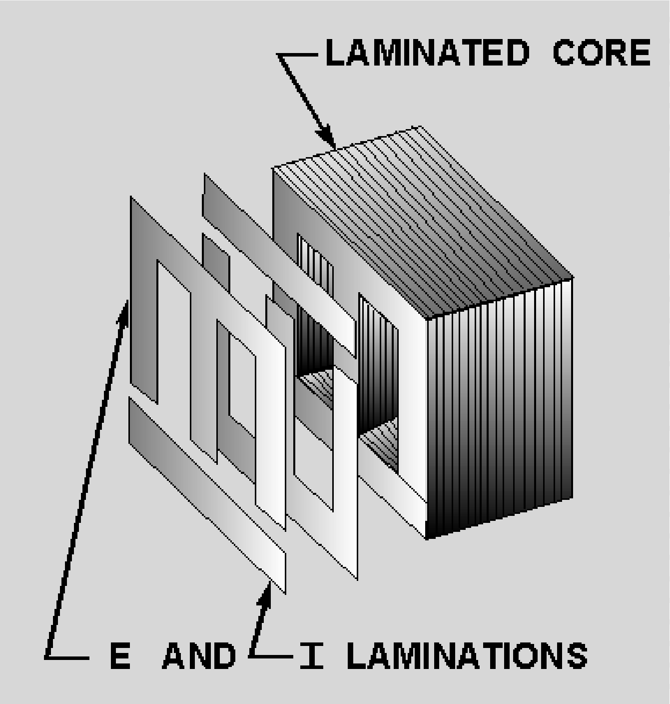


Transformer

Construction

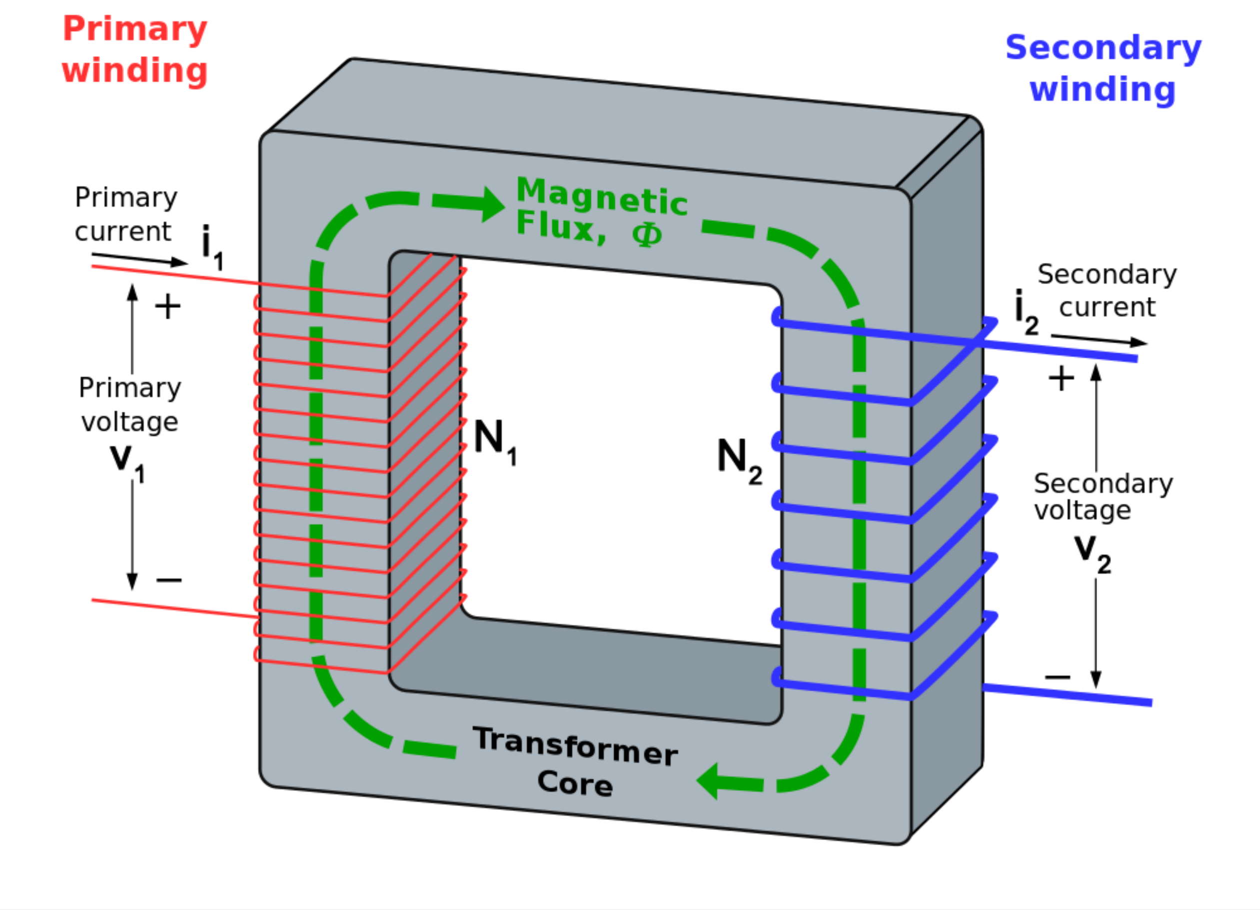
Silicon-Steel Laminated core



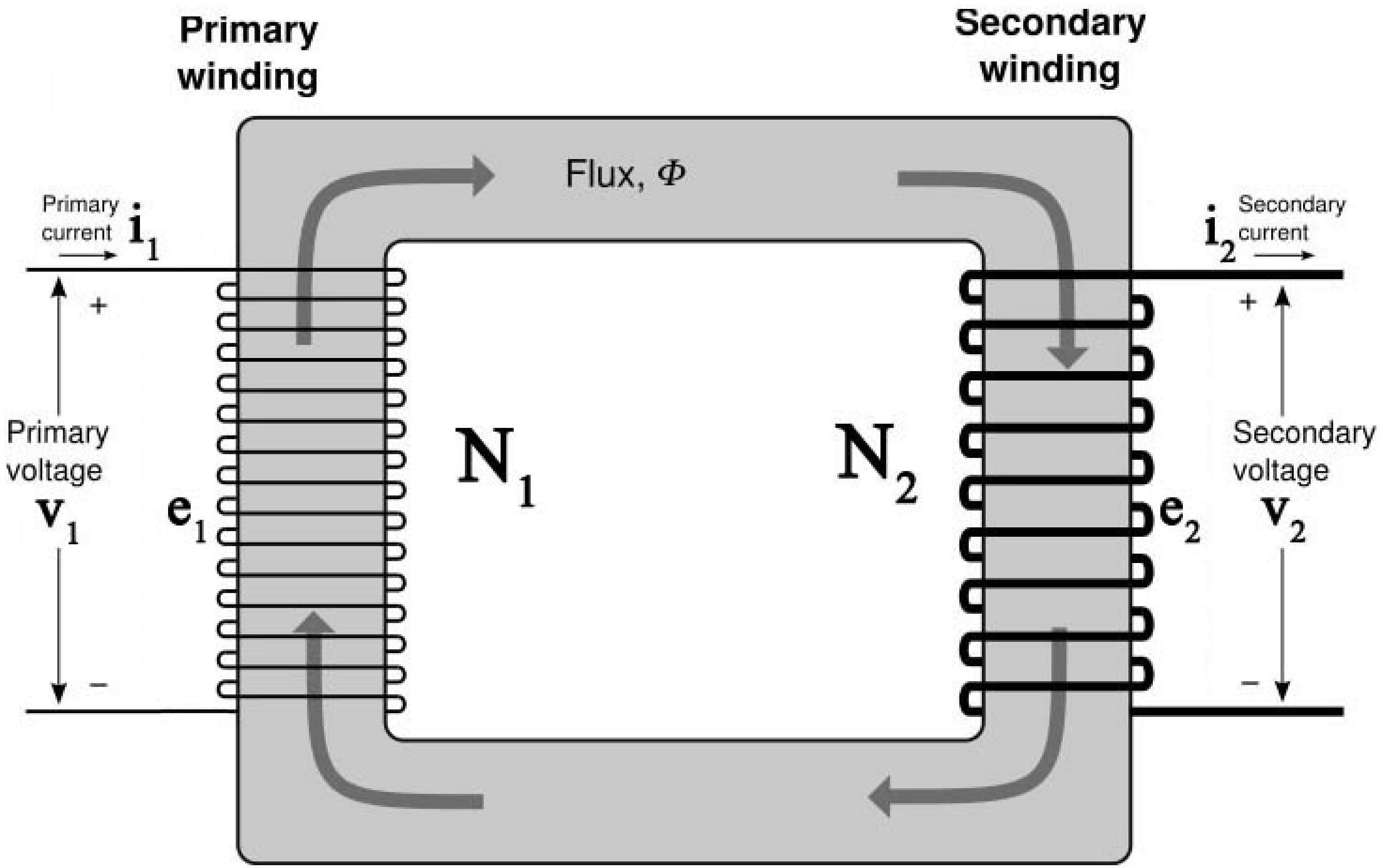


Transformer

Single-phase transformer



Transformer



Transformer

Ideal Transformer

- R_{coil} = zero
- μ → ∞
- No leakage flux
- No core losses

$$\boxed{v_1 = N_1 \frac{d\phi_1}{dt}}$$

$$\boxed{v_2 = N_2 \frac{d\phi_2}{dt}}$$

Since no leakage

$$\boxed{\phi_1 = \phi_2}$$

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$

$$\boxed{\frac{e_1}{e_2} = \frac{N_1}{N_2}}$$

Transformer

Ideal Transformer

$$\boxed{V_{1m}\sin(\omega t) = N_1\frac{d\phi}{dt}} \Rightarrow \boxed{\phi = \frac{V_{1m}}{\omega N_1}\sin(\omega t - \frac{\pi}{2})}$$

$$\boxed{\phi_m = \frac{V_{1m}}{\omega N_1}} \Rightarrow \boxed{V_{1rms} = \frac{2\pi f N_1 \phi_m}{\sqrt{2}}}$$

$$\boxed{V_1 = 4.44 f \phi_m N_1} \qquad \boxed{V_2 = 4.44 f \phi_m N_2}$$

$$\boxed{E_1 = 4.44 f \phi_m N_1} \qquad \boxed{E_2 = 4.44 f \phi_m N_2}$$

Transformer

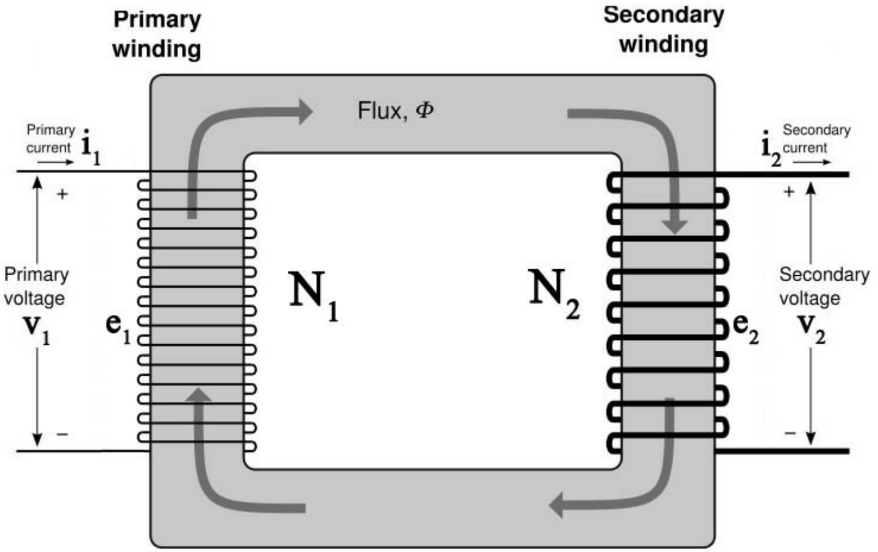
Ideal Transformer

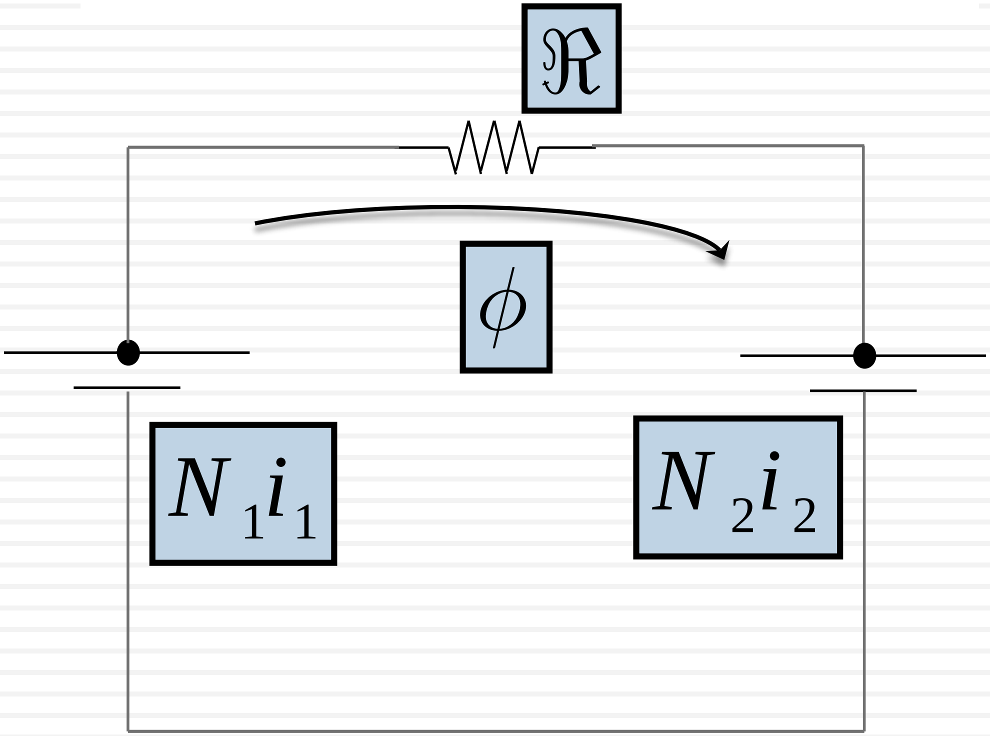
$$\boxed{N_1 i_1 - N_2 i_2 = \phi \Re}$$

Since μ → ∞

$$\boxed{\Re = zero}$$

$$\Rightarrow \boxed{\frac{N_1}{N_2} = \frac{i_2}{i_1}}$$





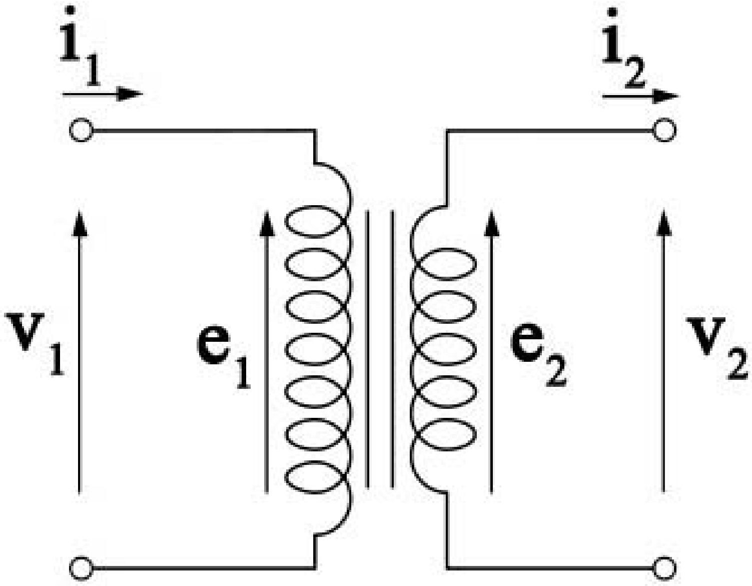
Transformer

Ideal Transformer

$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}}$$

$$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}}$$

$$\boxed{\frac{I_2}{I_1} = \frac{N_1}{N_2}}$$



Transformer

Ideal Transformer

$$S = V_1 I_1 = V_2 I_2$$

$$S = V_2 I_2 = I_2^2 Z_L$$

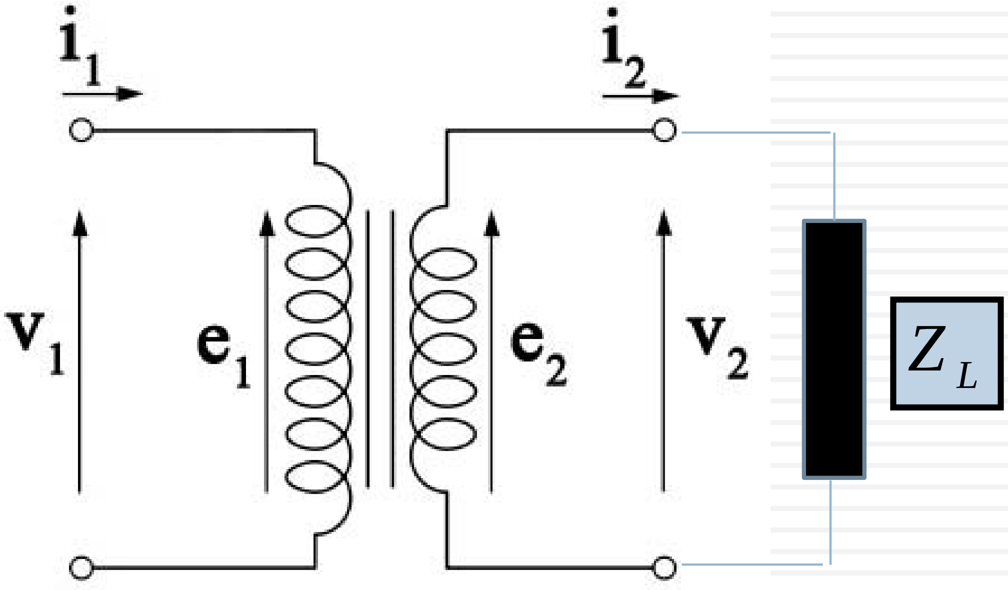
$$V_2 = I_2 Z_L$$

$$V_1 \frac{N_2}{N_1} = I_1 \frac{N_1}{N_2} Z_L$$

$$V_1 = I_1 Z_L \left(\frac{N_1}{N_2}\right)^2$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{I_2}{I_1} = \frac{N_1}{N_2}$$

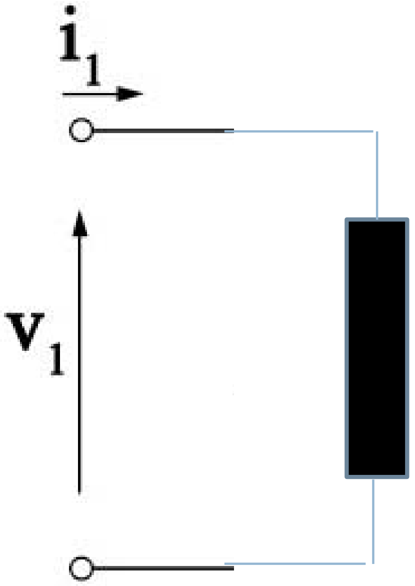


Transformer

Ideal Transformer

$$\boxed{V_1 = I_1 Z_L \left(\frac{N_1}{N_2}\right)^2}$$

$$\boxed{V_1 = I_1 Z_L'}$$



$$\boxed{Z_L' = Z_L \left(\frac{N_1}{N_2}\right)^2}$$

Load referred to the primary

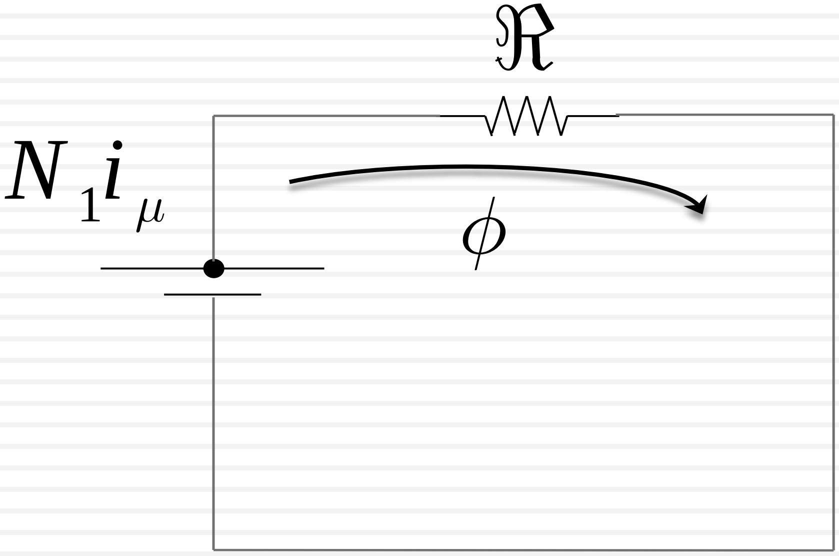
Transformer: Equivalent Circuit

Case B: - R_{coil} = zero - $\boxed{\mu \neq \infty}$

- No leakage flux - No core losses

At No-load: (i_2 = zero)

$$\boxed{N_1 i_\mu = \phi \Re}$$



From Faraday's Law

$$\boxed{\phi = \frac{V_{1m}}{\omega N_1} \sin(\omega t - \frac{\pi}{2})}$$

$$\boxed{i_\mu = \frac{V_{1m}}{\omega N_1^2} \Re \sin(\omega t - \frac{\pi}{2})}$$

Transformer: Equivalent Circuit

$$\boxed{i_{\mu} = \frac{V_{1m}}{\omega L_{core}} \sin(\omega t - \frac{\pi}{2})}$$

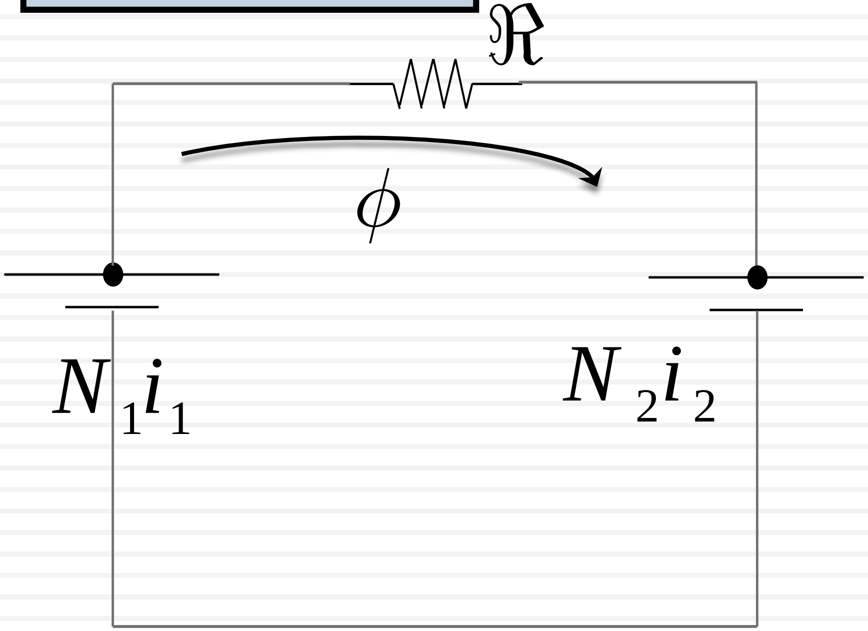
$$\boxed{I_{\mu} = \frac{V_1}{j\omega L_{core}}}$$

$$\boxed{X_{\mu} = \omega L_{core}}$$

If the transformer is loaded:

$$\boxed{N_1 i_1 - N_2 i_2 = \phi \Re}$$

$$\boxed{N_1 i_1 - N_2 i_2 = N_1 i_{\mu}}$$

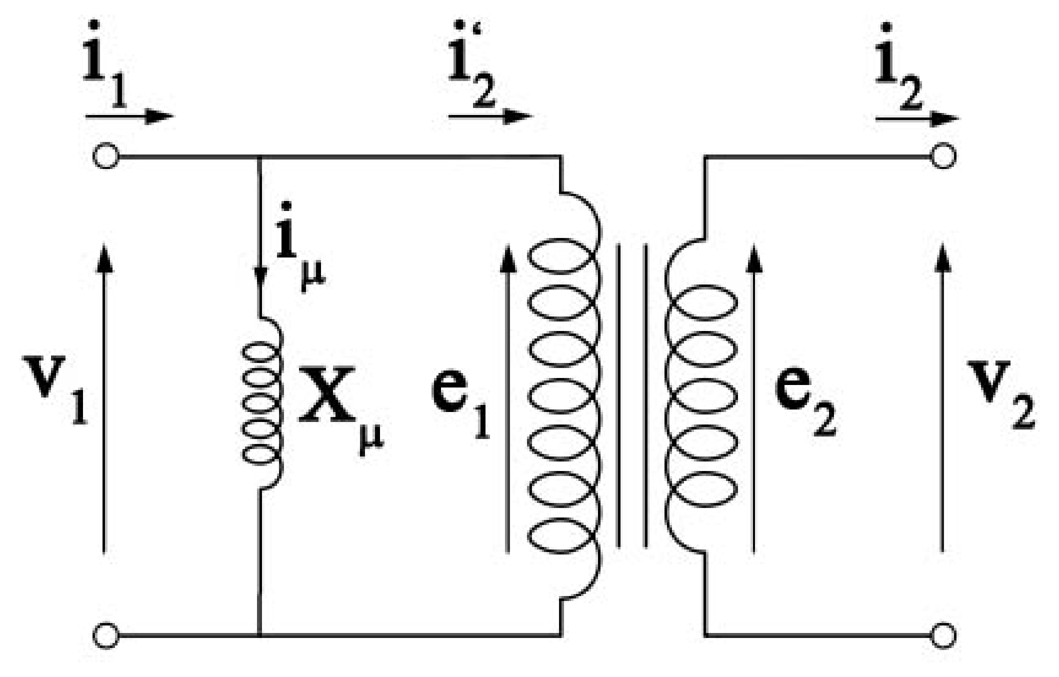


Transformer: Equivalent Circuit

$$\boxed{i_\mu = i_1 - \frac{N_2}{N_1} i_2}$$

$$\boxed{i_1 = i_\mu + \frac{N_2}{N_1} i_2}$$

$$\boxed{i'_2 = \frac{N_2}{N_1} i_2}$$



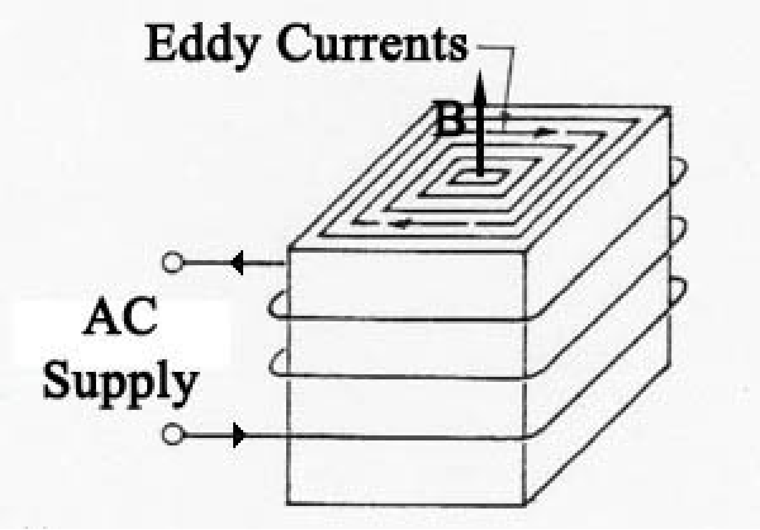
Transformer: Equivalent Circuit

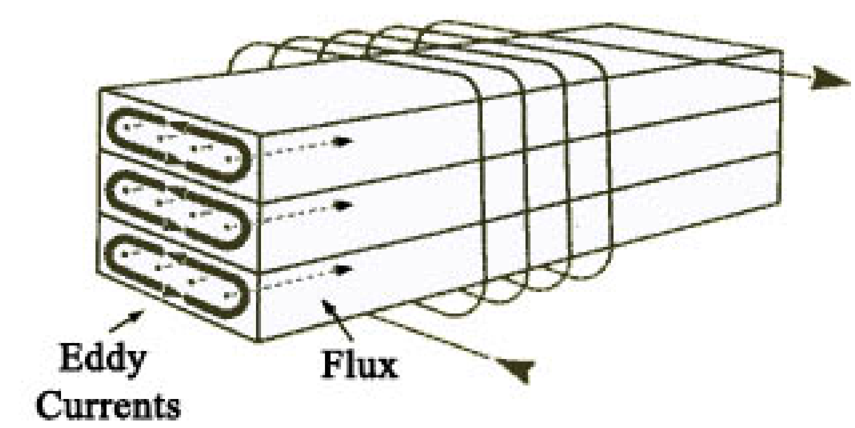
Case C: - R_{coil} = zero - $\mu \neq \infty$

- No leakage flux - core losses exist.

Core (iron) losses

1. Eddy Current Losses





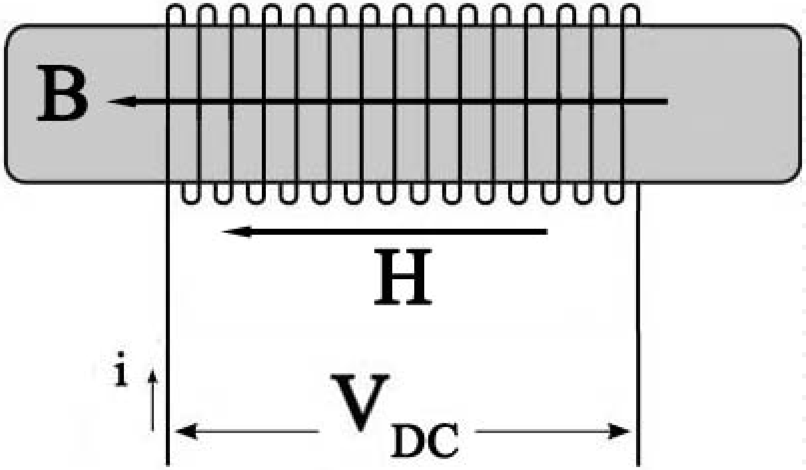
$$P_e \propto B_m^{\ 2} f^{\ 2} (thickness)^2$$

$$P_e = k_e B_m^{\ 2} f^{\ 2} (thickness)^2$$ W/m^3

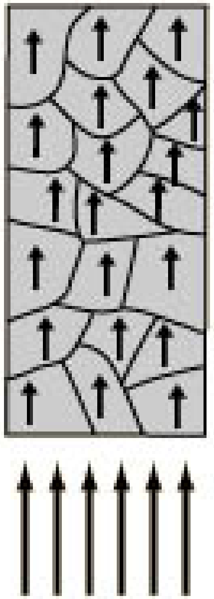
Transformer: Equivalent Circuit

Case C:

2. Hysteresis Losses



In bulk material the domains usually cancel, leaving the material unmagnetized.

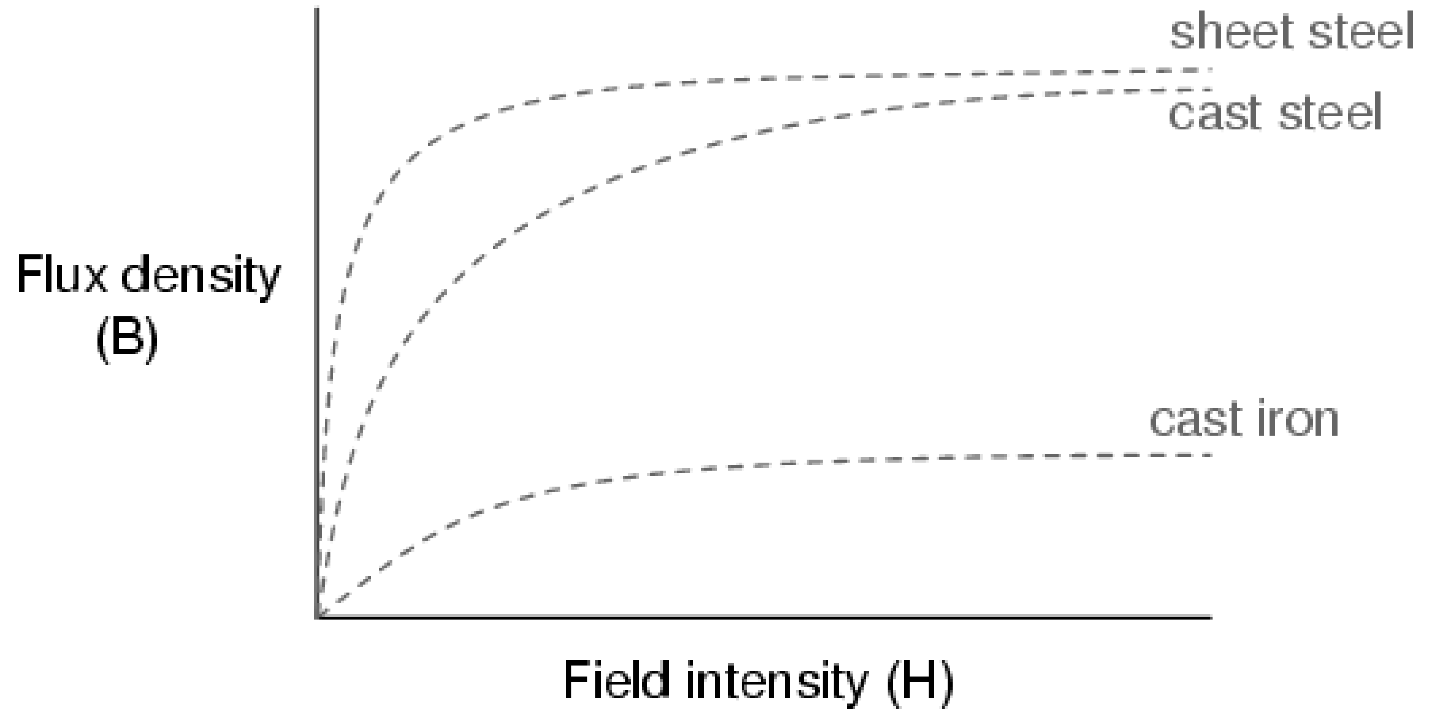


Externally applied magnetic field.

Transformer: Equivalent Circuit

Case C:

2. Hysteresis Losses

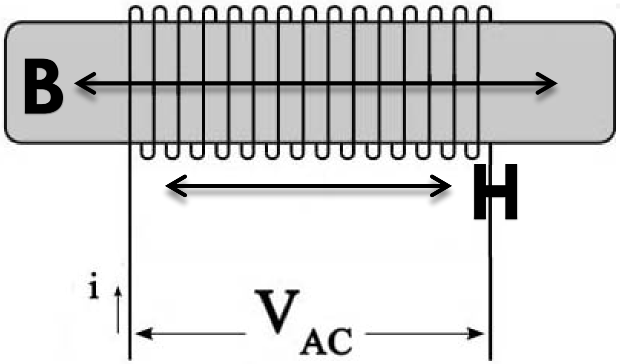


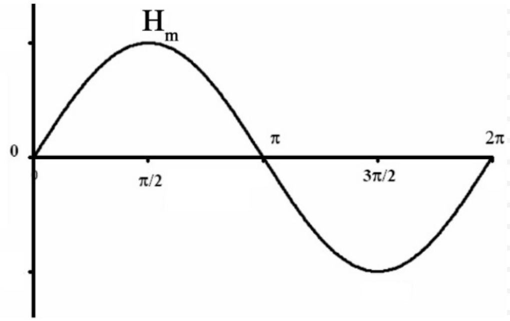
BH Curve (Magnetization Curve)

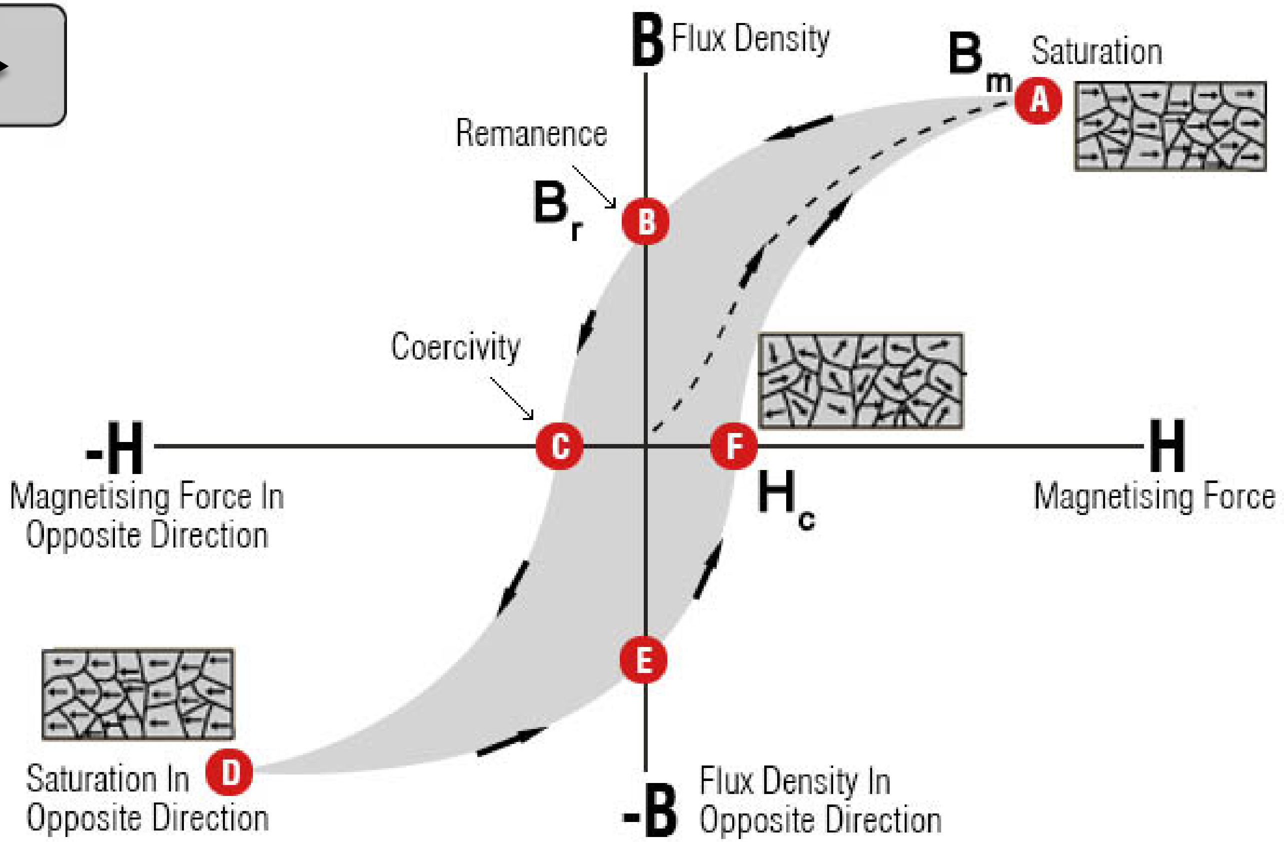
Transformer: Equivalent Circuit

Case C:

2. Hysteresis Losses







Transformer: Equivalent Circuit

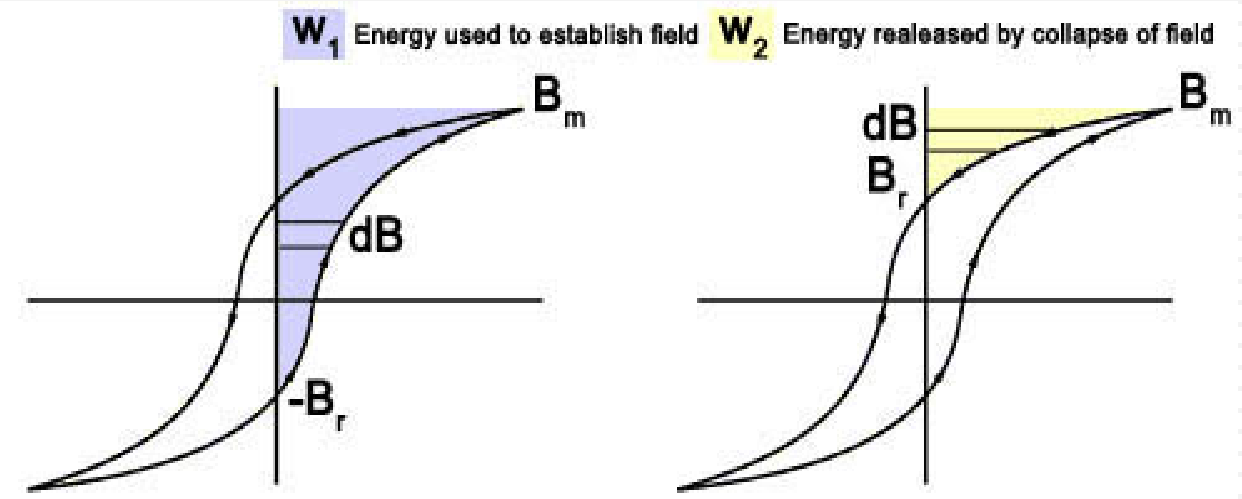
Case C:

2. Hysteresis Losses

$$H\,(\mathrm{A.m}^{-1})\times B\,(T\ = J.A^{-1}.m^{-2}) = J.m^{-3}$$

$$w_1 = \int_{-B_r}^{B_m} \underline{H}.\underline{dB}$$

$$w_2 = \int_{B_m}^{B_r} \underline{H}.\underline{dB}$$



Energy lost /unit volume/cycle = 2($w_1 - w_2$)= area of the loop

Transformer: Equivalent Circuit

Case C:

2. Hysteresis Losses

$$\boxed{P_h \propto B^n f\ (volume)} \qquad \boxed{P_h = k_h B^n f} \ \boxed{W/m^3}$$

***n* depends on the material (1.6 – 2.4)**

Total core losses:

$$\boxed{P_c = k_e B^2 f^2 (thickness)^2 + k_h B^n f} \ \boxed{W/m^3}$$

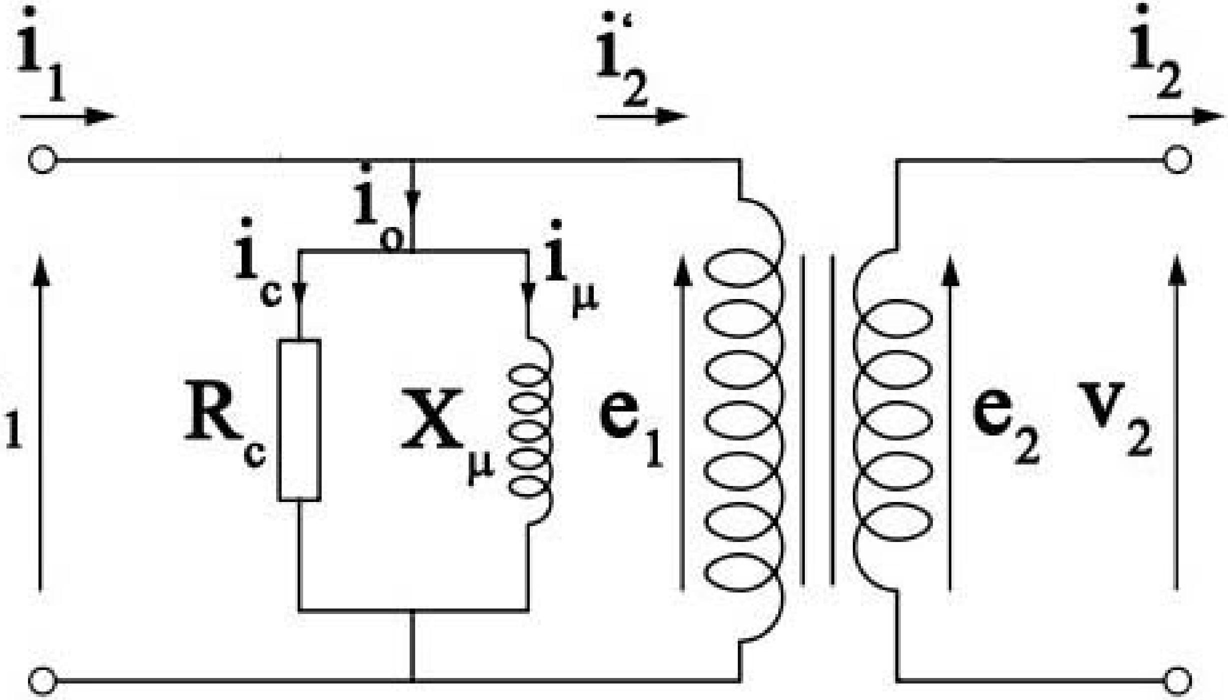
Since f is usually constant

$$\Rightarrow \boxed{P_c \propto B^2} \Rightarrow \boxed{P_c \propto \phi^2} \Rightarrow \boxed{P_c \propto V^2}$$

Transformer: Equivalent Circuit

Case C:

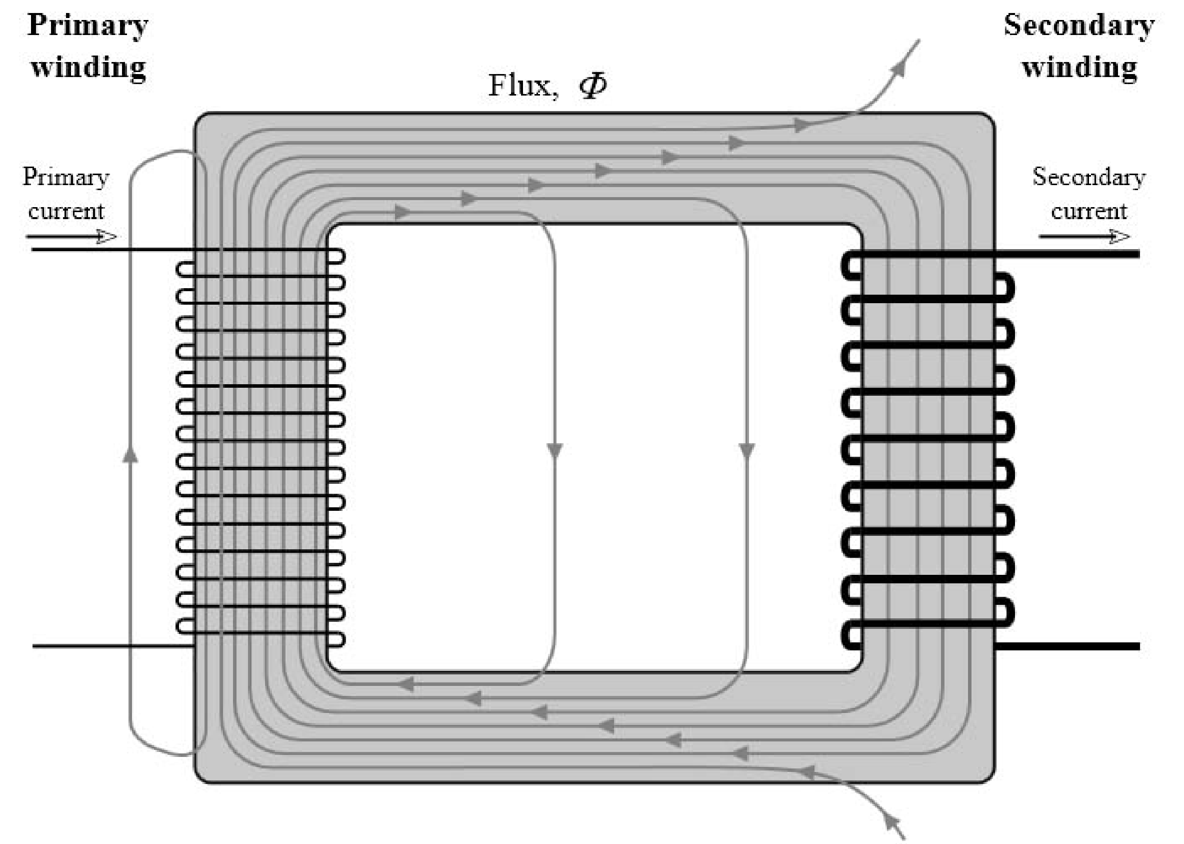
$$\boxed{P_c = \frac{V_1^2}{R_c}}$$



Transformer: Equivalent Circuit

Case D: - R_{coil} = zero - $\mu \neq \infty$

- Leakage flux exists. - core losses exist.



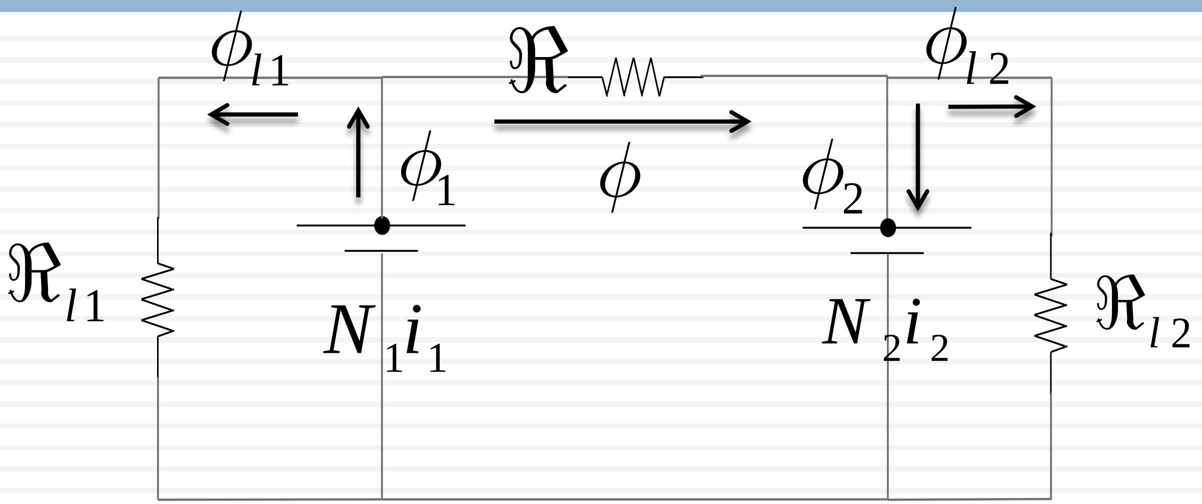
Transformer: Equivalent Circuit

Case D:

$$\phi_1 = \phi + \phi_{l1} \qquad (1)$$

$$\phi_2 = \phi - \phi_{l2} \qquad (2)$$

$$N_1 i_1 = \phi_{l1} \Re_{l1} \qquad (3)$$



$$N_2 i_2 = \phi_{l2} \Re_{l2} \qquad (4)$$

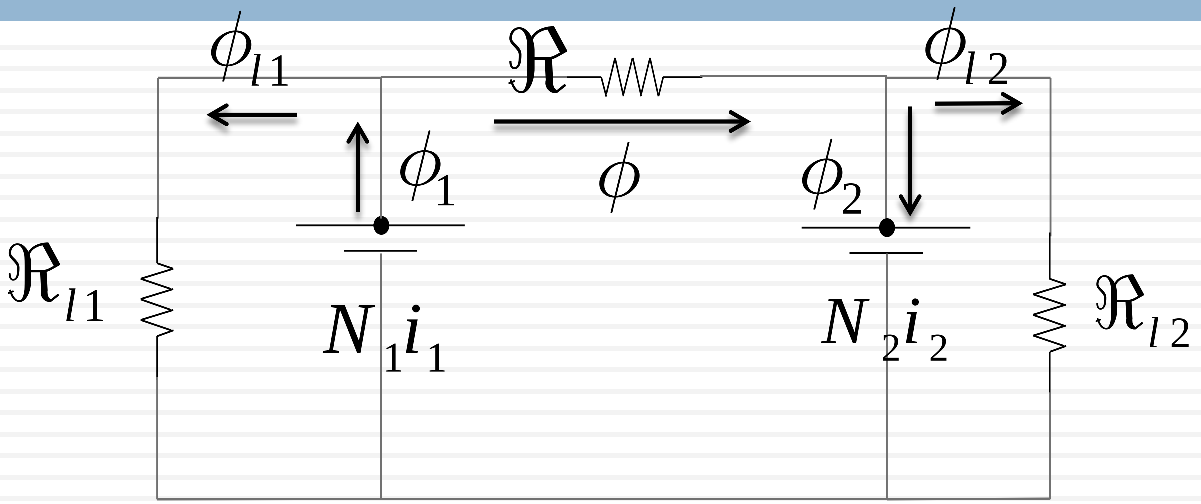
From (1) & (3)

$$N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi}{dt} + \boxed{N_1 \frac{d\phi_{l1}}{dt}} \Rightarrow v_{l1} = N_1 \frac{d\phi_{l1}}{dt} = \frac{N_1^2}{R_{l1}} \frac{di_1}{dt}$$

Transformer: Equivalent Circuit

Case D:

From (2) & (4)



$$N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi}{dt} - \boxed{N_2 \frac{d\phi_{l2}}{dt}} \Rightarrow v_{l2} = N_2 \frac{d\phi_{l2}}{dt} = \frac{N_2^2}{R_{l2}} \frac{di_2}{dt}$$

From Faraday's law

$$v_1 = N_1 \frac{d\phi_1}{dt} \qquad v_2 = N_2 \frac{d\phi_2}{dt}$$

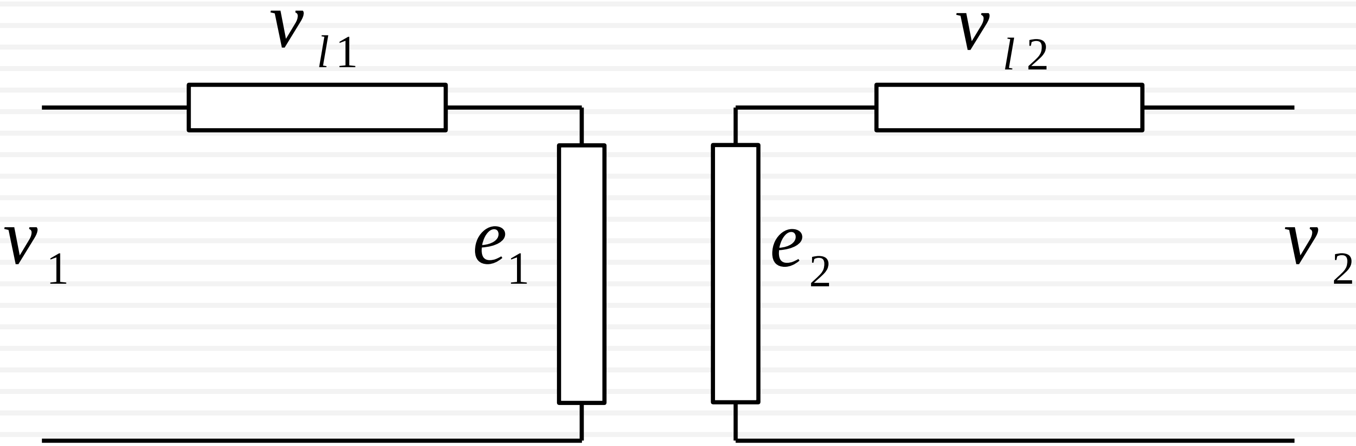
Transformer: Equivalent Circuit

Case D:

let

$$\boxed{e_1 = N_1 \frac{d\phi}{dt}} \qquad \boxed{e_2 = N_2 \frac{d\phi}{dt}}$$

⇨ $$\boxed{v_1 = e_1 + v_{l1}} \qquad \boxed{v_2 = e_2 - v_{l2}}$$



Transformer: Equivalent Circuit

Case D:

$$v_{l1} = \frac{N_1^2}{R_{l1}} \frac{di_1}{dt}$$

$$v_{l1} = L_{l1} \frac{di_1}{dt}$$

$$v_{l2} = \frac{N_2^2}{R_{l2}} \frac{di_2}{dt}$$

$$v_{l2} = L_{l2} \frac{di_2}{dt}$$

$$V_{l1} = j\omega L_{l1} I_1$$

$$V_{l1} = jX_{l1} I_1$$

$$V_{l2} = j\omega L_{l2} I_2$$

$$V_{l2} = jX_{l2} I_2$$

$$L_{l1} = \frac{N_1^2}{R_{l1}}$$

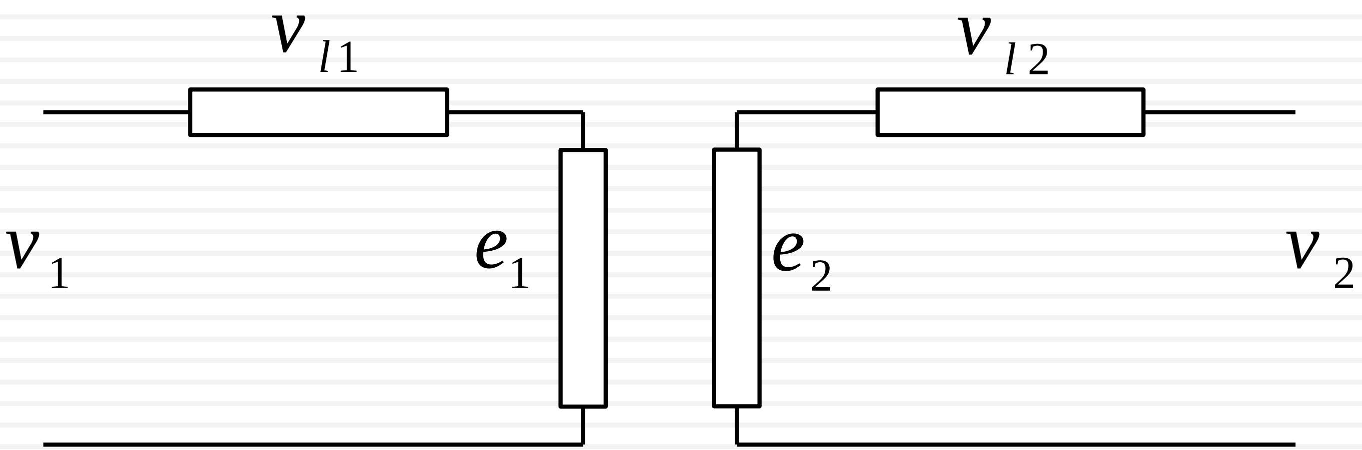
$$L_{l2} = \frac{N_2^2}{R_{l2}}$$

Transformer: Equivalent Circuit

Case D:

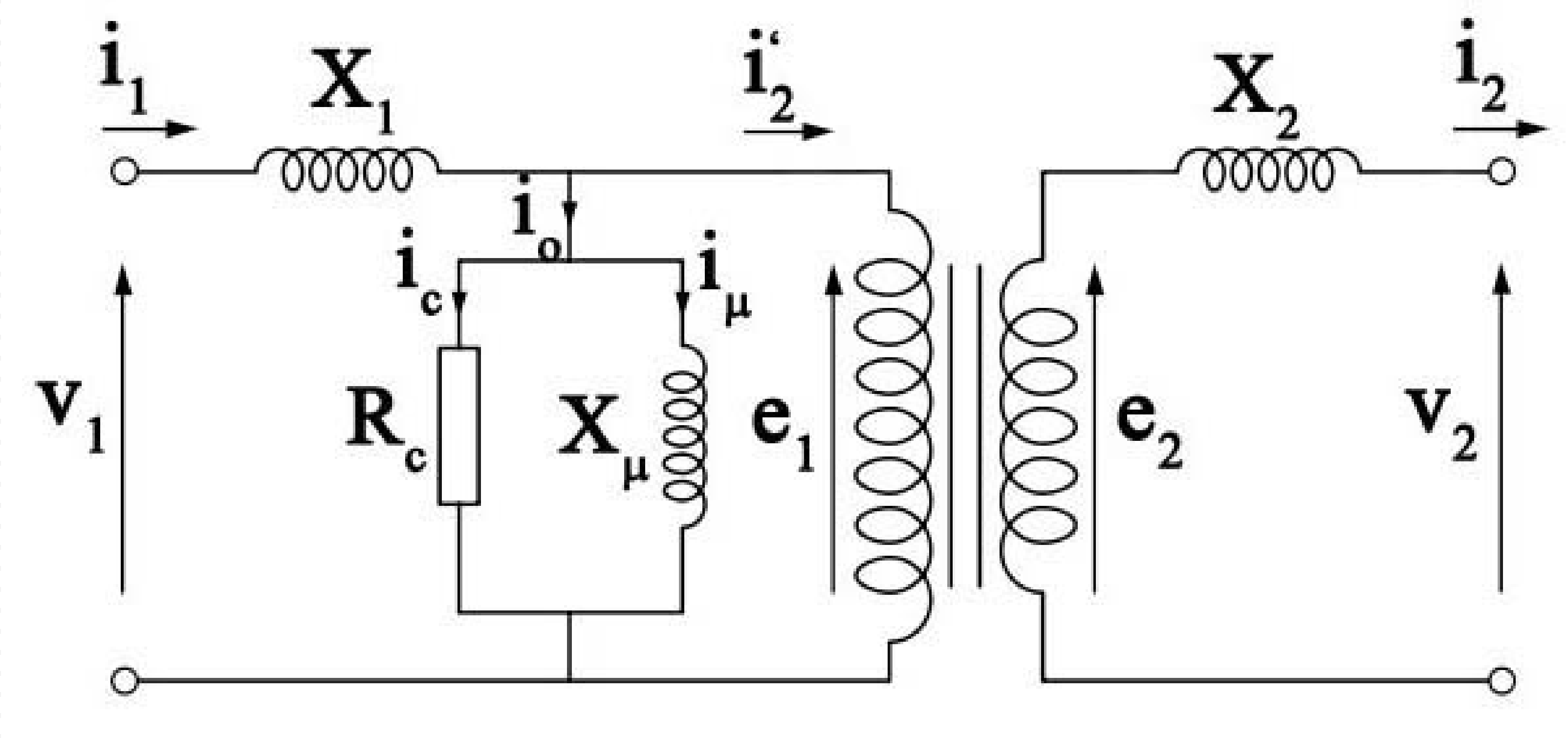
$$\boxed{V_{l1} = jX_{l1}I_1}$$

$$\boxed{V_{l2} = jX_{l2}I_2}$$



Transformer: Equivalent Circuit

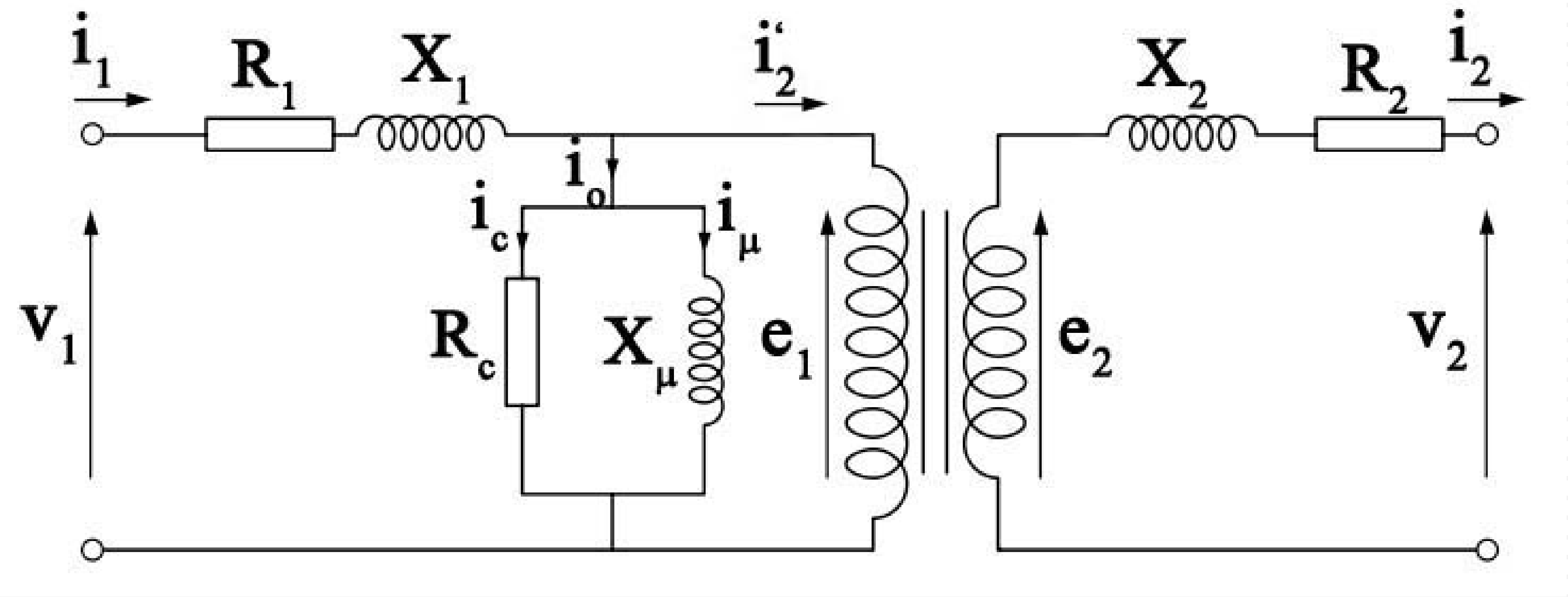
Case D:



Transformer: Equivalent Circuit

Case E: - $\boxed{R_{coil} \neq \text{zero}}$ - $\mu \neq \infty$

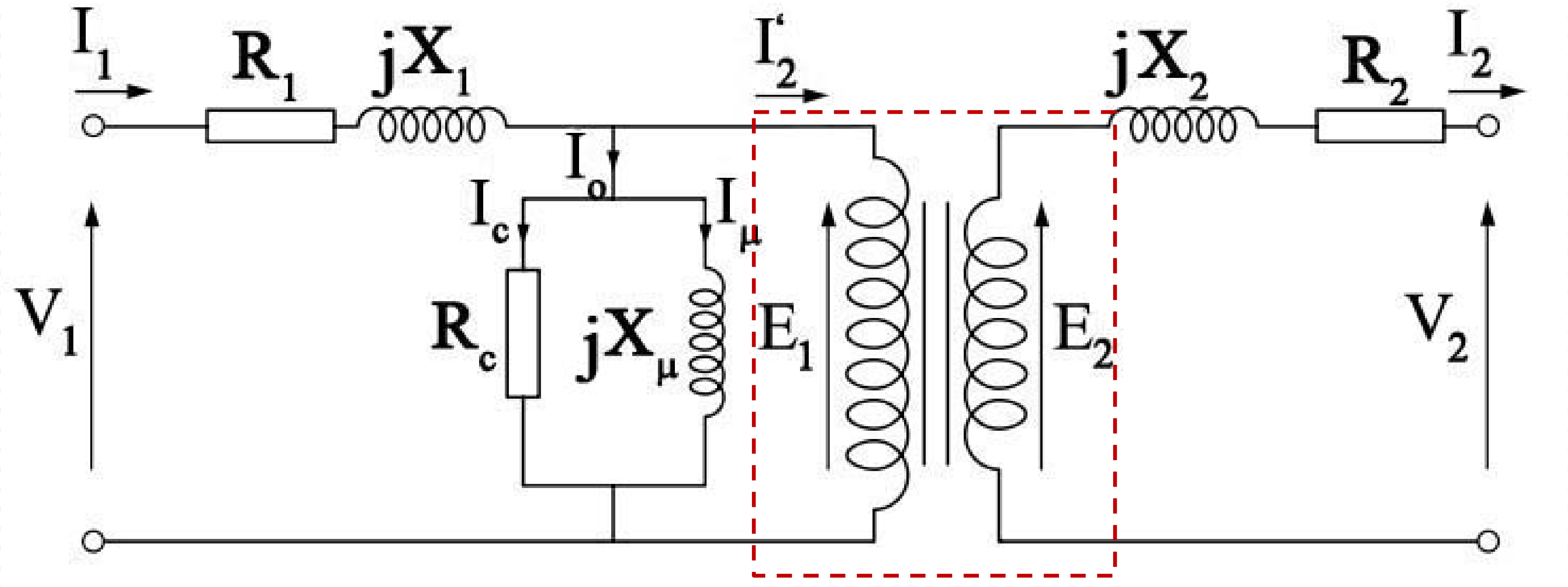
- Leakage flux exists. - core losses exist.



Exact Equivalent Circuit of the Transformer

Transformer: Equivalent Circuit

$$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}} \qquad \boxed{\frac{N_1}{N_2} = \frac{I_2}{I_2^{'}}}$$



Exact Equivalent Circuit of the Transformer

Transformer: Equivalent Circuit

R_1 = Resistance of the primary winding

R_2 = Resistance of the secondary winding

X_1 = Leakage Reactance of the primary winding

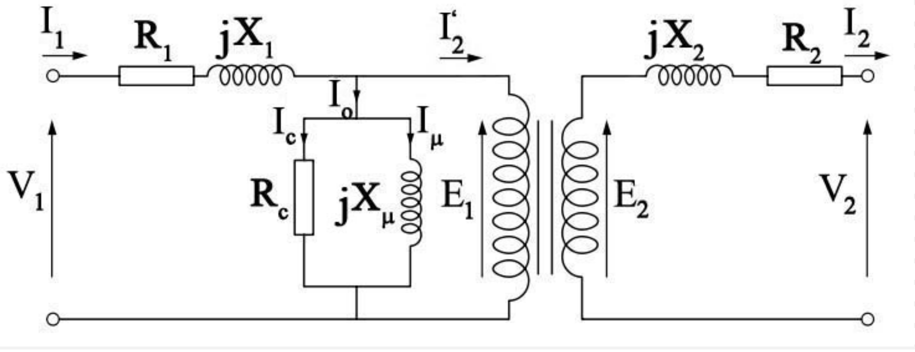
X_2 = Leakage Reactance of the secondary winding

R_C = Resistance representing the core (iron) losses

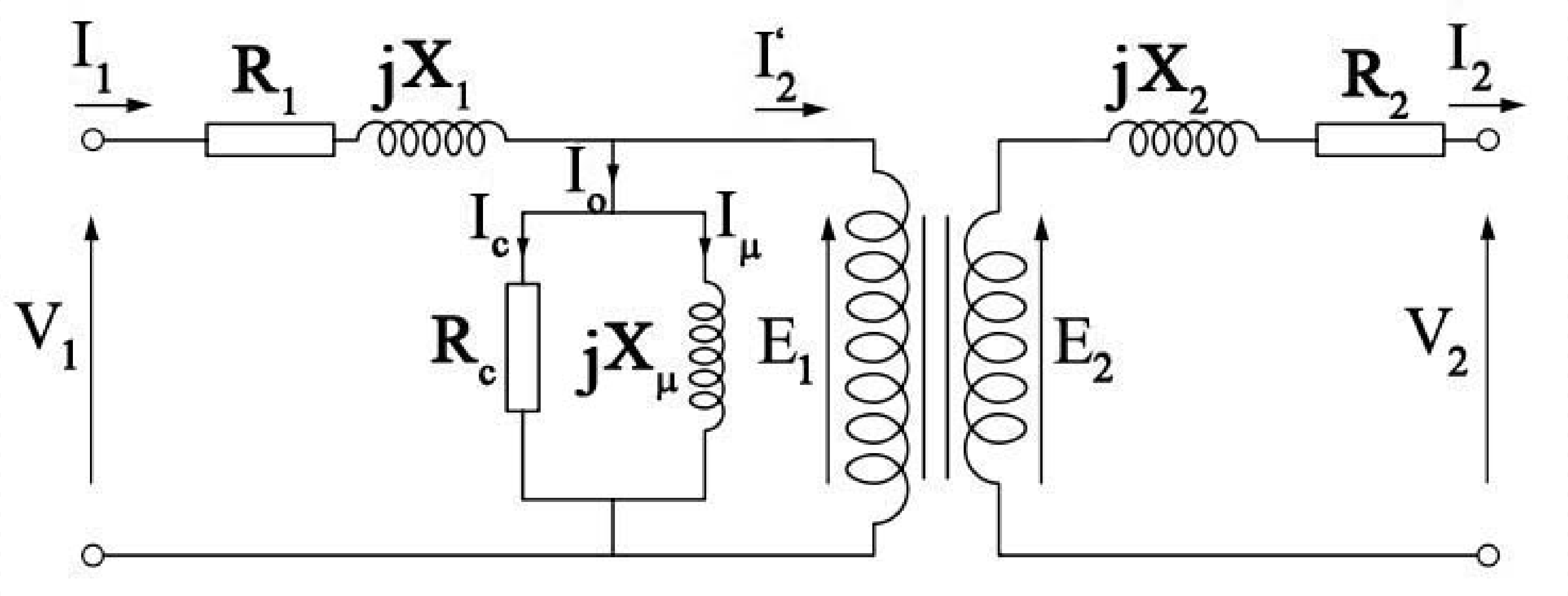
X_μ= reactance representing the main flux path

$$\boxed{R_C >> X_\mu}$$

$$\boxed{R_C \ \& X_\mu >> R_1, X_1 \ \& R_2, X_2}$$



Transformer: Equivalent Circuit



$$E_1 = 4.44 f \phi_m N_1$$

$$E_2 = 4.44 f \phi_m N_2$$

Transformer: Equivalent Circuit

At No-load:

$\boxed{I_2 = 0}$ → $\boxed{I_2' = 0}$ → $\boxed{E_2 = V_2}$

$\boxed{I_1 = I_o}$

$\boxed{I_o = 2 - 5\% I_{FL}}$

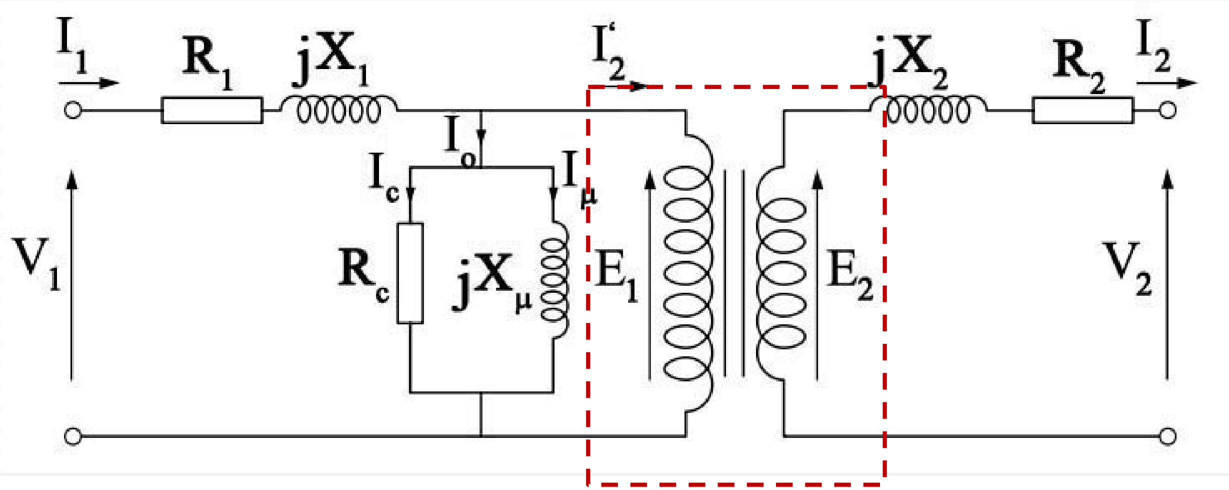
$$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}}$$

$$\boxed{\frac{N_1}{N_2} = \frac{I_2}{I_2'}}$$

→ $\boxed{I_o(R_1 + jX_1)}$ Small Voltage Drop

→ $\boxed{V_1 \cong E_1}$

→ $$\boxed{\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2}}$$

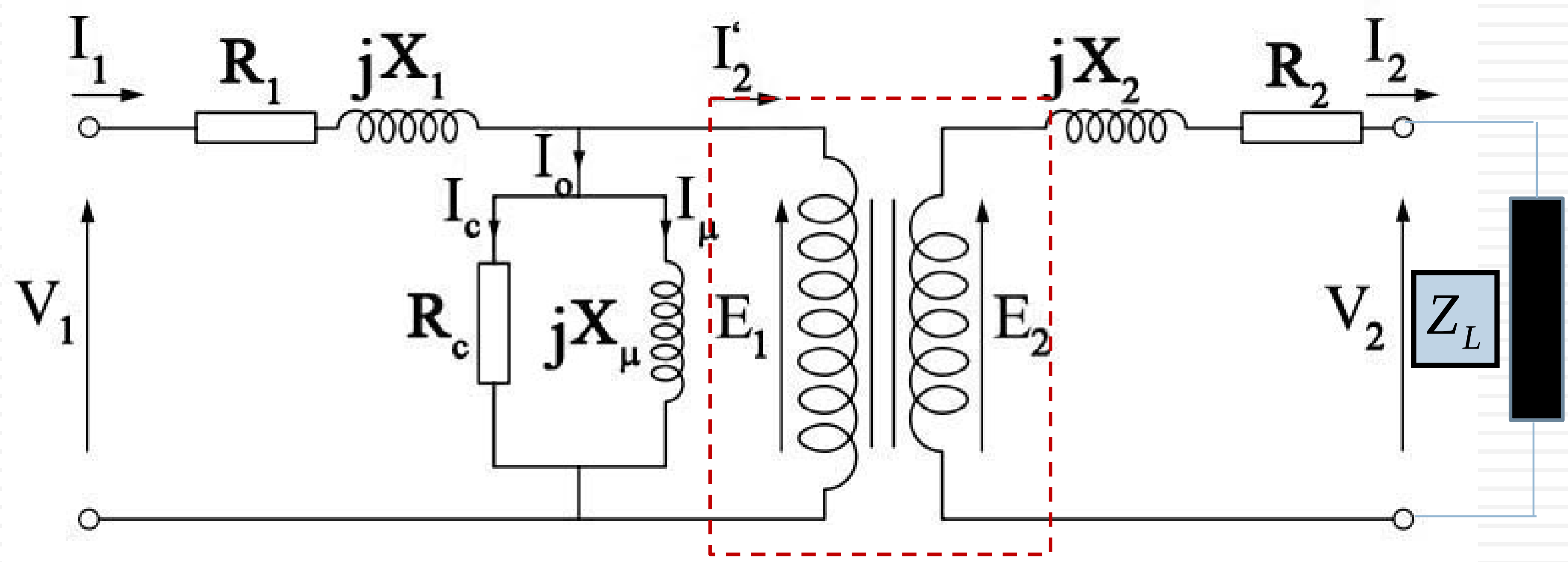


Transformer: Equivalent Circuit

At loading:

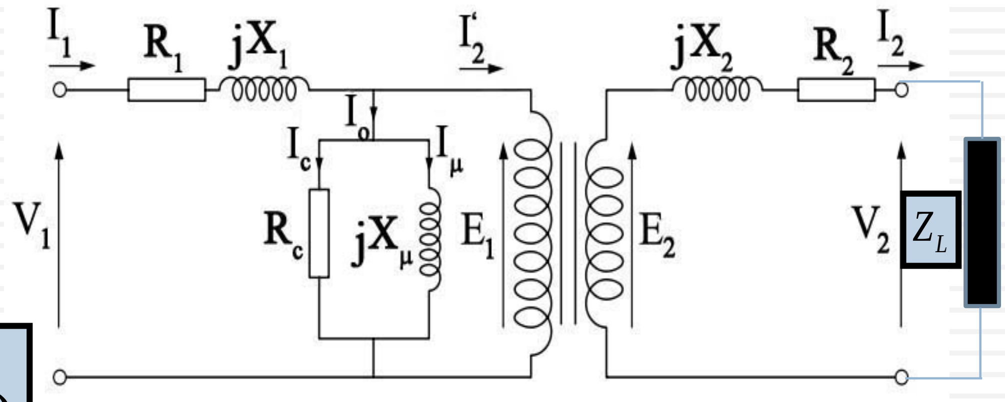
$$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}}$$

$$\boxed{\frac{N_1}{N_2} = \frac{I_2}{I_2'}}$$



Transformer: Equivalent Circuit

At loading:



$$E_2 = V_2 + I_2(R_2 + jX_2)$$

$$E_1 \frac{N_2}{N_1} = V_2 + I_2{}' \frac{N_1}{N_2}(R_2 + jX_2)$$

$$E_1 = V_2\left(\frac{N_1}{N_2}\right) + I_2{}'\left(\frac{N_1}{N_2}\right)^2 (R_2 + jX_2)$$

$$E_1 = V_2{}' + I_2{}'(R_2{}' + jX_2{}')$$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$\frac{N_1}{N_2} = \frac{I_2}{I_2{}'}$$

$$\frac{V_2{}'}{V_2} = \frac{N_1}{N_2}$$

$$R_2{}' = R_2\left(\frac{N_1}{N_2}\right)^2$$

$$X_2{}' = X_2\left(\frac{N_1}{N_2}\right)^2$$

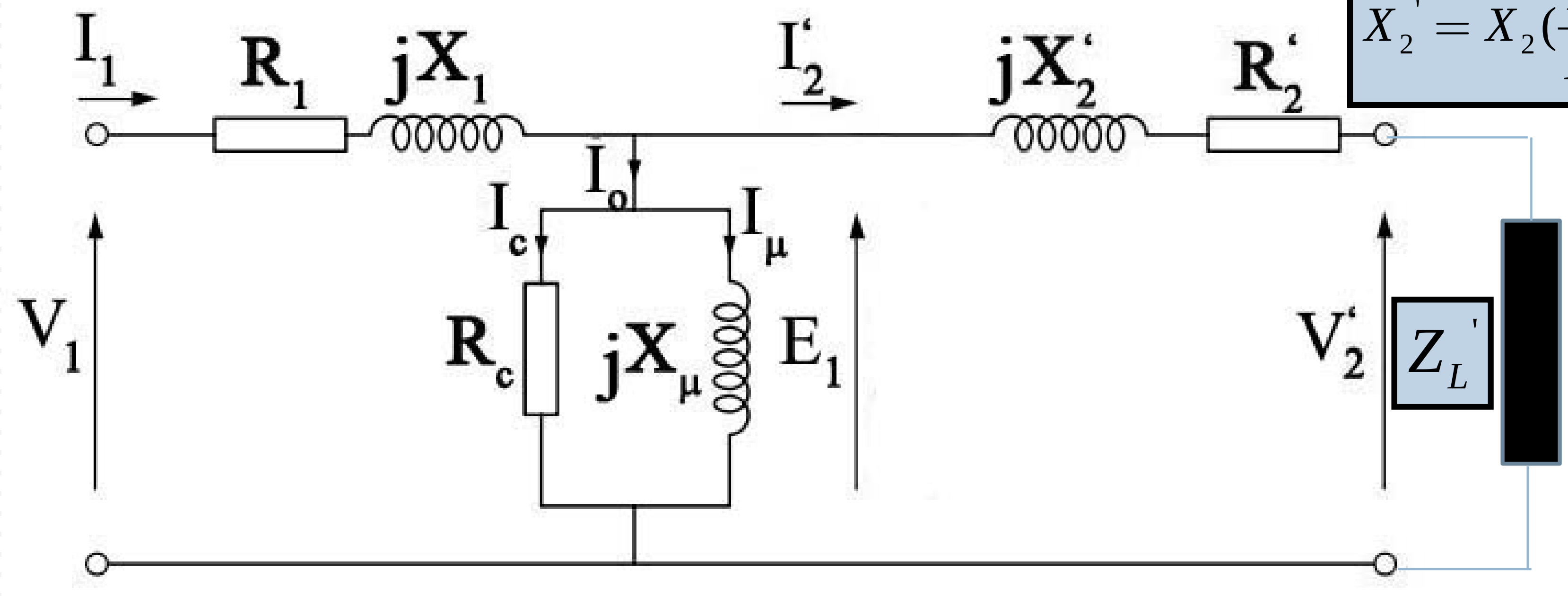
Transformer: Equivalent Circuit

$$E_1 = V_2^{'} + I_2^{'}(R_2^{'} + jX_2^{'})$$

$$\frac{V_2^{'}}{V_2} = \frac{N_1}{N_2}$$

$$R_2^{'} = R_2\left(\frac{N_1}{N_2}\right)^2$$

$$X_2^{'} = X_2\left(\frac{N_1}{N_2}\right)^2$$



Exact Equivalent Circuit referred to Primary

Transformer: Equivalent Circuit

$$\frac{V_1{}'}{V_1} = \frac{N_2}{N_1}$$

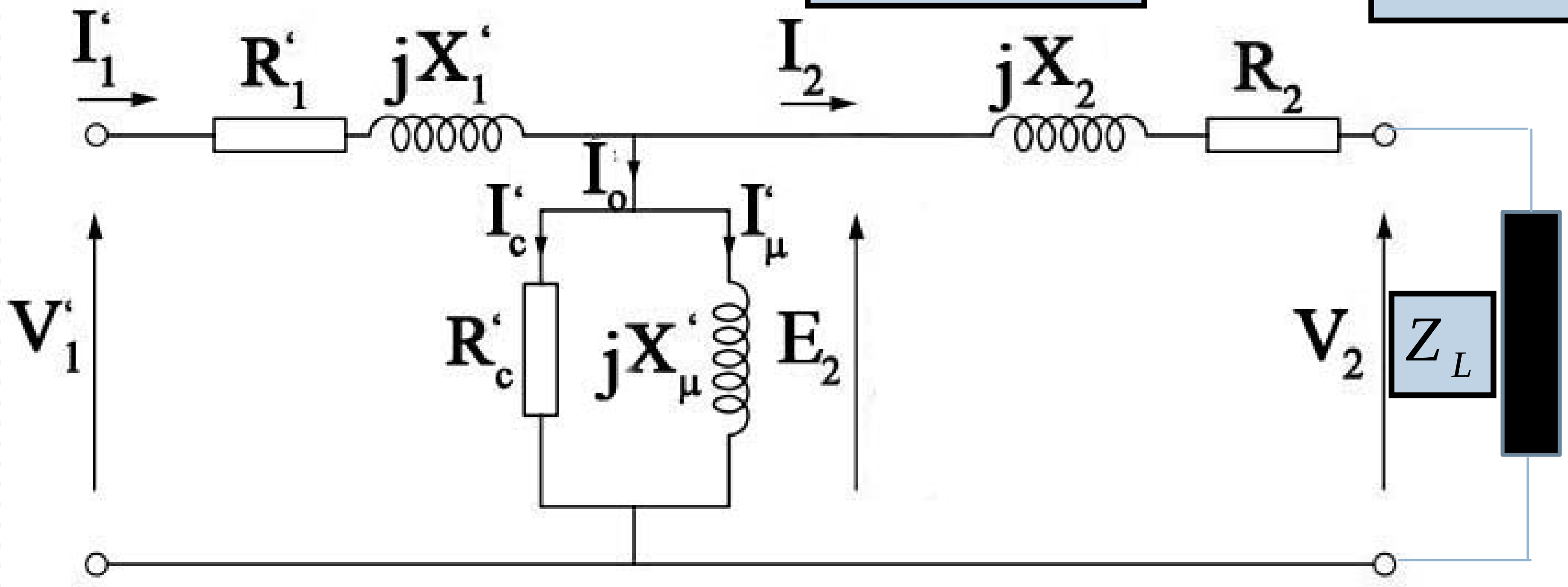
$$\frac{I_1{}'}{I_1} = \frac{N_1}{N_2}$$

$$R_c{}' = R_c(\frac{N_2}{N_1})^2$$

$$X_\mu{}' = X_\mu(\frac{N_2}{N_1})^2$$

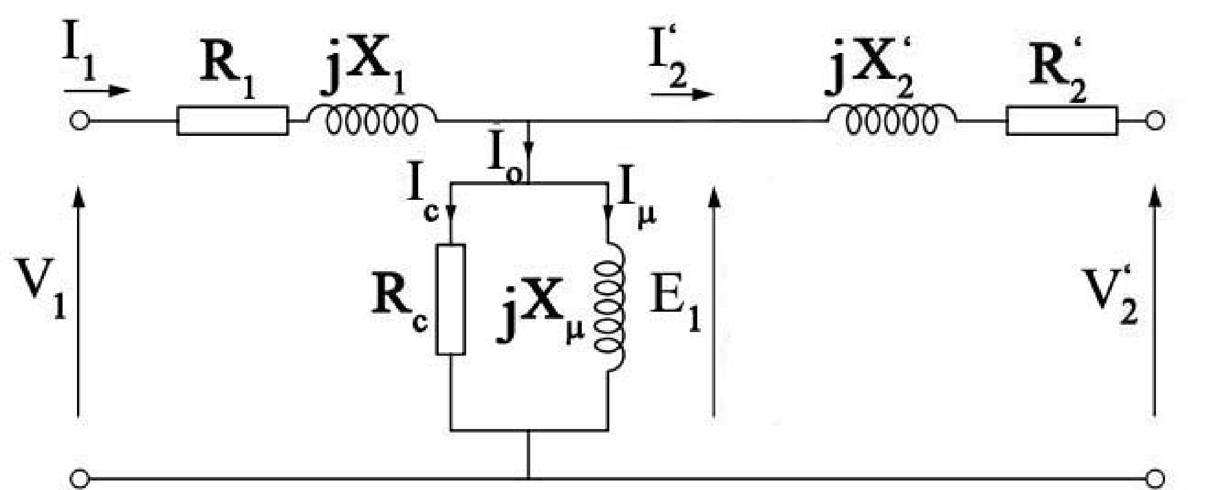
$$R_1{}' = R_1(\frac{N_2}{N_1})^2$$

$$X_1{}' = X_1(\frac{N_2}{N_1})^2$$



Exact Equivalent Circuit referred to Secondary

Transformer: Equivalent Circuit



$$\boxed{V_1 = E_1 + I_1(R_1 + jX_1)}$$

$$\boxed{I_1 = I_o + I_2'}$$

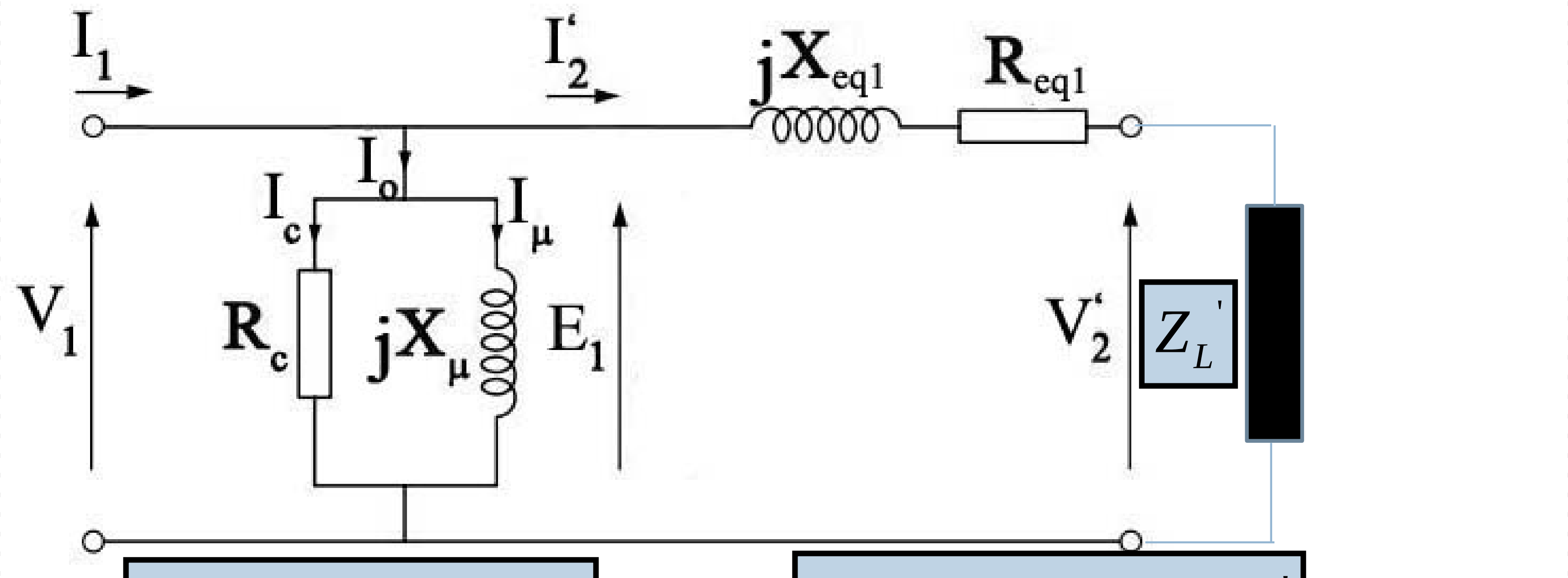
$$\boxed{V_1 = E_1 + (I_o + I_2')(R_1 + jX_1)} \Rightarrow \boxed{E_1 = V_2' + I_2'(R_2' + jX_2')}$$

$$\Rightarrow \boxed{V_1 = V_2' + \boxed{I_o(R_1 + jX_1)} + I_2'[(R_1 + jX_1) + (R_2' + jX_2')]}$$

Small Voltage Drop

Transformer: Equivalent Circuit

$$V_1 \cong V_2' + I_2'[(R_1 + jX_1) + (R_2' + jX_2')]$$



$$R_{eq1} = R_1 + R_2'$$

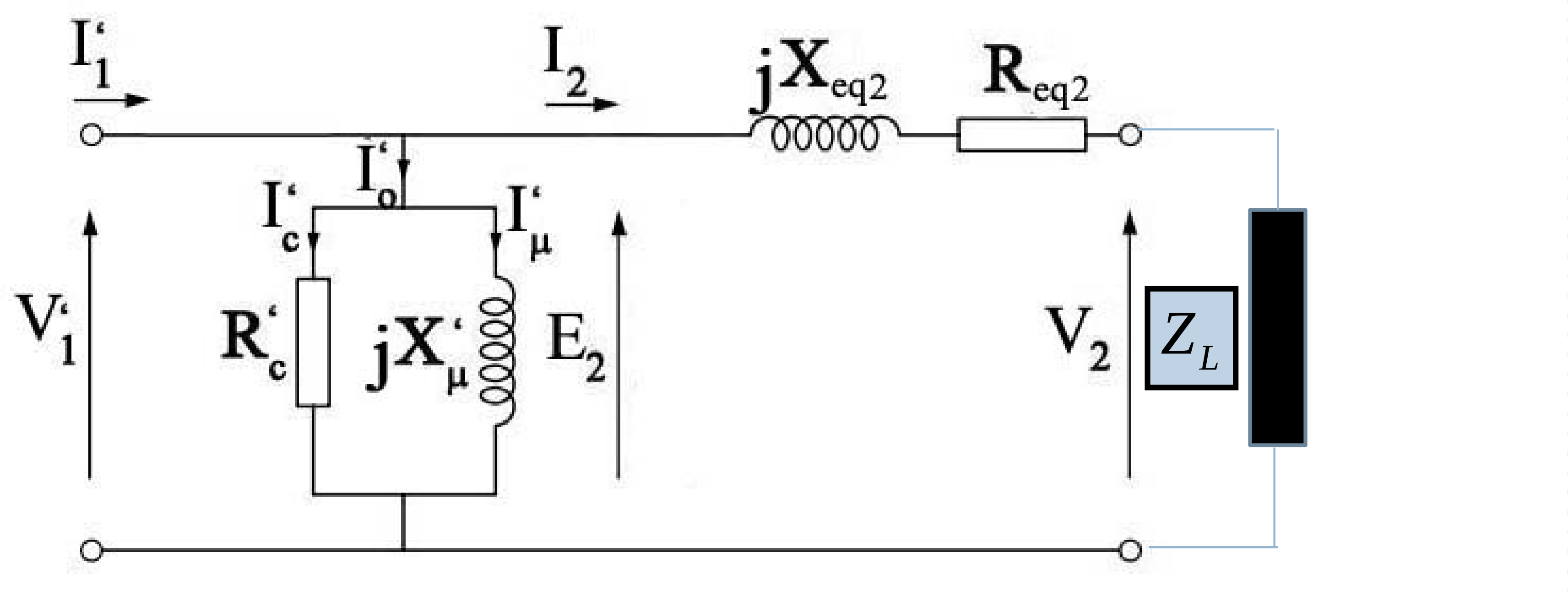
$$X_{eq1} = X_1 + X_2'$$

Equivalent resistance referred to the primary

Equivalent leakage reactance referred to the primary

Approximate Eq. Ct. referred to Primary

Transformer: Equivalent Circuit



$$R_{eq2} = R_1^{'} + R_2$$

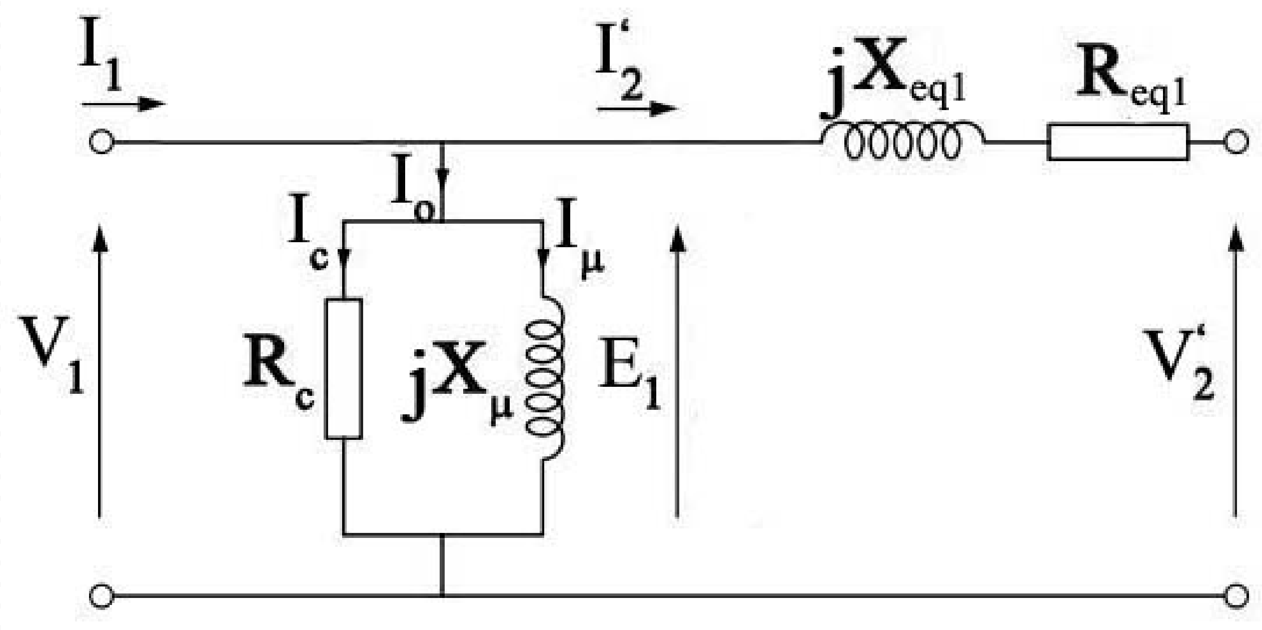
Equivalent resistance referred to the secondary

$$X_{eq2} = X_1^{'} + X_2$$

Equivalent leakage reactance referred to the secondary

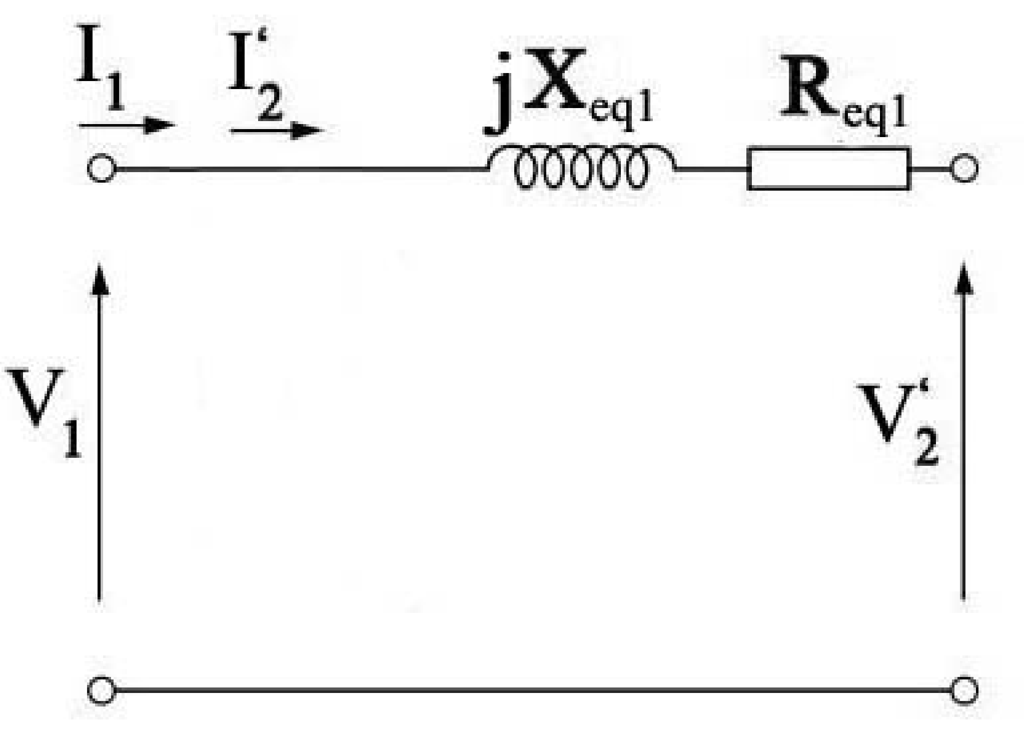
Approximate Eq. Ct. referred to Secondary

Transformer: Equivalent Circuit



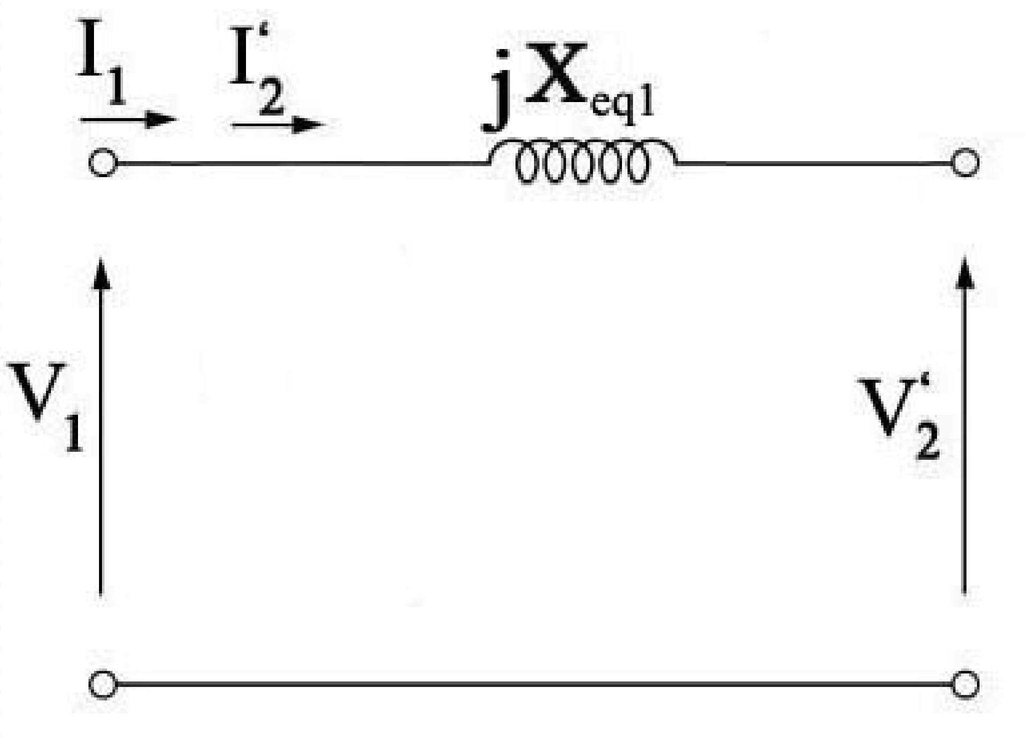
Eq. Ct. for small transformers (up to 100 kVA)

Transformer: Equivalent Circuit



Eq. Ct. for transformers (100-500 kVA)

Transformer: Equivalent Circuit



Eq. Ct. for large transformers (larger than 500 kVA)

Transformer: Equivalent Circuit

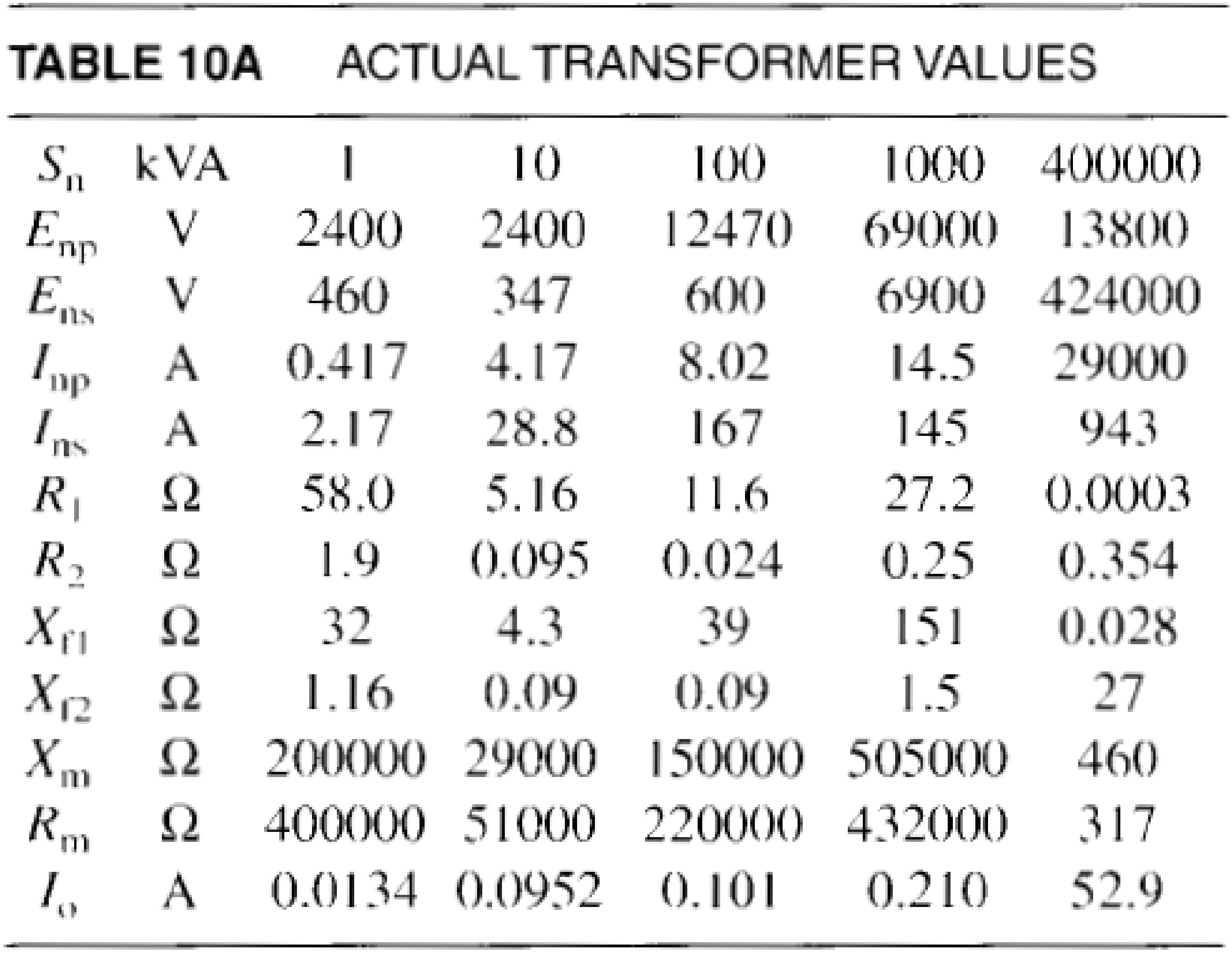


TABLE 10A ACTUAL TRANSFORMER VALUES

S_n	kVA	1	10	100	1000	400000
E_{np}	V	2400	2400	12470	69000	13800
E_{ns}	V	460	347	600	6900	424000
I_{np}	A	0.417	4.17	8.02	14.5	29000
I_{ns}	A	2.17	28.8	167	145	943
R_1	Ω	58.0	5.16	11.6	27.2	0.0003
R_2	Ω	1.9	0.095	0.024	0.25	0.354
X_{f1}	Ω	32	4.3	39	151	0.028
X_{f2}	Ω	1.16	0.09	0.09	1.5	27
X_m	Ω	200000	29000	150000	505000	460
R_m	Ω	400000	51000	220000	432000	317
I_o	A	0.0134	0.0952	0.101	0.210	52.9