

Introduction: Induction Motor

Fundamentals of AC Machines

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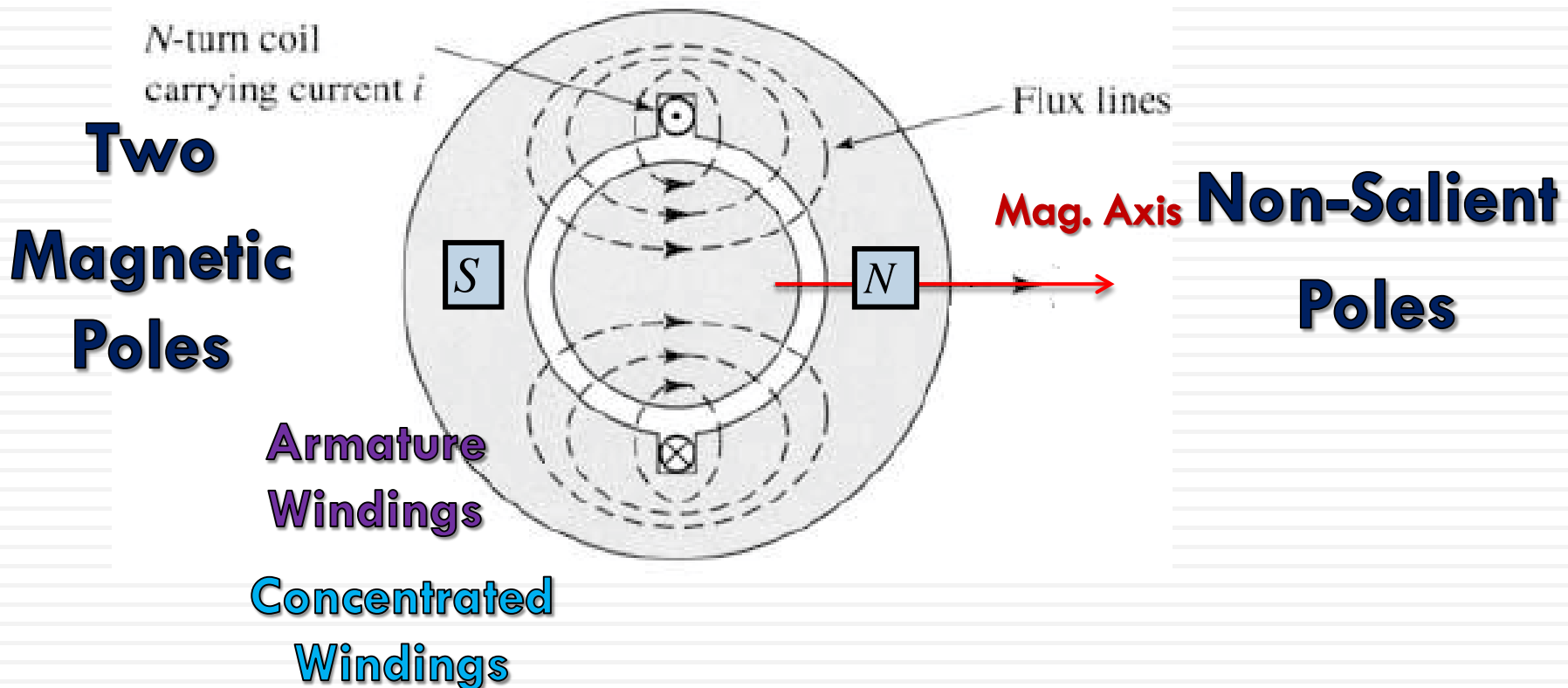
- Magnetic Field Production and Distribution
 - DC Machines
 - Synchronous Machines
 - Induction Machines
- Generated (induced) Voltages
 - Synchronous Machines
 - Induction Machines

Magnetic Field Production & Distribution

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Induction Machines

Single-phase machines



Magnetic Field Production & Distribution

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Induction Machines

Single-phase machines

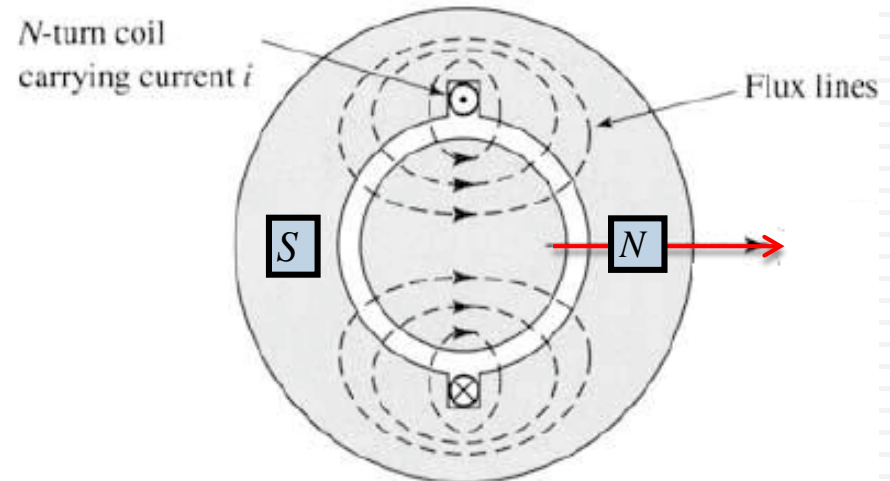
$$\oint_c \underline{H} \cdot \underline{dl} = I_{en}$$

$$\oint_c \underline{H} \cdot \underline{dl} = Ni$$

$$H \times g + H \times g \simeq Ni$$

$$H_g g = \frac{Ni}{2} = M$$

$$M_f = \frac{B_g}{\mu_o} g$$

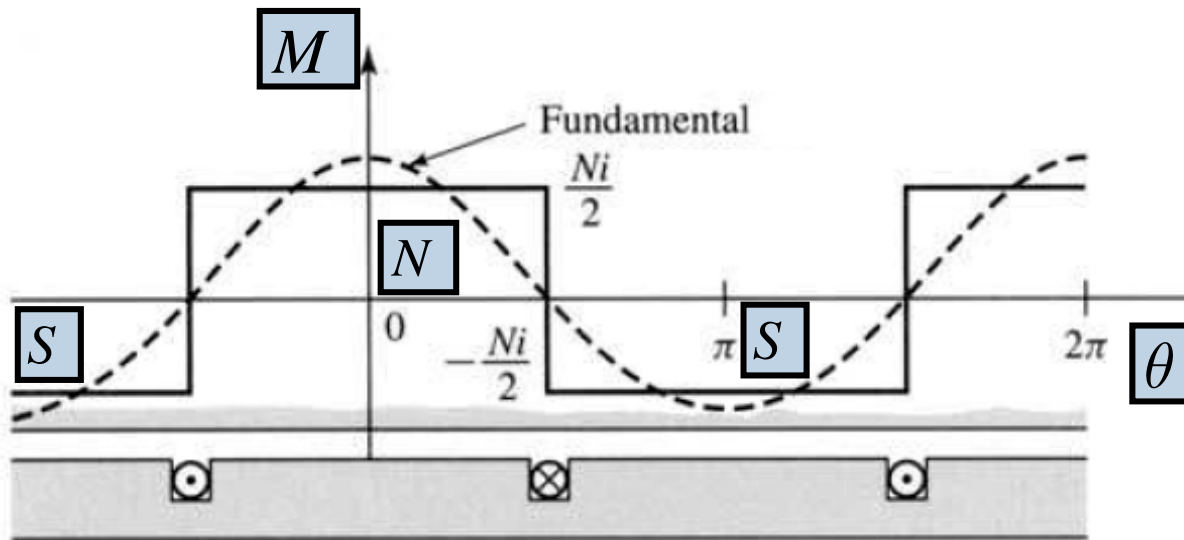


Magnetic Field Production & Distribution

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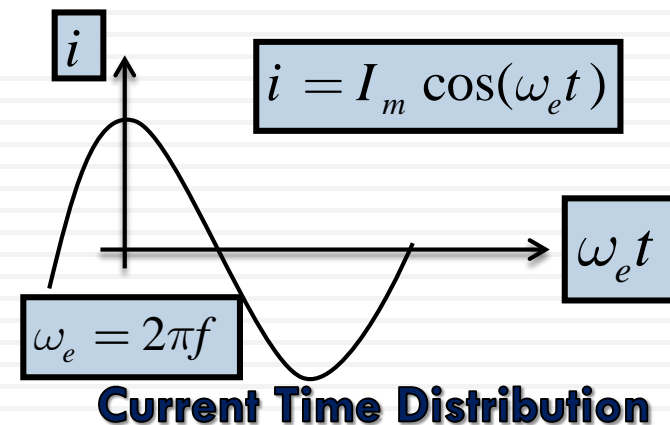
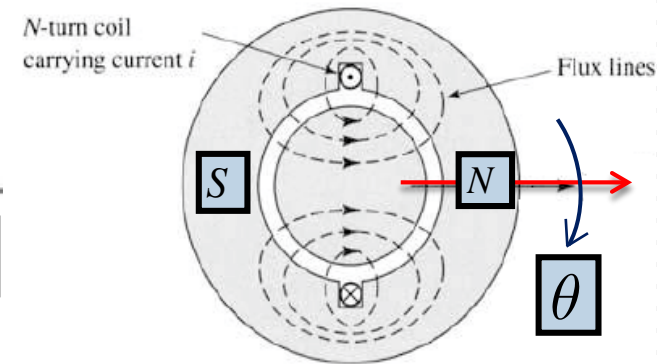
Induction Machines

Single-phase machines



MMF Space Distribution

$$M(\theta) = \frac{4}{\pi} \frac{Ni}{2p} \sum_{h=1,3,5,\dots} \frac{1}{h} \cos(h\theta)$$



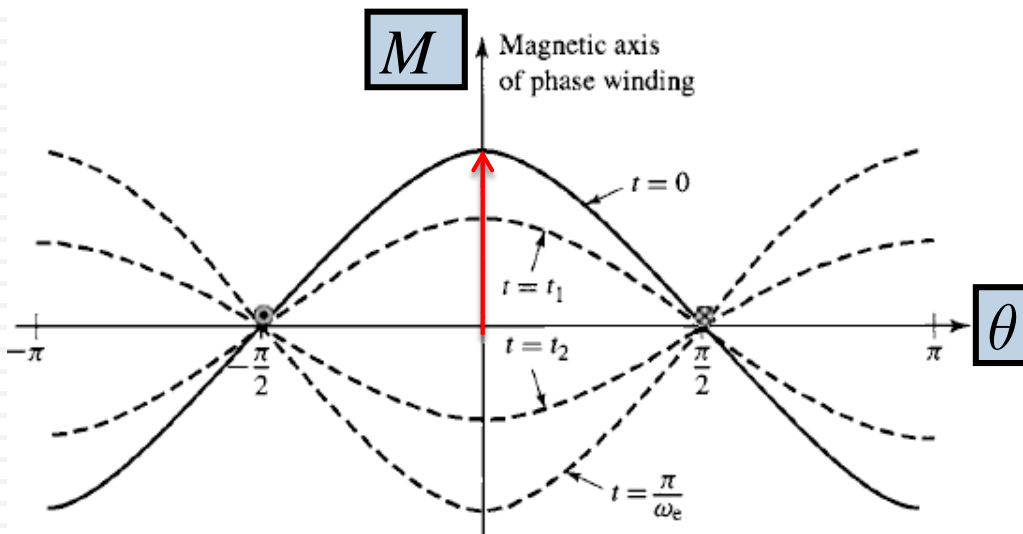
Current Time Distribution

Magnetic Field Production & Distribution

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Induction Machines

Single-phase machines



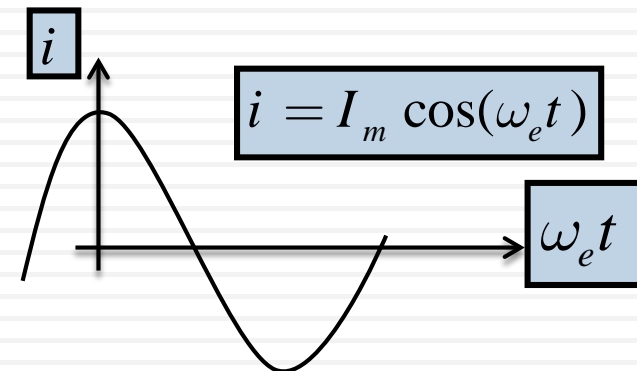
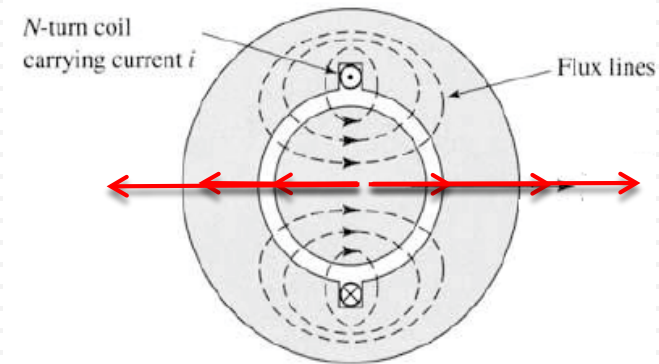
$$M_{fund} = \frac{4 Ni}{\pi 2} \cos \theta$$

MMF Space Distribution

AC Current



Pulsating Field



Current Time Distribution

Magnetic Field Production & Distribution

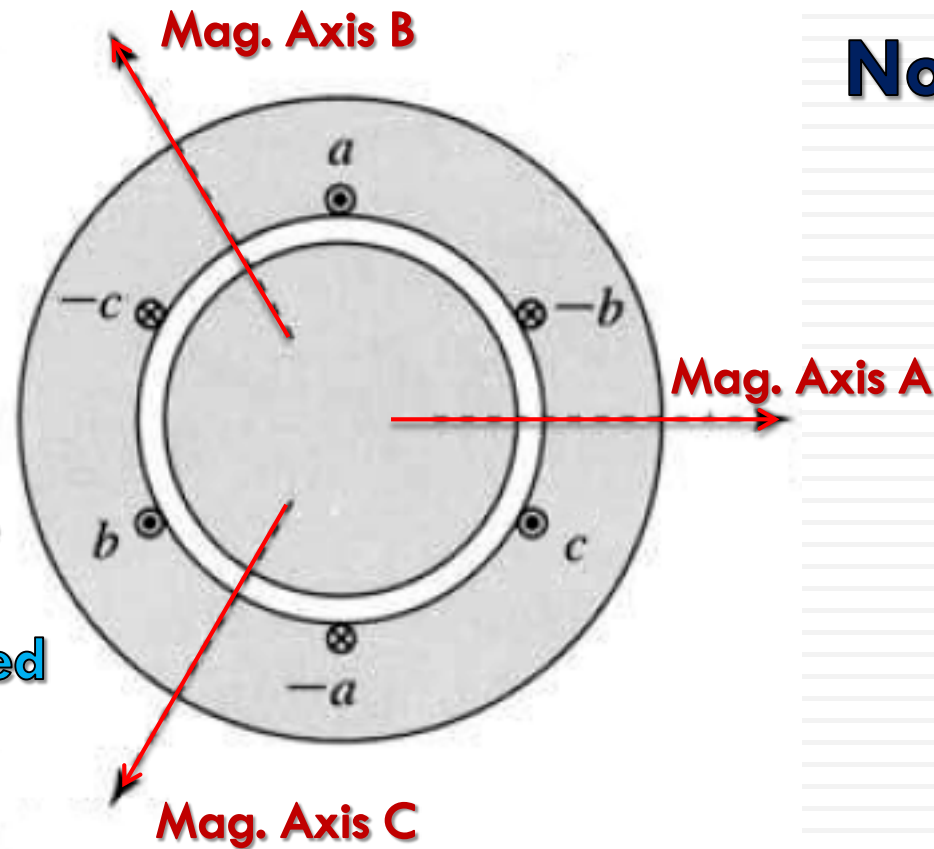
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Induction Machines

Three-phase machines

**Two
Magnetic
Poles**

**Armature
Windings
Concentrated
Windings**



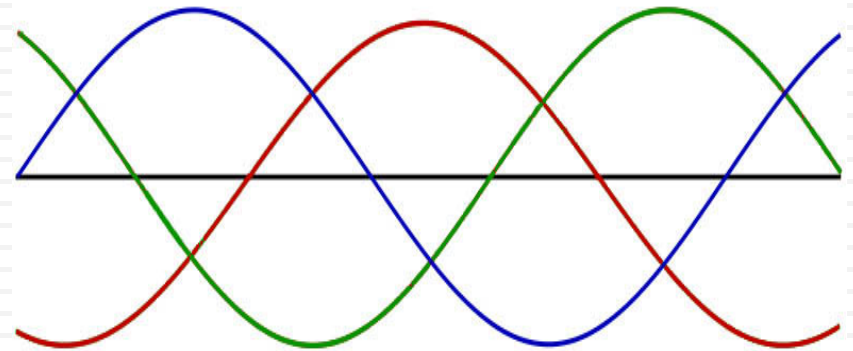
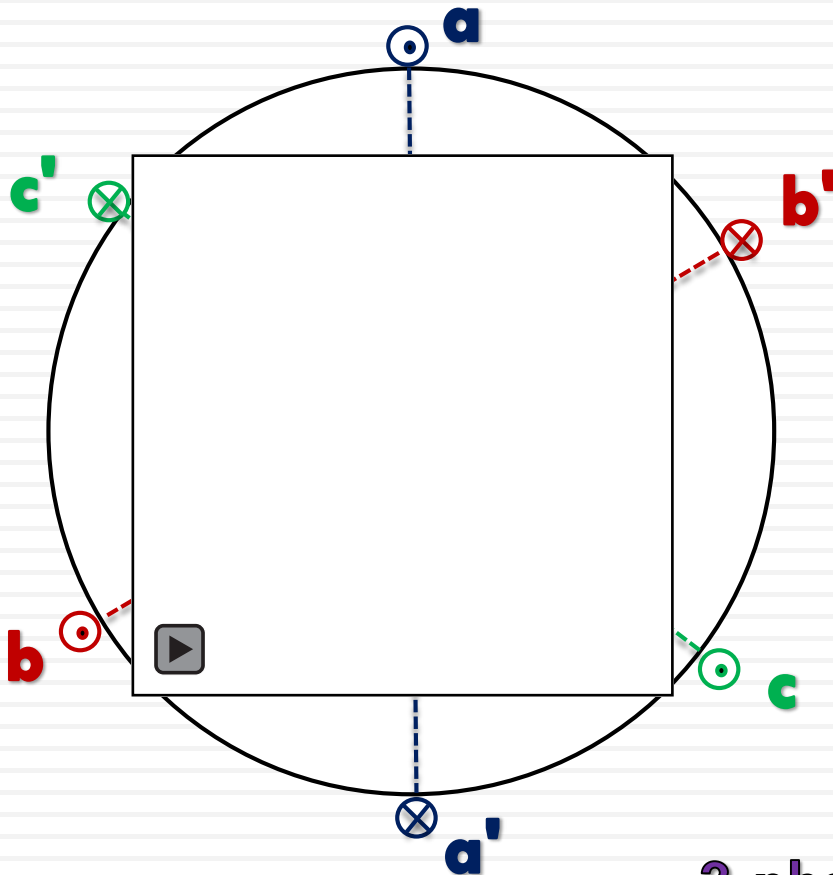
**Non-Salient
Poles**

Magnetic Field Production & Distribution

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Induction Machines

Three-phase machines

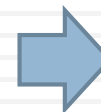


$$i_a = I_m \cos(\omega_e t)$$

$$i_b = I_m \cos(\omega_e t - 120)$$

$$i_c = I_m \cos(\omega_e t + 120)$$

3-phase Current



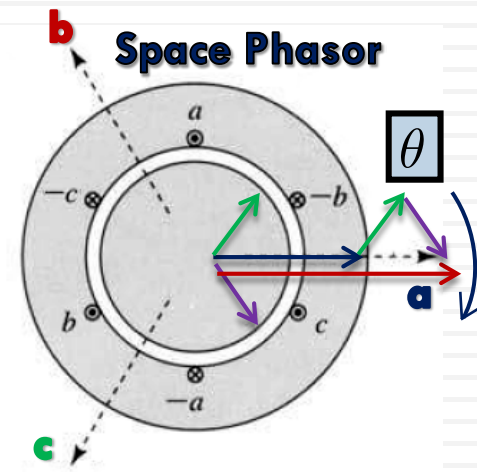
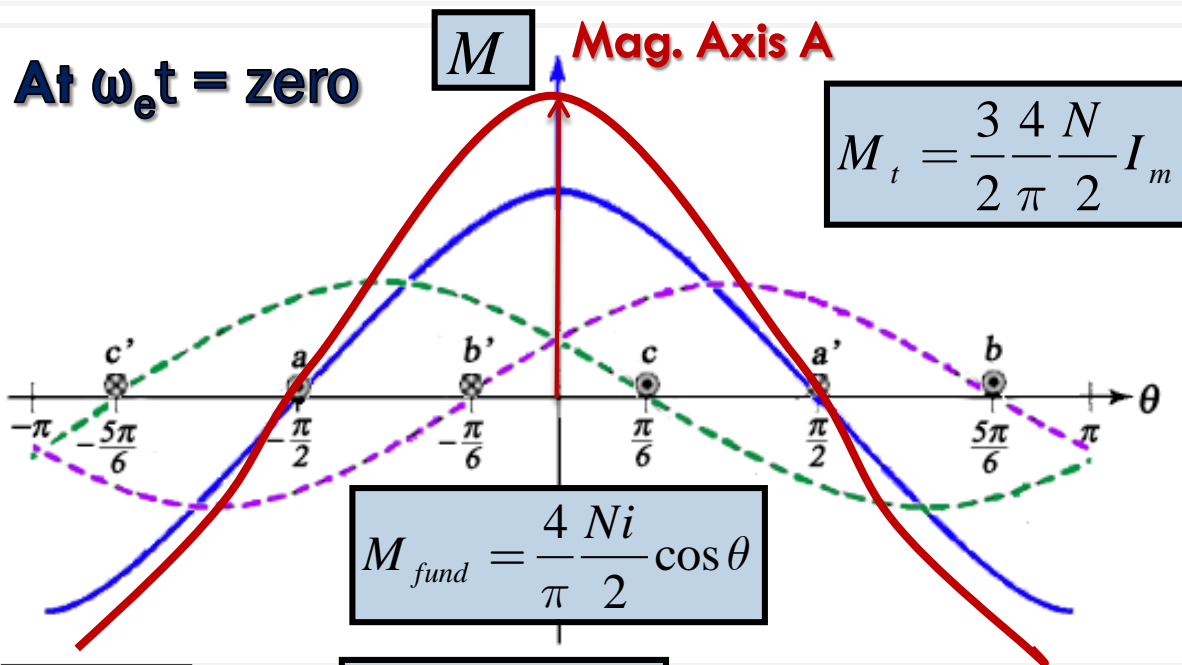
Rotating Field

Magnetic Field Production & Distribution

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Induction Machines

Three-phase machines



$$i_a = I_m$$

$$M_a = \frac{4N}{\pi} I_m$$

$$i_b = -0.5I_m$$

$$M_b = \frac{4N}{\pi} (-0.5I_m)$$

$$i_c = -0.5I_m$$

$$i_a = I_m \cos(\omega_e t)$$

$$i_b = I_m \cos(\omega_e t - 120)$$

$$i_c = I_m \cos(\omega_e t + 120)$$

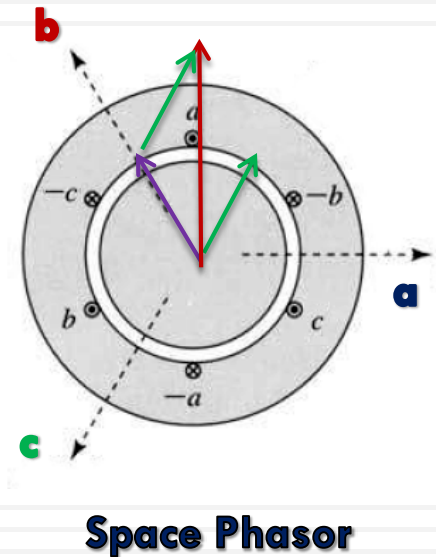
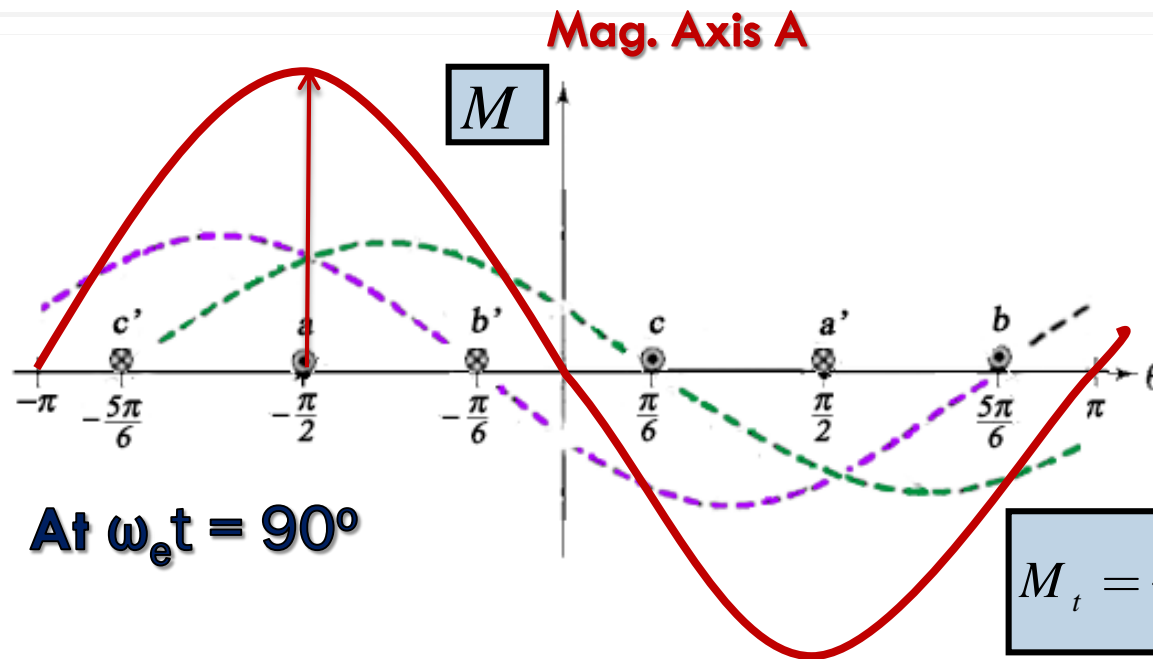
$$M_c = \frac{4N}{\pi} (-0.5I_m)$$

Magnetic Field Production & Distribution

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Induction Machines

Three-phase machines



$$M_t = \frac{3}{2} \frac{4N}{\pi} \frac{I_m}{2}$$

$$i_a = 0$$

$$M_a = 0$$

$$i_b = \frac{\sqrt{3}}{2} I_m$$

$$M_b = \frac{4N}{\pi} \frac{I_m}{2} \left(\frac{\sqrt{3}}{2} I_m \right)$$

$$i_c = -\frac{\sqrt{3}}{2} I_m$$

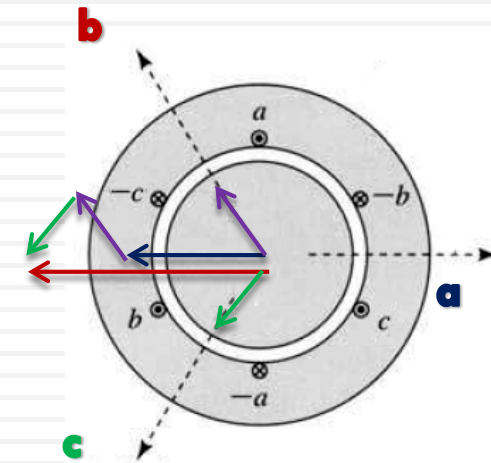
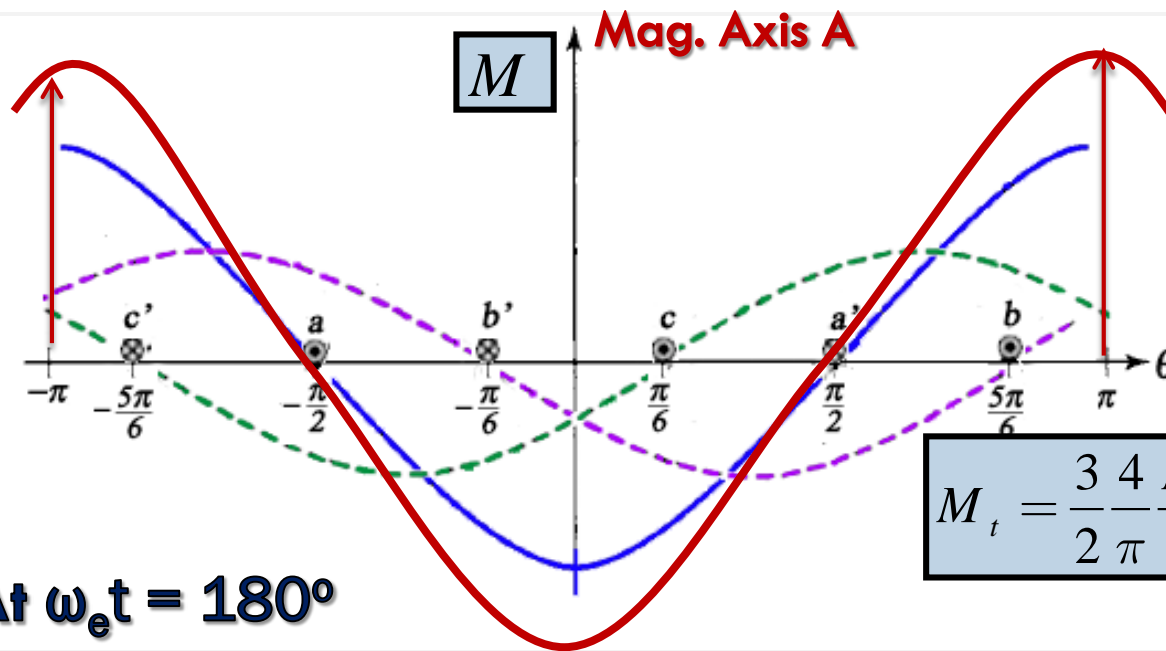
$$M_c = \frac{4N}{\pi} \frac{I_m}{2} \left(-\frac{\sqrt{3}}{2} I_m \right)$$

Magnetic Field Production & Distribution

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Induction Machines

Three-phase machines



$$M_t = \frac{3}{2} \frac{4N}{\pi} \frac{I_m}{2}$$

Space Phasor

At $\omega_e t = 180^\circ$

$$i_a = -I_m$$

$$M_a = \frac{4N}{\pi} \frac{I_m}{2} (-I_m)$$

$$i_b = 0.5I_m$$

$$M_b = \frac{4N}{\pi} \frac{I_m}{2} (0.5I_m)$$

$$i_c = 0.5I_m$$

$$M_c = \frac{4N}{\pi} \frac{I_m}{2} (0.5I_m)$$

Magnetic Field Production & Distribution

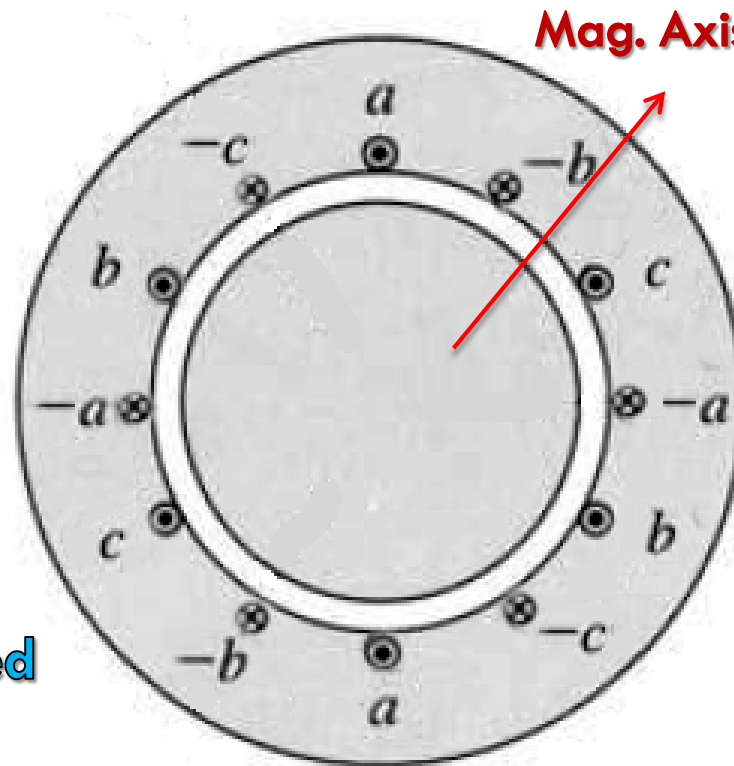
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Induction Machines

Three-phase machines

**Four
Magnetic
Poles**

Armature
Windings
Concentrated
Windings



**Non-Salient
Poles**

Magnetic Field Production & Distribution

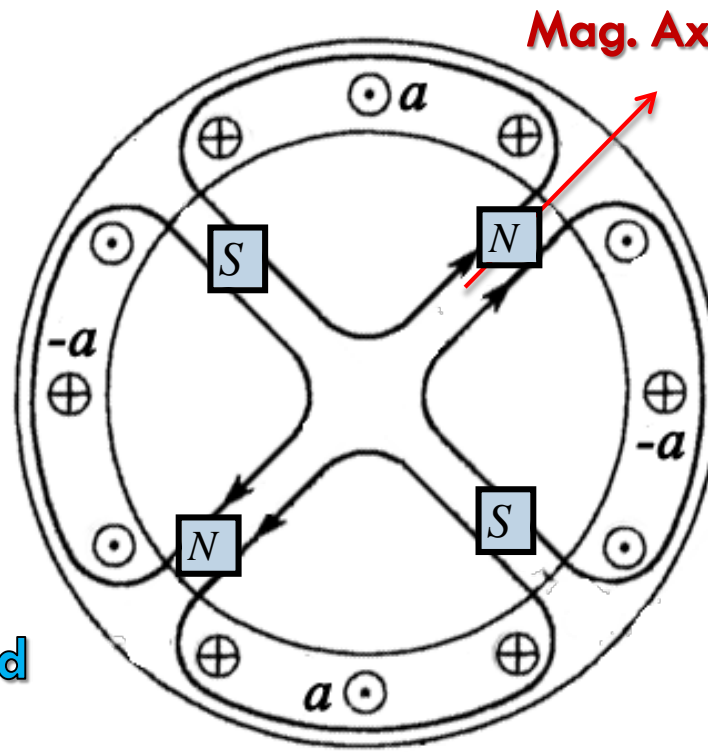
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Induction Machines

Three-phase machines

**Four
Magnetic
Poles**

Armature
Windings
Concentrated
Windings



**Non-Salient
Poles**

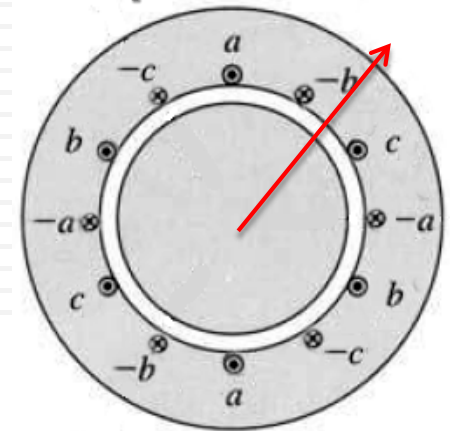
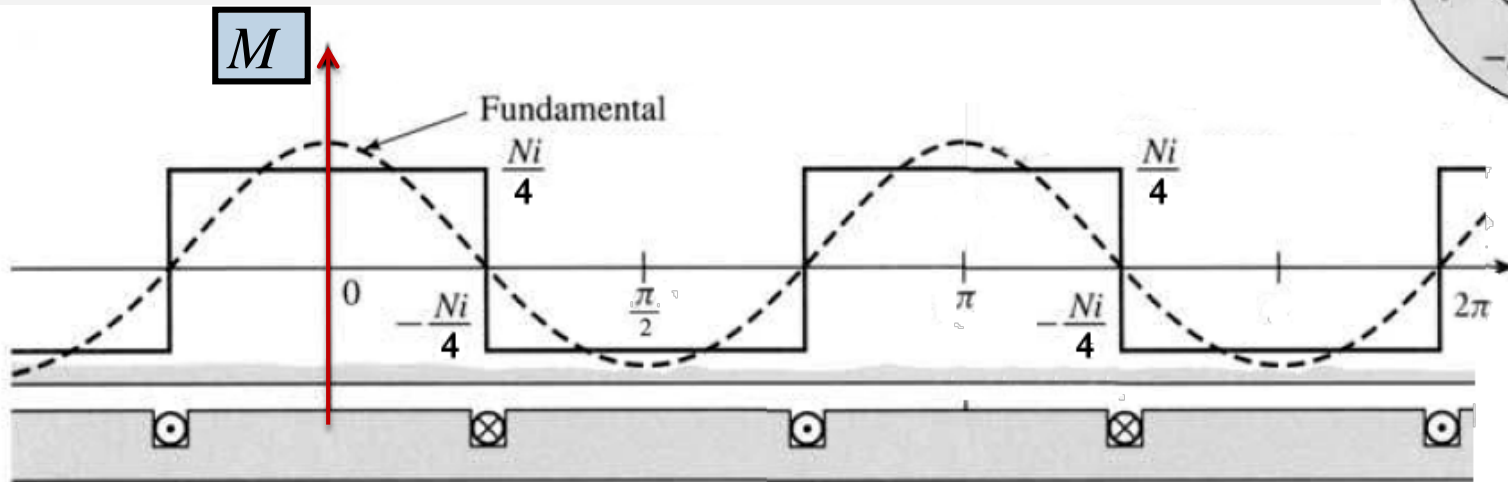
Magnetic Field Production & Distribution

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Induction Machines

Three-phase machines: Four magnetic poles

Space Phasor



MMF Space Distribution

Magnetic Field Production & Distribution

Induction Machines

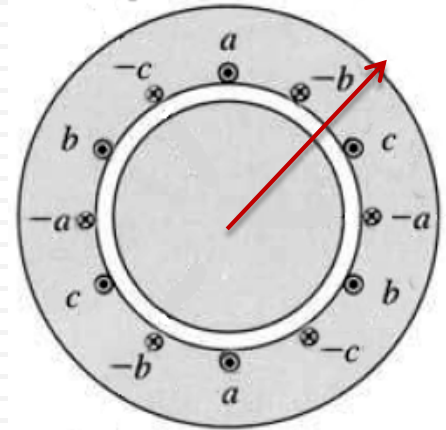
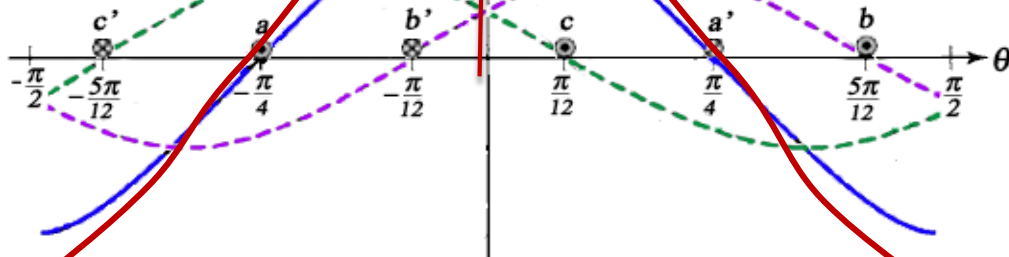
Three-phase machines: Four magnetic poles

Space Phasor

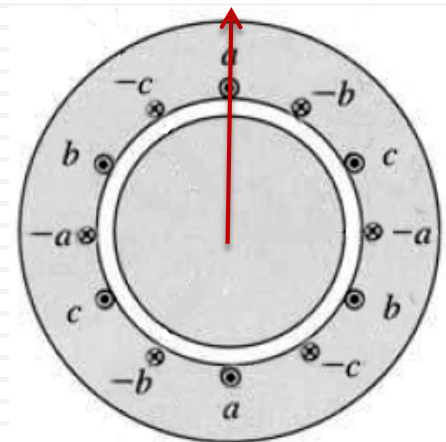
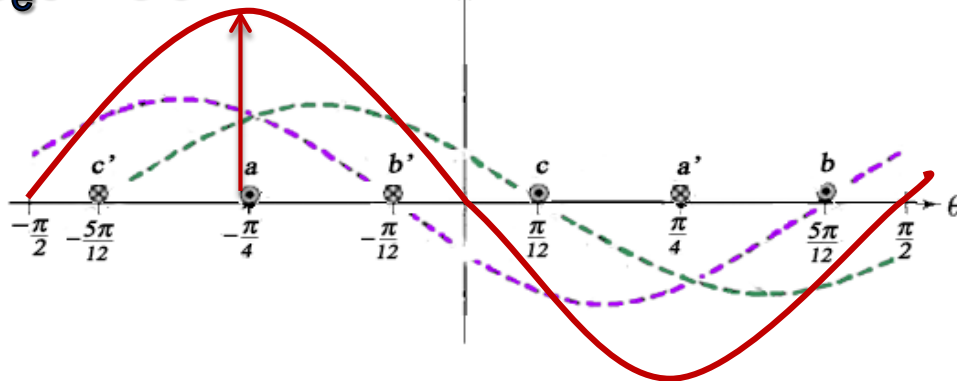
At $\omega_e t = \text{zero}$

M Mag. Axis A

$$M_t = \frac{3}{2} \frac{4N}{\pi} \frac{I_m}{4}$$



At $\omega_e t = 90^\circ$



Magnetic Field Production & Distribution

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Induction Machines

Speed of the Rotating Field

No. of poles	Elect. Angle	Mech. Angle
	No. of time cycles ($\omega_e t = \theta_e$)	No. of mech. Revolutions ($\theta = \theta_m$)
2	1 (2π)	1 ($2\pi(360^\circ)$)
4	1 (2π)	$\frac{1}{2}$ ($\pi(180^\circ)$)
6	1	$\frac{1}{3}$ ($2\pi/3$ (120°))
$2p$	f (cycles/sec)	f/p (rev/sec)

$$\theta_e = p\theta_m$$

p = no. of pair poles

Synchronous Speed

$$n_s = \frac{60f}{p}$$

rpm

$$\omega_s = \frac{n_s}{60} 2\pi = \frac{2\pi f}{p}$$

Rad/s

Magnetic Field Production & Distribution

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Induction Machines

The Rotating Field Equation

$$i_a = I_m \cos(\omega_e t)$$

$$i_b = I_m \cos(\omega_e t - 120)$$

$$i_c = I_m \cos(\omega_e t + 120)$$

$$M_a = \frac{4 N}{\pi 2p} i_a \cos p\theta$$

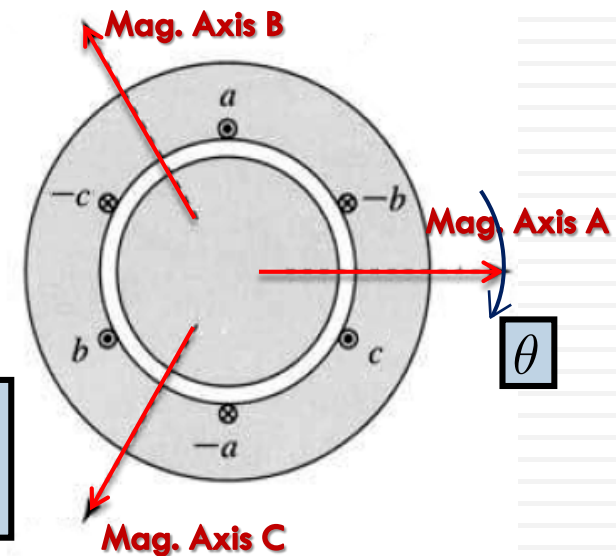
$$M_a = \frac{4 N}{\pi 2p} I_m \cos(\omega_e t) \cos p\theta$$

$$M_b = \frac{4 N}{\pi 2p} i_b \cos(p\theta - 120)$$

$$M_b = \frac{4 N}{\pi 2p} I_m \cos(\omega_e t - 120) \cos(p\theta - 120)$$

$$M_c = \frac{4 N}{\pi 2p} i_c \cos(p\theta + 120)$$

$$M_c = \frac{4 N}{\pi 2p} I_m \cos(\omega_e t + 120) \cos(p\theta + 120)$$



Magnetic Field Production & Distribution

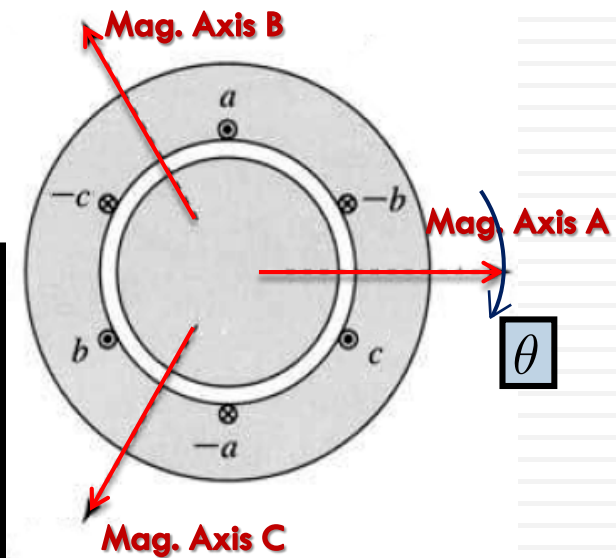
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Induction Machines

The Rotating Field Equation

$$M_t = M_a + M_b + M_c$$

$$M_t = \frac{4}{\pi} \frac{N}{2p} \frac{I_m}{2} [\cos(\omega_e t - p\theta) + \cos(\omega_e t + p\theta) + \cos(\omega_e t - p\theta) + \cos(\omega_e t + p\theta - 240) + \cos(\omega_e t - p\theta) + \cos(\omega_e t + p\theta + 240)]$$



$$M_t(\theta, t) = \frac{3}{2} \frac{4}{\pi} \frac{N}{2p} I_m \cos(\omega_e t - p\theta)$$

**Rotating
Field**

Magnetic Field Production & Distribution

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Induction Machines

Speed of the Rotating Field

$$M_t(\theta, t) = \frac{3}{2} \frac{4}{\pi} \frac{N}{2p} I_m \cos(\omega_e t - p\theta)$$

For a particular point:

$$\omega_e t - p\theta = \text{const.}$$

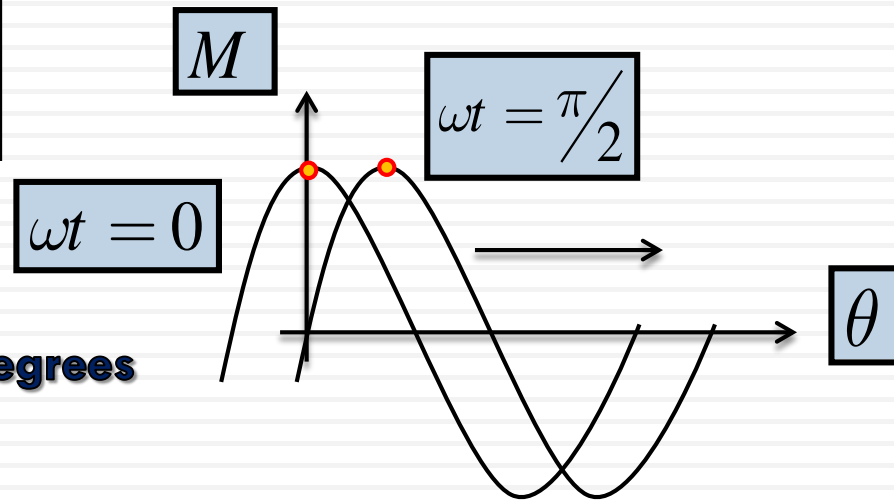
$$\omega_e - p \frac{d\theta}{dt} = \text{zero}$$

$$\text{Speed} = \frac{d\theta}{dt} = \frac{\omega_e}{p} = \frac{2\pi f}{p}$$

Rad/sec

$$n_s = \frac{60f}{p} \text{ rpm}$$

Electrical degrees



Magnetic Field Production & Distribution

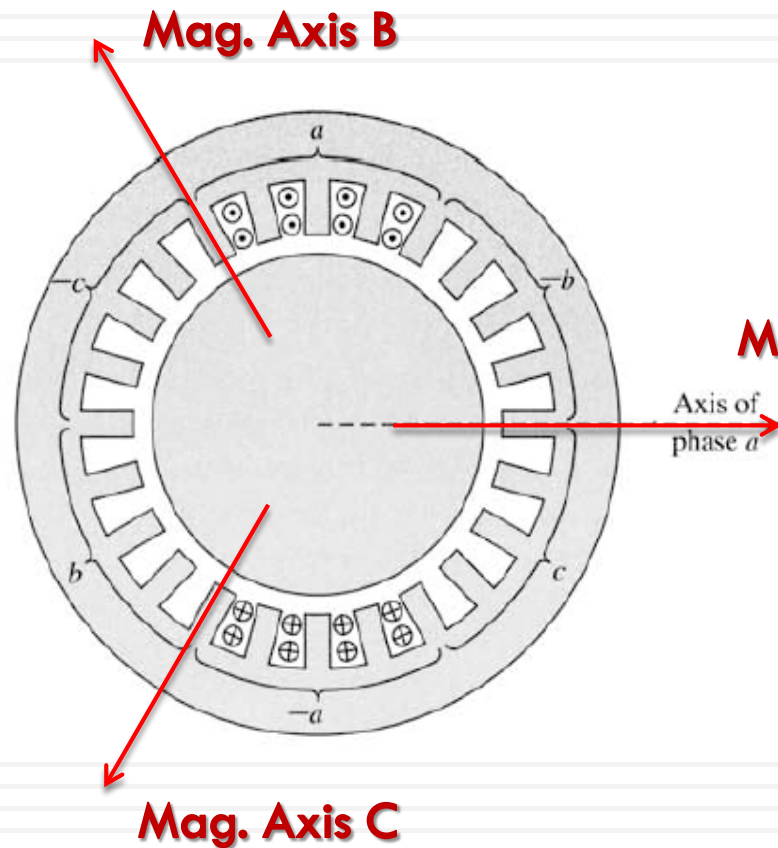
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Induction Machines

Three-phase machines

**Two
Magnetic
Poles**

**Armature
Windings
Distributed
Windings**

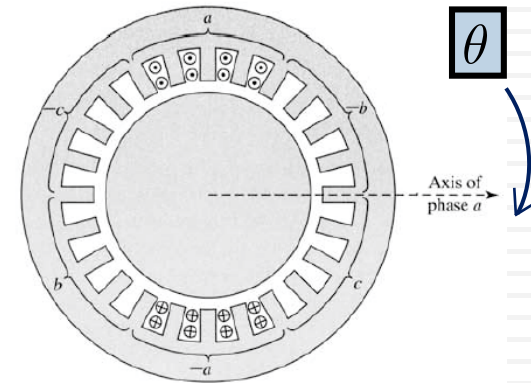
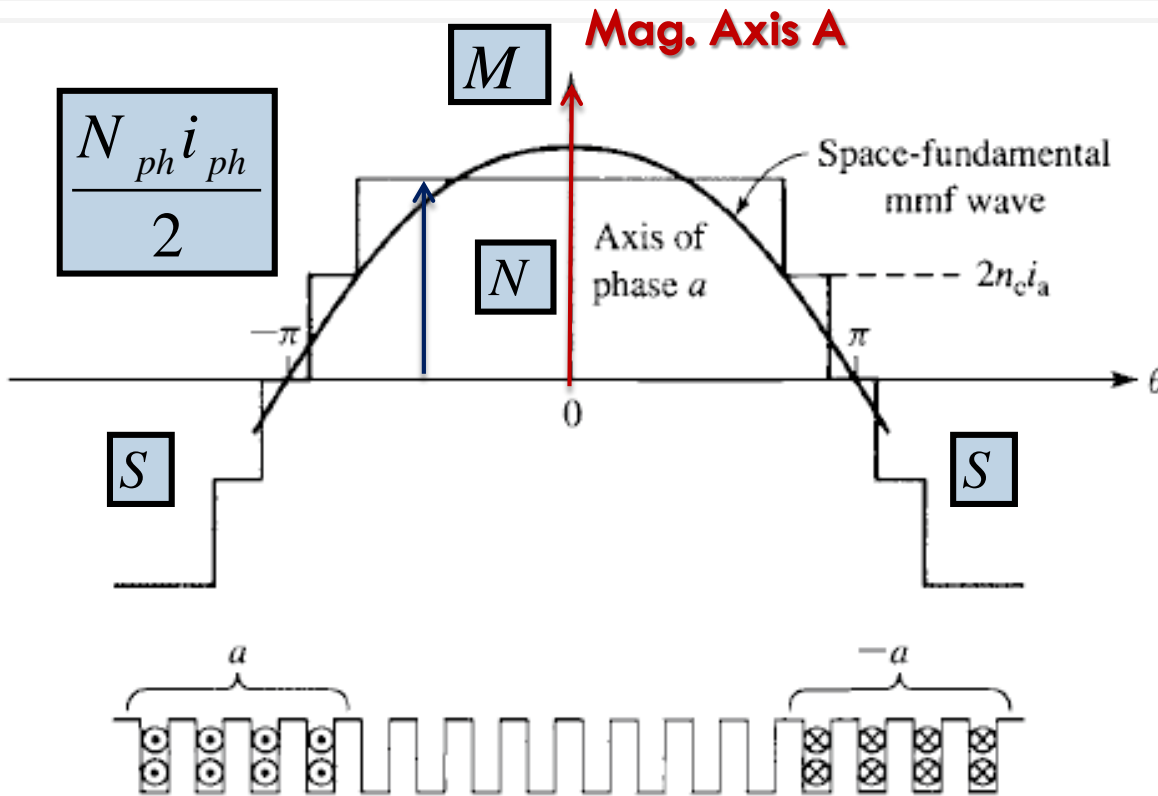


**Non-Salient
Poles**

Magnetic Field Production & Distribution

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Induction Machines



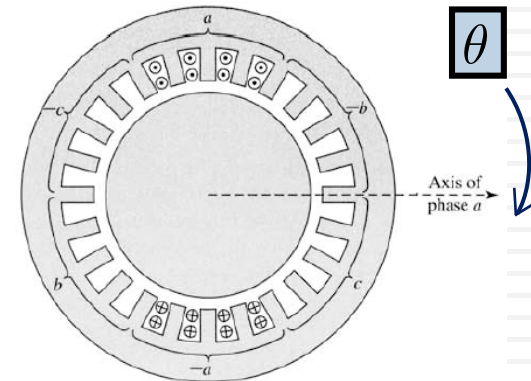
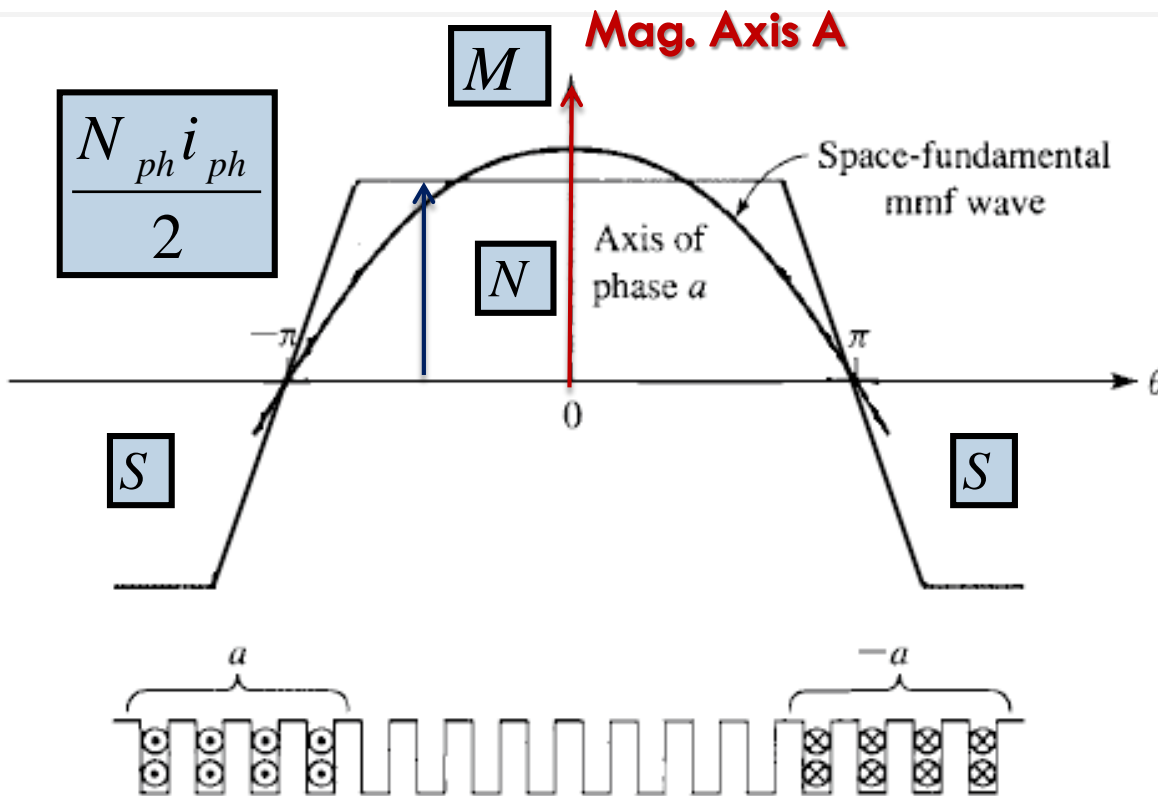
$$\frac{N_{ph} i_{ph}}{2p}$$

For any number of poles ($2p$)

Magnetic Field Production & Distribution

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Induction Machines



For any number of poles ($2p$)

$$\frac{N_{ph} i_{ph}}{2p}$$

Magnetic Field Production & Distribution

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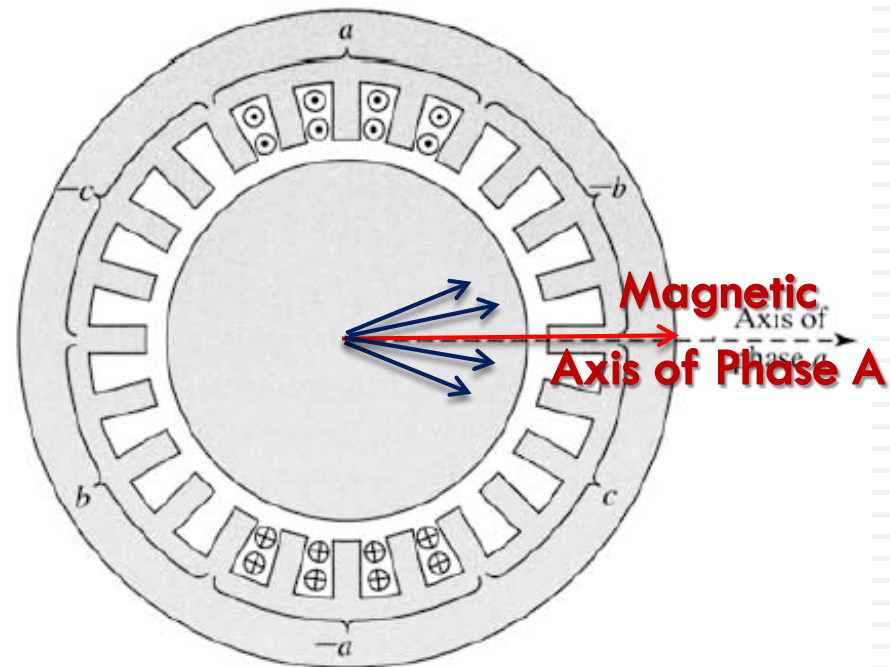
Induction Machines

$$M(\theta) = \frac{4}{\pi} \frac{N_{ph} i}{2p} k_w \sum_{h=1,3,5,\dots} \frac{1}{h} \cos(hp\theta)$$

Rotating Field

$$M_t(\theta, t) = \frac{3}{2} \frac{4}{\pi} \frac{N_{ph} k_w}{2p} I_m \cos(\omega_e t - p\theta)$$

Fundamental



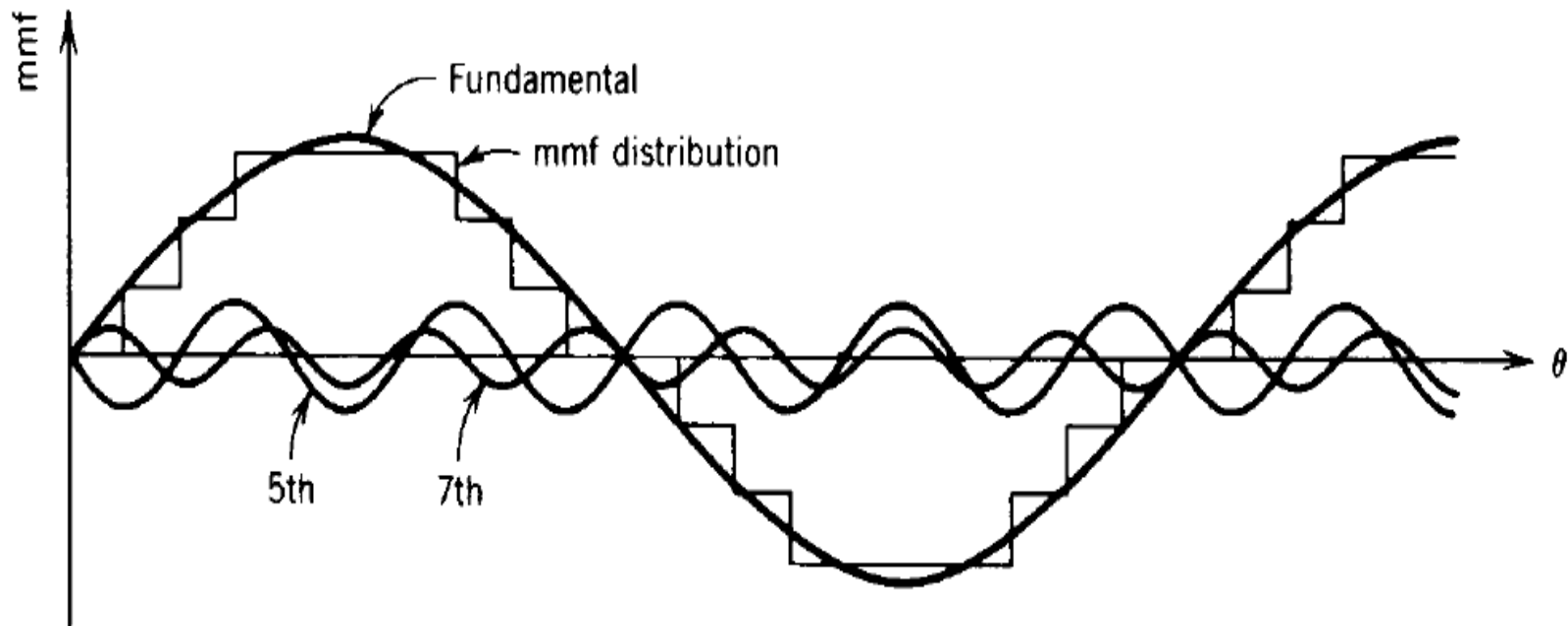
Magnetic Field Production & Distribution

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Induction Machines

Space Harmonics

$$M_f = M_1 \cos \theta + M_3 \cos 3\theta + M_5 \cos 5\theta + \dots$$



Magnetic Field Production & Distribution

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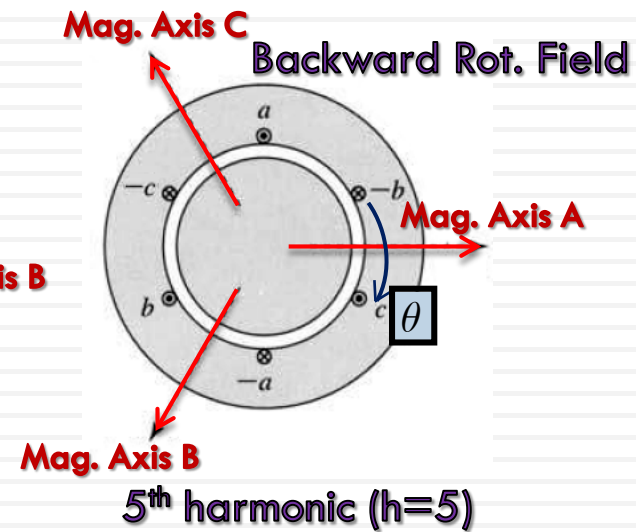
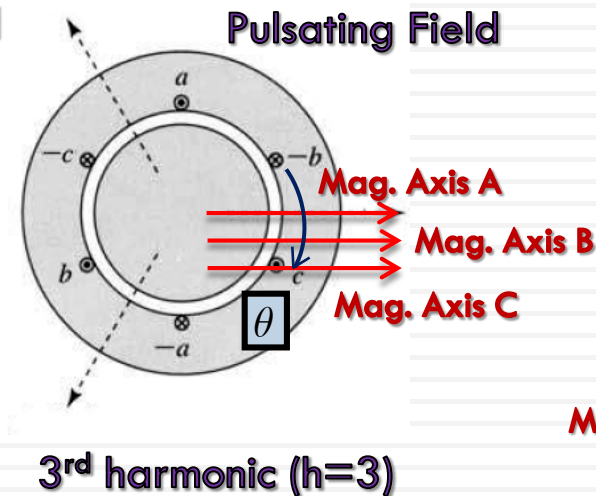
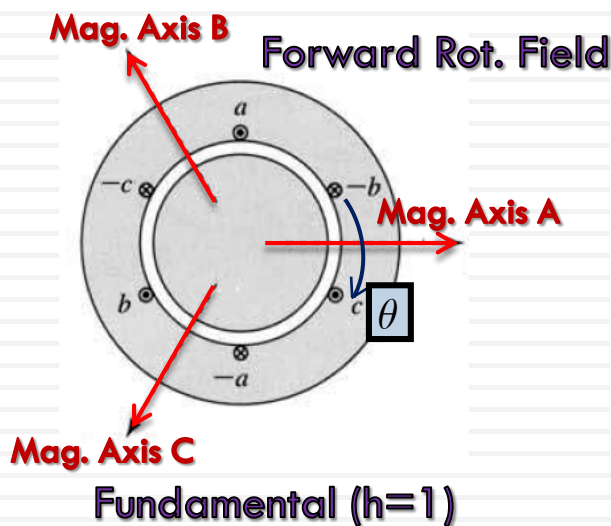
Induction Machines

Space Harmonics

$$M_{ah} = \frac{4}{\pi} \frac{N}{2hp} I_m \cos(\omega_e t) \cos hp\theta$$

$$M_{bh} = \frac{4}{\pi} \frac{N}{2hp} I_m \cos(\omega_e t - 120) \cos(h(p\theta - 120))$$

$$M_{ch} = \frac{4}{\pi} \frac{N}{2hp} I_m \cos(\omega_e t + 120) \cos(h(p\theta + 120))$$



Magnetic Field Production & Distribution

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Induction Machines

Space Harmonics

$$M_{th}(\theta, t) = \frac{3}{2} \frac{4}{\pi} \frac{N}{2hp} I_m \cos(\omega_e t \mp hp\theta)$$

$$n_s = \pm \frac{60f}{hp}$$

+ve for h=1,7,13, ...

-ve for h=5,11,17,...

