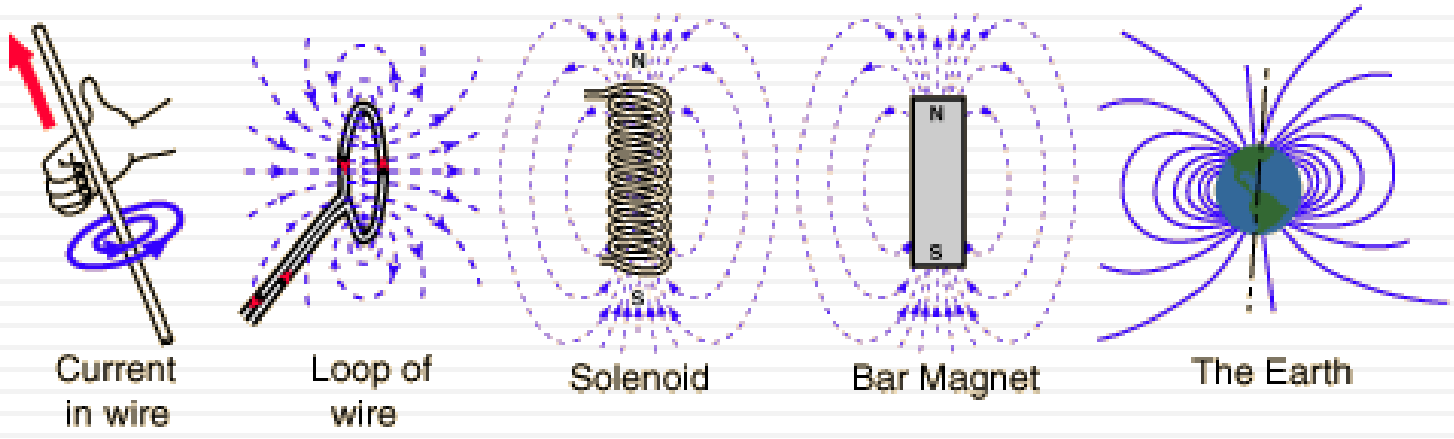
Revision: Basic Principles

- AC Circuits
- Three-phase Circuits
- Magnetic Fields
- Time-varying Fields: Faraday's Law
- Magnetic Circuits

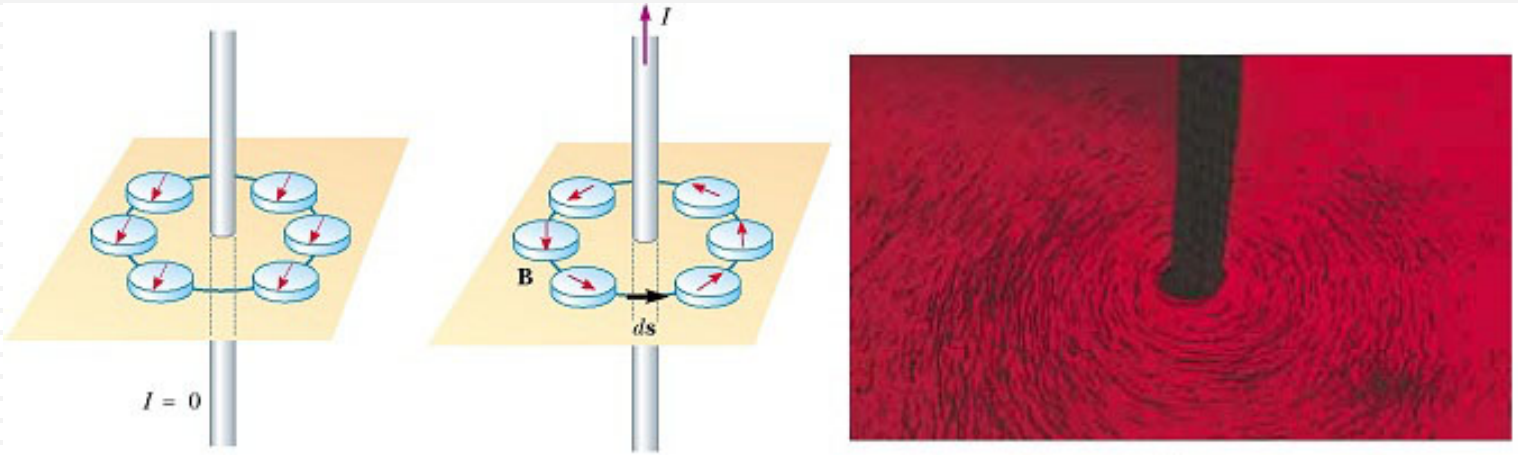
Revision: Basic Principles

Magnetic Fields

Sources Of Magnetic Fields



Magnetic Field Sources

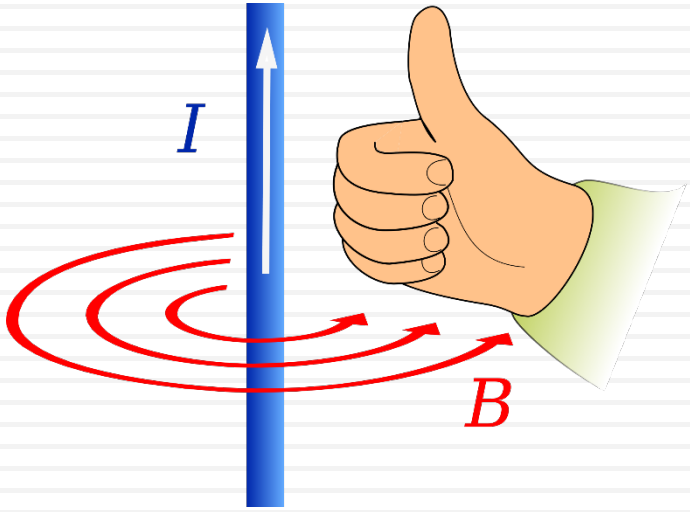


Revision: Basic Principles

Magnetic Fields

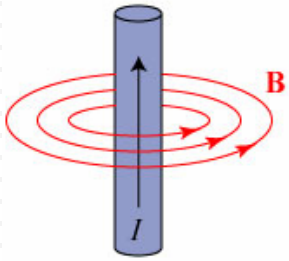
Direction Of Magnetic Fields:

Ampere's Right Hand Rule

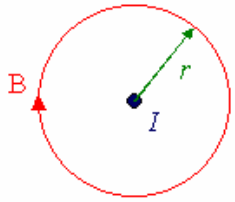


Ampere's Law

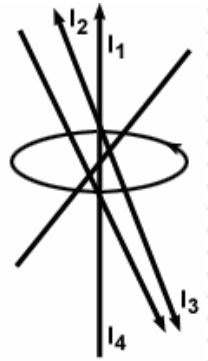
$$\oint_C \underline{H}.\underline{dl} = \sum I_{en}$$



Amperian loop of a straight wire



Constant B on Perimeter

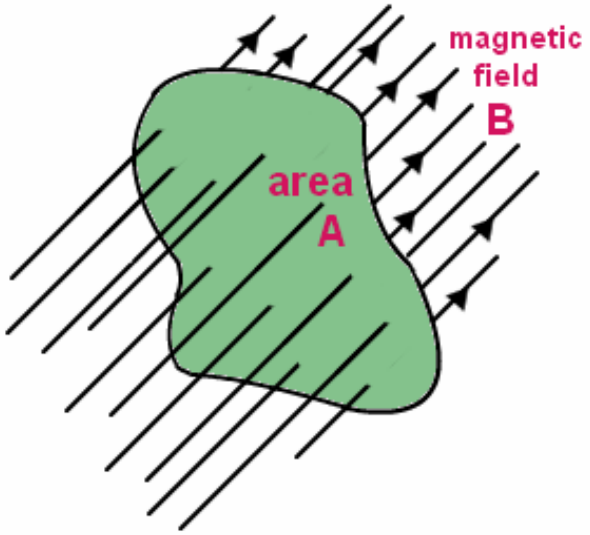


A closed path enclosing 4 wires

Revision: Basic Principles

Magnetic Fields

$$\underline{B} = \mu \underline{H} \qquad \mu = \mu_o \mu_r \qquad B = \frac{\phi}{A}$$



B Magnetic flux density (Tesla)

H Magnetic field intensity (A/m)

μ_o permeability of free space

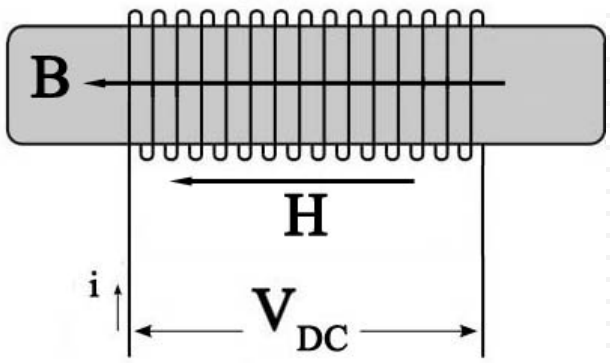
μ_r relative permeability of a medium

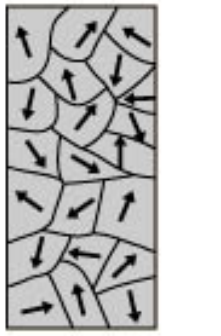
φ Magnetic Flux (Weber – Lines)

Revision: Basic Principles

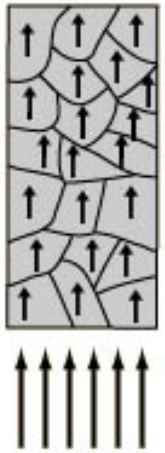
Magnetic Fields

Magnetic Materials

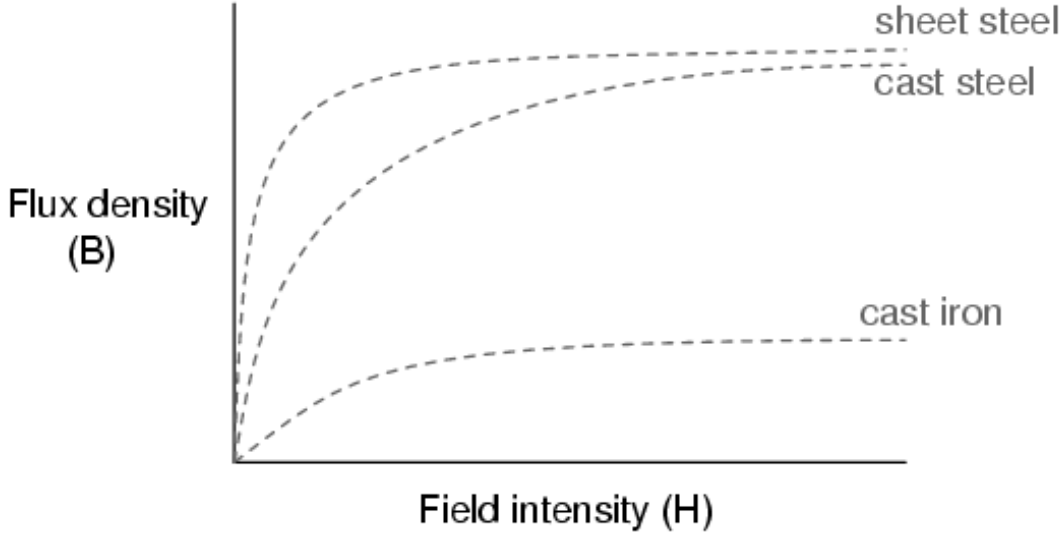




In bulk material the domains usually cancel, leaving the material unmagnetized.



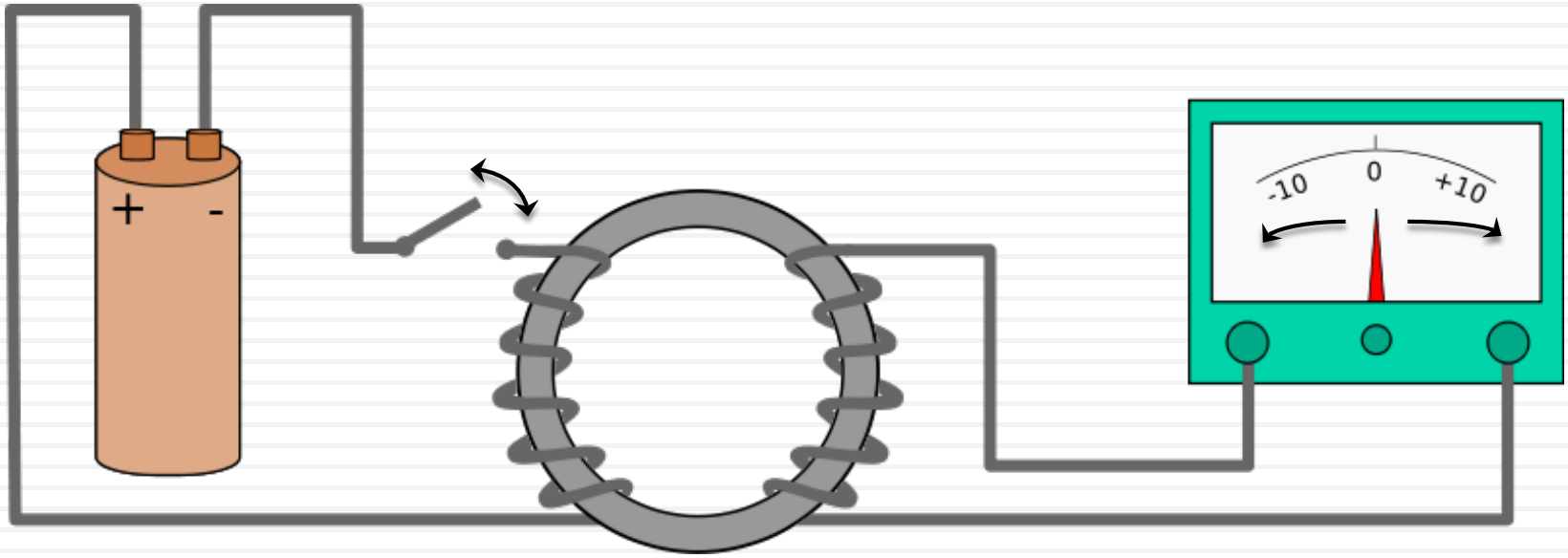
Externally applied magnetic field.



Revision: Basic Principles

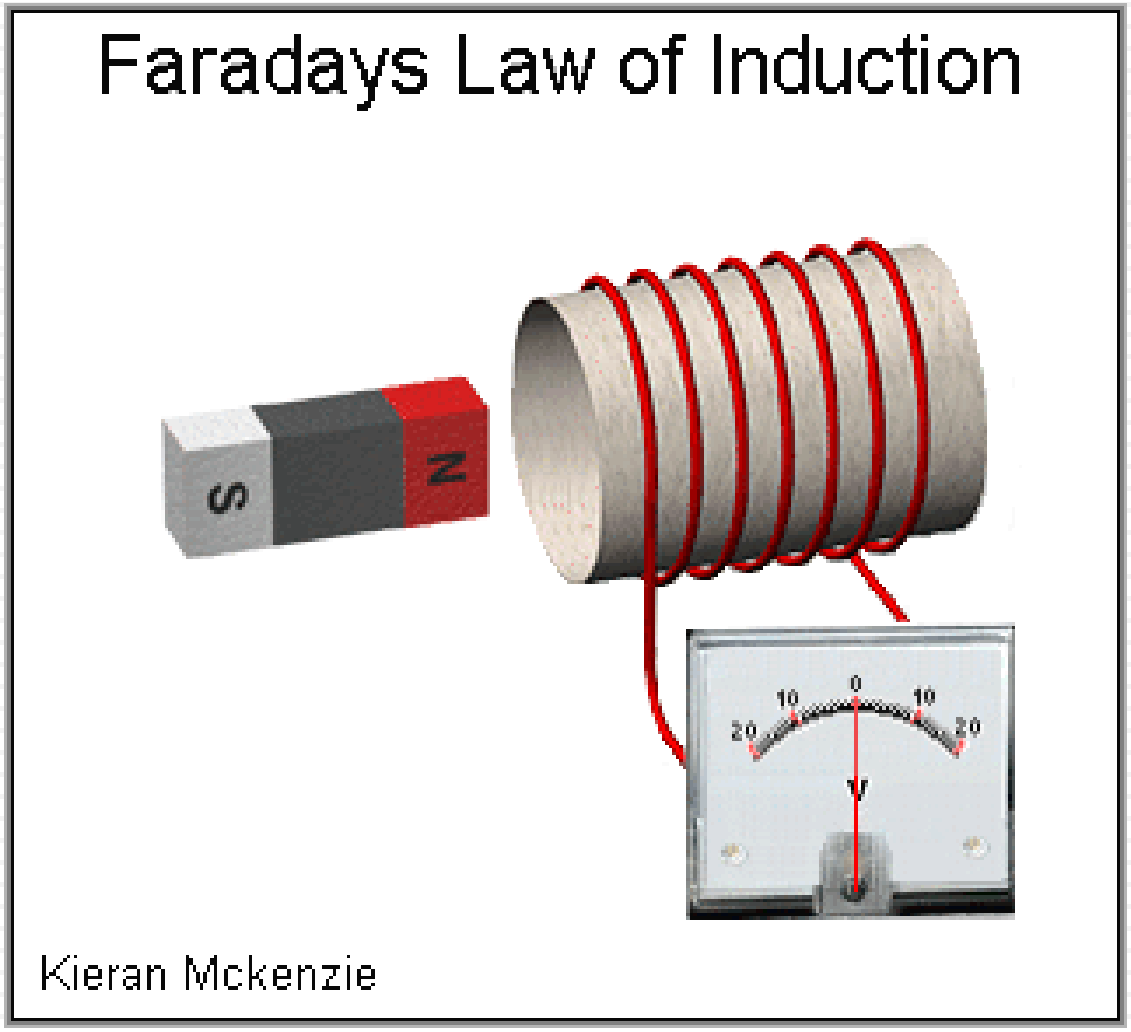
Time-Varying Fields

Faraday's Experiment (1831)



Revision: Basic Principles

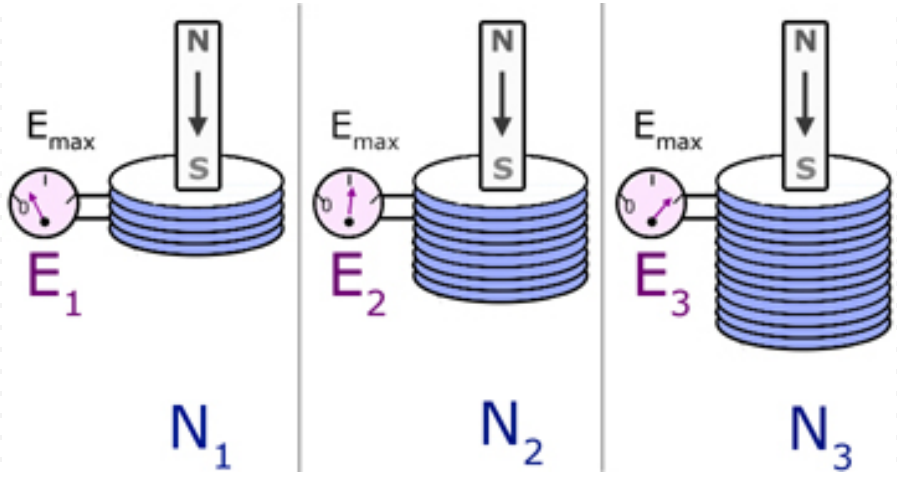
Time-Varying Fields



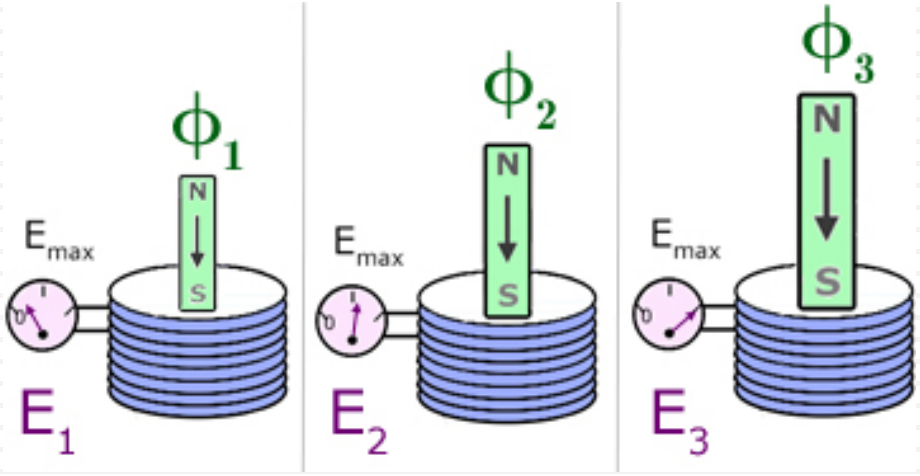
Revision: Basic Principles

Time-Varying Fields

$$E \propto N$$



$$E \propto \frac{d\phi}{dt}$$



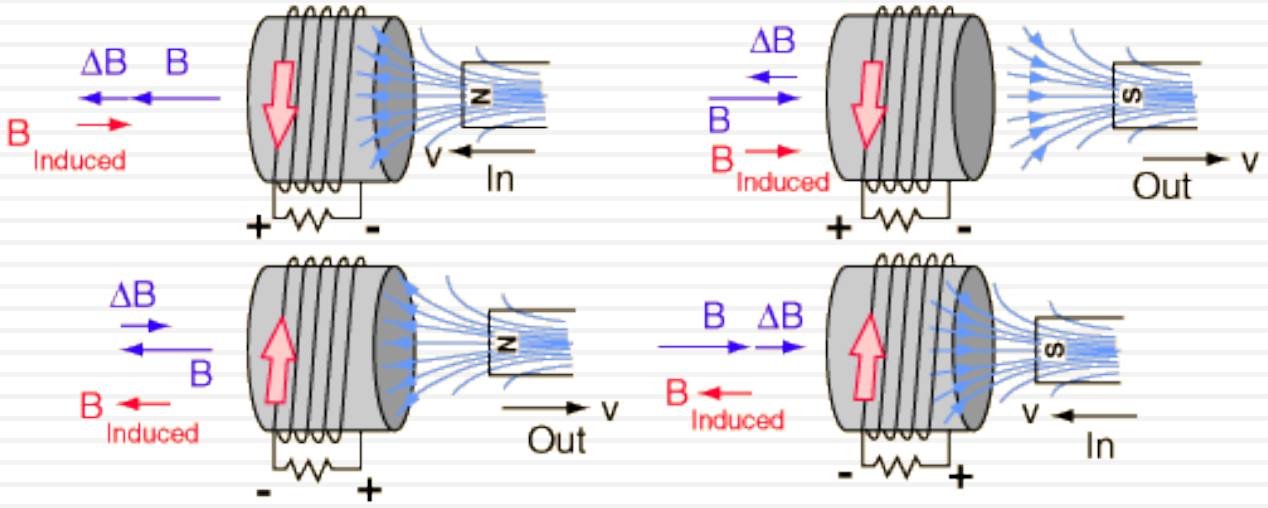
Revision: Basic Principles

Time-Varying Fields

Lenz's Law

$$\frac{d\phi}{dt} > 0$$

$$\frac{d\phi}{dt} < 0$$

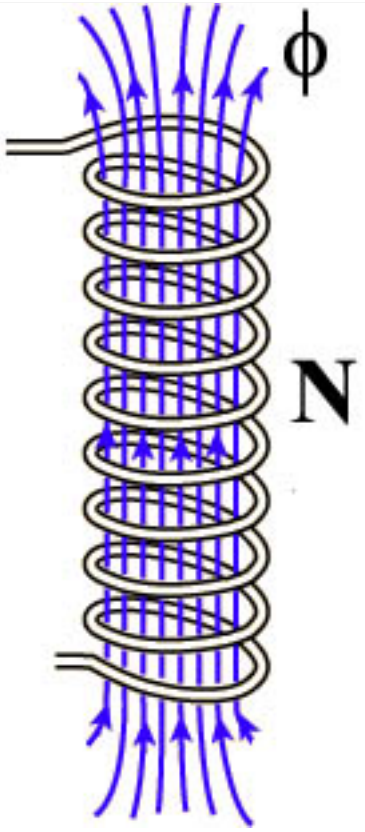


Revision: Basic Principles

Time-Varying Fields

Faraday's Law for Induction

$$induced\ \ emf = -N\frac{d\phi}{dt}$$



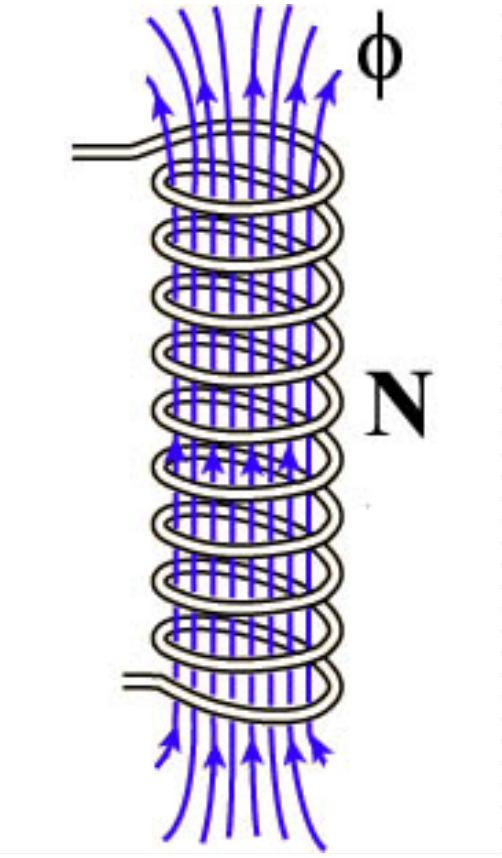
Revision: Basic Principles

Time-Varying Fields

Transformer EMF

$$e = -N\frac{d\phi}{dt}$$

$$e = -NA\frac{dB}{dt}$$



Revision: Basic Principles

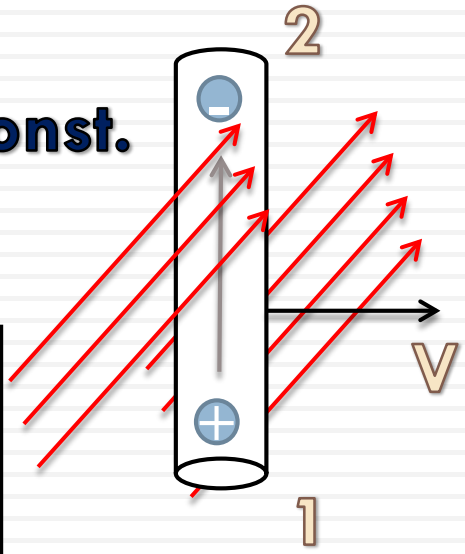
Time-Varying Fields

For a moving conductor:

$$\underline{F}_m = q\underline{v} \times \underline{B}$$

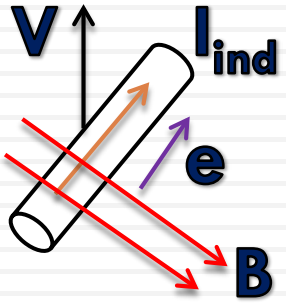
$$\frac{\underline{F}_m}{q} = \underline{v} \times \underline{B} = \underline{E}_{ind}$$

$$e_{21} = \int_1^2 \underline{v} \times \underline{B} . \underline{dl}$$



If the magnetic field is perpendicular to the conductor:

$$e = Blv$$



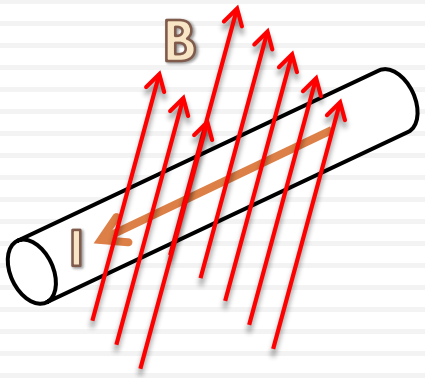
Revision: Basic Principles

Time-Varying Fields

Electromagnetic Force

For a current carrying conductor:

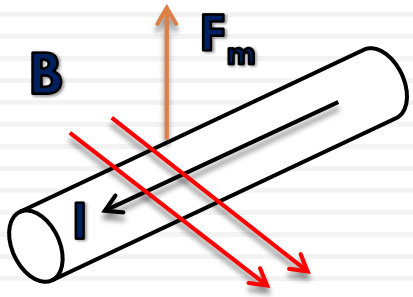
$$\underline{F}_m = I \int \underline{dl} \times \underline{B}$$



where dl is taken in the direction of current

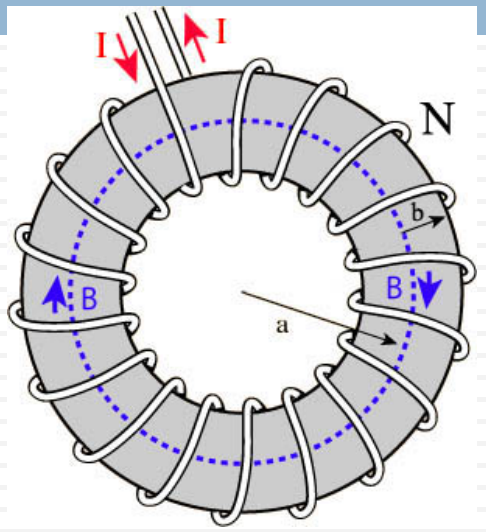
If the magnetic field is perpendicular to the conductor:

$$F_m = BIL$$



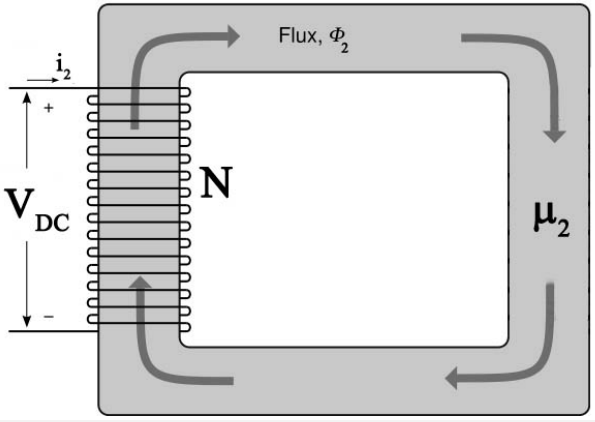
Concept of Magnetic Circuits

Magnetic Circuits



What's a magnetic circuit ?

- *They are basically ferromagnetic structures (mostly iron) with coils wound around them.*
- *Because of the material high permeability, most of the magnetic flux is confined inside the magnetic circuit.*
- *Examples: Transformers, Actuators, Electromagnets and Electric machines.*



Concept of Magnetic Circuits

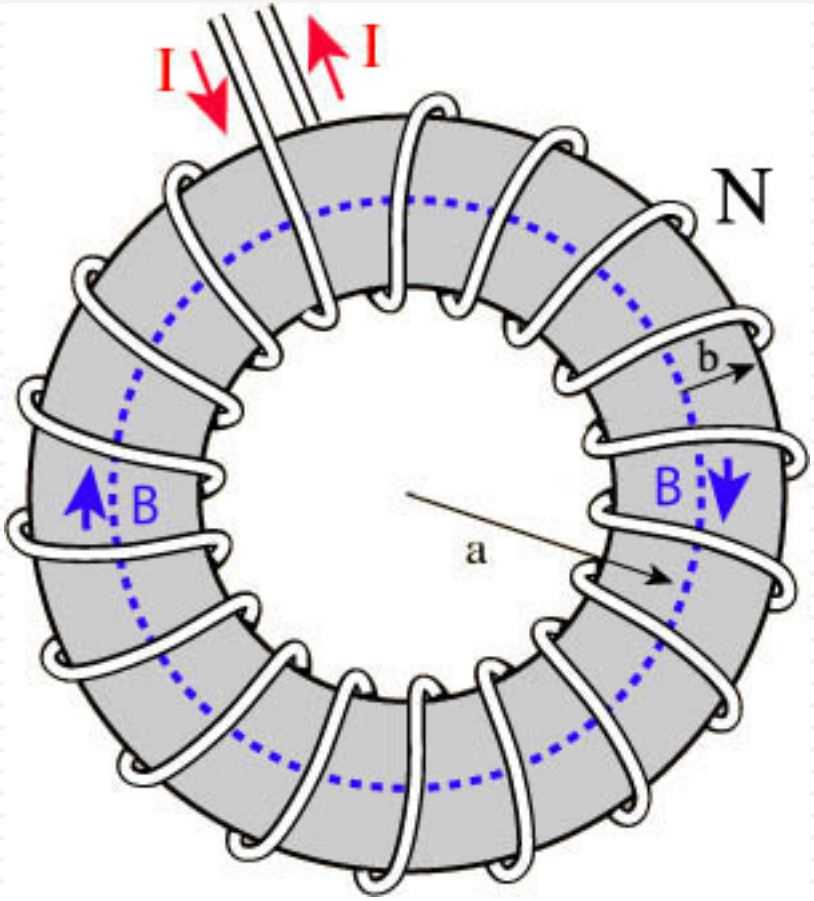
Magnetic Circuits

It's an approximate technique to compute magnetic flux.

$$H = \frac{NI}{2\pi a} \Rightarrow B = \frac{\mu NI}{2\pi a}$$

$$\phi = BA \Rightarrow \phi = \frac{\mu NI}{2\pi a}(\pi b^2)$$

$$\Rightarrow \phi = \frac{NI}{\frac{2\pi a}{\mu(\pi b^2)}}$$



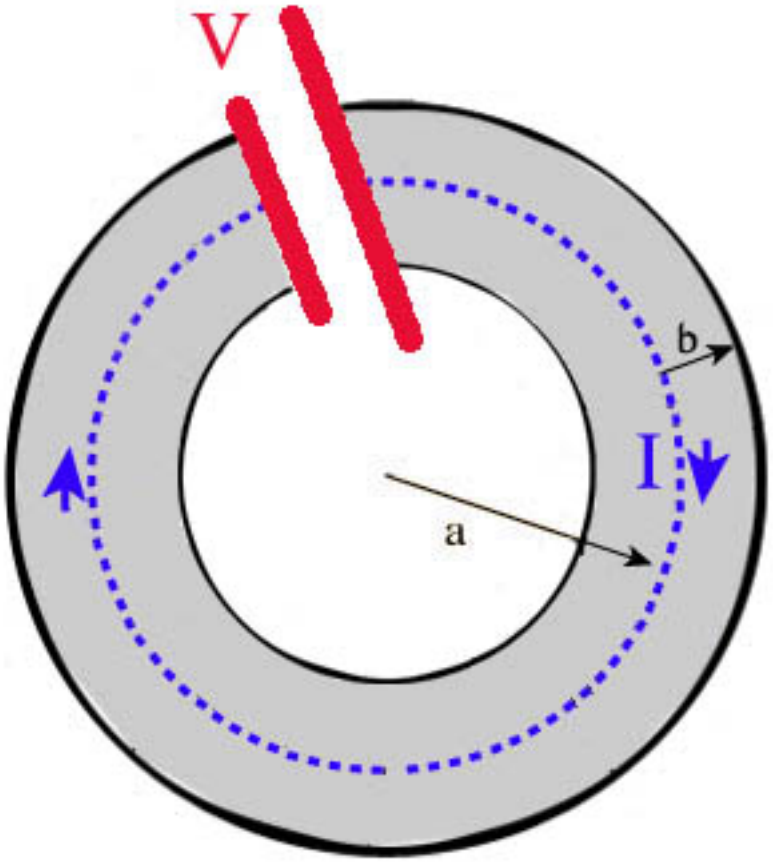
Concept of Magnetic Circuits

Magnetic Circuits

$$I = \frac{V}{R} \qquad R = \frac{l}{\sigma A}$$

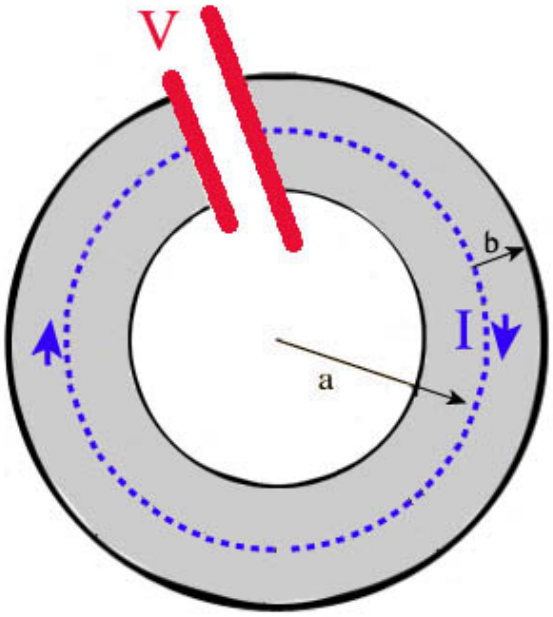
$$\Rightarrow \quad R = \frac{2\pi a}{\sigma(\pi b^2)}$$

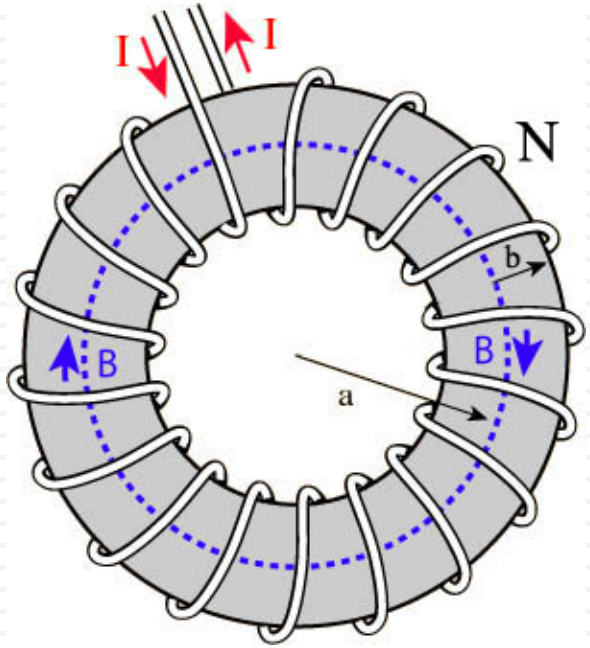
$$\Rightarrow \quad I = \frac{V}{\frac{2\pi a}{\sigma(\pi b^2)}}$$



Concept of Magnetic Circuits

Magnetic Circuits





$$I = \frac{V}{\frac{2\pi a}{\sigma(\pi b^2)}}$$

$$\phi = \frac{NI}{\frac{2\pi a}{\mu(\pi b^2)}}$$

Concept of Magnetic Circuits

Magnetic Circuits

$I \Leftrightarrow \phi$

$V\ (EMF) \Leftrightarrow NI\ (MMF)$

$\sigma \Leftrightarrow \mu$

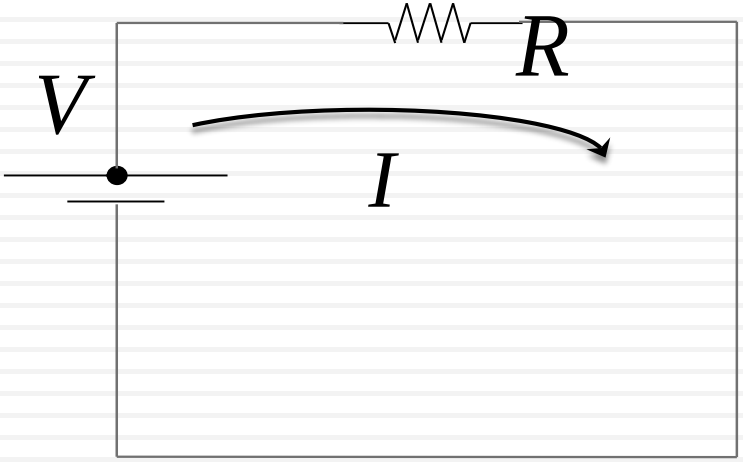
$$R = \frac{l}{\sigma A} \Leftrightarrow \Re = \frac{l_{mc}}{\mu A}$$

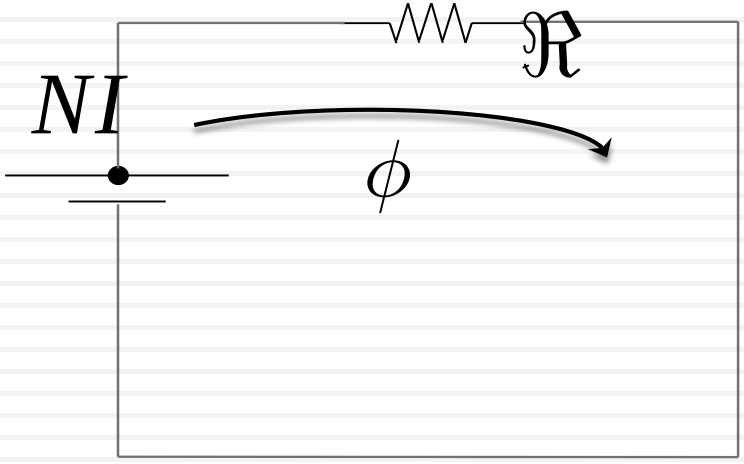
l_{mc}: Mean core length

Reluctance

Concept of Magnetic Circuits

Magnetic Circuits





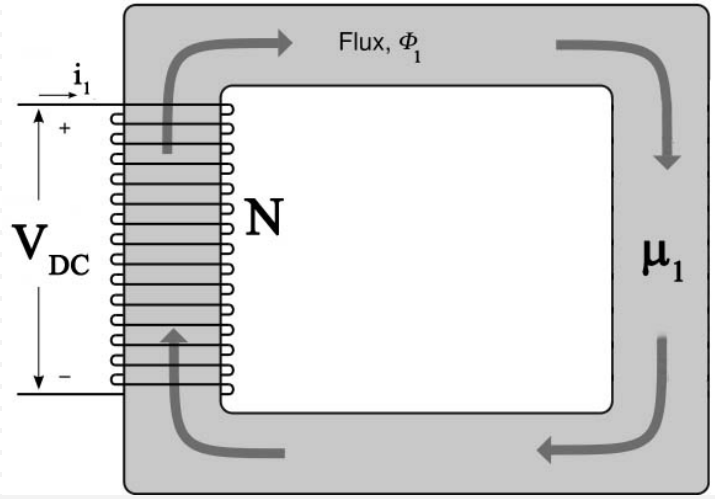
$$V = IR$$

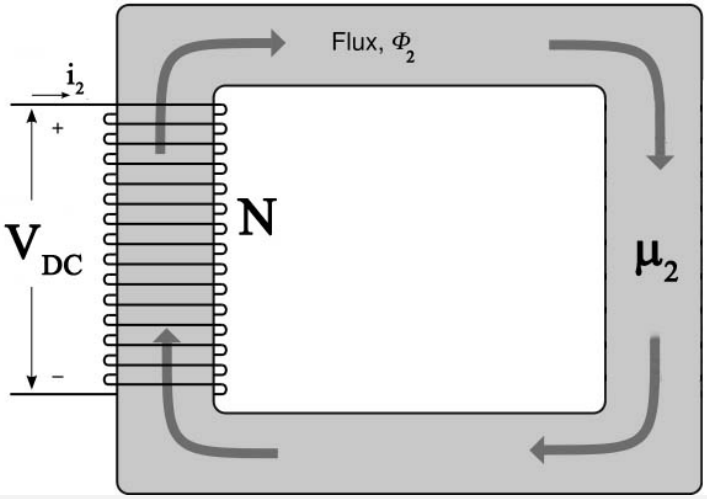
$$NI = \phi\Re$$

Concept of Magnetic Circuits

Magnetic Circuits

DC Magnetic Circuits: the supply is DC, magnetic circuit laws can be used directly.





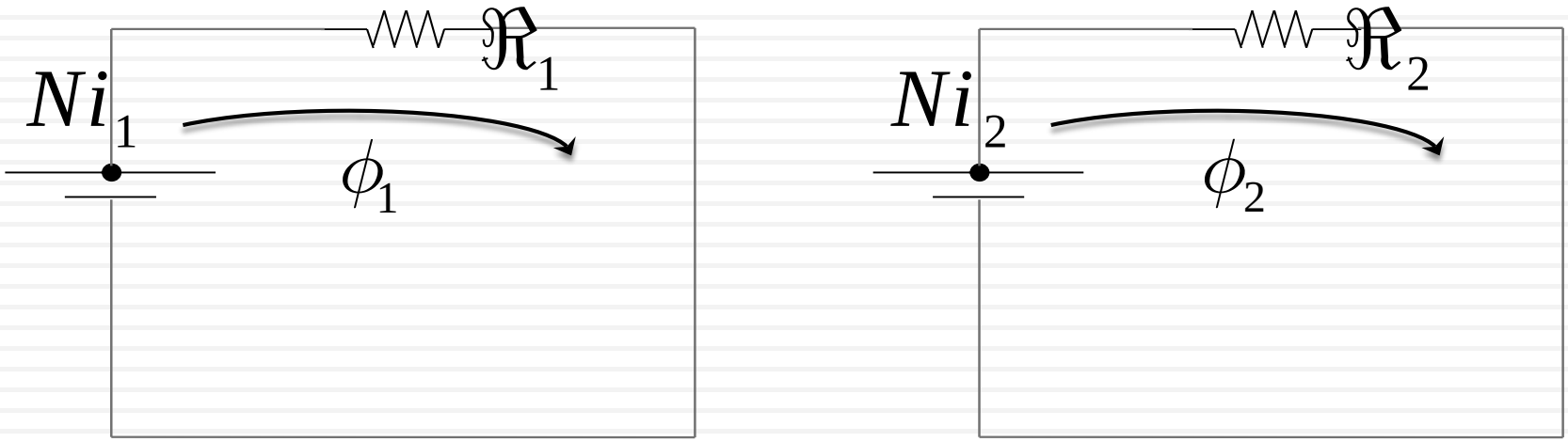
$$i_1 = \frac{V_{DC}}{R_{coil}}$$

$$i_2 = \frac{V_{DC}}{R_{coil}}$$

Concept of Magnetic Circuits

Magnetic Circuits

DC Magnetic Circuits



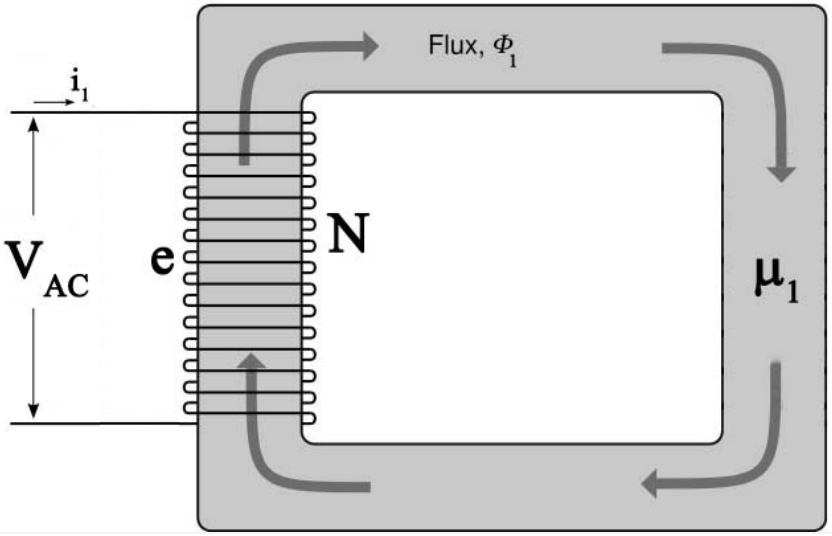
$$i_1 = i_2 \Rightarrow Ni_1 = Ni_2 \Rightarrow \phi_1 \Re_1 = \phi_2 \Re_2$$

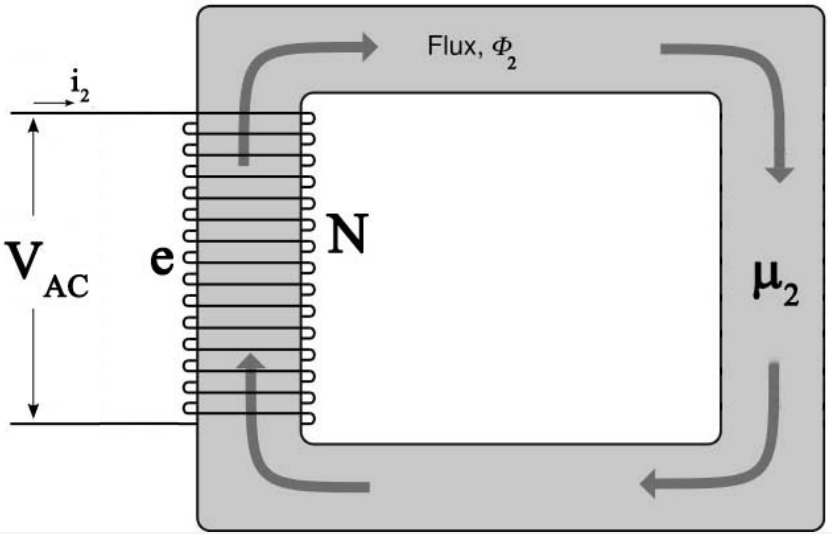
$$\Rightarrow \phi_1 \frac{l_{mc}}{\mu_1 A_c} = \phi_2 \frac{l_{mc}}{\mu_2 A_c} \Rightarrow \frac{\phi_1}{\phi_2} = \frac{\mu_1}{\mu_2}$$

Concept of Magnetic Circuits

Magnetic Circuits

AC Magnetic Circuits: the supply is AC, Faraday's law must be considered first.





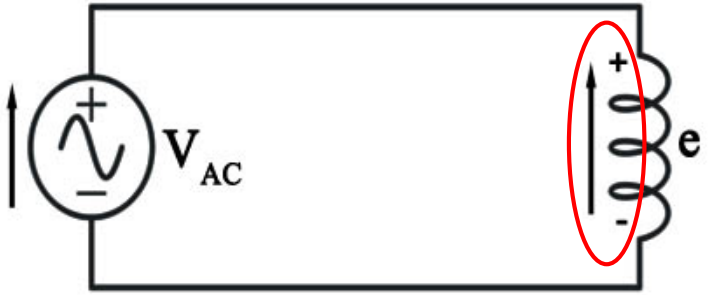
Concept of Magnetic Circuits

Magnetic Circuits

AC Magnetic Circuits:

$$\sum V = zero$$

$$e = -N \frac{d\phi}{dt}$$



(Neglecting Coil Resistance)

$$v_{AC} - |e| = zero$$

$$v_{AC} = |e|$$

Concept of Magnetic Circuits

Magnetic Circuits

AC Magnetic Circuits

$$v_{AC} = N \frac{d\phi}{dt}$$

$$V_m \sin(\omega t) = N \frac{d\phi}{dt}$$

By Integration

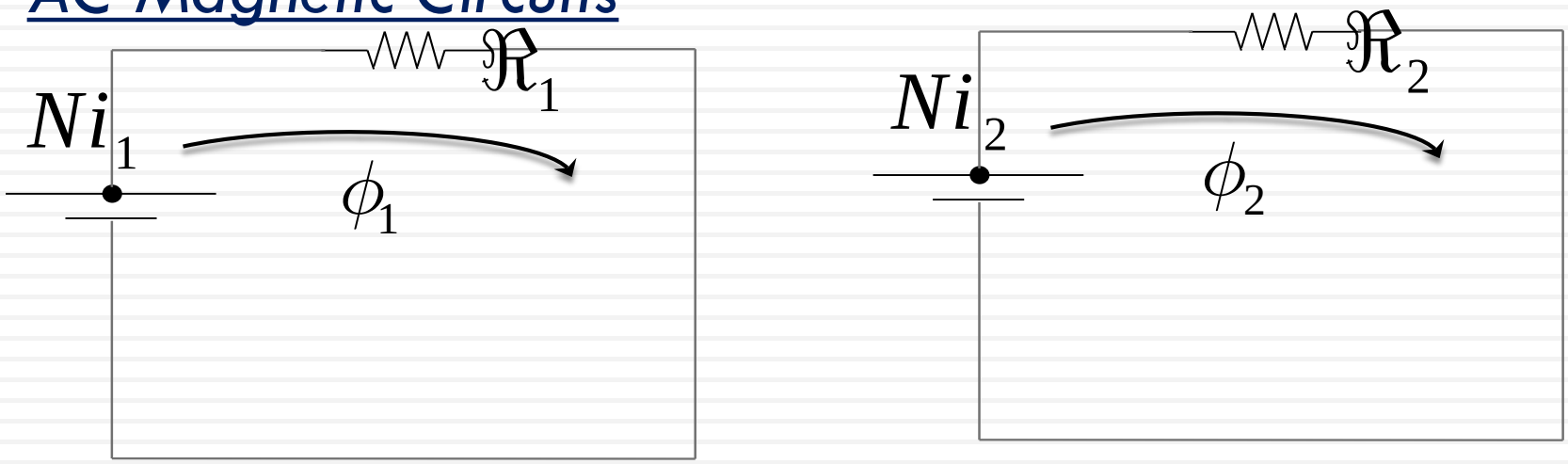
$$\phi = -\frac{V_m}{\omega N}\cos(\omega t) = \frac{V_m}{\omega N}\sin(\omega t - \frac{\pi}{2})$$

Φ is dependent on V and f

Concept of Magnetic Circuits

Magnetic Circuits

AC Magnetic Circuits



$$\phi_1 = \phi_2 = \phi \Rightarrow Ni_1 = \phi \frac{l_{mc}}{\mu_1 A_c} \qquad Ni_2 = \phi \frac{l_{mc}}{\mu_2 A_c}$$

$$\Rightarrow \frac{Ni_1 \mu_1 A_c}{l_{mc}} = \frac{Ni_2 \mu_2 A_c}{l_{mc}} \Rightarrow \frac{i_1}{i_2} = \frac{\mu_2}{\mu_1}$$

Concept of Magnetic Circuits

Magnetic Circuits

AC Magnetic Circuits

To get the current

$$i = \phi \frac{l_{mc}}{N\,\mu A_c}$$

$$\Rightarrow \quad i = \left(\frac{l_{mc}}{N\,\mu A_c}\right)\frac{V_m}{\omega N}\sin\left(\omega t - \frac{\pi}{2}\right)$$

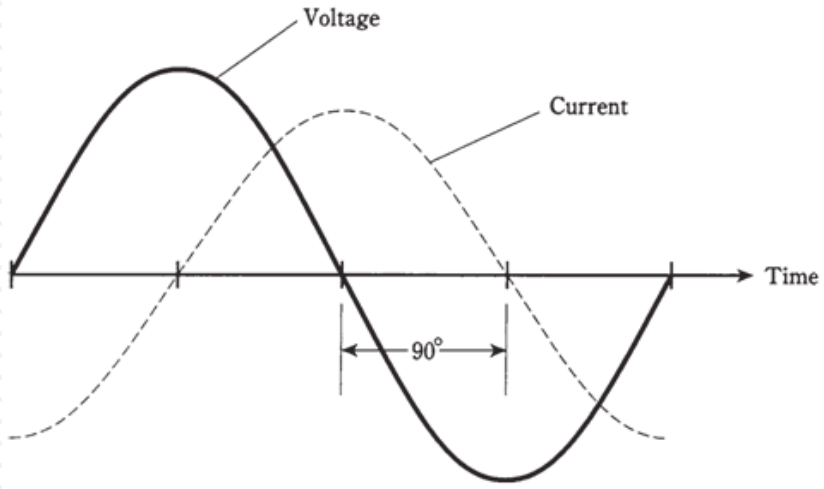
$$\Rightarrow \quad i = \frac{V_m}{\omega\left(\frac{N^2 \mu A_c}{l_{mc}}\right)}\sin\left(\omega t - \frac{\pi}{2}\right)$$

Concept of Magnetic Circuits

Magnetic Circuits

AC Magnetic Circuits

$$I = \frac{V}{j\omega\left(\frac{N^2 \mu A_c}{l_{mc}}\right)}$$



$$I = \frac{V}{j\omega L_{core}} \Rightarrow L_{core} = \frac{N^2}{\frac{l_{mc}}{\mu A_c}} \qquad \Re = \frac{l_{mc}}{\mu A_c}$$

$$\Rightarrow L = \frac{N^2}{\Re}$$

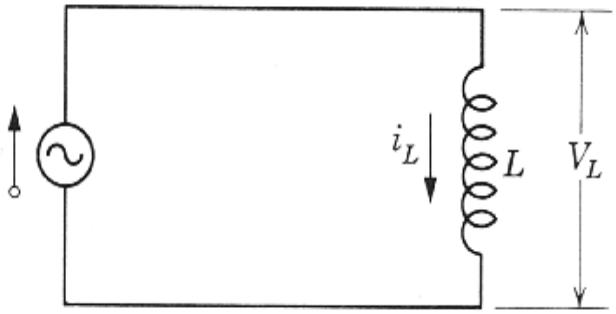
Self Inductance

Concept of Magnetic Circuits

Magnetic Circuits

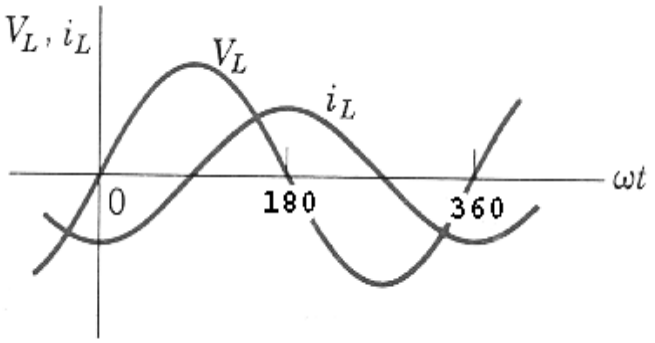
AC Magnetic Circuits

$$v_{AC} = N \frac{d\phi}{dt}$$



From magnetic circuits:

$$Ni = \phi \Re$$



$$v_{AC} = N \frac{d}{dt}\left(\frac{Ni}{\Re}\right)$$

$$v_{AC} = \frac{N^2}{\Re}\frac{di}{dt}$$

$$v_{AC} = L \frac{di}{dt}$$

Example

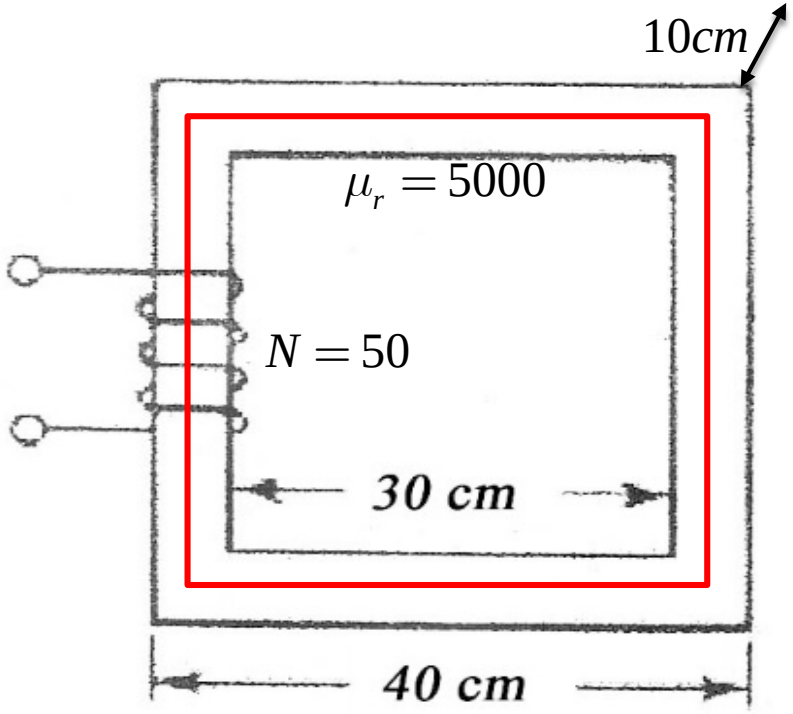
Find the value of the current that will produce a flux of 0.0025 Wb.

$$NI = \phi \Re$$

$$\Re = \frac{l_{mc}}{\mu A_c}$$

$$l_{mc} = 4(30 + 2 \times 2.5) = 140cm$$

$$A_c = 5 * 10 = 50cm^2$$



Example

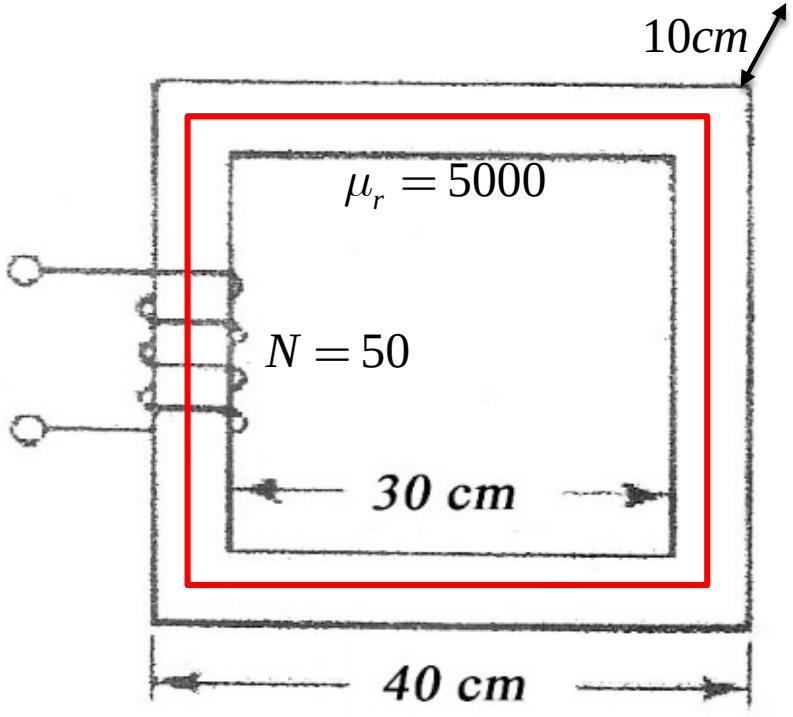
Find the value of the current that will produce a flux of 0.0025 Wb.

$$\Re = \frac{1.4}{(5000)(4\pi \times 10^{-7})(50 \times 10^{-4})}$$

$$\Re = 44563.38 \ H^{-1}$$

$$NI = \phi \Re$$

$$I = 2.228 \ A$$



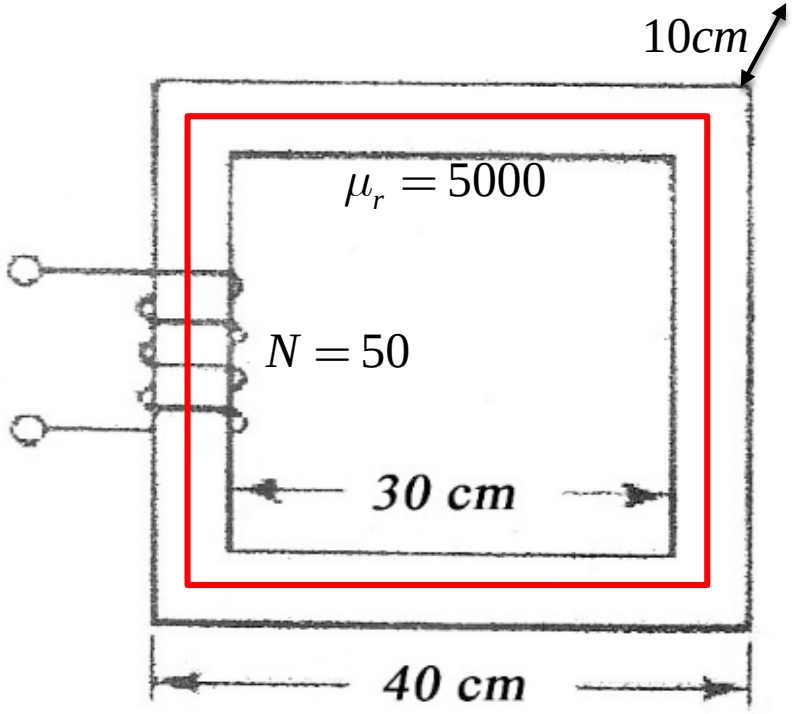
Example

Find the inductance of the coil.

$$\Re = 44563.38\ H^{-1}$$

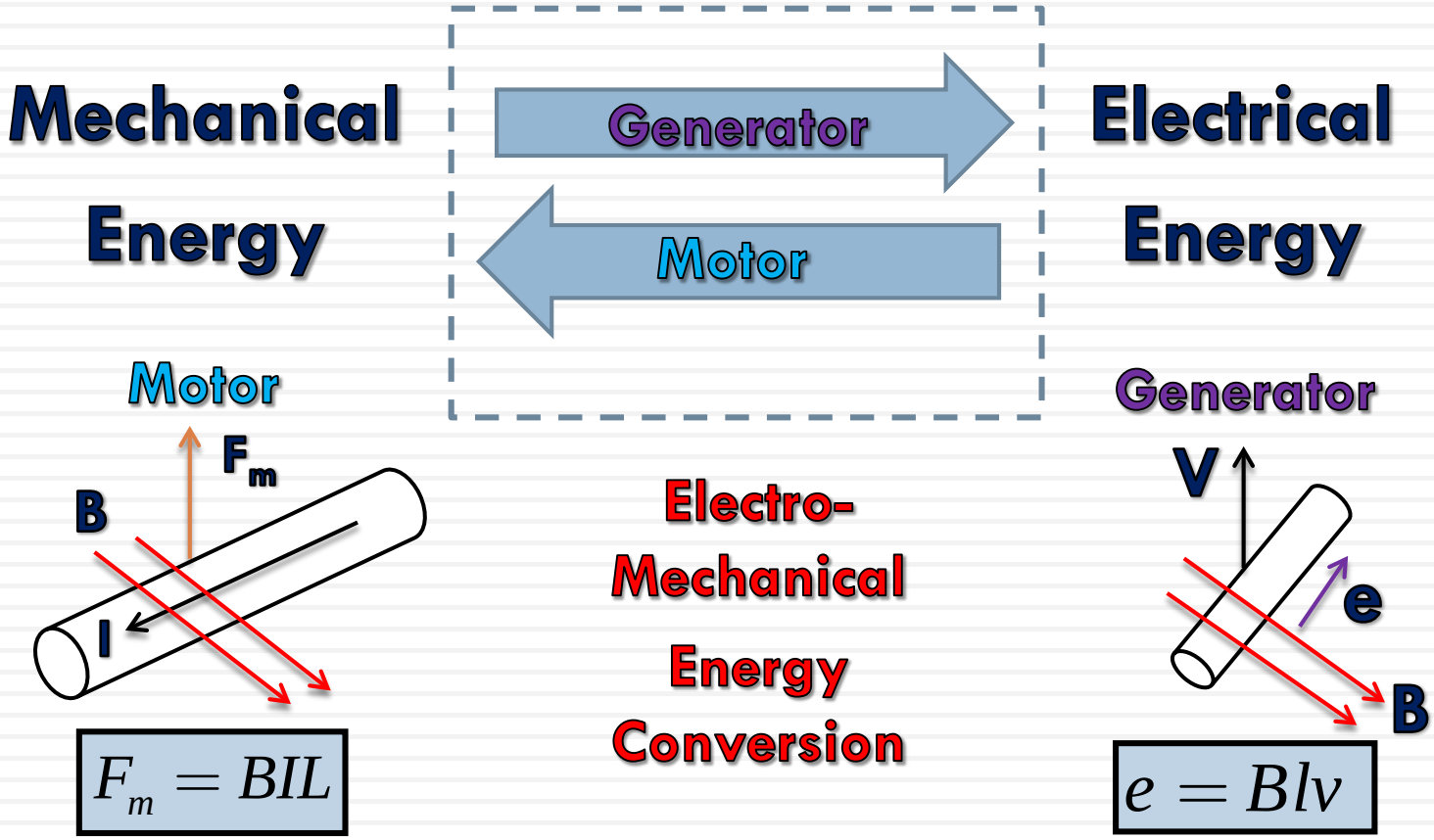
$$L = \frac{N^{2}}{\Re}$$

$$L = 0.056H$$



Magnetic Circuits: Electric Machines

An electrical machine is a device which converts mechanical energy into electrical energy or vice versa.

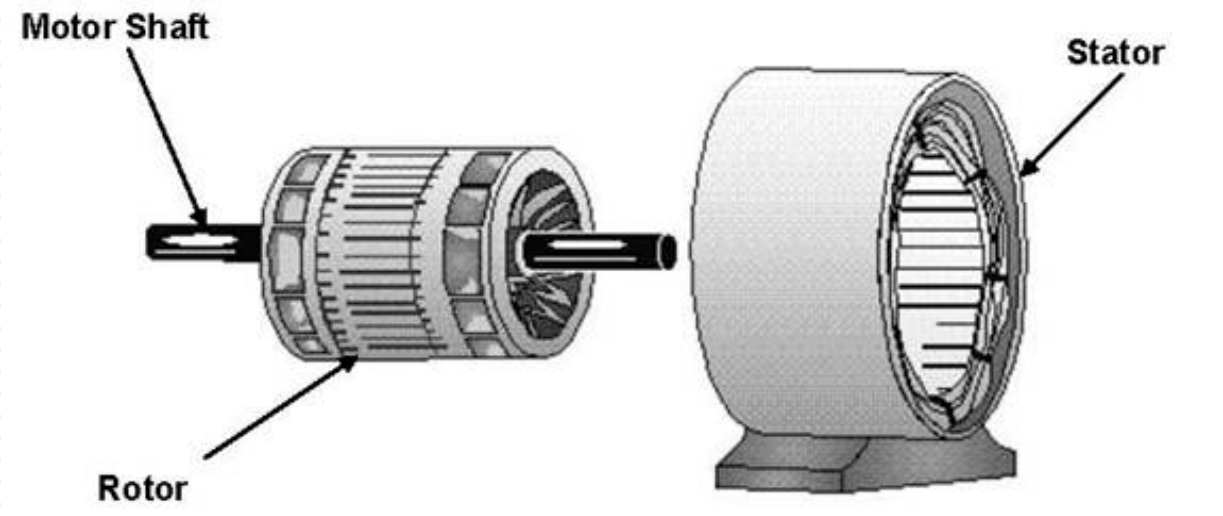


Magnetic Circuits: Electric Machines

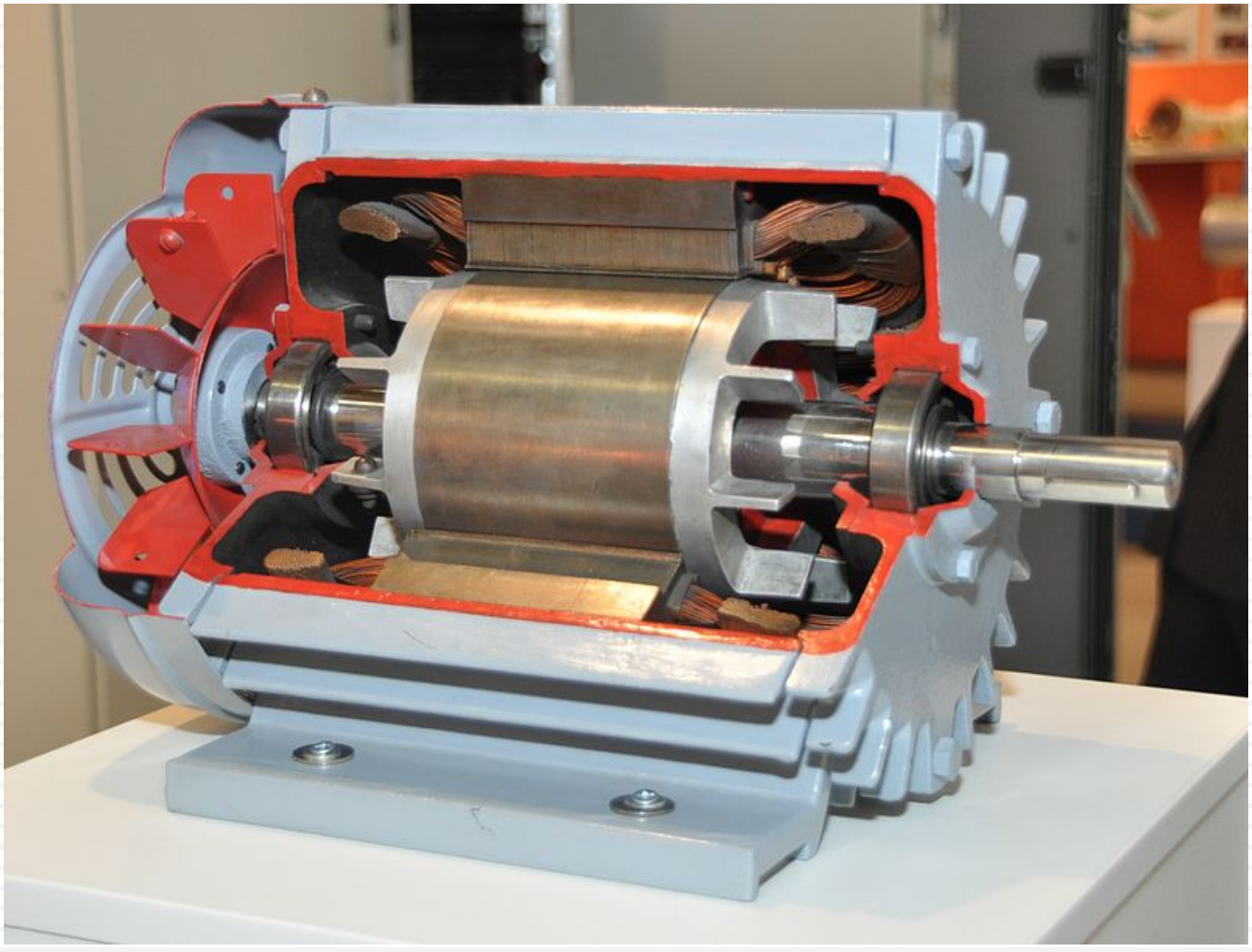
Electrical machines have generally two basic parts named "Stator" and "Rotor".

The stator is the stationary member. The rotor is the rotating member.

A small air gap exists between the two member.



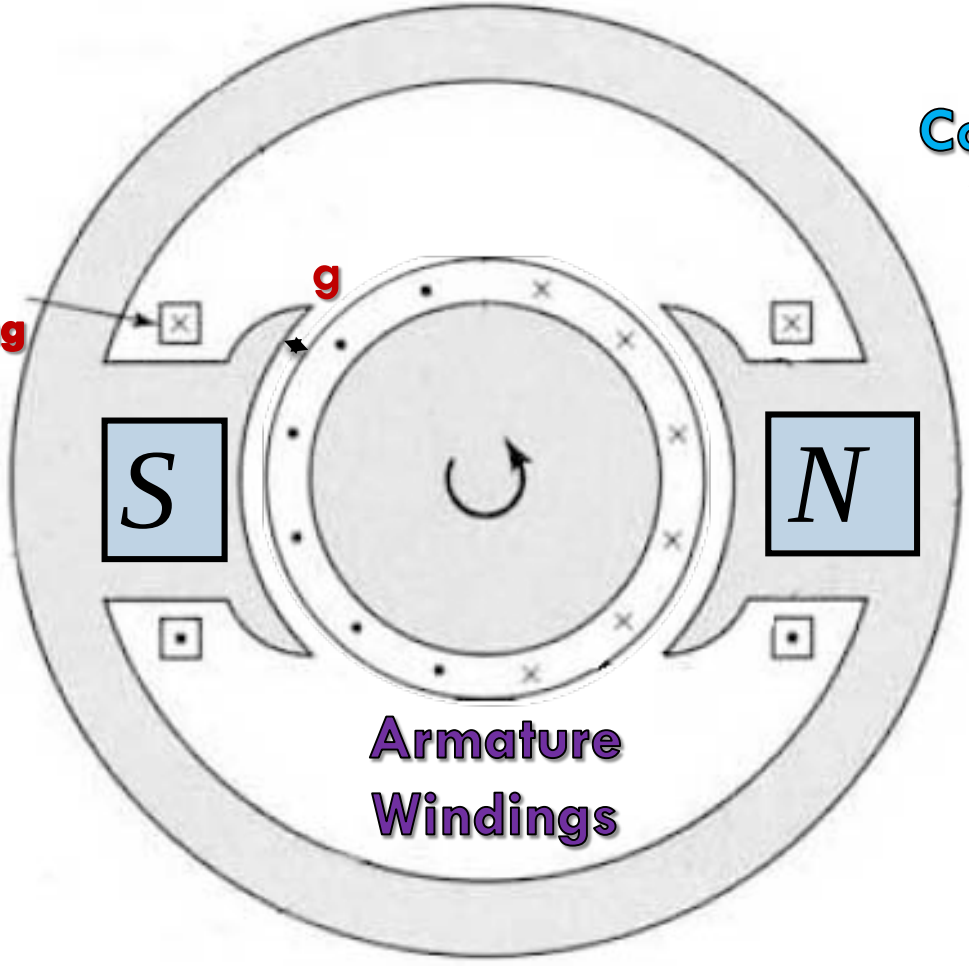
Magnetic Circuits: Electric Machines



Magnetic Circuits: Electric Machines

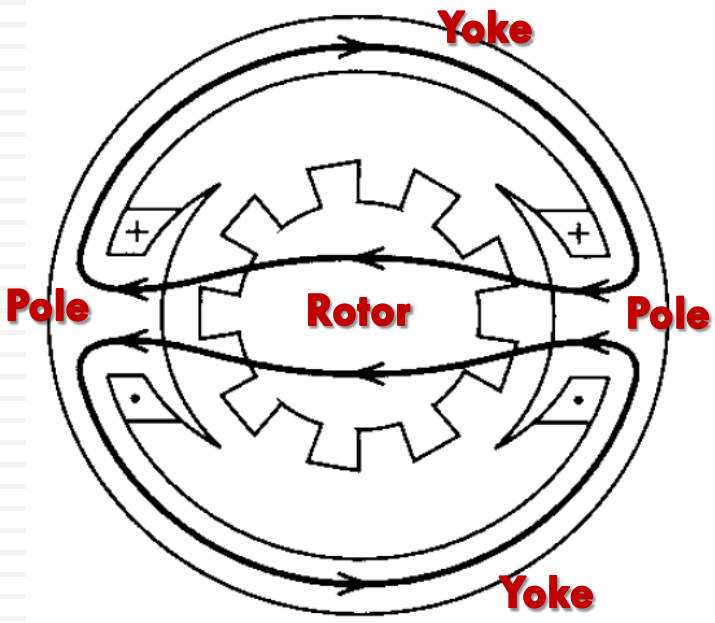
DC Machines

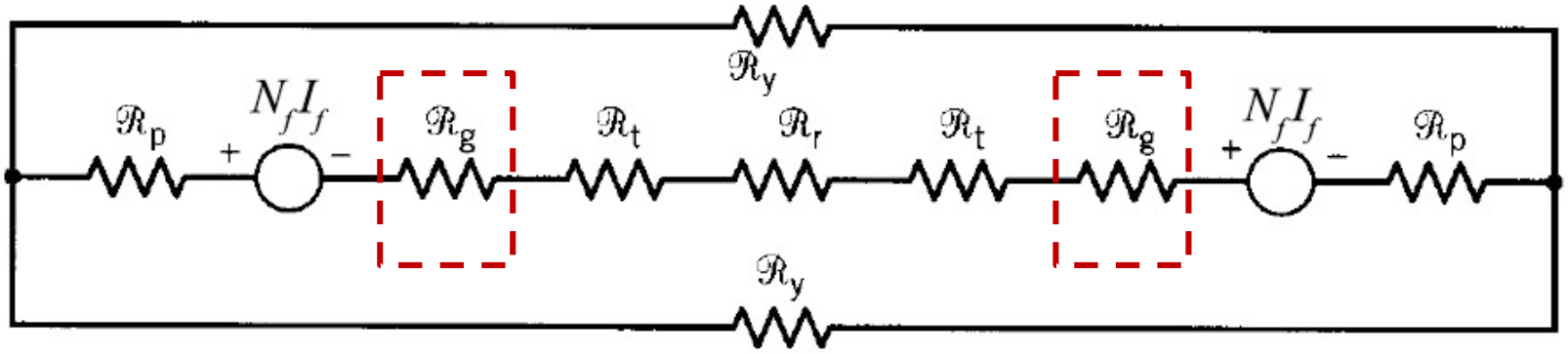
Salient Poles Stator



Magnetic Circuits: Electric Machines

DC Machines





Magnetic Circuits: Electric Machines

DC Machines

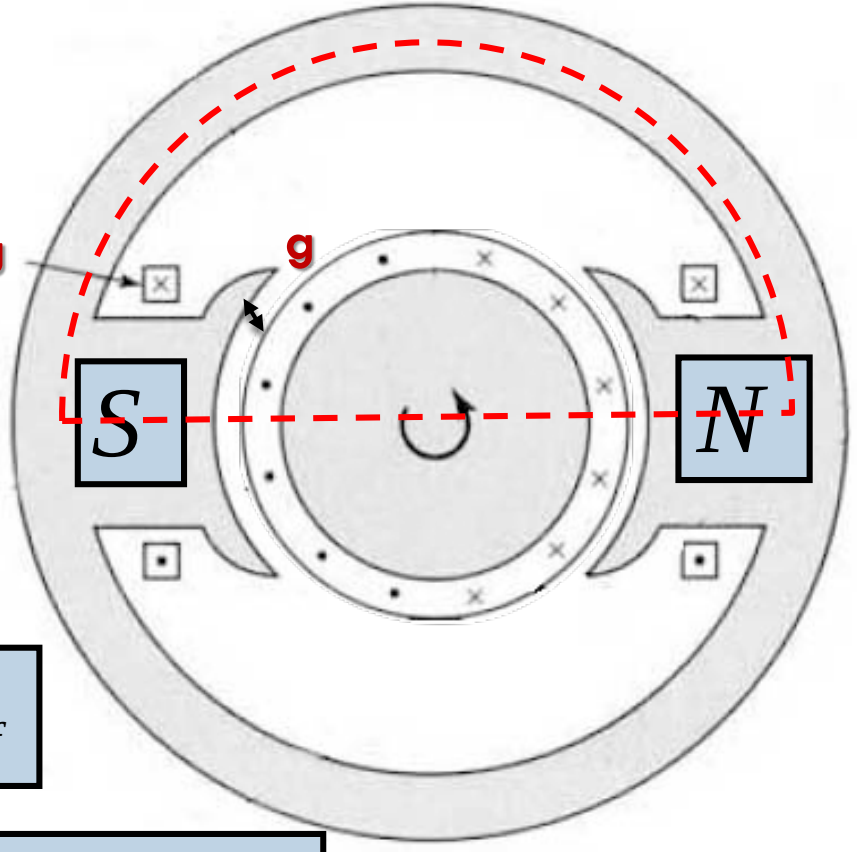
$$\oint_c \underline{H}.\underline{dl} = I_{en}$$

$$\oint_c \underline{H}.\underline{dl} = 2N_f I_f$$

$$H \times g + H \times g \simeq 2N_f I_f$$

Neglecting the reluctance of the steel parts

$$H_g\, g = N_f I_f = MMF = M_f$$



$$M_f = B_g \frac{g}{\mu_o}$$

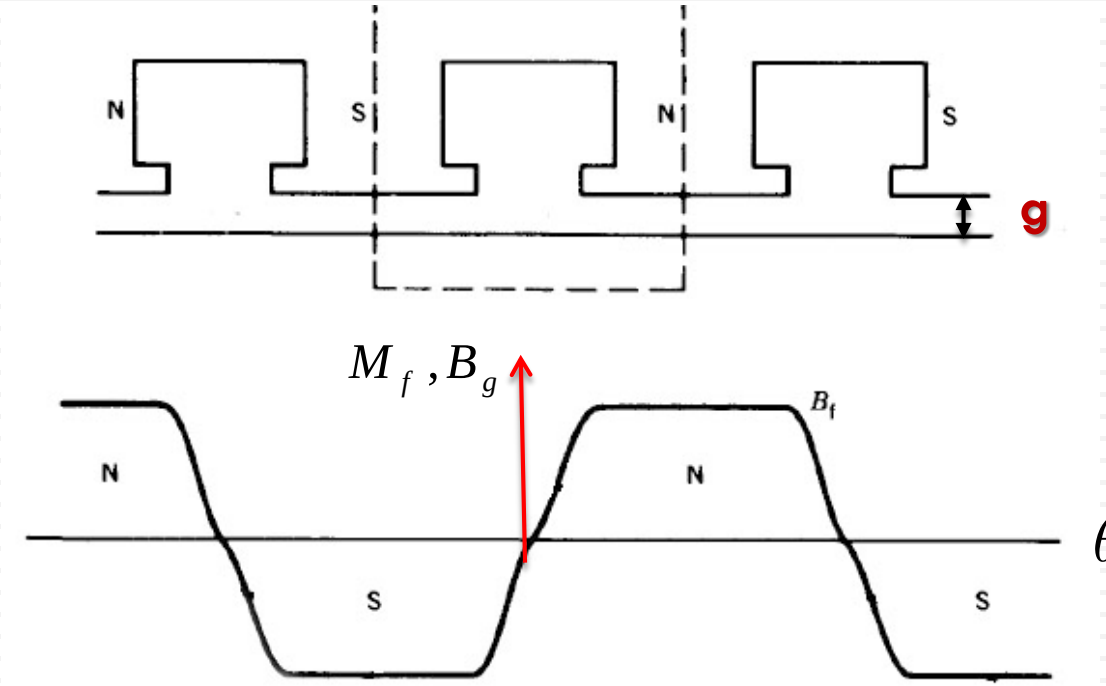
Air Gap
MMF

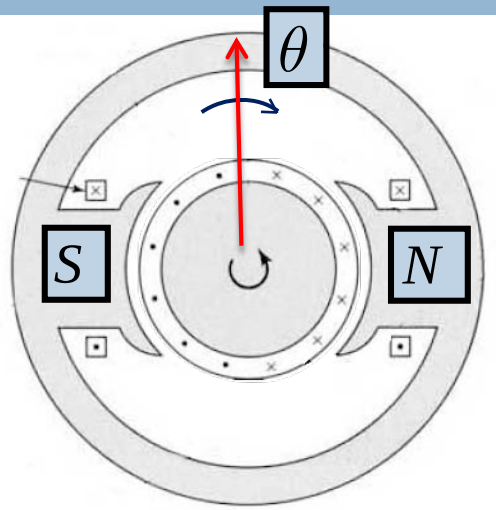
$$B_g = \mu_o \frac{M_f}{g}$$

Air Gap
Flux Density

Magnetic Circuits: Electric Machines

DC Machines





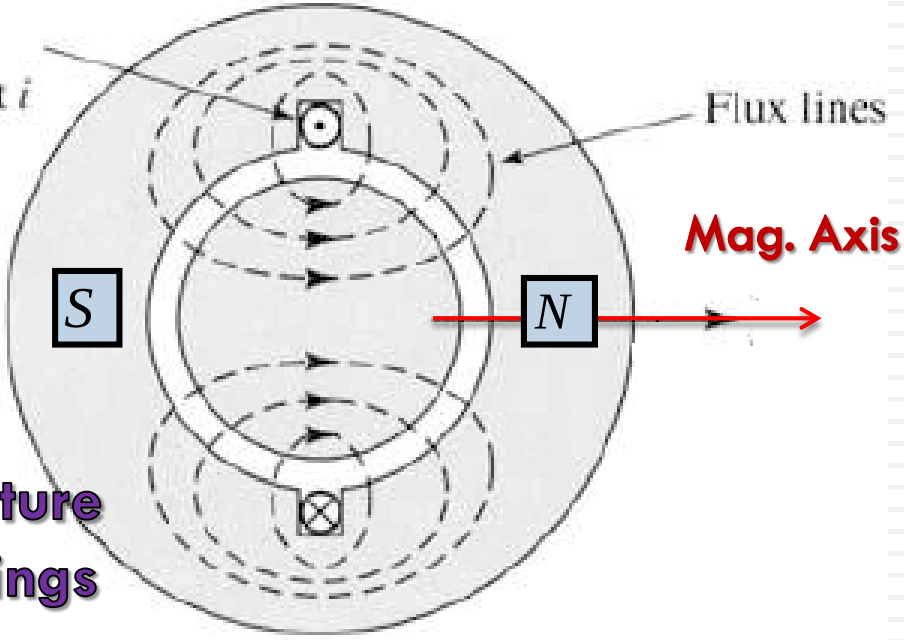
Air Gap MMF (flux density) Distribution

I_f **DC Current** → B_g **Stationary Field**

Magnetic Circuits: Electric Machines

AC Machines

Single-phase machines



Two Magnetic Poles

Non-Salient Poles

Armature Windings

Concentrated Windings

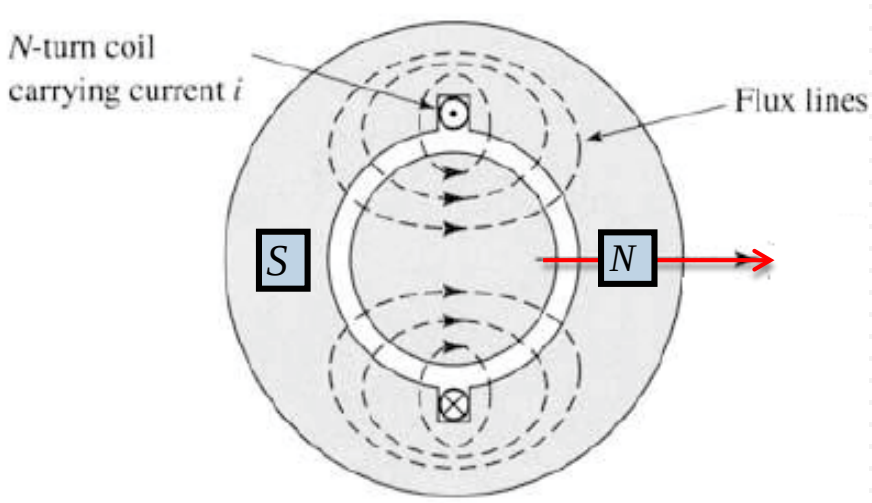
Magnetic Field Production & Distribution

AC Machines

Single-phase machines

$$\oint_c \underline{H}.\underline{dl} = I_{en}$$

$$\oint_c \underline{H}.\underline{dl} = Ni$$



$$H \times g + H \times g \simeq Ni$$

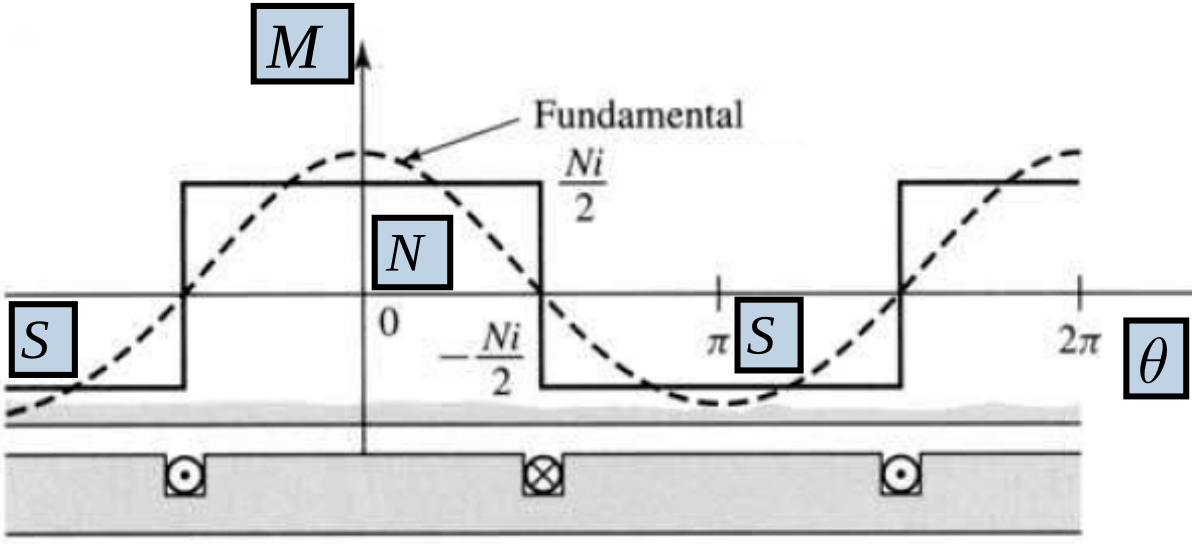
$$H_g\, g = \frac{Ni}{2} = M$$

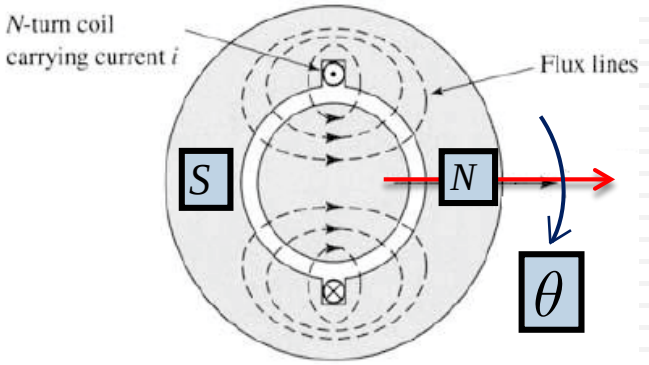
$$M_f = \frac{B_g}{\mu_o} g$$

Magnetic Circuits: Electric Machines

AC Machines

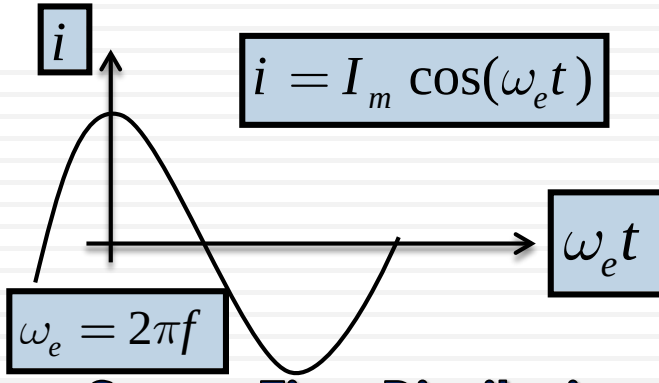
Single-phase machines





MMF Space Distribution

$$M(\theta) = \frac{4}{\pi}\frac{Ni}{2p}\sum_{h=1,3,5,..}\frac{1}{h}\cos(h\theta)$$

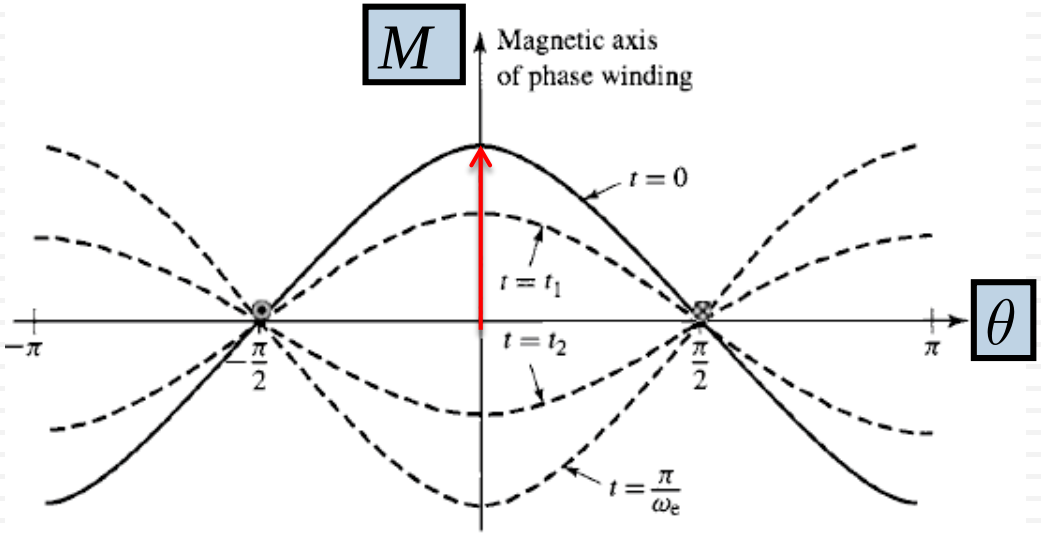


Current Time Distribution

Magnetic Circuits: Electric Machines

AC Machines

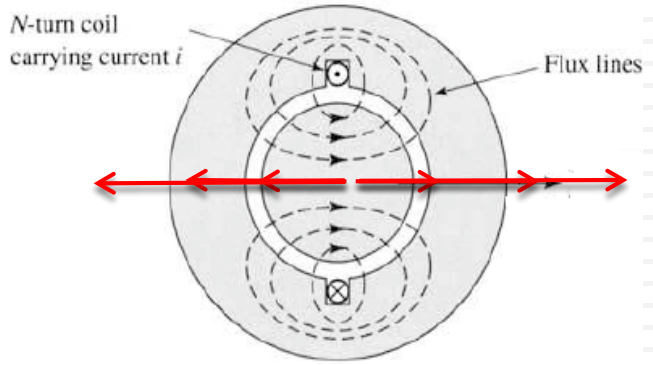
Single-phase machines

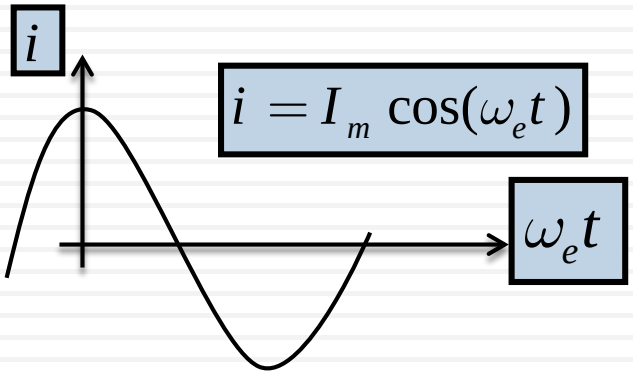


$$M_{fund} = \frac{4}{\pi}\frac{Ni}{2}\cos\theta$$

MMF Space Distribution

AC Current ⇨ **Pulsating Field**





Current Time Distribution

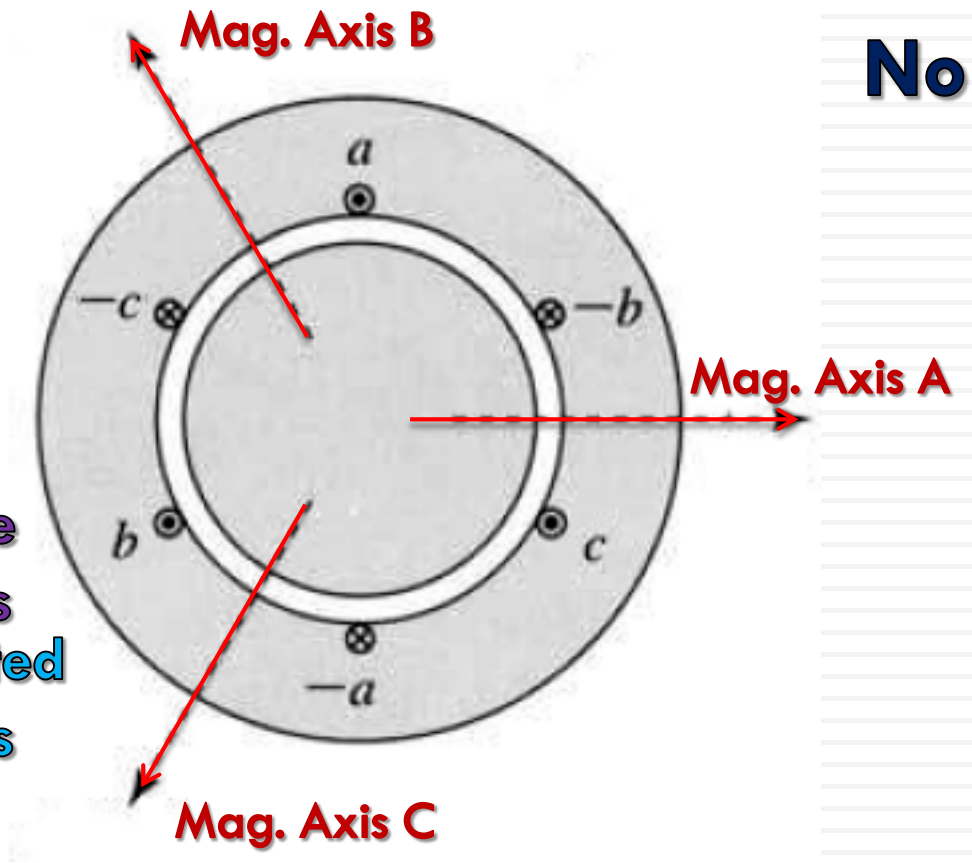
Magnetic Circuits: Electric Machines

AC Machines

Three-phase machines

Two Magnetic Poles

Non-Salient Poles

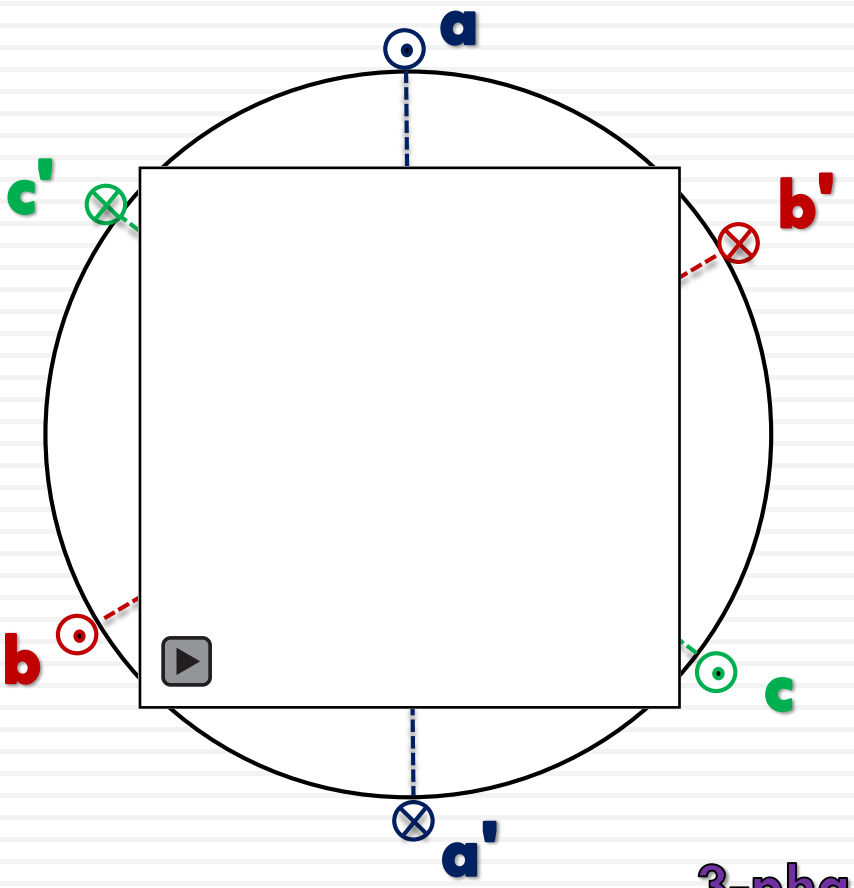


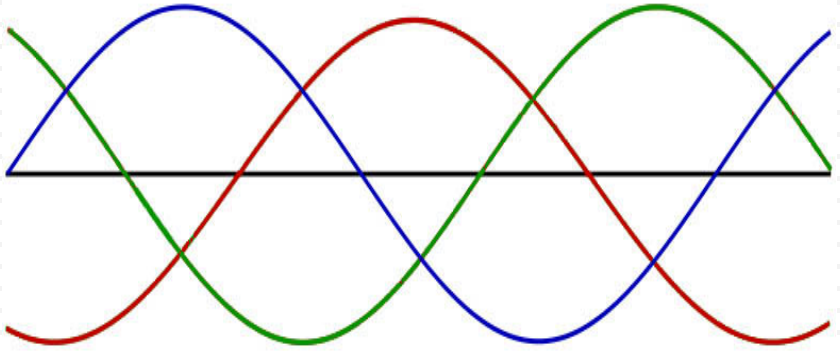
Armature Windings Concentrated Windings

Magnetic Circuits: Electric Machines

AC Machines

Three-phase machines





$$i_a = I_m \cos(\omega_e t)$$

$$i_b = I_m \cos(\omega_e t - 120)$$

$$i_c = I_m \cos(\omega_e t + 120)$$

3-phase Current ⇒ **Rotating Field**