Fourier Method for Waveform Analysis II

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OUTLINE

- Average Power and RMS Values.
- Exponential Fourier Series
- Harmonic Generation and Total Harmonic Distortion (THD)
- Solved Examples

Trigonometric Fourier Series

Average Power and RMS Values

By writing the input voltage and current in amplitude-phase form

$$V(t) = V_{dc} + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \phi_n)$$

$$I(t) = I_{dc} + \sum_{n=1}^{\infty} I_n \cos(n\omega_0 t + \theta_n)$$

the average power can be calculated as

$$P_{av} = \frac{1}{T}\int_0^T V(t)I(t)dt = V_{dc}I_{dc} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\phi_n - \theta_n)$$

Trigonometric Fourier Series

Average Power and RMS Values

$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \phi_n)$$

The r.m.s. (effective) value of a function f is calculated using the formula

$$f_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2 dt}$$

$$= \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2}$$

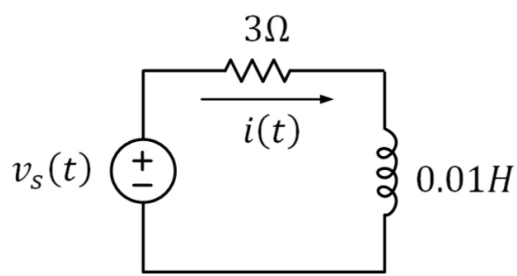
Trigonometric Fourier Series

Example (4)

If the supply is given by

$$v_s(t) = 100\sin(300t) + 10\sin(600t)$$

find $i(t)$ and I_{rms}.



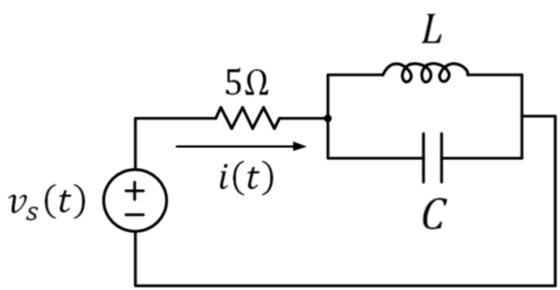
Trigonometric Fourier Series

Example (5)

For the circuit shown, if

$$v_s(t) = 50 + 20\sin(500t) + 10\sin(1000t)$$

and at $\omega = 500 rad/s$ $X_L = 2\Omega$ and $X_C = 8\Omega$. Calculate $i(t)$, I_{rms} and P_{av}.



Exponential Fourier Series

Represent the sine and cosine functions in the exponential form using Euler's identity

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}, \qquad F_n = \frac{1}{T}\int_0^T f(t) e^{-jn\omega_0 t}\, dt$$

<u>Line Spectra:</u>

$|F_n|$ is the magnitude spectrum

$\angle F_n$ is the phase spectrum

$|F_n|^2$ is the power spectrum

<u>Transformation from trigonometric to exponential</u>

$$a_0 = F_0$$

$$a_n = F_n + F_{-n}$$

$$b_n = j[F_n - F_{-n}]$$

$$F_n = \frac{1}{2}[a_n - jb_n]$$

Exponential Fourier Series

Average Power

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

Using Parseval's theorem

$$P_{av} = \sum_{n=-\infty}^{\infty} |F_n|^2$$

Exponential Fourier Series

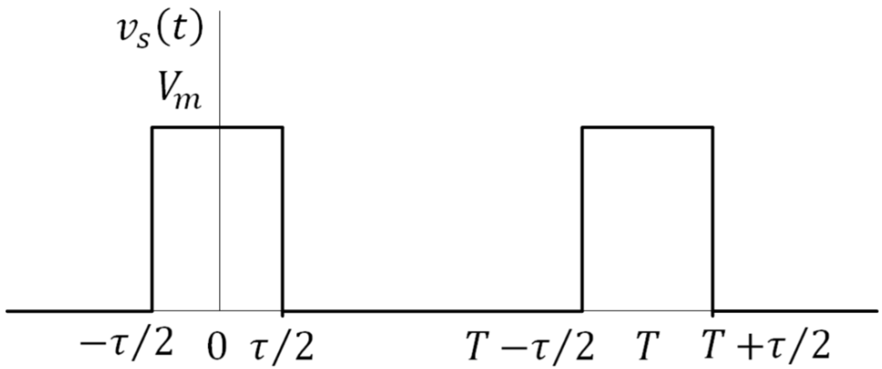
Example (6)

Find the exponential Fourier spectrum of the following:

a) Sawtooth signal and compare it with the trigonometric form.

b) $f(t) = \sin(100t)$

c) The periodic gate function



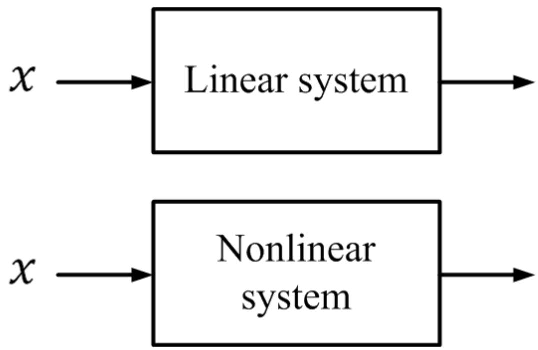
Exponential Fourier Series

Example (7)

Find the power in the signal $f(t) = 2\sin(100t)$

Total Harmonic Distortion

The THD is used to measure the linearity of a system



$y = ax \; = aA\cos(\omega_0 t)$

$y = ax + bx^2$

$x = A\cos(\omega_0 t)$

$y = aA\cos(\omega_0 t) + b(A\cos(\omega_0 t))^2$

Hint: $cos^2\theta = \frac{1}{2}[1 + cos(2\theta)]$

$$= \frac{1}{2}bA^2 + aA\cos(\omega_0 t) + \frac{1}{2}bA^2\cos(2\omega_0 t)$$

Total Harmonic Distortion

$$THD = \left.\frac{\text{sum of the powers of all harmonic components}}{\text{the power of the fundamental frequency}}\right|_{output}$$

$$THD = \frac{\sum_{n=2}^{\infty} \frac{1}{2}(a_n^2 + b_n^2)}{\frac{1}{2}(a_1^2 + b_1^2) + a_0^2} = \frac{\sum_{n=2}^{\infty} \frac{1}{2} A_n^2}{\frac{1}{2} A_1^2 + a_0^2}$$

$$Linear\ Gain = \frac{\text{Amplitude of the fundamental at output}}{\text{Amplitude of the fundamental at input}}$$

$$Linear\ Gain = \frac{A_{1out}}{A_{in}}$$

Total Harmonic Distortion

Example (8)

A given amplifier is tested with an input 500 Hz sinusoid of 2 mV peak amplitude. The output is found to be composed of parabolic sections given as follows:

$$y(t) = \begin{cases} 1 - 4t^2, & -0.5 \leq t \leq 0.5 \\ 4(t-1)^2 - 1, & 0.5 \leq t \leq 1.5 \end{cases}$$

Calculate:

a) THD.

b) The linear gain of the amplifier.