Fourier Method for Waveform Analysis

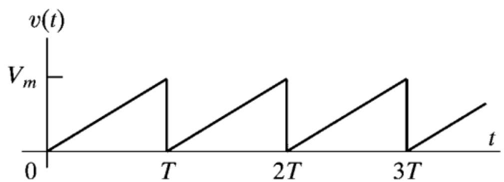
Dr. Mohamed Refky Amin
Electronics and Electrical Communications Engineering Department (EECE)
Cairo University
elc.n112.eng@gmail.com
http://scholar.cu.edu.eg/refky/

OUTLINE

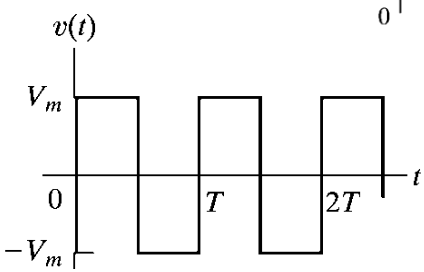
- Introduction
- Fourier Series Representation of Periodic Signals: A Review
- Trigonometric Fourier Series.
- Solved Examples

Introduction

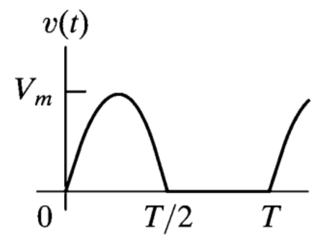
- Up to now, we studied and analysis circuits with either DC or sinusoidal sources.
- Now we consider the case when the source is a non-sinusoidal periodic signals



Sawtooth Signal



Square wave



Half-wave rectifier output

Introduction

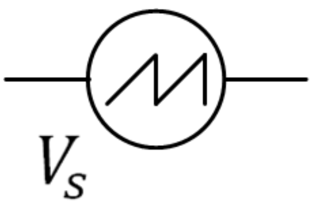
- A periodic function is one that repeats itself every T seconds

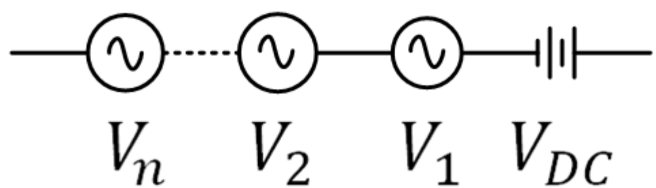
$$f(t) = f(t + nT)$$

where n is an integer and T is the **Period** of the function f

- To deal with periodic sources, Fourier series is used.

- Fourier series represents any periodic function as a sum of sinusoids.

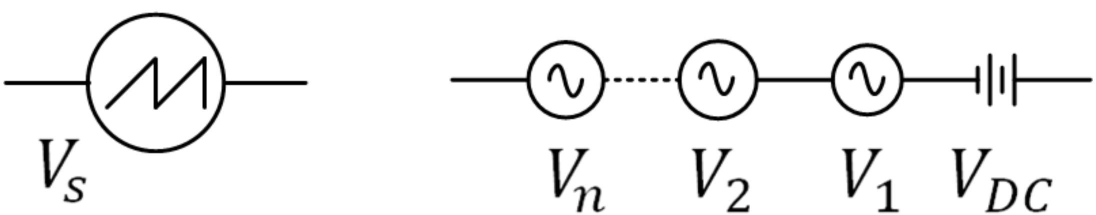




Trigonometric Fourier Series

- The Fourier series of a periodic function $x(t)$ is a representation that resolves $x(t)$ into a DC component and an AC component comprising an infinite series of harmonic sinusoids.

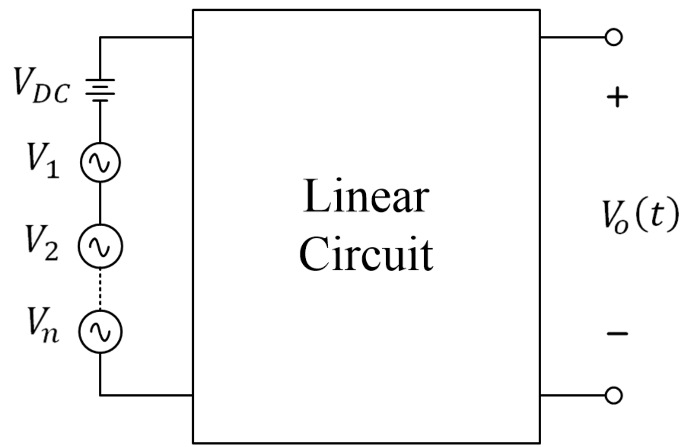
$$x(t) = \underbrace{a_0}_{DC} + \underbrace{\sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)}_{AC}$$



Trigonometric Fourier Series

- Because of the linearity of the circuits we deal with, superposition theorem can be used to determine the output of the circuit.

$$V_0 = V_0\Big|_{V_{dc}} + V_0\Big|_{V_1} + V_0\Big|_{V_2} + \cdots + V_0\Big|_{V_n}$$



Trigonometric Fourier Series

$$x(t) = \underbrace{a_0}_{DC} + \underbrace{\sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)}_{AC}$$

- $\omega_0 = \frac{2\pi}{T}$ is called the **fundamental frequency**
- The sinusoid $\cos(n\omega_0 t)$ or $\sin(n\omega_0 t)$ is called the **nth harmonic** of f
  - An odd harmonic if n is odd
  - An even harmonic if n is even
- The constants a_n and b_n are called the **Fourier coefficients**
- The coefficient a_0 is the DC component or the average value of x.

Trigonometric Fourier Series

$$a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, dt$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos(n\omega_0 t)\, dt$$

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\sin(n\omega_0 t)\, dt$$

Trigonometric Fourier Series

Symmetry Considerations

- Even Symmetry

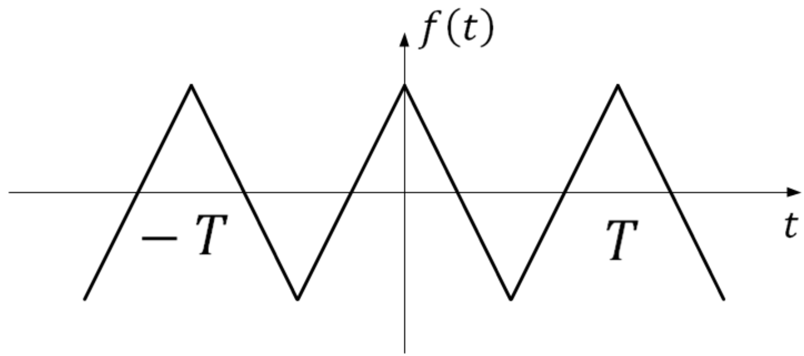
A function $f(t)$ is even if its plot is symmetrical about the vertical axis

$$f(t) = f(-t)$$

$$a_0 = \frac{2}{T}\int_0^{\frac{T}{2}} f(t)\,dt$$

$$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos(n\omega_0 t)\,dt$$

$$b_n = 0$$



Trigonometric Fourier Series

Symmetry Considerations

- Odd Symmetry

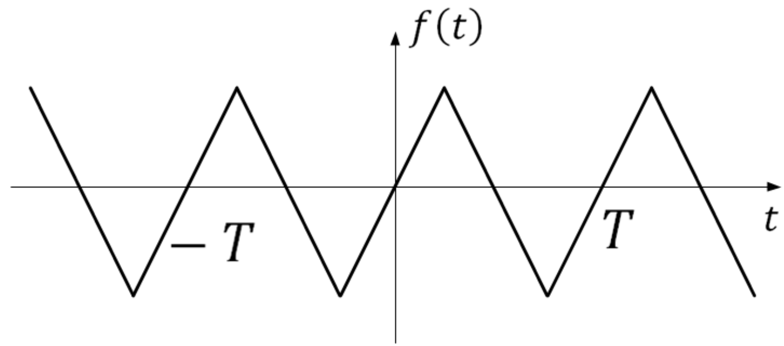
A function $f(t)$ is odd if its plot is symmetrical about the origin

$$f(t) = -f(-t)$$

$$a_0 = 0$$

$$a_n = 0$$

$$b_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\sin(n\omega_0 t)\,dt$$



Trigonometric Fourier Series

Symmetry Considerations

- Half-Wave Symmetry

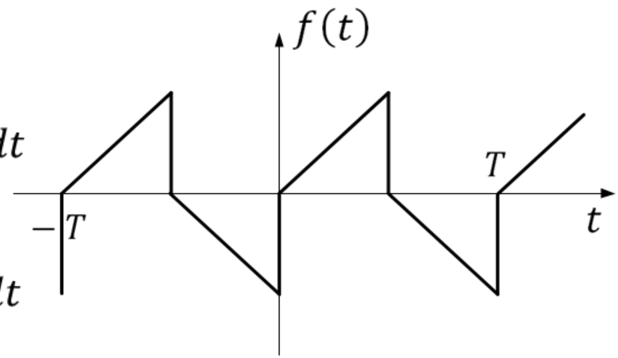
A function $f(t)$ is half-wave symmetry if

$$f(t) = -f\left(t + \frac{T}{2}\right)$$

$$a_0 = a_{2n} = b_{2n} = 0$$

$$a_{2n+1} = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos\big((2n+1)\omega_0 t\big)\,dt$$

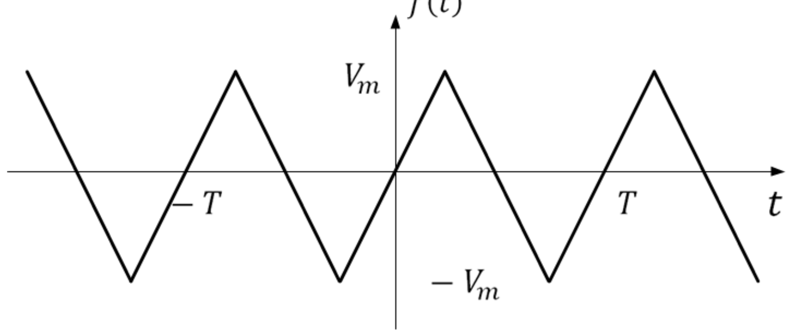
$$b_{2n+1} = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\sin\big((2n+1)\omega_0 t\big)\,dt$$



Trigonometric Fourier Series

Example (1)

1. Find the Fourier series representation of the triangular waveform shown in the figure



Hint: $$\int x\sin(ax)\,dx = \frac{1}{a^2}\sin(ax) - \frac{1}{a}x\cos(ax)$$

$$\int x\cos(ax)\,dx = \frac{1}{a^2}\cos(ax) - \frac{1}{a}x\sin(ax)$$

Trigonometric Fourier Series

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$= a_0 + \sum_{n=1}^{\infty} (a_n - jb_n) \cos(n\omega_0 t)$$

Thus, an alternative Fourier series form is the **Amplitude-Phase** form

$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \phi_n)$$

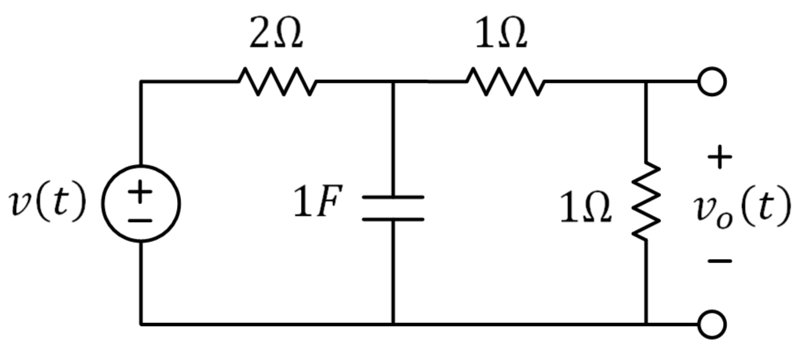
$$A_n = \sqrt{a_n^2 + b_n^2}, \qquad \phi_n = -tan^{-1}\left(\frac{b_n}{a_n}\right)$$

Trigonometric Fourier Series

Example (2)

Calculate the steady-state voltage $v_o(t)$ if the input voltage is given by:

$$v(t) = \sum_{n=1}^{\infty} \left(\frac{20}{n\pi} \sin(2nt) - \frac{40}{n^2\pi^2} \cos(2nt) \right), \qquad n\ odd$$



Trigonometric Fourier Series

Example (3)

Determine the expression for the steady state current $i(t)$ if the input voltage is given by

$$v_s(t) = \frac{20}{\pi} + \sum_{n=1}^{\infty}\left(\frac{-40}{\pi(4n^2-1)}\cos(2nt)\right)$$