Transformers

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Outline of This Lecture

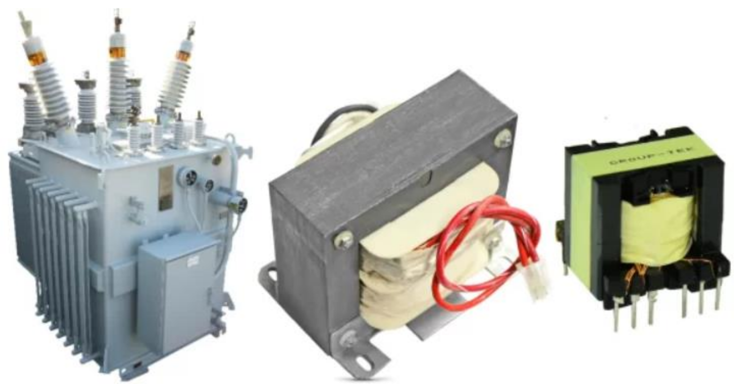
- Introduction
- Linear Transformer
- Ideal Transformer

Transformers

Introduction

A transformer is a magnetic device that takes advantage of the phenomenon of mutual inductance.

A transformer is generally a four-terminal device comprising two magnetically coupled coils.



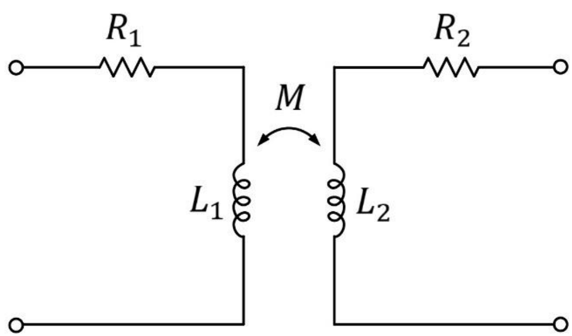
Transformers

Linear Transformer

A transformer is linear if the coils are wound on a magnetically linear material (has a constant magnetic permeability).

Most materials are magnetically linear (air, plastic, Bakelite, and wood).

Linear transformers are used in radio and TV sets.



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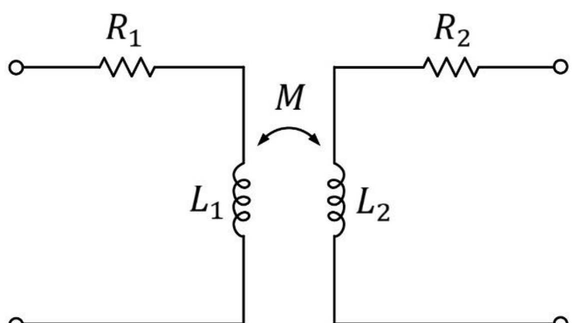
Linear Transformer

The coil that is directly connected to the voltage source is called the primary coil.

The coil connected to the load is called the secondary coil.

R_1 and R_2 are included to account for the power dissipation in the coils.

Primary coil



Secondary coil

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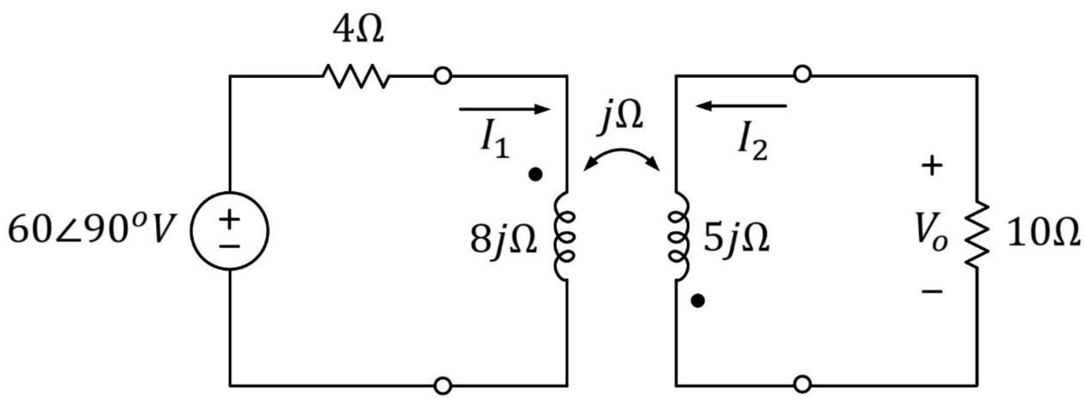
Example (1)

For the linear transformer shown below, find I_1, I_2, and V_o using

- The direct solution method
- the T-equivalent circuit



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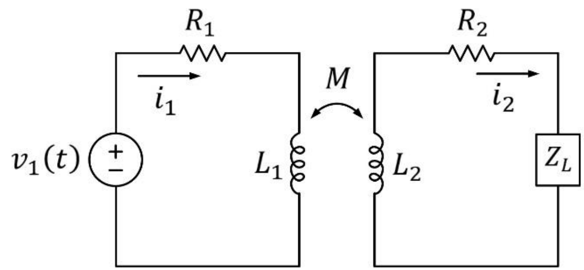
Linear Transformer

Input Impedance & Reflected Impedance



$$V_1 = I_1(R_1 + j\omega L_1) \pm j\omega M I_2$$

$$0 = I_2(R_2 + Z_L + j\omega L_2) \pm j\omega M I_1$$

Z_{in} governs the behavior of the primary circuit.

$$Z_{in} = \frac{V_1}{I_1} = (R_1 + j\omega L_1) \pm j\omega M \frac{I_2}{I_1}$$

Form the second equation

$$I_2 = \frac{\mp j\omega M I_1}{R_2 + Z_L + j\omega L_2} = \frac{\mp j\omega M I_1}{Z_{Secondary}}$$

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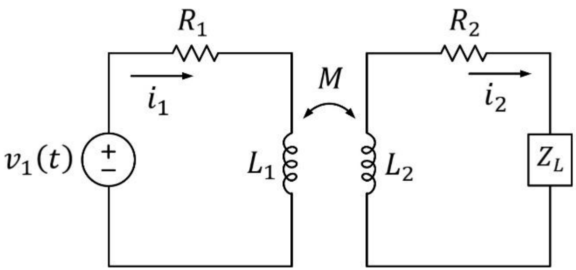
Linear Transformer

Input Impedance & Reflected Impedance

$$Z_{in} = (R_1 + j\omega L_1) + \frac{\omega^2 M^2}{Z_{Secondary}}$$

$$= Z_{Primary} + \frac{\omega^2 M^2}{Z_{Secondary}}$$

$$= Z_{Primary} + Z_{Reflected}$$



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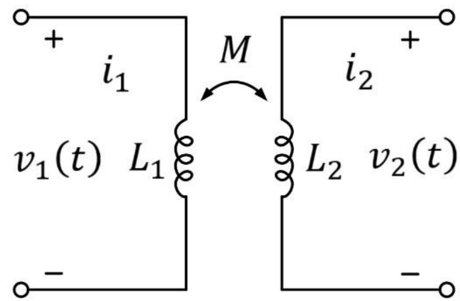
Ideal Transformer

Ideal transformer is the limiting case of the two coupled ideal inductors ($R_1 = R_2 = 0$) and the coupling is perfect ($k = 1$).

Because $k = 1$, when a sinusoidal voltage is applied, the same magnetic flux goes through both coils.

According to Faraday's law, the voltage across the primary coil is given by

$$v_1(t) = N_1 \frac{d\phi}{dt}$$



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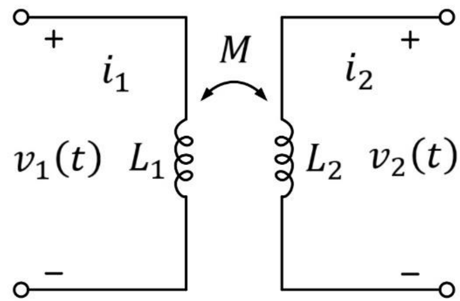
Ideal Transformer

According to Faraday's law, the voltage across the secondary coil is given by

$$v_2(t) = N_2 \frac{d\phi}{dt}$$

$$\frac{v_1(t)}{v_2(t)} = \frac{V_1}{V_2} = \frac{N_1}{N_2} = a$$



From the conservation of power

$$S_1 = S_2 \quad \rightarrow \quad V_1 I_1^* = V_2 I_2^*$$

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = a$$

Transformers

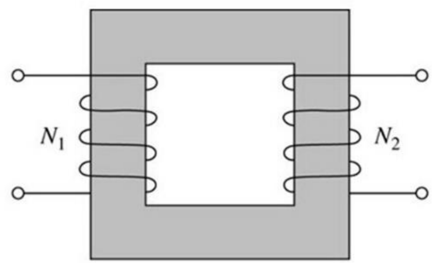
Ideal Transformer

The assumptions of the ideal transformer are

- The coils are lossless (resistance of the cables are zero).
- The coils are perfectly coupled ($k = 1$).
- $L_1,\ L_2,\ M \rightarrow \infty$

An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.

Iron-core transformers are close approximations to ideal transformers.



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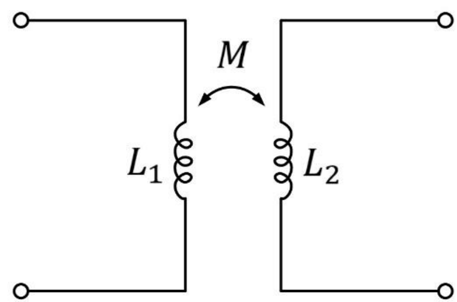
Ideal Transformer

The assumptions of the ideal transformer are

- The coils are lossless (resistance of the cables are zero).
- The coils are perfectly coupled ($k = 1$).
- $L_1,\ L_2,\ M \rightarrow \infty$

The vertical lines between the coils indicate an iron core as distinct from the air core used in linear transformers.



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Ideal Transformer

Step-up and Step-down Transformer

A step-up transformer is one whose secondary voltage is greater than its primary voltage

$$V_2 > V_1 \quad \rightarrow \quad a < 1 \quad \rightarrow \quad N_1 < N_2 \quad \rightarrow \quad I_2 < I_1$$

A step-down transformer is one whose secondary voltage is less than its primary voltage

$$V_2 < V_1 \quad \rightarrow \quad a > 1 \quad \rightarrow \quad N_1 > N_2 \quad \rightarrow \quad I_2 > I_1$$

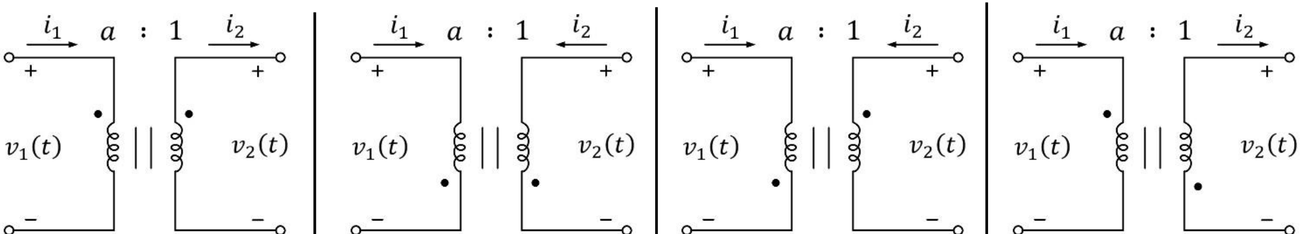
Transformers

Ideal Transformer

Voltages Polarity and Currents Direction

If V_1 and V_1 are both positive or both negative at the dotted terminals, the ratio is positive. Otherwise, the ratio is negative.

If I_1 and I_2 both enter into or both leave the dotted terminals, the ratio is negative. Otherwise, the ratio is positive.



$\frac{V_1}{V_2} = a \quad \frac{I_2}{I_1} = a$

$\frac{V_1}{V_2} = a \quad \frac{I_2}{I_1} = -a$

$\frac{V_1}{V_2} = -a \quad \frac{I_2}{I_1} = a$

$\frac{V_1}{V_2} = -a \quad \frac{I_2}{I_1} = -a$

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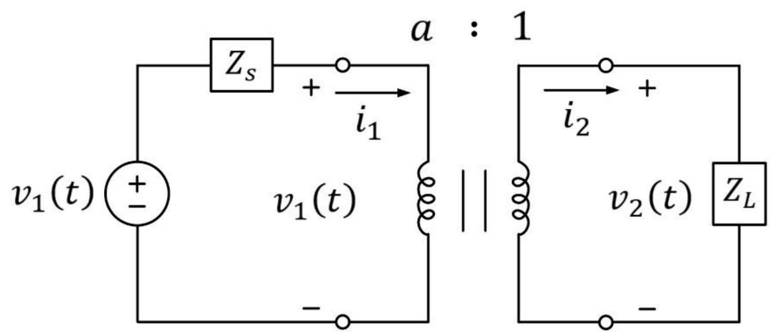
Ideal Transformer

Referred Impedance – Refer to Primary

$$Z_{in} = \frac{V_1}{I_1} = \frac{a \times V_2}{I_2/a} = a^2 \frac{V_2}{I_2} = a^2 Z_L$$



$$Z_L = \frac{V_2}{I_2}$$

Z_L appeares in the primary circuit as $a^2 Z_L$

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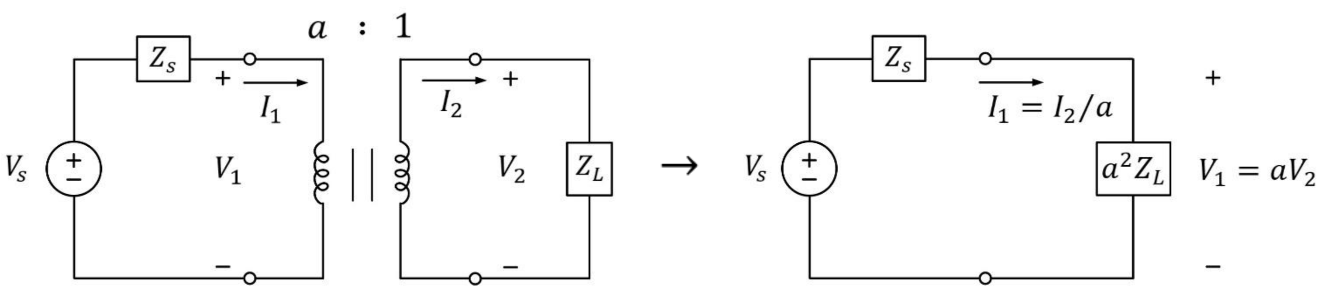
Ideal Transformer

Referred Impedance – Refer to Primary

In refer to primary, the primary and the secondary are connected as one circuit and the secondary elements are replaced as

$$Z_L \rightarrow a^2 Z_L, \qquad V_2 \rightarrow V_1 = aV_2, \qquad I_2 \rightarrow I_1 = I_2/a$$



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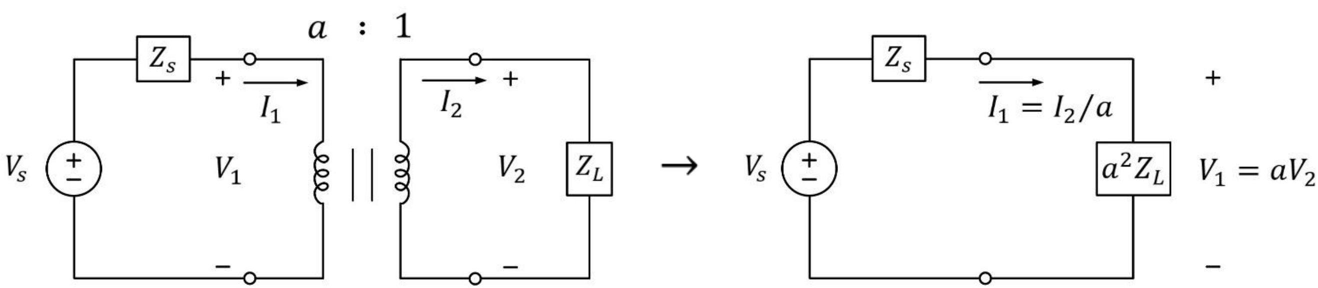
Ideal Transformer

Referred Impedance – Refer to Primary

Load power is given by

$$P_{Z_L} = |I_2|^2 \times Re\{Z_L\} = |I_1|^2 \times Re\{a^2 Z_L\}$$



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Ideal Transformer

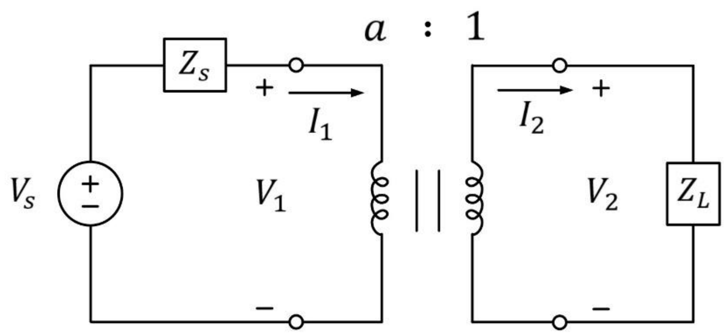
Referred Impedance – Refer to Secondary

$$Z_{th} = \left.\frac{V_2}{I_2}\right|_{V_S=0, Z_L=\infty}$$

$$= \frac{V_1/a}{a \times I_1}$$

$$= \frac{1}{a^2}\frac{V_1}{I_1}$$

$$= Z_S/a^2$$



$$Z_s = \frac{V_1}{I_1}$$

Z_S appeares in the secondary circuit as Z_S/a^2

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Ideal Transformer

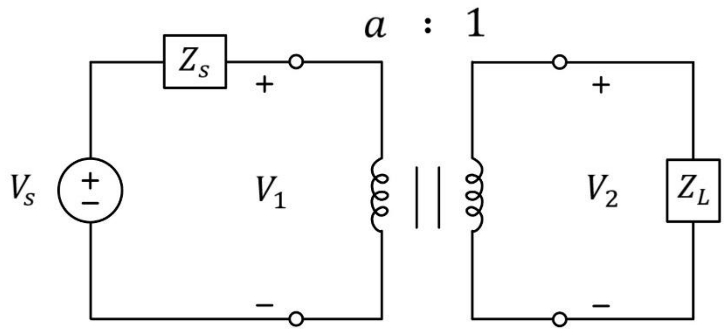
Referred Impedance – Refer to Secondary

$$V_{th} = V_2 \Big|_{I_2=0}$$

$$I_2 = 0 \quad \rightarrow \quad I_1 = 0$$

$$V_1 = V_s$$

$$V_2 = \frac{V_1}{a} = \frac{V_s}{a}$$



V_S appeares in the secondary circuit as V_S/a

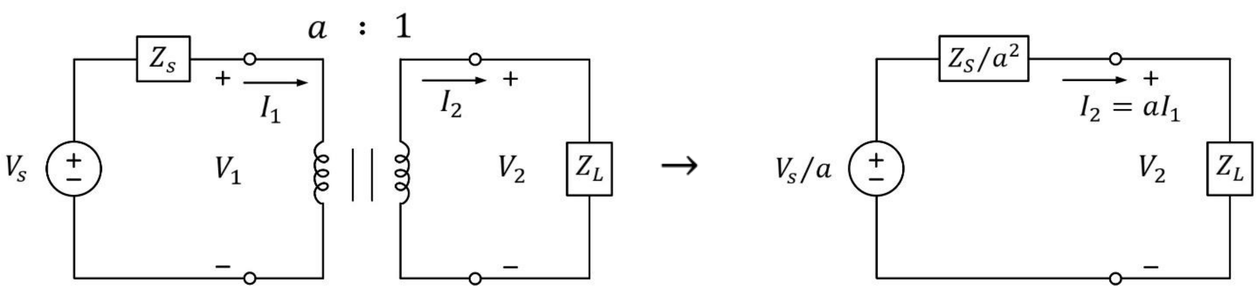
Transformers

Ideal Transformer

Referred Impedance – Refer to Secondary

In refer to secondary, the primary and the secondary are connected as one circuit and the secondary elements are replaced as

$$Z_S \rightarrow Z_S/a^2 , \qquad V_S \rightarrow V_S/a , \qquad I_1 \rightarrow I_2 = a \times I_1$$



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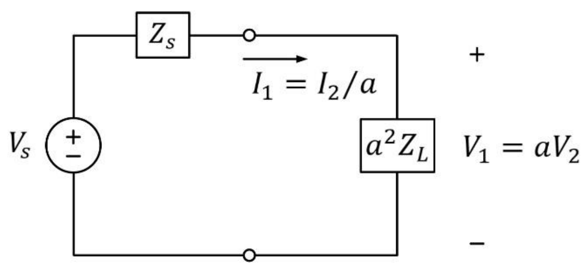
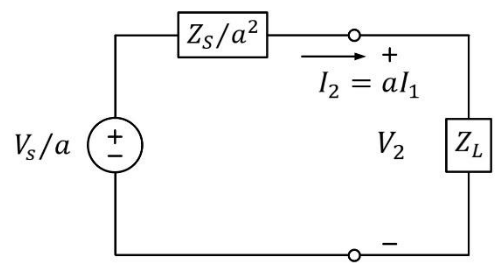
Ideal Transformer

Referred Impedance

Referred impedance (either refer to primary or refer to secondary) simplifies the solution of the transformer circuit and simplifies the maximum power transform calculations.





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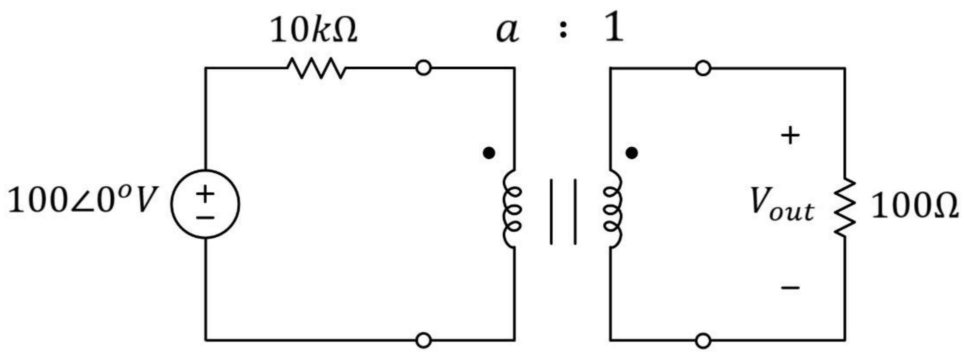
Example (2)

For the circuit shown below, find a, P_{max}, and V_{out} to get a maximum power for the load 100Ω.



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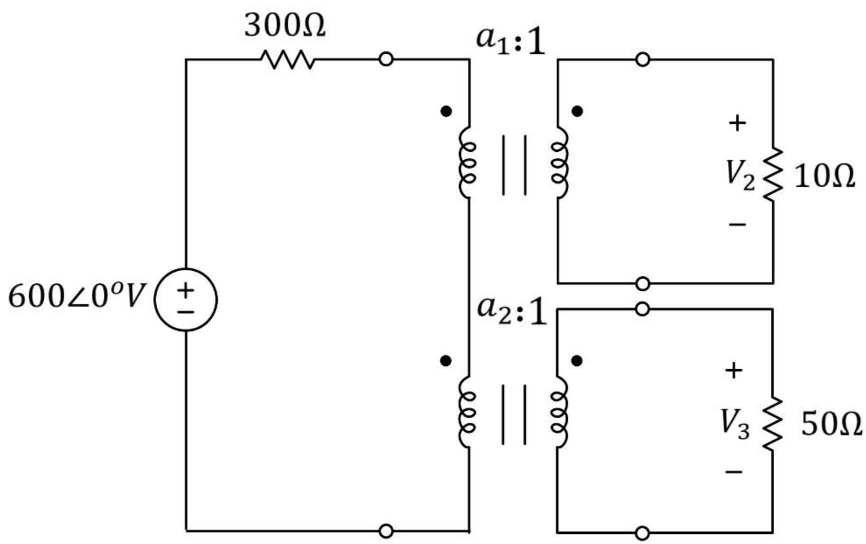
Example (3)

For the circuit shown below, find a_1, and a_2, such that $P_{10\Omega} = 5P_{50\Omega}$ using matching transform. Then, get V_2 and V_3.



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Example (4)

For the circuit shown below, find a_1, and a_2, such that Z_L get maximum power. Then, get V_1 and V_4.