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Investigation of soliton solutions with different wave structures to the (2 + 1)-dimensional Heisenberg ferromagnetic spin chain equation

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Abstract

The principal objective of this article is to construct new and further exact soliton solutions of the (2 + 1)-dimensional Heisenberg ferromagnetic spin chain equation which investigates the nonlinear dynamics of magnets and explains their ordering in ferromagnetic materials. These solutions are exerted via the new extended FAN sub-equation method. We successfully obtain dark, bright, combined bright-dark, combined dark-singular, periodic, periodic singular, and elliptic wave solutions to this equation which are interesting classes of nonlinear excitation presenting spin dynamics in classical and semi-classical continuum Heisenberg systems. 3D figures are illustrated under an appropriate selection of parameters. The applied technique is suitable to be used in gaining the exact solutions of most nonlinear partial/fractional differential equations which appear in complex phenomena.

Keywords: soliton solutions, Heisenberg ferromagnetic equation, FAN sub-equation method

(Some figures may appear in colour only in the online journal)

1. Introduction

Solitons have been widely studied in theory and experiment in recent years. Nowadays, the investigation of the soliton solutions of a number of complex nonlinear equations plays a considerable role due to the expectant effectuation in the real world, especially in different aspects of mathematical and physical phenomena [1–9]. Most complex phenomena arising in applied science, such as nuclear physics, chemical reactions, signal processing, optical fibers, fluid mechanics, plasma, nonlinear optics and ecology, can be sometimes modeled and described by these equations. Hereby, a massive number of mathematicians and physicists have attempted to invent various approaches by which one can obtain the soltion

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solutions of such equations. Among several present methods, we mention the Riccati-Bernoulli sub-ODE method [10, 11], exp-function method [12, 13], sine-cosine method [14, 15], tanh-sech method [16, 17], extended tanh-method [18, 19], F-expansion method [20–22], homogeneous balance method [23, 24], Jacobi elliptic function method [25, 26], the unified method and its generalized form [27–33], and so on. This work is established to utilize the extended Fan Sub-equation technique [34, 35] in determining the soliton and elliptic solutions of the (2+1)-dimensional Heisenberg ferromagnetic spin chain (HFSC) equation [36–40].

The HFSC equation [36–40] is given by:

$$\mathrm{i}\psi_t + \varrho_1\psi_{xx} + \varrho_2\psi_{yy} + \varrho_3\psi_{xy} - \varrho_4|\psi|^2\psi = 0. \tag{1}$$

Here, $\psi = \psi(x, y, t)$ is a complex valued function, x, y and t denote the scaled spatial and time coordinates, respectively and the coefficients ϱ_j for $j = 1, 2, 3, 4$; are real constants given by [7, 39]

$$\varrho_1 = \kappa^4(\Lambda + \Lambda_2),\ \varrho_2 = \kappa^4(\Lambda_1 + \Lambda_2),$$
$$\varrho_3 = 2\kappa^4\Lambda_2,\ \varrho_4 = 2\kappa^4\Omega,$$

where the parameters Λ, Λ_1 represent the coefficients of bilinear exchange interactions in the xy-plane, Λ_2 denotes the neighboring interaction along the diagonal, Ω is the uniaxial crystal field anisotropy parameter, and κ is a lattice parameter.

Heisenberg ferromagnetic spin chain equation with different magnetic interactions in the classical and semi-classical continuum limit have been identified as interesting nonlinear model systems exhibiting integrability properties including soliton spin excitations. This equation can be used to depict the propagation of long waves, which has many applications in the percolation of water.

The rest of this continuing article is methodized as follows: In section 2, we propound the formation of the extended Fan Sub-equation method and we implement this technique to find new soliton and elliptic solutions of the HFSC equation. The physical behavior of the solutions together with their graphical illustration is within section 3. Finally, section 4 is comprised of conclusions in a suitable manner.

2. Mathematical analysis

To solve equation (1), we first need to apply the traveling wave transformation

$$\psi = \mathcal{V}(\xi)\mathrm{e}^{\mathrm{i}\Phi}, \quad \xi = ax + by - \mu t, \quad \Phi = px + qy - rt, \tag{2}$$

where a, b, μ, p, q, and r are constants to be determined.

Utilizing the wave transformation (2) in equation (1), we attain the following imaginary and real parts, respectively:

$$\mu = 2a\varrho_1 p + 2b\varrho_2 q + \varrho_3(bq + aq), \tag{3}$$

$$\delta_1\mathcal{V}'' + \delta_2\mathcal{V}^3 + \delta_3\mathcal{V} = 0, \tag{4}$$

where

$$\delta_1 = \varrho_4 a^2 + \varrho_2 b^2 + \varrho_3 ab,$$
$$\delta_2 = -\varrho_4,$$
$$\delta_3 = r - \varrho_1 p^2 - q(\varrho_2 q + \varrho_3 p).$$

By applying the homogeneous balance to equation (4), we have $n = 1$. Suppose equation (4) has the solution of the form

$$\mathcal{V} = a_0 + a_1\phi(\xi), \tag{5}$$

where ϕ satisfies the following general elliptic equation,

$$\left(\frac{\mathrm{d}\phi(\xi)}{\mathrm{d}\xi}\right)^2 = \zeta_0 + \zeta_1\phi(\xi) + \zeta_2\phi^2(\xi) + \zeta_3\phi^3(\xi) + \zeta_4\phi^4(\xi), \tag{6}$$

ζ_i $(i = 0, 1, 2, 3, 4)$ are real constants.

Substituting (5) along (6) in (4) and collecting the coefficients of $\phi^j\phi^{(k)}$,

$$a_0(a_0^2\delta_2 + \delta_3) + \frac{1}{2}a_1\delta_1\zeta_1 = 0,$$
$$a_1(3a_0^2\delta_2 + \delta_3) + a_1\delta_1\zeta_2 = 0,$$
$$3a_0a_1^2\delta_2 + \frac{3}{2}a_1\delta_1\zeta_3 = 0,$$
$$a_1^3\delta_2 + 2a_1\delta_1\zeta_4 = 0,$$

we select variables suitably, to have the most of ζ_i, $(i = 0, 1, 2, 3, 4)$,

$$\zeta_1 = -\frac{2a_0(a_0^2\delta_2 + \delta_3)}{a_1\delta_1},$$
$$\zeta_2 = -\frac{3a_0^2\delta_2 + \delta_3}{\delta_1},$$
$$\zeta_3 = -\frac{2a_0a_1\delta_2}{\delta_1},$$
$$\zeta_4 = -\frac{a_1^2\delta_2}{2\delta_1},$$

which give

$$a_0 = \frac{\sqrt{\delta_1(-\zeta_2) - \delta_3}}{\sqrt{3}\sqrt{\delta_2}},$$
$$a_1 = \frac{\sqrt{2}\sqrt{-\delta_1}\sqrt{\zeta_4}}{\sqrt{\delta_2}},$$

therefore,

$$\psi = (a_0 + a_1\phi(\xi))\mathrm{e}^{\mathrm{i}\Phi}. \tag{7}$$

We have following solutions, for more details see also [34, 35].

Case I.

If $\zeta_0 = \vartheta_3^2$, $\zeta_1 = 2\vartheta_1\vartheta_3$, $\zeta_2 = 2\vartheta_2\vartheta_3 + \vartheta_1^2$, $\zeta_3 = 2\vartheta_1\vartheta_2$, $\zeta_4 = \vartheta_2^2$, where ϑ_1, ϑ_2, and ϑ_3 are arbitrary constants. The solutions of (1) are ψ_η^I, $(\eta = 1, 2, \ldots, 24)$. Some of important solitons are listed below.

Type I: when $\vartheta_1^2 - 4\vartheta_2\vartheta_3 > 0,\ \vartheta_1\vartheta_2 \neq 0,\ \vartheta_2\vartheta_3 \neq 0$. The following family of dark solitons is obtained as

$$\psi_1^I(\xi) = [a_0 + a_1\left(-\frac{\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\tanh\left(\frac{1}{2}\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\right) + \vartheta_1}{2\vartheta_2}\right)\Bigg]e^{i\Phi}. \quad (8)$$

$$\psi_{20}^I(\xi) = \left[a_0 + a_1\left(-\frac{2\vartheta_3\cos\left(\frac{1}{2}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right)}{\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\sin\left(\frac{1}{2}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right) + \vartheta_1\cos\left(\frac{1}{2}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right)}\right)\right]e^{i\Phi}, \quad (13)$$

The family of combined bright-dark soliton is obtained as,

$$\psi_3^I(\xi) = \Bigg[a_0 - \frac{a_1}{2\vartheta_2}(\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3} \times (i\ \mathrm{sech}\,(\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}) + \tanh(\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3})) + \vartheta_1)]e^{i\Phi}. \quad (9)$$

The family of combined dark-singular solitons is obtained as

$$\psi_5^I(\xi) = \Bigg[a_0 - \frac{a_1}{2\vartheta_2}(\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3} \times \left(\tanh\left(\frac{1}{4}\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\right) + \coth\left(\frac{1}{4}\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\right)\right) + \vartheta_1\Bigg)\Bigg]e^{i\Phi}. \quad (10)$$

The family of solitons is obtained as

$$\psi_{10}^I(\xi) = [a_0 + a_1(2\cosh(\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}) \times (\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}\sinh(\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}) - (\vartheta_1\cosh(\xi\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}) \pm i\sqrt{\vartheta_1^2 - 4\vartheta_2\vartheta_3}))^{-1})]e^{i\Phi}. \quad (11)$$

$$\psi_5^{II}(\xi) = [a_0 + a_1\left(-\frac{\sqrt{-6qr}\left(\tanh\left(\frac{1}{4}\xi\sqrt{-6qr}\right) + \coth\left(\frac{1}{4}\xi\sqrt{-6qr}\right)\right) + 2\sqrt{-2qr}}{4q}\right)\Bigg]e^{i\Phi}, \quad (16)$$

Type II: when $\vartheta_1^2 - 4\vartheta_2\vartheta_3 < 0,\ \vartheta_1\vartheta_2 \neq 0,\ \vartheta_2\vartheta_3 \neq 0$. The following families of periodic solitons are obtained

$$\psi_{13}^I(\xi) = [a_0 + a_1\left(-\frac{\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\tan\left(\frac{1}{2}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right) - \vartheta_1}{2\vartheta_2}\right)\Bigg]e^{i\Phi}, \quad (12)$$

$$\psi_{24}^I(\xi) = \Bigg[a_0 + a_1\Bigg(\left(4r\sin\left(\frac{1}{4}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right) \times \cos\left(\frac{1}{4}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right)\right) \times \left(2\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\cos^2\left(\frac{1}{4}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right) - 2\vartheta_1\sin\left(\frac{1}{4}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right) \times \cos\left(\frac{1}{4}\xi\sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right) - \sqrt{4\vartheta_2\vartheta_3 - \vartheta_1^2}\right)^{-1}\Bigg)\Bigg]e^{i\Phi}. \quad (14)$$

Case II.

If $\zeta_0 = \vartheta_3^2$, $\zeta_1 = 2\vartheta_1\vartheta_3$, $\zeta_2 = 0$, $\zeta_3 = 2\vartheta_1\vartheta_2$, $\zeta_4 = \vartheta_2^2$, the solutions of (1) are ψ_η^{II}, (η = 1, 2,…,12). A family of dark soliton is obtained

$$\psi_1^{II}(\xi) = [a_0 + a_1\left(-\frac{\sqrt{-6\vartheta_2\vartheta_3}\tanh\left(\frac{1}{2}\xi\sqrt{-6\vartheta_2\vartheta_3}\right) + \sqrt{-2\vartheta_2\vartheta_3}}{2\vartheta_2}\right)\Bigg]e^{i\Phi}. \quad (15)$$

Another form of dark-singular soliton is obtained

Case III.

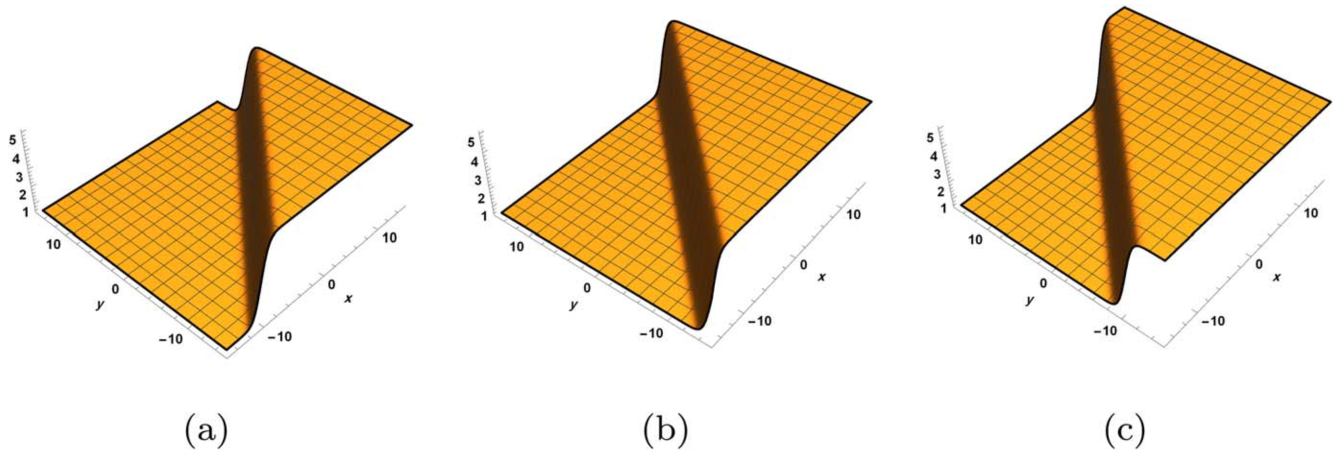

Figure 1. $|\psi_1^I(x, y, t)|$: The complex solitary wave solution when (a) $t = -0.5$ (b) $t = 0$ (c) $t = 0.5$.

If $\zeta_0 = \zeta_1 = 0$, we have the following solution of (1) in the form ψ_η^{III}, $(\eta = 1, 2,\ldots,10)$.

Type I: $\zeta_2 = 1$, $\zeta_3 = \frac{-2\lambda_3}{\lambda_1}$, $\zeta_4 = \frac{\lambda_3^2 - \lambda_2^2}{\lambda_1^2}$, where λ_1, λ_2, λ_3 are arbitrary constants.

$$\psi_1^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{sech}(\xi)}{\lambda_2 \operatorname{sech}(\xi) + \lambda_3}\right)\right]\mathrm{e}^{\mathrm{i}\Phi}. \quad (17)$$

Type II: $\zeta_2 = 1$, $\zeta_3 = \frac{-2\lambda_3}{\lambda_1}$, $\zeta_4 = \frac{\lambda_3^2 + \lambda_2^2}{\lambda_1^2}$, where λ_1, λ_2, λ_3 are arbitrary constants.

$$\psi_2^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{csch}(\xi)}{\lambda_2 \operatorname{csch}(\xi) + \lambda_3}\right)\right]\mathrm{e}^{\mathrm{i}\Phi}. \quad (18)$$

$$\psi_4^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{csch}^2(\xi)}{\lambda_2 \coth(\xi) + \lambda_3 + \lambda_4 \operatorname{csch}^2(\xi)}\right)\right]\mathrm{e}^{\mathrm{i}\Phi}. \quad (22)$$

In particular, if we consider $\lambda_2 = \lambda_4$; another family of dark and singular solitons are obtained as follows

$$\psi_4^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{csch}^2(\xi)}{\lambda_2 \coth(\xi) + \lambda_3 + \lambda_2 \operatorname{csch}^2(\xi)}\right)\right]\mathrm{e}^{\mathrm{i}\Phi}. \quad (23)$$

Type V: $\zeta_2 = -1$, $\zeta_3 = \frac{2\lambda_3}{\lambda_1}$, $\zeta_4 = \frac{\lambda_3^2 - \lambda_2^2}{\lambda_1^2}$, where λ_1, λ_2, λ_3 are arbitrary constants.

$$\psi_6^{III}(\xi) = \left[a_0 + a_1\left(-\frac{\lambda_1(\sinh(\lambda_1\xi) + \cosh(\lambda_1\xi))(\sinh(\lambda_1\xi) + \cosh(\lambda_1\xi) + \lambda_2)}{\lambda_3}\right)\right]\mathrm{e}^{\mathrm{i}\Phi}. \quad (24)$$

In particular, if we take $\lambda_2 = 0$ in above equations (17)–(18). We obtain the families of bright and singular solitons as follows

$$\psi_1^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{sech}(\xi)}{\lambda_3}\right)\right]\mathrm{e}^{\mathrm{i}\Phi}. \quad (19)$$

$$\psi_2^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{csch}(\xi)}{\lambda_3}\right)\right]\mathrm{e}^{\mathrm{i}\Phi}. \quad (20)$$

Type III: $\zeta_2 = 4$, $\zeta_3 = -\frac{4(2\lambda_2 + \lambda_4)}{\lambda_1}$, $\zeta_4 = \frac{4\lambda_2^2 + 4\lambda_4\lambda_2 + \lambda_3^2}{\lambda_1^2}$, where λ_1, λ_2, λ_3, λ_4 are arbitrary constants.

$$\psi_3^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \operatorname{sech}^2(\xi)}{\lambda_2 \tanh(\xi) + \lambda_3 + \lambda_4 \operatorname{sech}^2(\xi)}\right)\right]\mathrm{e}^{\mathrm{i}\Phi}. \quad (21)$$

Type IV: $\zeta_2 = 4$, $\zeta_3 = \frac{4(\lambda_4 - 2\lambda_2)}{\lambda_1}$, $\zeta_4 = \frac{4\lambda_2^2 - 4\lambda_4\lambda_2 + \lambda_3^2}{\lambda_1^2}$, where λ_1, λ_2, λ_3, λ_4 are arbitrary constants.

Type VI: $\zeta_2 = 4$, $\zeta_3 = \frac{-2\lambda_3}{\lambda_1}$, $\zeta_4 = \frac{\lambda_3^2 - \lambda_2^2}{\lambda_1^2}$, where λ_1, λ_2, λ_3 are arbitrary constants.

$$\psi_8^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \csc(\xi)}{\lambda_2 \csc(\xi) + \lambda_3}\right)\right]\mathrm{e}^{\mathrm{i}\Phi}. \quad (25)$$

Type VII: $\zeta_2 = -4$, $\zeta_3 = \frac{4(2\lambda_2 + \lambda_4)}{\lambda_1}$, $\zeta_4 = -\frac{4\lambda_2^2 + 4\lambda_4\lambda_2 - \lambda_3^2}{\lambda_1^2}$, where λ_1, λ_2, λ_3, λ_4 are arbitrary constants.

$$\psi_9^{III}(\xi) = \left[a_0 + a_1\left(\frac{\lambda_1 \sec^2(\xi)}{\lambda_2 \tan(\xi) + \lambda_3 + \lambda_4 \sec^2(\xi)}\right)\right]\mathrm{e}^{\mathrm{i}\Phi}. \quad (26)$$

Case IV.

If $\zeta_1 = \zeta_3 = 0$, we have the following solutions of (1) in the form ψ_η^{IV}, $(\eta = 1, 2,\ldots,16)$ [34, 35].

For $\zeta_0 = \frac{1}{4}$, $\zeta_2 = \frac{1 - 2m^2}{2}$, $\zeta_4 = \frac{1}{4}$, the solution of (1) is of the form

$$\psi_3^{IV}(\xi) = [a_0 + a_1(\operatorname{cn}\xi)]\mathrm{e}^{\mathrm{i}\Phi}, \quad (27)$$

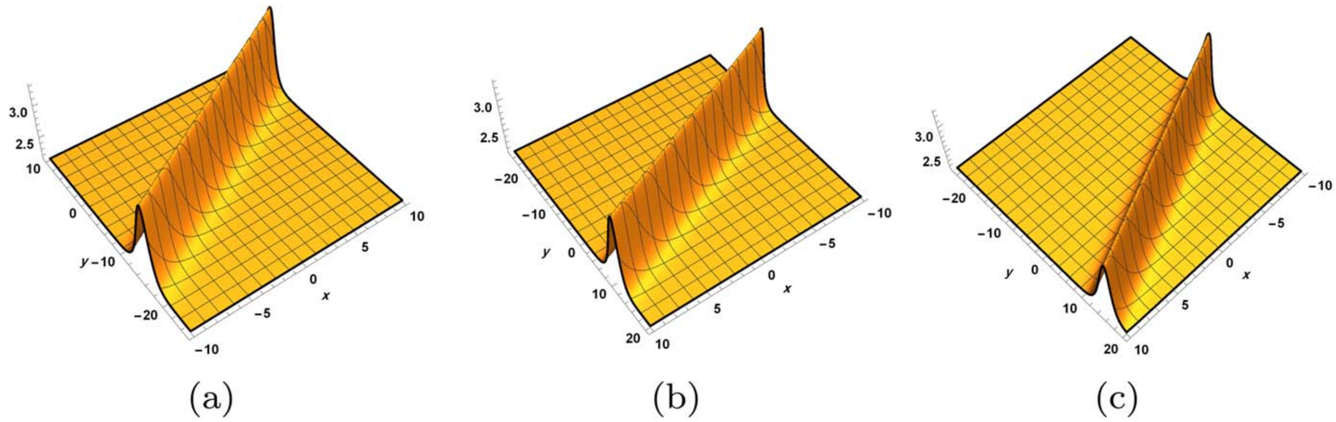

Figure 2. $|\psi_1^{III}(x,\, y,\, t)|$: The complex bright soliton wave solution when (a) $t = -0.5$ (b) $t = 0$ (c) $t = 0.5$.

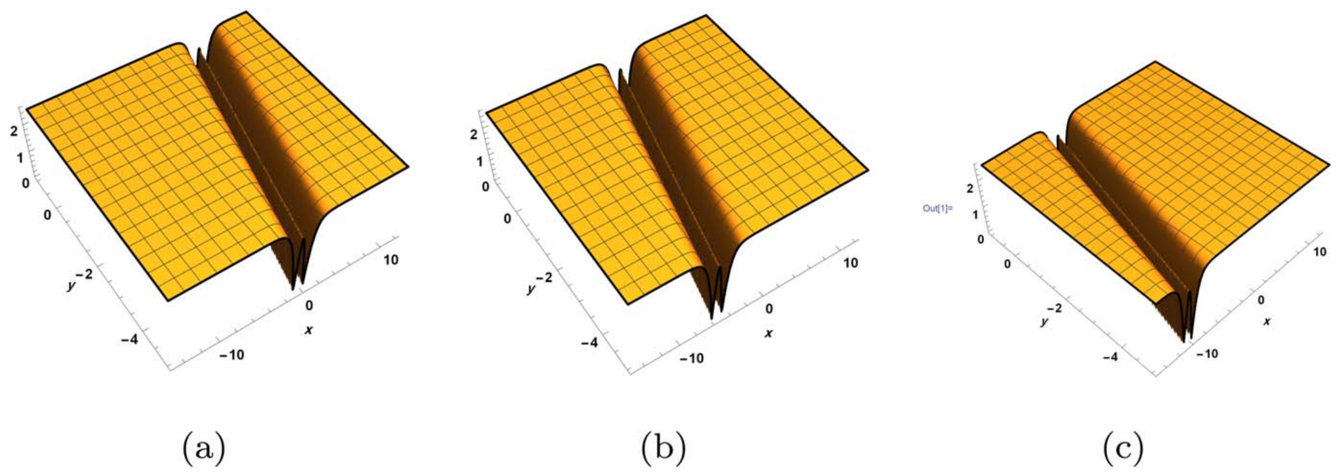

Figure 3. $|\psi_3^{III}(x,\, y,\, t)|$: The complex dark soliton wave solution when (a) $t = -0.5$ (b) $t = 0$ (c) $t = 0.5$.

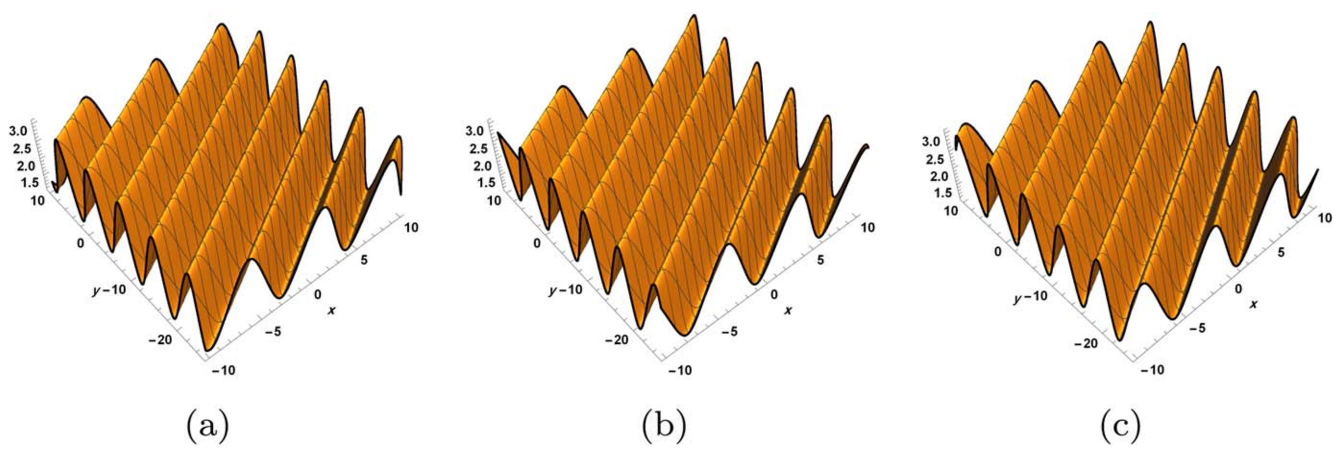

Figure 4. $|\psi_3^{IV}(x,\, y,\, t)|$: The complex elliptic wave solution when (a) $t = -0.5$ (b) $t = 0$ (c) $t = 0.5$.

gives the bright soliton for $m \to 1$,

$$\psi_3^{IV}(\xi) = [a_0 + a_1 \operatorname{sech}(\xi)]\mathrm{e}^{\mathrm{i}\Phi}, \tag{28}$$

and the periodic singular solution for $m \to 0$,

$$\psi_3^{IV}(\xi) = [a_0 + a_1 \cos(\xi)]\mathrm{e}^{\mathrm{i}\Phi}, \tag{29}$$

for $\zeta_0 = \frac{1}{4}$, $\zeta_2 = \frac{1-2m^2}{2}$, $\zeta_4 = \frac{1}{4}$, the solution of (1) is of the form

$$\psi_{13}^{IV}(\xi) = [a_0 + a_1(\mathrm{ns}\xi \pm \mathrm{cs}\xi)]\mathrm{e}^{\mathrm{i}\Phi}, \tag{30}$$

gives the combined dark-singular wave solution for $m \to 1$,

$$\psi_{13}^{IV}(\xi) = [a_0 + a_1(\coth(\xi) + \operatorname{csch}(\xi))]\mathrm{e}^{\mathrm{i}\Phi}, \tag{31}$$

and the periodic singular solution for $m \to 0$,

$$\psi_{13}^{IV}(\xi) = [a_0 + a_1(\cot(\xi) + \csc(\xi))]e^{i\Phi}. \quad (32)$$

3. Physical description

The graphical representation of solitons has been illustrated in the following figures, for various values of the parameters. Mathematica 11 is used to carry out simulations and to visualize the behavior of nonlinear waves observed by the equation (1).

Figures 1(a), (b), and (c) illustrate the 3D chart of the absolute value of $\psi_1^I(x, y, t)$ established in Case I (Type I) when $t = -0.5$, $t = 0$, and $t = 0.5$ respectively. Figure 1 represents complex solitary wave solution with the parameters $\vartheta_1 = 1, \vartheta_2 = -1, \vartheta_3 = 1, \varrho_1 = 1, \varrho_2 = 3, \varrho_3 = 4, \varrho_4 = -1, a = 1, b = -1, p = -2, q = 1$, and $r = -3$.

Figures 2(a), (b), and (c) show the 3D chart of the absolute value of $\psi_1^{III}(x, y, t)$ established in Case III (Type I) when $t = -0.5$, $t = 0$, and $t = 0.5$ respectively. Figure 2 represents complex bright soliton wave solution with the parameters $\lambda_1 = -1$, $\lambda_2 = -1$, $\lambda_3 = -2$, $\varrho_1 = 1$, $\varrho_2 = 3$, $\varrho_3 = 4$, $\varrho_4 = -1$, $a = 1$, $b = -1$, $p = -2$, $q = 1$, and $r = -3$.

Figures 3(a), (b), and (c) show the 3D chart of the absolute value of $\psi_3^{III}(x, y, t)$ established in Case III (Type III) when $t = -0.5$, $t = 0$, and $t = 0.5$ respectively. Figure 3 represents complex dark soliton wave (a 'W ' shape wave) solution with the parameters $\lambda_1 = 1$, $\lambda_2 = -1$, $\lambda_3 = -2$, $\lambda_4 = 1$, $\varrho_1 = 1$, $\varrho_2 = 3$, $\varrho_3 = 4$, $\varrho_4 = -1$, $a = 1$, $b = -1$, $p = -2$, $q = 1$, and $r = -3$.

Figures 4(a), 4(b), and 4(c) show the 3D chart of the absolute value of $\psi_3^{IV}(x, y, t)$ established in Case IV when $t = -0.5$, $t = 0$, and $t = 0.5$ respectively. Figure 4 represents complex elliptic wave solution with the parameters $\lambda_1 = -1$, λ_2=-1, $\lambda_3 = -2$, $\varrho_1 = 1$, $\varrho_2 = 3$, $\varrho_3 = 4$, $\varrho_4 = -1$, $\zeta_0 = \frac{1}{4}$, $\zeta_2 = \frac{1-2m^2}{2}$, $\zeta_4 = \frac{1}{4}$, $m = \frac{1}{3}$, $a = 1$, $b = -1$, $p = -2$, $q = 1$, and $r = -3$.

4. Conclusions

In this study, new soliton and elliptic wave solutions with different wave structures for the Heisenberg ferromagnetic spin chain equation have been constructed via the extended FAN sub-equation method. A set of new exact solutions is found corresponding to various parameters. The graphical representations of the solutions are also demonstrated by figures 1–4, to investigate the behavior of the nonlinear model. Moreover, it is observed that the proposed approach can also be applied to other types of more complex models of contemporary science.

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