AIP Advances

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Cite as: AIP Advances **10**, 045212 (2020); https://doi.org/10.1063/5.0002879
Submitted: 28 January 2020 . Accepted: 15 March 2020 . Published Online: 09 April 2020

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ABSTRACT

This article presents a survey on the exact and numerical solutions of the nonlinear long–short wave interaction system. The system performs an optical domain, which does not alter during multiplication according to a ticklish equipoise between nonlinear and linear influences in elastic surrounding (the medium that can alter the figure due to the existence of a deforming strength and comes back to its original shape in the absence of this force). The wave in this medium is obtained by vibrations that are the outcomes of the acoustic power. The modified auxiliary equation and the quintic B-spline approaches are investigated in our model to obtain a bundle of solutions to discuss new physical behaviors for this model. Moreover, the stability property is discussed for the analytical solutions via the properties related to the Hamiltonian system to show the range of the ability of solutions to be used in the applications of the model. These novel properties are explained by different types of figures. Finally, the convergence and the absolute error between the obtained solutions are discussed in a table.

I. INTRODUCTION

The optical treatise is scrutinized as one of the most notable studies in this era according to its different applications in many fields. Mathematicians have been deriving many approaches to create different classes of solutions that are used to clarify many physical interpretations of the optical soliton waves. This kind of waves comprises of a special type of optical field, which does not alter through multiplication.[1–8] The optical soliton is divided into two types as follows:

1. Spatial solitons: The nonlinear influence can balance the diffraction. The electromagnetic field will adjust the medium's refractive index while it propagates, thus, creating an architecture identical to a graded index fiber.[9–13]
2. Impermanent solitons: If the electromagnetic field is already spatially constrained, pulses that do not change their form can be transmitted as the nonlinear impacts match the dispersion.[14–18]

The interaction of a nonlinear long–short wave paradigm depicts the interaction between one long longitudinal wave and one short transverse wave propagating in a generalized elastic medium. This system comes in the following form:

$$\begin{cases} i\,\Phi_t + \Phi_{xx} - \Phi\,\Theta = 0, \\ \Theta_t + \Theta_x + \left(|\Phi|^2\right)_x = 0, \end{cases} \tag{1.1}$$

where $\Phi(x, t)$ and $\Theta(x, t)$ reflect the slowly changing envelope of the short transverse and the long longitudinal waves, respectively. x is the positional harmonize and t is the time. In plasma, waves are investigated as an interrelated collection of fields and particles, which spread in a periodically duplicating manner. Plasma waves have the EM structure in two forms: electrostatic and electromagnetic, with oscillating species in the form of electrons and ions. Examples for dispersion relation of plasma waves in electrostatic and electromagnetic fields are the following:

- Plasma oscillation: It is a swift oscillation of the electron intensity in propulsion media such as metals or plasmas in the ultraviolet territory.
- Upper hybrid oscillation: It is a condition where magnetized plasma is oscillating.
- Ion acoustic wave: It is one type of longitudinal oscillation of the electrons and ions in plasma.
- Electrostatic ion cyclotron wave: It is a longitudinal sway of the ions in magnetized plasma, almost a portrait of the magnetic field.
- Langmuir wave.
- Lower hybrid oscillation: It is a longitudinal inconstancy of ions and electrons in magnetized plasma.
- Light-wave: This consists of swung magnetic and electrical fields, including radio waves, visible light, microwaves, gamma rays, ultraviolet, x-rays, and infrared.

All these properties and abilities of the nonlinear partial differential equations are used to describe the natural phenomena. Under these investigations, many researchers have developed some approaches to get different solitary traveling wave solutions of these models.[19–32]

The arrangement of this paper is as follows: Sec. II constructed new optical soliton wave solutions and numerical solutions of the nonlinear long–short wave interaction system via the modified auxiliary equation[33–37] and quintic B-spline[38–43] methods, respectively. Furthermore, the stability behavior of the gained analytical solutions is studied through the properties of the Hamiltonian system. Section III gives the conclusion.

II. IMPLEMENTATION AND DISCUSSION

Here, we use the modified auxiliary equation method, which is the most general analytical method and quintic B-spline method as a numerical approach to the nonlinear long–short wave interaction scheme.[44–48] This model is specified by

$$\begin{cases} i\,\Phi_t + \Phi_{xx} - \Phi\,\Theta = 0, \\ \Theta_t + \Theta_x + \left(|\Phi|^2\right)_x = 0. \end{cases} \tag{2.1}$$

Using wave transformation, $\Phi(x, t) = e^{i\,\eta}\,u(\xi)$ and $\Theta(x, t) = v(\xi)$, where $\eta = (\rho\,x + c\,t)$ and $\xi = (a\,x + b\,t)$, we reduce the nonlinear partial differential Eq. (2.1) into a one-dimensional ordinary differential equation

$$\begin{cases} (b + 2\,a\,\rho)\,i\,u - \left(\rho^2 + c\right)u + a^2\,u'' - u\,v = 0, \\ (a + b)\,v' + a\left(u^2\right)' = 0. \end{cases} \tag{2.2}$$

When the complex part equals zero, we get

$$b = -2\,a\,\rho. \tag{2.3}$$

The second equation of the system (2.2) after integration becomes

$$v = \frac{-1}{1 - 2\,\rho}\,u^2, \tag{2.4}$$

where the integration constant is zero for simplicity. Substituting (2.3) and (2.4) in the first equation in the system (2.2) yields

$$a^2\,u'' - \left(\rho^2 + c\right)u + \frac{1}{1 - 2\,\rho}\,u^3 = 0. \tag{2.5}$$

The balance rule gives $n = 1$.

A. Soliton waves

Applying the modified auxiliary equation method to the nonlinear long–short wave interaction system and using the balance value, we get to write the solution of Eq. (2.5) as

$$u(\xi) = a_1 K^{f(\xi)} + a_0 + b_1 K^{-f(\xi)}, \tag{2.6}$$

where a_0, a_1, b_1, and K ($K > 0$, $K \neq 1$) are arbitrary constants, and $f(\xi)$ is the solution function of the following auxiliary equation:

$$f'(\xi) = \frac{\beta + \alpha K^{-f(\xi)} + \sigma K^{f(\xi)}}{\ln(K)}, \tag{2.7}$$

where α, β, and σ are arbitrary constants to be determined later. Substituting Eq. (2.7) and its derivatives in Eq. (2.5) and collecting all terms with the same power give a system of algebraic equations. Solving this system by using any computer software yields

Family I

$$a_0 = -\frac{a\beta\sqrt{2\rho - 1}}{\sqrt{2}},\, a_1 = -\sqrt{2}a\sqrt{2\rho - 1}\sigma,$$
$$b_1 = 0,\, c = \frac{1}{2}\left(4\alpha a^2\sigma - a^2\beta^2 - 2\rho^2\right).$$

Thus, the solitary wave solutions of Eq. (2.1) can be obtained from the following formulas:

When $\beta^2 - 4\alpha\sigma < 0$ & $\sigma \neq 0$,

$$\Phi_1(x,t) = -a\sqrt{\rho - \frac{1}{2}}\sqrt{4\alpha\sigma - \beta^2}\tan\left(\frac{1}{2}\sqrt{4\alpha\sigma - \beta^2}(ax + bt)\right) \times \exp\left(i\rho(x - \rho t) - \frac{1}{2}ia^2 t\left(\beta^2 - 4\alpha\sigma\right)\right), \tag{2.8}$$

$$\Phi_2(x,t) = -a\sqrt{\rho - \frac{1}{2}}\sqrt{4\alpha\sigma - \beta^2}\cot\left(\frac{1}{2}\sqrt{4\alpha\sigma - \beta^2}(ax + bt)\right) \times \exp\left(i\rho(x - \rho t) - \frac{1}{2}ia^2 t\left(\beta^2 - 4\alpha\sigma\right)\right). \tag{2.9}$$

When $\beta^2 - 4\alpha\sigma > 0$ & $\sigma \neq 0$,

$$\Phi_3(x,t) = a\sqrt{\rho - \frac{1}{2}}\sqrt{\beta^2 - 4\alpha\sigma}\tanh\left(\frac{1}{2}\sqrt{\beta^2 - 4\alpha\sigma}(ax+bt)\right) \times \exp\left(i\rho(x-\rho t) - \frac{1}{2}ia^2t\left(\beta^2 - 4\alpha\sigma\right)\right), \tag{2.10}$$

$$\Phi_4(x,t) = a\sqrt{\rho - \frac{1}{2}}\sqrt{\beta^2 - 4\alpha\sigma}\coth\left(\frac{1}{2}\sqrt{\beta^2 - 4\alpha\sigma}(ax+bt)\right) \times \exp\left(i\rho(x-\rho t) - \frac{1}{2}ia^2t\left(\beta^2 - 4\alpha\sigma\right)\right). \tag{2.11}$$

When $\alpha\sigma > 0$ & $\alpha \neq 0$ & $\sigma \neq 0$ & $\beta = 0$,

$$\Phi_5(x,t) = -2a\sqrt{\rho - \frac{1}{2}}\sqrt{\alpha\sigma}e^{i\left(2\alpha a^2\sigma t + \rho(x-\rho t)\right)}\tan\left(\sqrt{\alpha\sigma}(ax+bt)\right), \tag{2.12}$$

$$\Phi_6(x,t) = a\sqrt{4\rho - 2}\sqrt{\alpha\sigma}e^{i\left(2\alpha a^2\sigma t + \rho(x-\rho t)\right)}\cot\left(\sqrt{\alpha\sigma}(ax+bt)\right). \tag{2.13}$$

When $\alpha\sigma < 0$ & $\alpha \neq 0$ & $\sigma \neq 0$ & $\beta = 0$,

$$\Phi_7(x,t) = a\sqrt{4\rho - 2}\sqrt{-\alpha\sigma}e^{i\left(2\alpha a^2\sigma t + \rho(x-\rho t)\right)} \times \tanh\left(\sqrt{-\alpha\sigma}(ax+bt)\right), \tag{2.14}$$

$$\Phi_8(x,t) = a\sqrt{4\rho - 2}\sqrt{-\alpha\sigma}e^{i\left(2\alpha a^2\sigma t + \rho(x-\rho t)\right)} \times \coth\left(\sqrt{-\alpha\sigma}(ax+bt)\right). \tag{2.15}$$

When $\beta = 0$ & $\alpha = -\sigma$,

$$\Phi_9(x,t) = a\alpha\sqrt{4\rho - 2}e^{-i\left(2a^2\alpha^2 t + \rho(\rho t - x)\right)}\coth(\alpha(ax+bt)). \tag{2.16}$$

When $\beta = \sigma = \kappa$ & $\alpha = 0$,

$$\Phi_{10}(x,t) = \frac{a\kappa\sqrt{\rho - \frac{1}{2}}\left(e^{\kappa(ax+bt)} + 1\right)e^{-\frac{1}{2}i\left(a^2\kappa^2 t + 2\rho(\rho t - x)\right)}}{e^{\kappa(ax+bt)} - 1}. \tag{2.17}$$

When $\alpha = 0$ & $\beta \neq 0$ & $\sigma \neq 0$,

$$\Phi_{11}(x,t) = \frac{a\beta\sqrt{\rho - \frac{1}{2}}e^{-\frac{1}{2}i\left(a^2\beta^2 t + 2\rho(\rho t - x)\right)}\left(\sigma e^{\beta(ax+bt)} + 2\right)}{\sigma e^{\beta(ax+bt)} - 2}. \tag{2.18}$$

When $\beta = 0$ & $\alpha = \sigma$,

$$\Phi_{12}(x,t) = -2a\alpha\sqrt{\rho - \frac{1}{2}}e^{i\left(2a^2\alpha^2 t + \rho(x-\rho t)\right)}\tan(a\alpha x + \alpha bt + C). \tag{2.19}$$

When $\beta^2 - 4\alpha\sigma = 0$,

$$\Phi_{13}(x,t) = \frac{8a\alpha\sqrt{\rho - \frac{1}{2}}\sigma e^{i\rho(x-\rho t)}}{\beta^2(ax+bt)}. \tag{2.20}$$

Family II

$$a_0 = -\frac{a\beta\sqrt{2\rho - 1}}{\sqrt{2}}, a_1 = 0, b_1 = -\sqrt{2}a\alpha\sqrt{2\rho - 1},$$
$$c = \frac{1}{2}\left(4\alpha a^2\sigma - a^2\beta^2 - 2\rho^2\right).$$

Thus, the solitary wave solutions of Eq. (2.1) can be obtained from the following formulas:

When $\beta^2 - 4\alpha\sigma < 0$ & $\sigma \neq 0$,

$$\Phi_{14}(x,t) = a\sqrt{\rho - \frac{1}{2}} \times \left(\frac{4\alpha\sigma}{\beta - \sqrt{4\alpha\sigma - \beta^2}\tan\left(\frac{1}{2}\sqrt{4\alpha\sigma - \beta^2}(ax+bt)\right)} - \beta\right) \times \exp\left(i\rho(x-\rho t) - \frac{1}{2}ia^2t\left(\beta^2 - 4\alpha\sigma\right)\right), \tag{2.21}$$

$$\Phi_{15}(x,t) = a\sqrt{\rho - \frac{1}{2}} \times \left(\frac{4\alpha\sigma}{\beta - \sqrt{4\alpha\sigma - \beta^2}\cot\left(\frac{1}{2}\sqrt{4\alpha\sigma - \beta^2}(ax+bt)\right)} - \beta\right) \times \exp\left(i\rho(x-\rho t) - \frac{1}{2}ia^2t\left(\beta^2 - 4\alpha\sigma\right)\right). \tag{2.22}$$

When $\beta^2 - 4\alpha\sigma > 0$ & $\sigma \neq 0$,

$$\Phi_{16}(x,t) = a\sqrt{\rho - \frac{1}{2}} \times \left(\frac{4\alpha\sigma}{\sqrt{\beta^2 - 4\alpha\sigma}\tanh\left(\frac{1}{2}\sqrt{\beta^2 - 4\alpha\sigma}(ax+bt)\right) + \beta} - \beta\right) \times \exp\left(i\rho(x-\rho t) - \frac{1}{2}ia^2t\left(\beta^2 - 4\alpha\sigma\right)\right), \tag{2.23}$$

$$\Phi_{17}(x,t) = a\sqrt{\rho - \frac{1}{2}} \times \left(\frac{4\alpha\sigma}{\sqrt{\beta^2 - 4\alpha\sigma}\coth\left(\frac{1}{2}\sqrt{\beta^2 - 4\alpha\sigma}(ax+bt)\right) + \beta} - \beta\right) \times \exp\left(i\rho(x-\rho t) - \frac{1}{2}ia^2t\left(\beta^2 - 4\alpha\sigma\right)\right). \tag{2.24}$$

When $\alpha\sigma > 0$ & $\alpha \neq 0$ & $\sigma \neq 0$ & $\beta = 0$,

$$\Phi_{18}(x,t) = -2a\sqrt{\rho - \frac{1}{2}}\sqrt{\alpha\sigma}e^{i\left(2\alpha a^2\sigma t + \rho(x-\rho t)\right)}\cot\left(\sqrt{\alpha\sigma}(ax+bt)\right), \tag{2.25}$$

$$\Phi_{19}(x,t) = a\sqrt{4\rho - 2}\sqrt{\alpha\sigma}e^{i\left(2\alpha a^2\sigma t + \rho(x-\rho t)\right)}\tan\left(\sqrt{\alpha\sigma}(ax+bt)\right). \tag{2.26}$$

When $\alpha\sigma < 0$ & $\alpha \neq 0$ & $\sigma \neq 0$ & $\beta = 0$,

$$\Phi_{20}(x,t) = -2a\sqrt{\rho - \frac{1}{2}}\sqrt{-\alpha\sigma}e^{i\left(2\alpha a^2\sigma t+\rho(x-\rho t)\right)} \times \coth\left(\sqrt{-\alpha\sigma}(ax+bt)\right), \tag{2.27}$$

$$\Phi_{21}(x,t) = -2a\sqrt{\rho - \frac{1}{2}}\sqrt{-\alpha\sigma}e^{i\left(2\alpha a^2\sigma t+\rho(x-\rho t)\right)} \times \tanh\left(\sqrt{-\alpha\sigma}(ax+bt)\right). \tag{2.28}$$

When $\beta = 0$ & $\alpha = -\sigma$,

$$\Phi_{22}(x,t) = -a\alpha\sqrt{4\rho - 2}e^{-i\left(2a^2\alpha^2 t+\rho(\rho t-x)\right)}\tanh(\alpha(ax+bt)). \tag{2.29}$$

When $\beta = \frac{\alpha}{2} = \kappa$ & $\sigma = 0$,

$$\Phi_{23}(x,t) = -\frac{a\kappa\sqrt{\rho - \frac{1}{2}}\left(e^{\kappa(ax+bt)}+2\right)e^{-\frac{1}{2}i\left(a^2\kappa^2 t+2\rho(\rho t-x)\right)}}{e^{\kappa(ax+bt)}-2}. \tag{2.30}$$

When $\beta = \sigma = 0$ & $\alpha \neq 0$,

$$\Phi_{24}(x,t) = -\frac{2a\sqrt{\rho - \frac{1}{2}}e^{i\rho(x-\rho t)}}{ax+bt}. \tag{2.31}$$

When $\beta = 0$ & $\alpha = \sigma$,

$$\Phi_{25}(x,t) = -2a\alpha\sqrt{\rho - \frac{1}{2}}e^{i\left(2a^2\alpha^2 t+\rho(x-\rho t)\right)}\cot(a\alpha x + \alpha bt + C). \tag{2.32}$$

When $\sigma = 0$ & $\beta \neq 0$ & $\alpha \neq 0$,

$$\Phi_{26}(x,t) = -\frac{a\beta\sqrt{\rho - \frac{1}{2}}e^{-\frac{1}{2}i\left(a^2\beta^2 t+2\rho(\rho t-x)\right)}\left(\beta e^{\beta(ax+bt)}+\alpha\right)}{\beta e^{\beta(ax+bt)}-\alpha}. \tag{2.33}$$

When $\beta^2 - 4\alpha\sigma = 0$,

$$\Phi_{27}(x,t) = -\frac{a\beta\sqrt{4\rho - 2}e^{i\rho(x-\rho t)}}{a\beta x + b\beta t + 2}. \tag{2.34}$$

Figures 1–3 depict the 3D and 2D charts of the absolute, real, and imaginary parts of $\Phi_i(x, t)$, when $i = 3, 4, 10$.

B. Stability property

Here, we examine the stability property of the nonlinear long–short wave interaction system via a Hamiltonian system. Consider the formula of the Hamiltonian system as

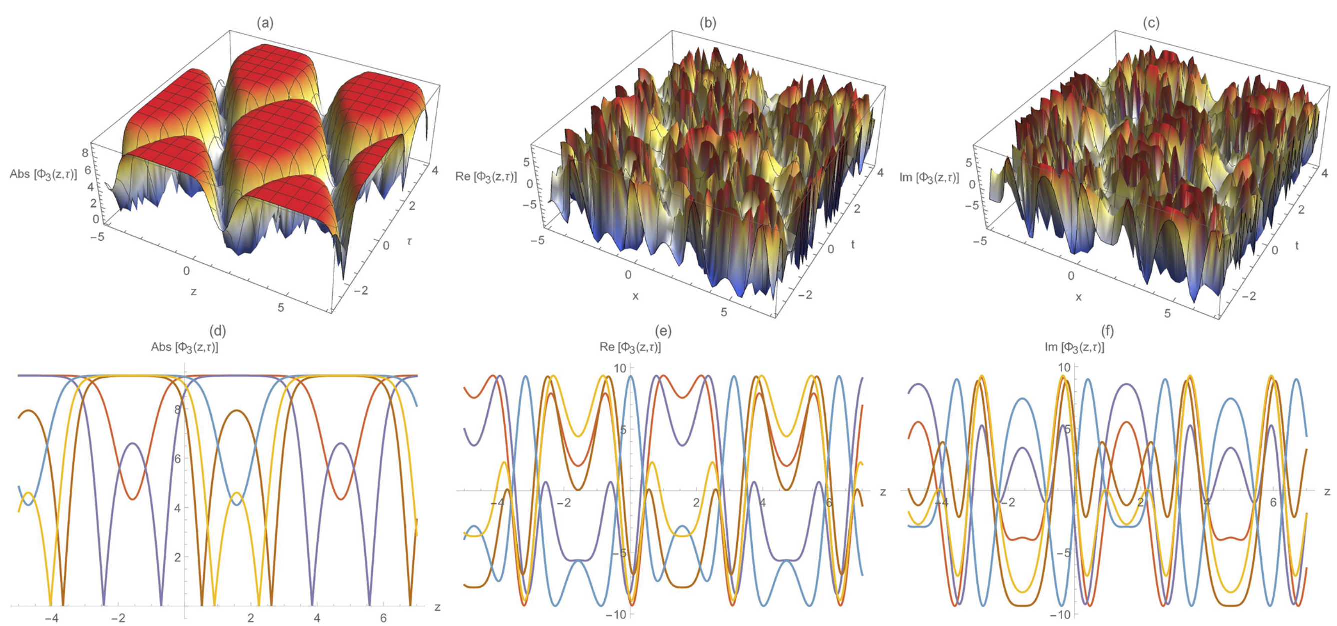

FIG. 1. [(a) and (d)] Absolute, [(b) and (e)] real, and [(c) and (f)] imaginary plots in two- and three-dimensional sketches of Eq. (2.10), respectively, when $b = 6$, $\alpha = 2$, $a = 5$, $\beta = 3$, $\rho = 4$, and $\sigma = 1$.

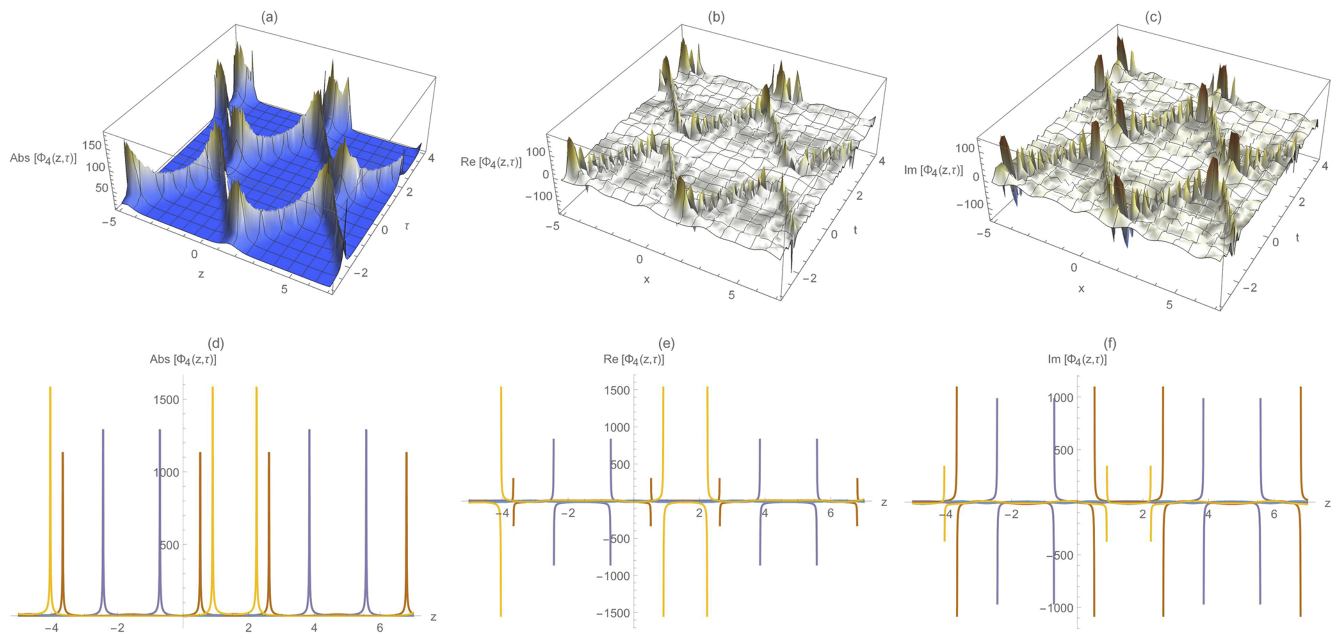

FIG. 2. [(a) and (d)] Absolute, [(b) and (e)] real, and [(c) and (f)] imaginary plots in two- and three-dimensional sketches of Eq. (2.11), respectively, when $b = 6$, $\alpha = 2$, $a = 5$, $\beta = 3$, $\rho = 4$, and $\sigma = 1$.

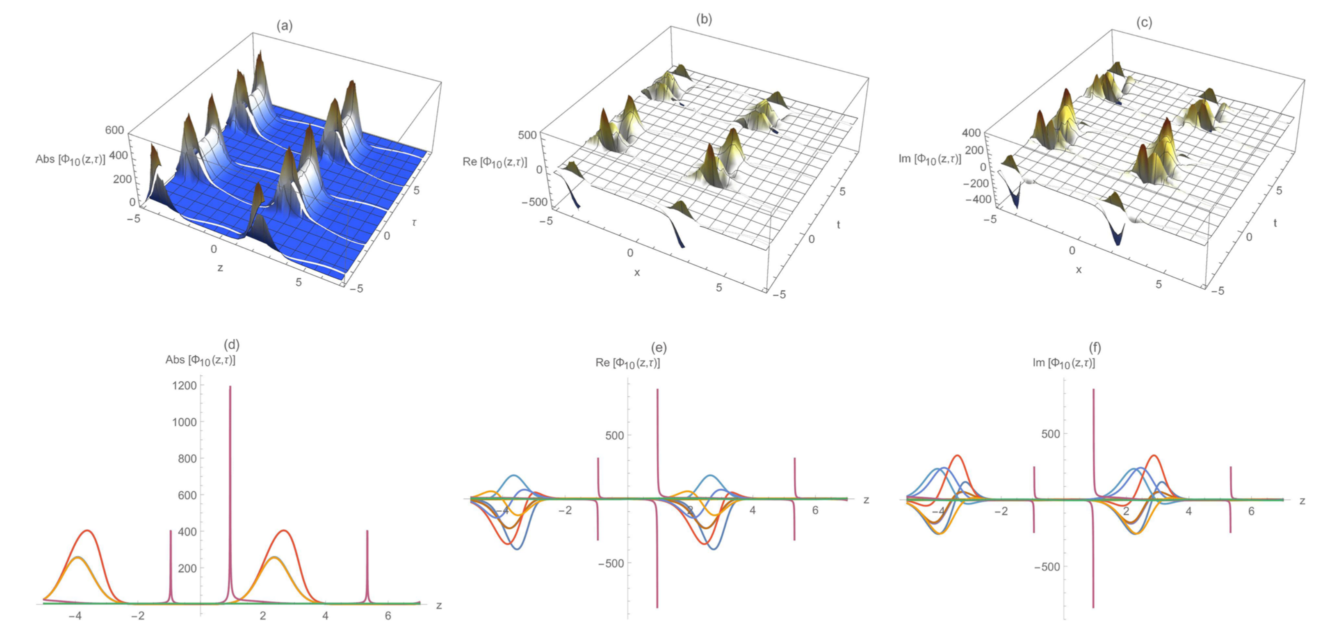

FIG. 3. [(a) and (d)] Absolute, [(b) and (e)] real, and [(c) and (f)] imaginary plots in three-dimensional sketches of Eq. (2.17), respectively, when $a = 1$, $b = 4$, $\kappa = 3$, and $\rho = 2$.

$$M = \frac{1}{2}\int_{-\backepsilon}^{\backepsilon}\Phi^2(\xi)d\xi, \tag{2.35}$$

where $\backepsilon$ is a constant. The condition for stability exists when

$$\left.\frac{\partial M}{\partial c}\right|_{b=\mathfrak{I}} > 0, \tag{2.36}$$

where b, $\mathfrak{I}$ are constants.

For an example of studying the stability of the solution of Eq. (2.5) by using (2.10) with the following values of the constants $\left[\alpha = 2,\ a = 5,\ \beta = 3,\ \rho = 4.5,\ \sigma = 1\right]$ yields

$$M = \frac{200.b + 3.2\log(1.\cosh(2.5\,b) - 1.\sinh(2.5\,b)) - 3.2\log(1.\sinh(2.5\,b) + 1.\cosh(2.5\,b))}{b}. \tag{2.37}$$

Thus, we obtain

$$\left.\frac{\partial M}{\partial b}\right|_{b=6} = 2.203\,77 > 0. \tag{2.38}$$

This means that this solution has a stable property. We can apply the previous steps to the other gained solutions.

C. Numerical simulation

Applying the quintic B-spline method to Eq. (2.5) yields the numerical solution by the following formula:

$$u(\xi) = \sum_{i=-1}^{n+1} c_i\, B_i, \tag{2.39}$$

where c_i and B_i satisfy the following conditions, respectively:

$$L\,u(\xi) = f(x_i, u(\xi_i)) \texttt{ where } (i = 0, 1, \ldots, n) \tag{2.40}$$

and

$$B_i(\xi) = \frac{1}{h^5}\begin{cases} (\xi - \xi_{i-3})^5, & \xi \in [\xi_{i-3}, \xi_{i-2}],\\ (\xi - \xi_{i-3})^5 - 6(\xi - \xi_{i-2})^5, & \xi \in [\xi_{i-2}, \xi_{i-1}],\\ (\xi - \xi_{i-3})^5 - 6(\xi - \xi_{i-2})^5 + 15(\xi - \xi_{i-1})^5, & \xi \in [\xi_{i-1}, \xi_i],\\ (\xi_{i+3} - \xi)^5 - 6(\xi_{i+2} - \xi)^5 + 15(\xi_{i+1} - \xi)^5, & \xi \in [\xi_i, \xi_{i+1}],\\ (\xi_{i+3} - \xi)^5 - 6(\xi_{i+2} - \xi)^5, & \xi \in [\xi_{i+1}, \xi_{i+2}],\\ (\xi_{i+3} - \xi)^5, & \xi \in [\xi_{i+2}, \xi_{i+3}],\\ 0, & \text{otherwise,} \end{cases} \tag{2.41}$$

where $i \in [-2, n+2]$. Thus, we obtain

$$u_i(\xi) = c_{i-2} + 26\,c_{i-1} + 66\,c_i + 26\,c_{i+1} + c_{i+2}. \tag{2.42}$$

Substituting Eq. (2.42) and its derivatives in Eq. (2.5) yields $n + 5$ of equations. Solving these equations, we get the values of c_i as shown in Table I.

TABLE I. Analytical, numerical, and absolute errors by using a quintic spline scheme in Eq. (2.5).

Value of ξ	Value of analytical	Value of numerical	Value of absolute error
0.000	0.000 000 0	$4.336\,81 \times 10^{-19}$	$4.336\,81 \times 10^{-19}$
0.010	0.009 999 9	0.009 999 9	$3.840\,11 \times 10^{-9}$
0.020	0.019 999 3	0.019 999 3	$9.679\,18 \times 10^{-9}$
0.030	0.029 997 8	0.029 997 8	$1.487\,11 \times 10^{-8}$
0.040	0.039 994 7	0.039 994 7	$1.987\,65 \times 10^{-8}$
0.050	0.049 989 6	0.049 989 6	$2.405\,67 \times 10^{-8}$
0.060	0.059 982 0	0.059 982 0	$2.679\,86 \times 10^{-8}$
0.070	0.069 971 4	0.069 971 5	$2.680\,21 \times 10^{-8}$
0.080	0.079 957 4	0.079 957 4	$2.338\,47 \times 10^{-8}$
0.090	0.089 939 3	0.089 939 3	$1.155\,88 \times 10^{-8}$
0.100	0.099 916 7	0.099 916 7	0

III. CONCLUSION

This research successfully applied the modified auxiliary equation and quintic B-spline methods on the nonlinear long–short wave interaction system for obtaining analytical and numerical solutions of this system, respectively. Moreover, the stability property is discussed for the obtained analytical solutions by using the Hamiltonian system. The physical properties are explained by sketching of some obtained analytical solutions. Moreover, the comparison between the distinct obtained types of solutions is given by calculating the absolute error between the accuracy and power of the used methods in this research paper. The acquired results demonstrate that the recommended analytical method with the numerical approach is accurate, efficient, and versatile in mathematical physics to solve other NLEEs. In addition, the reported results are valuable for the practical use of the plasma waves in electrostatic and electromagnetic fields.

ACKNOWLEDGMENTS

This work was funded by the Basic Science Research Unit, Scientific Research Deanship at Majmaah University, Project No. 1439-20. The authors are extremely grateful to the Deanship of Scientific Research and Basic Science Research Unit, Majmaah University, and the China Scholarship Council (CSC) (No. 201908320301).

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