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IDENTIFICATION OF NATURAL FREQUENCIES AND DAMPING RATIOS: STOCK PRICE SYSTEM COMPARISON

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Abstract

Dynamic data system (DDS) approach is applied to modeling and identification of time series as stochastic dynamic systems in particular stock price data. $\text{ARMA}(n, n-1)$ models are identified by natural frequencies and damping ratios, which are used to compare company performance assessed through stability of stock prices in two different time periods. Characteristic roots of the $\text{ARMA}(n, n-1)$ models are computed together with their corresponding natural frequencies and damping ratios. Dominant roots (DRs) for the autoregressive models are determined and their contributions to variance components and power are computed. The impulse response function (IRF) is used as a memory of the system to describe its dynamic characteristic. Fourier transforms are used to find the autospectrum which is a representation of the variance distribution in the frequency domain (FD). Future behavior of stock prices is related to past values through head-step forecasting.

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1. Introduction

One of the goals of time series analysis is to develop suitable models and obtain accurate predictions for measurements over time of various kinds of phenomena. In statistics and econometrics, and in particular in time series analysis, an autoregressive integrated moving average (ARIMA) model is a generalization of an autoregressive moving average (ARMA) model. These models are fitted to time series data either to better understand the data or to predict future points in the series forecasting. Traditionally, the statistical dependence in the time series data is expressed by the correlation or autocorrelation between successive observations. Therefore, the existing methods of time series analysis are based on empirical or estimated autocorrelation or Fourier transform autospectrum. This fact makes the techniques of time series analysis based on such estimates difficult and cumbersome requiring heavy reliance on trial and error procedures. Usually, statisticians and economists approximate their models using difference equations from plots of empirical autocorrelations and autospectrum.

The modern dynamic data system (DDS) approach brings together time series and system analyses in a quantitative analysis that avoids the considerable trial and error procedures presently needed in both fields which vastly improve their applications. Time series is treated as a realization of the response of a stochastic dynamic system to uncorrelated or independent "white noise" input. The mathematical model for the dynamic system can be perceived as a system with uncorrelated "white noise" input and correlated stochastic-process or time series output. The purpose of DDS is to introduce $\mathrm{ARMA}(n,\ n-1)$ model in the form of stochastic linear difference equations to express this dependence. The basic characteristics of $\mathrm{ARMA}(n,\ n-1)$ models are impulse response function (IRF), also called *Green's function* and referred to in the literature as weighting function [3, 22, 27]. This function summarizes the dependence (correlation) and plays the essential role in modeling and system analysis which can express conditions of stability of the autoregressive AR(n) model. We will use the name IRF or Green's function to emphasize the basic theory of difference equations, [20] and [21].

Natural frequencies and damping ratios are essential parameters required to comparing the behavior of different systems or for the same system in different time periods to evaluate performance through stability. Conventionally, the identification of the natural frequencies and damping ratios has been done by estimation of non-parametric frequency responses, and curve fitting procedure using a linear or non-linear least square method. In this paper, the DDS approach using autoregressive moving average $\mathrm{ARMA}(n, n-1)$ models are applied in order to overcome the estimation problems associated with fast Fourier transform (FFT)-based methods and to identify the model parameters directly from the fitted models. The DDS approach is applied to an analysis of non-experimental data obtained from a stock price system. It is explained how an $\mathrm{ARMA}(n, n-1)$ model of a discrete system in the form of difference equations could be used to identify the natural frequencies and damping ratios of the stock price.

In this study, the DDS approach was applied as one of the modern approaches to the time series analysis on the stock price data for Mobinil Telecommunication Company, Egypt in two different periods according to the following steps: Modeling the time series of the stock price data in the form of stochastic dynamic models of autoregressive moving averages $\mathrm{ARMA}(n, n-1)$, where the appropriate model was chosen to represent the data and to find characteristic roots. Identification of $\mathrm{ARMA}(n, n-1)$ models by calculating the natural frequencies and damping ratios for each autoregressive $\mathrm{AR}(n)$ models characteristic roots to determine the dominant roots (DR) have more effects on system response. The extent to which the dominant roots contribute to the variance components is determined by calculating the power and the sharings of these dominant roots in the total variance of the model. The impulse response function IRF was used as a representation of the essential dynamic characteristic and the memory of the system to investigate the stability, by calculating the weights for all real and complex conjugate roots for $\mathrm{ARMA}(n, n-1)$ models. The impact of disturbances on the system return to the equilibrium state quickly has been checked. The parametric spectral analysis was implemented directly from

$\text{ARMA}(n, n-1)$ to find the autospectrum. This requires taking Fourier transform of the autocovariance function which was obtained from the double convolution of IRF in the time domain. Autospectrum describes how the total variance distributed through all frequency bandwidths in the frequency domain (FD). The conditional expectation and probability limits forecasting technique were used to predict the future behavior of stock price. The main concern was the analysis of a single series of data or univariate time series.

2. Dynamic Data System (DDS)

The DDS simplifies the two-dimensional $\text{ARMA}(n, m)$ to one-dimensional $\text{ARMA}(n, n-1)$. Pandit and Wu [21] pointed out that each stable random sequence can be approximated with the required precision with an $\text{ARMA}(n, n-1)$ model where n is large enough. Accordingly, to this, the determination of the dimensions can be dealt with along the linear direction. The DDS approach uses non-linear least square techniques to fit a series of difference/differential equations to the observed data until “statistically adequate” approximation is reached. The data was reduced to an uncorrelated residual series of white noise impulses that are free of any dynamics by the autoregressive moving average ARMA model function. The function achieving the reduction is called the *impulse response function*; Pandit and Wu [21]. The model itself contains the dynamics of the system and can be analyzed to find these dynamics and the corresponding properties of the system. No prior knowledge of the system is necessary for modeling it. DDS in the form of $\text{ARMA}(n, n-1)$ models shows that the Green’s function or IRF, and the autocovariance function can be represented by a linear combination of exponentials and damped sinusoidal related to the physical phenomena generating the signal. Pandit and Wu [21] found that Green’s function lends itself to more fundamental interpretation and analysis. It can reduce the memory of a system to a single random disturbance. This method has been used to help understand the system dynamics of a variety of systems [23, 24, 28]. The DDS approach has been applied in many different fields in the industrial and management systems, engineering area, business, operations research, quality control, human factors, and ergonomics, for more details, see [1, 4-9, 11-20, 24-26, 28-30].

3. DDS Application for Stock Prices Data

The non-experimental data represented in two different periods of Mobinil Telecommunication Company stock prices data (see Appendix for sample data) called *period* (1) and *period* (2) were analyzed using ARMA models. The objective was to evaluate the company's performance through stability of stock prices due to the nature of the characteristic roots and the dominant roots have more effects on the system response than other roots. Therefore, the next step in the analysis was to develop the time series function to capture the differences observed in a plot of the raw data. Time series data for two different data sets comprising period (1) and period (2) were modeled with DDS approach. For the statistically adequate $\mathrm{ARMA}(n,\ n-1)$ models with all unified autocorrelations (#UAC) within the ± 1.96 possible band, the residuals a_t's can be approximately taken as independent confirmed by *F*-test criteria. A FORTRAN program was specially written for this purpose. Table 1 shows $\mathrm{ARMA}(n,\ n-1)$ modeling for two different periods, $\mathrm{ARMA}(8,\ 7)$ and $\mathrm{ARMA}(10,\ 9)$ for the period (1) and period (2), respectively, are given by:

$$y_t = k_1 y_{t-1} + k_2 y_{t-2} + \cdots + k_8 y_{t-8} + a_t - h_1 a_{t-1} - h_2 a_{t-2} - \cdots - h_7 a_{t-7}, \tag{1}$$

$$y_t = k_1 y_{t-1} + k_2 y_{t-2} + \cdots + k_{10} y_{t-10} + a_t - h_1 a_{t-1} - h_2 a_{t-2} - \cdots - h_9 a_{t-9}, \tag{2}$$

where the stock price at the time *t* is y_t and y_{t-i} is the past value of stock price at *i* time intervals before t, k_i is the autoregressive parameter and h_i is the moving average parameter, $i = 1, 2, 3, \ldots$. The random disturbance or noise at time t is given by a_t. The random disturbances a_t's are normally and independently distributed with mean zero and variance σ_a^2, $a_t \sim NID(0, \sigma_a^2)$.

Table 1. ARMA$(n, n-1)$ modeling for Mobinil Telecommunication stock prices

Parameters	**Period (1)**	**Period (2)**
	ARMA(8, 7) **model**	ARMA(10, 9) **model**
k_1	0.69874	0.82175
k_2	0.33427	–0.11793
k_3	0.57997	0.09450
k_4	–0.88072	0.42775
k_5	–0.26245	–0.19569
k_6	0.03194	–0.15886
k_7	0.83698	0.41280
k_8	–0.35675	–0.83338
k_9		0.37477
k_{10}		0.13673
h_1	–0.04377	–0.03527
h_2	0.01622	–0.23895
h_3	0.72350	–0.23326
h_4	–0.43291	0.35181
h_5	–0.53793	0.07798
h_6	–0.45818	0.04316
h_7	0.47717	0.37280
h_8		–0.80129
h_9		–0.19689
μ	**122.20026 ± 56.32124**	**170.66957 ± 20.61416**
Variance	**19.512300**	**8.461760**
RSS	**4761.001093**	**2090.054709**

Figures 1 and 2 show the plot of ARMA$(8, 7)$ and ARMA$(10, 9)$ for period (1) and period (2), respectively.

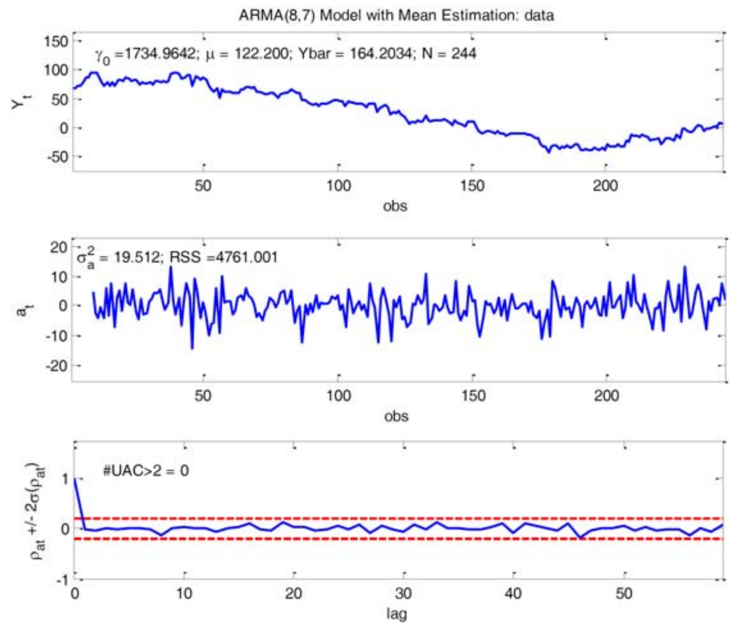

Figure 1. ARMA$(8, 7)$ model for the period (1).

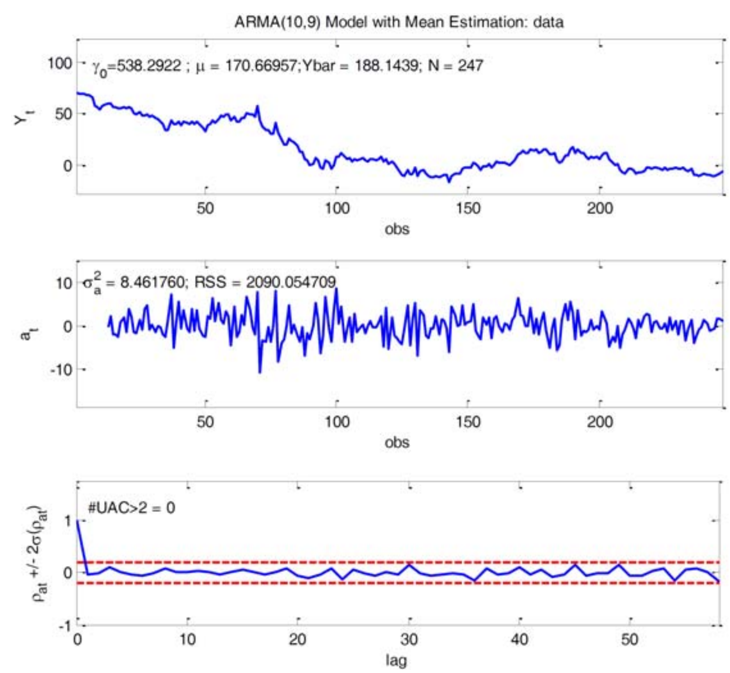

Figure 2. ARMA$(10, 9)$ model for the period (2).

Tables 2 and 3 show the characteristic roots of the autoregressive part of Mobinil Telecommunication stock prices data with their natural frequencies ω_n and damping ratios ξ used to determine the dominant roots (DRs) which cause more effects in the system response through describing the variance contributions d_i (which represent the contributions to the variance γ_0 by the discrete roots λ_i's). The power of roots (which represents the percentage of the variance in the data attributed to the roots), and sharing this roots in total variance of $\text{ARMA}(n, n-1)$ models for two different periods (1) and (2), respectively, we will discuss in a later part.

Table 2. Model characteristics of autoregressive operator for the period (1)

ARMA(8, 7) Model						
Discrete complex roots		Natural frequency (HZ) ω_n	Damping ratio ξ	Variance component d_i	Power	Share of variance
REAL	IMAG					
0.4864		0.1147	———	0.4513729	1.496317	0.2919613
0.7672 ± 0.5082		0.0940	0.1406	0.1736874	0.5757798	0.1123461
–0.3633 ± 0.9317		0.3092	0.0000	0.045389667	0.1504683	0.029359378
–0.7919 ± 0.4994		0.4106	0.0256	0.08978590	4 0.2976434	0.058076177
0.9881		**0.0019**	———	**29.40536**	**97.47979**	**19.02026**

From Table 2, the real discrete autoregressive root takes bold expression **(0.9881)** with lower natural frequencies $\omega_n = (\mathbf{0.0019})$ with undamping ratios and high variance component $d_i = \mathbf{29.40536},$ has the major power percentage **(97.47979%)** and sharing in the total variance **(19.02026)** of $\text{ARMA}(8, 7)$ model, this root called the *dominant root* (*DR*) on the system response than other roots. We can arrange the discrete roots corresponding to *lower natural frequencies* ω_n and *high damping ratio* ξ with respect to real and complex conjugate roots as (0.4864), (0.7672 ± 0.5082), (–0.3633 ± 0.9317), (–0.7919 ± 0.4994). These roots have high variance component, power, and sharing in the total variance as shown in Table 2 above for stock prices for $\text{ARMA}(8, 7)$ model for a period (1).

Table 3. Model characteristics of autoregressive operator for the period (2)

ARMA(10, 9) model						
Discrete complex roots		Natural frequency (HZ) ω_n	Damping ratio ξ	Variance component d_i	Power	Share of variance
REAL	IMAG					
–0.8933 ± 0.3766		0.4365	0.0113	0.3163202	1.286600	0.1088690
–0.2280 ± 0.9512		0.2875	0.0122	0.092244618	0.3751954	0.031748135
0.3482 ± 0.8789		0.1902	0.0470	0.4995376	2.031817	0.1719275
–0.2314		0.5000	——	0.014255964	0.057984661	0.0049065226
0.8105 ± 0.3076		0.0620	0.3663	0.6554567	2.666002	0.2255907
0.9783		**0.0035**	——	**23.00793**	**93.58240**	**7.918717**

Similarly, from Table 3, the real discrete autoregressive root takes bold expression **(0.9783)** with lower natural frequencies $\omega_n = (\mathbf{0.0035})$ with undamping ratios of high variance component $d_i = 23.00793,$ the major power percentage (93.58240 %) and sharing of the total variance (7.918717) of ARMA(10, 9) model, this root called the dominant root DR on the system response than other roots. We can arrange the discrete roots corresponding to *lower natural frequencies* ω_n and *high damping ratio* ξ with respect to real and complex conjugate roots as (0.8105 ± 0.3076), (0.3482 ± 0.8789), (–0.2280 ± 0.9512), (–0.8933 ± 0.3766), and (–0.2314). These roots have high variance component, power, and sharing in total variance as shown in Table 3 above for stock prices for ARMA(10, 9) model for a period (2).

Green's function acts as an impulse response function for random impulses of the $\mathrm{ARMA}(n, n-1)$ models in the form of a linear combination of exponentials and damped sinusoidal can be related the physical phenomenon of the behavior of the Mobinil Telecommunication Company stock price. The complete Green's function for the non-experiment data associated with stock prices data for the period (1) is given by:

$$G(j) = \begin{bmatrix} 0.21940 * (0.48641) * J + 0.13767 * (0.92030) \\ ** J * COS[2 * P1 * 0.09312 * DELTA * J + (2.01668)] \\ + 0.04045 * (1.00001) ** J * COS[2 * P1 * 0.30917 \\ * DELTA * J + (2.33124)] + 0.06273 * (0.93616) \\ ** J * COS[2 * P1 * 0.41045 * DELTA * J + (0.87128)] \\ + 0.82746 * (\mathbf{0.9881}) * * J, \text{ for } j \geq 0 \end{bmatrix}. \quad (3)$$

It is clear from the above function that the dominant root (DR) contribution comes from the last bolded expression (**0.9881**) given in Table 2 for a period (1), hence Green's function can more accurately represent mathematically the dynamic characteristic of stock prices system. These features are clearly reflected in Green's function plot represented in Figure 3-A.

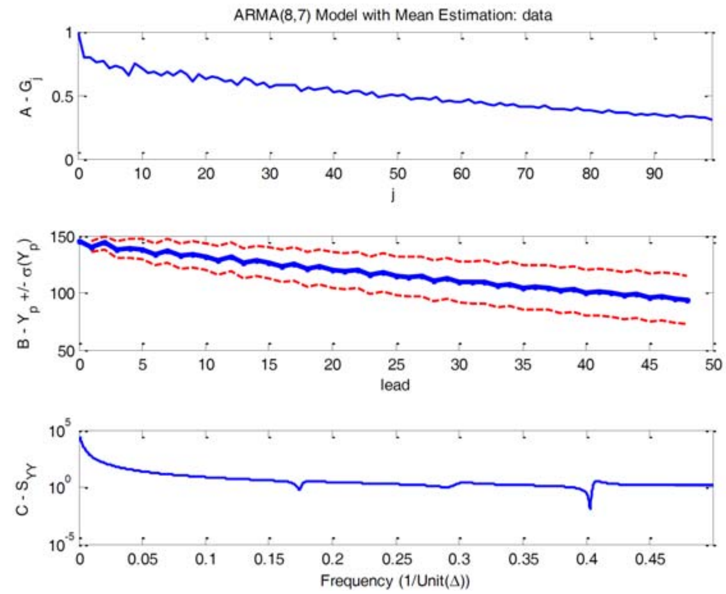

Figure 3. Decomposition $\text{ARMA}(8,\ 7)$ model.

Notice that in the case of the period (2), there are two dominant terms in Green's function given by the following equation:

$$G(j) = \begin{bmatrix} 0.03723 * (\mathbf{0.96941}) ** J * COS[2 * P1 * 0.43650 * DELTA \\ * J + (-2.34187)] + 0.04776 * (0.97814) ** J * COS[2 * P1 \\ * 0.28745 * DELTA * J + (-3.12299)] + 0.19205 * (0.94540) \\ ** J * COS[2 * P1 * 0.18997 * DELTA * J + (2.04310)] \\ + 0.00955 * (-0.23141) * * J + 0.37539 * (0.86694) \\ ** J * COS[2 * P1 * 0.05773 * DELTA * J + (1.08123)] \\ + 0.97500 * (\mathbf{0.9783}) ** J, \text{ for } j \geq 0 \end{bmatrix}. \quad (4)$$

There are significant characteristic roots, one is real with an exponential decay has the highest contribution due to the expression $(\mathbf{0.9783})^j$, and second is complex conjugate (–0.8933 ± 0.3766) term or an oscillatory term with absolute value **(0.96941)** has a low damping coefficient $\xi = 0.0113$, see Table 3 at the natural frequency $\omega_n = 0.4365$. This low damping coefficient implies that the oscillatory term has narrow frequency band repeats at the frequency over a time period. These features were clearly reflected in Green's function plot represented in Figure 4-A.

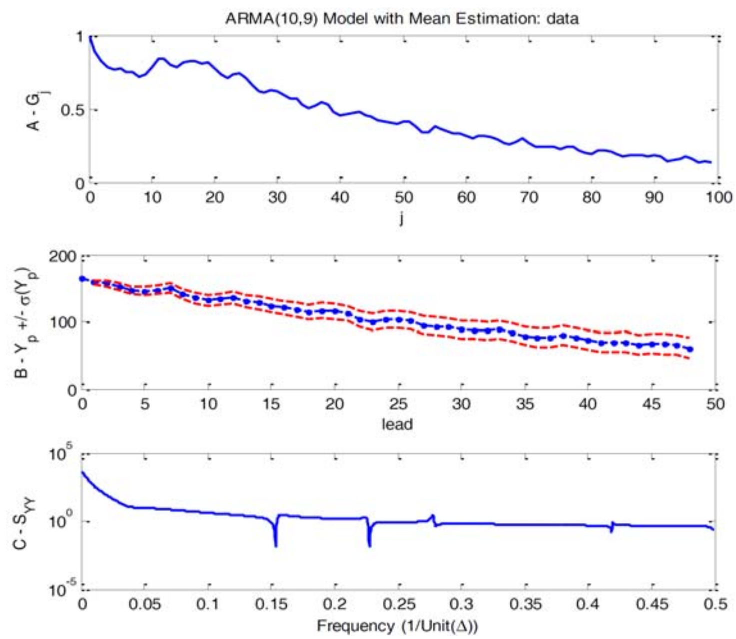

Figure 4. Decomposition ARMA(10, 9) model.

The time series is an ordered set of data of a certain variable. A characteristic of time series model is the use of the information on the variable to be forecasted which in this case is Mobinil Telecommunication Company stock price. The main objective of the time series analysis is to establish how the future behavior of variables is related to their past values [11]. The time series analysis is based on finding the mathematical model for a stochastic dynamic system in the form of ARMA$(n, n-1)$ models, the variable is treated as the response of a stochastic dynamic system to uncorrelated disturbances or independent input. From Figures 3-B and 4-B, the middle line is the predicted value; the upper and lower lines are the 95% confidence interval. We can know that the predicted value has smooth and closed from the limits of the confidence interval.

It is clear from Figures 3-B and 4-B that all the prediction values fall within the forecast period for ahead steps forecasting, as that not widen in prediction period, [2, 3, 23]. This refers that $\text{ARMA}(8, 7)$ and $\text{ARMA}(10, 9)$ models are appropriated to predict for stock price accurately. The forecasting functions appear with high values of predictions declining gradually. The prediction within the sample also appears close to the prediction values. By using data dependent systems, the results arrived at the suitable models which should be fitted for the stock price data.

Figures 3-C and 4-C present the plot of the autospectrum of $\text{ARMA}(n, n-1)$ models for stock prices data of two different periods (1) and (2). Clearly, the autospectrum indicates the dominant of the low-frequency roots with a narrow spread and peaks at a lower frequency. The autospectrum falls off rapidly indicating the high damping coefficient of the dominant roots and also due to the negligible contribution from the high-frequency roots in the data.

4. Discussion

The Green's function as the main characteristic of a dynamic system can be interpreted in two ways. The first is indicator of how well the shocks a_{t-j} are remembered by the system j time units back for $j = 0, 1, 2, \ldots, t$. The second is how quickly the system will return to its equilibrium. The plot of the impulse response or Green's function G_j for the $\text{ARMA}(8, 7)$ and $\text{ARMA}(10, 9)$ models of the stock price system is shown in Figures 3-B and 4-B. Note that the figures show the value $G_0 = 1$ and trend of $G_j \to 0$ when $j \to \infty$. Figure 3-B shows Mobinil Telecommunication Company stock price, the rate of impulse response which has a slow decay and it exhibits a strong memory behavior, so the Green's function appears to be "asymptotically stable". Figure 4-B demonstrates that the system in a period (2) returns to balance rapidly after transient oscillation; hence Mobinil Telecommunication company stock price system is stable.

By definition of Green's function, if the curve approaches a finite value, then it is called *stable*, wherefore if the value of Green's function happens to be zero, then it is asymptotically stable [1, 7, 16]. In fact, it can be shown that the closing value of characteristic roots to unity (stability boundary), the Green's function more closely represents the stability trends overall in the data. The real roots represent the decay in the process in terms of exponentials, and imaginary roots represent the oscillatory in terms of damped sinusoidal for the data as a function of time. We note in the period (1) a system with a much longer memory (longer time to decay to zero), this means that a single random disturbance entered into the system will be remembered for a long time, and hence less stability as compared to a period (2) (shorter time to decay to zero) and hence more stability this is consistent with Green's function plot for the period (2) appear to be smoother in shape and decay to zero faster than period (1).

Frequency decomposition and autospectrum analysis did not reveal some differences between the two periods (1) and (2). The stock price data for period (1) and period (2) are characterized by a high damping ratio and low natural frequency that appears as a large share in Mobinil Telecommunication company market. Its long-standing prestige and reputation are taken into account for the high damping ratio showing that the disturbance in its stock prices does not have a lasting effect. It damped quickly. ARMA model clearly captures the driving frequency of the stock prices data for periods (1) and (2) as dominant roots which account for the largest component of variance d_i (area under the autospectrum) in the process, and also the smooth curve associated with the stock prices data period (1) and period (2) in terms of a single dominant low frequency root. This result is consistent with the fact that there are no other significant frequencies appearing to be present or superposed on the stock prices data for a period (1) and period (2). A real root corresponds to non-repetitive feature and a complex conjugate root will have a frequency associated with it.

Pandit and Wiener [19, 27] recall that the high damping ratio of the roots indicates that this frequency dies of rapidly in data (typically, a damping ratio of more than 10% is considered to be large). The damping coefficients are presented in Tables 2 and 3 corresponding to the spread of the specific frequency of a particular root in the frequency domain. Small damping coefficients correspond to a narrow frequency band whereas a high damping coefficient yields a broad frequency associated with the root in the frequency domain (FD).

5. Conclusion

Dynamic data system (DDS) approach was effective in modeling and the identification for non-experimental data for Mobinil Telecommunication Company, Egypt stock price in the form of autoregressive moving average $\mathrm{ARMA}(n,\ n-1)$. Modeling results indicated that $\mathrm{ARMA}(n,\ n-1)$ models for two different time periods were adequate with all lags of unified autocorrelation (#UAC) within the ±1.96 possible band. It should be noted that the DDS approach estimates the values of the characteristic roots directly instead of using a differencing operation before modeling. We noted that there is the reciprocal relation between natural frequencies and damping ratios, hence the roots with lower natural frequencies have undamping ratios (if real) and high damping ratios (if pairs of complex conjugate) and vice versa. The results indicated that for Mobinil Telecommunication stock price data, the high damping ratio and low natural frequencies corresponding to the dominant roots (DRs) have a high percentage of variance component, power, and sharing in the total variance of $\mathrm{ARMA}(n,\ n-1)$ models. This is true for the two different periods studied. The results also showed that the stock price system in the second period was more stable than the first. This is clear from Green's function or IRF damping to zero quickly; indicating the ineffectiveness of disturbances in stock price system in a second period; meaning that the system has a short memory thus became more stable.

The conditional expectation and probability limits technique for head-steps forecasting were used to analyze the trend of Mobinil Telecommunication Company stock price. The trend of the stock price is central to the understanding of marketing behavior. The fluctuation in stock prices has a direct effect on the increasing marketing of Mobinil Telecommunication Company shares. The results of the forecast indicated future stock price stability for the two different periods of the Mobinil Telecommunication Company, but the second period was characterized by a narrow forecasting confidence interval compared to a wide confidence interval in the first period. Both forecasting functions for two different periods were within the upper and lower limits of the confidence interval, so the forecasting in permissible limits. The results indicate that in the near future, by implementing the models arrived at in this paper, there will not be any increase in the level of significance in Mobinil Telecommunication stock price. The parametric spectral analysis for autospectrum with smooth curve does not appear any significant differences between the two different periods in the variance distribution through frequencies bandwidth, thus the peaks at low frequencies correspond to the high variance. The comparison between the two different periods shows that the performance of the company was good in both periods, but it is better in the second period. This is reflected in sales of the Mobinil Telecommunication Company, Egypt shares. The DDS approach not only provides a precise representation of the data but also makes it possible to evaluate the contribution of the stock price as a component in the development of Mobinil Telecommunication Company stock pricing marketing.

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Appendix

A general type of $\text{ARMA}(n, n-1)$ dependence is a model of nth order in an autoregressive part and $(n-1)$th order in moving average part in the form of difference equation given by:

$$y_t - k_1 y_{t-1} - k_2 t_{t-2} - \cdots - k_n y_{t-n} = a_t - h_1 a_{t-1} - h_2 a_{t-2} - \cdots - h_{n-1} a_{t-n+1}, \tag{1}$$

where k's is called *autoregressive parameters*, h's is called the *moving average parameter*, and a_t's are the residual deviations with probability normal distribution with zero mean and dispersion of σ_a^2, $a_t \sim NID(0, \sigma_a^2)$.

The Green's function G_j or impulse response function (IRF) summarizes the dependence (correlation) and plays the essentials role in the system modeling and analyses which can express conditions of stability of models. To express it more simply, we can introduce a backshift operator B in general mode defined as:

$$B \cdot y_t = y_{t-1} \quad \text{or} \quad B^j \cdot y_t = y_{t-j}. \tag{2}$$

Using the general $\text{ARMA}(n,\ n-1)$ model form, from equation (1) gets the form:

$$(1 - k_1 B - k_2 B^2 - \cdots - k_n B^n)\, y_t = (1 - h_1 B - h_2 B^2 - \cdots - h_{n-1} B^{n-1})\, a_t. \quad (3)$$

The Greens' function G_j of difference equation (3) can be used to express values of y_t as a linear combination of stochastic values of a_t as:

$$y_t = \sum_{j=0}^{\infty} G_j a_{t-j} = \left[\sum_{j=-\infty}^{t} G_{t-j} B^j\right] a_j$$

$$= G_0 a_t + G_1 a_{t-1} + G_2 a_{t-2} + \cdots + G_j a_{t-j}. \quad (4)$$

The Green's function shows the weight given in the present response of y_t to the disturbance a_{t-j}, which entered the system *j* time units back.

Equation (4) is called *Wold's decomposition*, it can be used to derive all statistical properties of the time series which gives the decomposition of y_t into an infinite number of orthogonal variables $G_j a_{t-j}$.

The condition of stability of $\text{ARMA}(n,\ n-1)$ model is generally in the form $|\lambda_k| < 1,$ for $k = 1, 2, \ldots, n,$ where λ_k are roots of the characteristic equation on the left-hand side of equation (1) in form of:

$$\lambda^n - k_1 \lambda^{n-1} - k_2 \lambda^{n-2} - \cdots - k^n = 0. \quad (5)$$

The Green's function can be obtained as a solution of the homogeneous difference equation with the initial condition. The solution of an *n*th order difference equation is the linear combination of the exponential function:

$$G_j = u_1 \lambda_1^j + u_2 \lambda_2^j + \cdots + u_n \lambda_n^j, \quad (6)$$

where λ's are the characteristic roots, and u_i are the weights corresponding each the characteristic roots given by:

$$u_i = \frac{(\lambda_i^{n-1} - h_1 \lambda_i^{n-2} - \cdots - h_{n-1})}{(\lambda_i - \lambda_1)(\lambda_i - \lambda_2)\cdots(\lambda_i - \lambda_{i-1})(\lambda_i - \lambda_{i+1})\cdots(\lambda_i - \lambda_n)}, \quad i = 1, 2, \ldots, n. \quad (7)$$

Each real root λ_i in equation (6) provides an exponentially decaying sinusoidal mode with frequency and decay rates. The u_i terms simply scale the magnitude of the response from the *i*th mode and can also introduce a phase shift when that mode is sinusoidal. To better clarify the role of complex conjugate pairs of roots each λ_i, λ_i^* and associated u_i, u_i^* can be expressed in the form:

$$u_i\lambda_i^j + u_i^*\lambda_i^{j*} = 2|u_i||\lambda_i^j|\cos(\omega_i j + \beta_i), \tag{8}$$

where the damped frequency ω_i and phase shift β_i come from the root λ_i and the corresponding scaling factor u_i, respectively, Pandit and Wu [21]. The damped frequency can further be expressed in terms of the damping ratio ξ and natural frequency ω_n as:

$$\omega_i = \omega_n\sqrt{1-\xi^2} = \cos^{-1}\frac{Re(\lambda_i)}{|\lambda_i|}, \tag{9}$$

where the damped angular frequency ω_i has expressed an angle per sampling interval. Thus, the dynamic pattern of the data is completely captured by Green's function with each real root or a complex conjugate pair of roots contributing a signal mode. After fitting the model, the variance (is often proportional to the total amount of power) can be written in terms of the roots as:

$$\gamma_0 = d_1 + d_2 + \cdots + d_n, \tag{10}$$

where d_i represents the contribution to γ_0 by λ_i. The variance component d_i can be found from:

$$d_i = \sum_{j=1}^{n}\frac{u_i u_j}{1-\lambda_i\lambda_j}, \quad i = 1, 2, \ldots, n. \tag{11}$$

Thus, the power of the roots represents the percentage of the variance in the data attributed to the root.

Fourier transform of the autocovariance function is known as autospectrum since it shows how the variance of the process $y(t)$ is distributed over the frequency bands [10]. By taking the Fourier transform of the autocovariance function, we can show that the autospectrum of an $\mathrm{ARMA}(n,\ n-1)$ process is given by:

$$S_Y(\omega) = \frac{\Delta\sigma_a^2}{2\pi} \frac{|\, e^{mi\omega\Delta} - h_1 e^{(m-1)i\omega\Delta} - \cdots - h_m\,|^2}{|\, e^{ni\omega\Delta} - h_1 e^{(n-1)i\omega\Delta} - \cdots - h_n\,|^2}, \quad -\frac{\pi}{\Delta} \leq \omega \leq \frac{\pi}{\Delta}, \quad (12)$$

where Δ is the sampling interval and ω is the angular frequency in radians per unit time. The natural frequency ω_n and damping ratios ξ are very basic parameters describing the characteristics of system identification and can be calculated from the parametric relation between continuous $y(t)$ (which sampled with a sampling interval Δ), and discrete y_t models. These characteristics of the dynamic system can be used to compare the behavior of different systems or for the same system in different periods given by:

$$\omega_n = \frac{1}{\Delta}\left[\sqrt{\frac{[\ln(\lambda_i\lambda_i^*)]^2}{4} + \left[\cos^{-1}\left(\frac{\lambda_i + \lambda_i^*}{\sqrt{2\lambda_i\lambda_i^*}}\right)\right]^2}\right], \quad (13)$$

$$\zeta = \sqrt{\frac{[\ln(\lambda_i\lambda_i^*)]^2}{[\ln(\lambda_i\lambda_i^*)]^2 + 4\left[\cos\left(\frac{\lambda_i + \lambda_i^*}{\sqrt{2\lambda_i\lambda_i^*}}\right)\right]^2}}. \quad (14)$$

One of the principal aims of system modeling or time series modeling is prediction or forecasting. It is often called *extrapolation* since it involves extrapolating the value $Y_{t+\ell}\ell$ steps ahead from the knowledge of the series Y_t, and its structure. A procedure for obtaining the forecasts recursively, together with their probability limits, is also given. For any ARMA model, the orthogonal decomposition of $Y_{t+\ell}$, which is to be predicted at time *t*, is given by:

$$Y_{t+\ell} = a_{t+\ell} + G_1 a_{t+\ell-1} + G_2 a_{t+\ell-2} + \cdots. \tag{15}$$

Therefore, the best linear forecast of based on $Y_t, Y_{t-1}Y_{t-2}, \ldots$ or equivalently, $a_{t-j},\ j = 0, 1, 2, 3, \ldots$ is the part of the decomposition and the remaining part is naturally the prediction error. Thus,

$$Y_{t+\ell} = \underbrace{a_{t+\ell} + G_1 a_{t+\ell-1} + G_2 a_{t+\ell-2}}_{\substack{e_t(\ell) \\ \text{Error}}} + \cdots + \underbrace{G_\ell a_t + G_{\ell+1} a_{t-1} + G_{\ell+2} a_{t-2} + \cdots}_{\substack{\hat{Y}_t(\ell) \\ \text{Forecast}}}. \tag{16}$$

These results can be generalized for an arbitrary $\text{ARMA}(n, m)$ model by equation

$$\begin{aligned} Y_{t+\ell} &= \hat{Y}_t(\ell) + e_t(\ell) \\ &= \hat{Y}_t(\ell) + (a_{t+\ell} + G_1 a_{t+\ell-1} + G_2 a_{t+\ell-2} + \cdots + G_{\ell-1} a_{t+1}) \end{aligned} \tag{17}$$

and therefore the conditional distribution of $Y_{t+\ell}$ given $Y_t, Y_{t-1}, Y_{t-2}, \ldots$ is specified by:

$$\begin{aligned} (Y_{t+\ell} | Y_t, Y_{t-1}, Y_{t-2}, \ldots) &\sim ND(\hat{Y}_t(\ell), V[e_t(\ell)) \\ &\sim ND(\hat{Y}_t(\ell), \sigma_a^2(1 + G_1^2 + G_2^2 + \cdots + G_{\ell-1}^2))]. \end{aligned} \tag{18}$$

The 95% probability limits on the forecasts are given by:

$$\hat{Y}_t(\ell) \pm 1.96\sigma_a(1 + G_1^2 + G_2^2 + \cdots + G_{\ell-1}^2)^{\frac{1}{2}}. \tag{19}$$

Here, $\hat{Y}_t(\ell)$ can be computed by using the rules for conditional expectation given by equation. For an arbitrary ARMA model, the ℓ step ahead forecast error is:

$$e_t(\ell) = a_{t+\ell} + G_1 a_{t+\ell-1} + G_2 a_{t+\ell-2} + \cdots + G_{\ell-1} a_{t+1}. \tag{20}$$

Then $\hat{Y}_t(\ell)$ is the best prediction and $e_t(\ell)$ is the prediction error with minimum variance. The conditional expectation is the same as orthogonal projection, Pandit and Wu [21].

Table 1. Mobinil Telecommunication stock prices (daily) January 2 through December 31, 2008 (period 1)

206.94	225.13	230.16	197.06	197	183.25	165.55	149.75	127.5	108	110	109	140.49
206.96	222.5	221.96	197.56	199.99	183.05	162	146	127.95	106.95	106.01	112.75	140.15
209.15	217.89	225.04	198	204.9	174.25	166.74	143.1	123.11	102.05	105	118.99	146.97
209.59	219.72	229.5	204	200	182.88	160	150	124.99	105	106.01	120	145
213.83	219.89	228.5	204	198	182	156	146.75	128.79	104	110	117	
224.08	212.13	210.01	207	197.99	181	150.66	144.03	127	107	105.07	116.01	
223.63	215.59	225.99	208	185	180	145	142.02	126.99	104	106	125	
232.37	215.03	225.01	206.58	185.11	180.06	148	140.09	128.11	105	116.01	119.5	
232.65	213.65	220	207	180.8	179	147	149	127	109	114.51	136	
232.67	215.81	224.9	200	180	176.93	150.01	149	127.55	109	126.01	136.01	
223.94	213.06	220	198.9	177	173	149.5	148.89	126	100	124	132	
217.12	217.85	209.98	197.03	179.98	180	149	140	126.95	100	124.11	130.15	
209.7	218.25	204	195	176.16	180	159.4	132	123.06	101	121	133	
215.45	216.91	200.25	196.01	180	180.01	149.84	128	120	100.05	125	138.7	
210.75	218.3	205	197	179	165.12	147.9	130	120.5	100	124.45	142.55	
216.06	215.84	190	196.99	179.99	172.1	150	131.75	109	103	122.5	141.01	
210.64	218.55	204.7	193.31	181	175.8	150	133	104.98	101.4	116	132.06	
219.48	230.79	199	190.05	183	178	151	130	106	99.99	117.1	136.11	
220	231.65	199.99	195	185	174	150.05	128	95.5	104	117	136.2	
218.15	232.11	199.99	197.06	184.88	160	152	132	105	100.05	119	139.11	

Observations read (column-wise)

Table 2. Mobinil Telecommunication stock prices (daily) January 3 through December 30, 2010 (period 2)

241.3	227.6	210	217.99	191.6	180	175.07	162.98	174.9	182	181.39	169.4	162.9
240.21	226.25	214	214	198	183.8	172.8	163	174.03	182	183	169	162.9
241	222.21	213	213.51	195	179.5	170.01	156.05	174	183.05	183	168	161.55
241	221	212	217	194.5	178	166.05	161.7	174	177.1	177.82	167	161.01
240.02	222.89	214	217	190.05	178.62	162.06	164	173.05	174.5	173.99	169.04	162
239.12	219.99	211.6	222	184.61	175.5	161.11	163.99	172.01	175.58	172.06	167.02	163.94
237	220	213.98	221.4	185	177.9	167.02	163.63	174.01	180.9	173	168.26	165.48
229	221	212	220.3	178	176.99	163.07	167.94	177	181.39	167.5	168	
227.89	218.55	208	219	176.12	177.5	164	169.32	181.97	187.45	165	169	
226.04	219.97	205	228.99	172.04	178.48	169.31	169.1	183.96	189	164	167.94	
229	217	209.9	214.9	173	176.34	161.25	170	185.5	183.11	167.88	168	
230.02	215.98	212	209.7	179	175	166	174.49	184.03	186.97	168.5	169.89	
231	213.97	214.99	206.07	177.03	177.49	166.94	173	183.55	184	167.74	166.75	
231	211.5	212.2	206	170.01	177.36	164	170.75	187	183	164	166	
228	206.25	216.61	202	168.25	177	161.5	167.08	185.9	179.99	165	168	
227.97	206	219.98	201.9	177	178	160.51	170.99	186.5	178	165	163.01	
226.11	214.99	219.01	212.3	174	179.99	160	171.1	184	179.88	166	162	
227.12	211.3	221	203	168	175.1	161.9	172.99	185	178.71	169	161.7	
228	211.5	221	196.2	172	178	160.12	170	187.5	180.01	170	162.01	
226.97	213.49	215	191.71	180	172.75	161.9	172	186.94	178	168.6	163.6	

Observations read (column-wise)