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RESEARCH ARTICLE

A Double Layer Arterial Wall Mathematical Model Subjected To Pulsating Blood Pressure

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Abstract

The behavior of a double layer arterial transversal section is investigated under the effect of radial pulsating pressure. The model considers that the radial pressure follows an exponential rise exponential decay biphasic periodic function, mathematically represented by a Fourier series, simulating the blood pressure on its inner wall. The dependence of the arterial strain response on viscoelastic properties of each layer is represented. The dissipated power and strain energy rate are computed. The stress-strain response is modeled for each layer. Furthermore, the storage and loss moduli are deduced for different set of viscoelastic parameters.

Nomenclature

l : section length
E_1, E_2: elasticity modulus of inner (intima media) and outer (adventitia) layers
R_{01} , R_{02} inner and outer radii
K_1\`, $K_2$\`: spring constants of inner and outer layers
B_1\` , $B_2$\`:friction coefficient of inner and outer layers
$\varepsilon_1(t)$, $\varepsilon_2(t)$: strain functions of inner and outer layers
$\rho_{\ell 1}$, $\rho_{\ell 2}$: linear mass densities for inner and outer layers.
B_1, B_2: normalized viscosity coefficients for inner and outer layers.
K_1,K_2: normalized elasticity constants for inner and outer layers.
T: periodic time of pressure pulse
P_o: pressure pulse amplitude
$P_1(t)$, $P_2(t)$, $P_3(t)$: blood pressure pulses acting on inner wall, interface and outer wall respectively.
$F_1(t)$, $F_2(t)$, $F_3(t)$: normalized radial force components acting on inner wall, interface and outer wall respectively.
$\epsilon_1(t)$, $\epsilon_2(t)$: strain function for inner and outer layers T_1, T_2: mechanical stresses acting on inner and outer wall
I_1, I_2, I_3: equivalent time dependent electric currents
V_1, V_2, V_3:equivalent time dependent electric voltages
q_1, q_2, q_3 : equivalent time dependent electric charges
Y_{eq} : equivalent admittance of electric circuit.
Req: equivalent resistance of electric circuit.
Xeq: equivalent reactance of electric circuit.
e_{diss}: dissipated energy per pulse per meter.
P_{diss}: overall power loss.
P_{str} :overall stored power

Introduction

The study of the viscoelastic properties and response of the blood vessels have been one of the most important and biomechanical field of research for quite a while. This study represents an essential step towards controlling the efficiency of the blood pulse propagation process. The blood pulse propagates along the axis of the blood vessels, namely arteries, with a radial component pulsating on the inner wall. This propagation is similar to the propagation of waves in media. The arterial pulse is considered as a pressure wave travelling down the multilayer wall associated with a strain wave. The arterial strain response is the main issue here as it can be considered as an energy storage element that pushes the blood spurts forward down the artery. The rigidity of the arteries causes the pressure to be very high during systole and it would fall to low values during diastole. The blood flow as a result of this would be intermittent. Furthermore, the reduced distensibility of the arterial wall has a direct effect on the efficiency of the heart making its attack more likely. It is thus extremely important to simulate the pressure wave propagation especially its pulsating radial component. The problem is complicated not only because of the non-linearity in blood vessels elasticity, but because of the varying nature of the pulse wave as well. The anisotropic, multilayer and nonlinear viscoelastic properties of the arterial walls lead to certain mathematical complications when the radial dimensions are considered under the effect of pulsating pressure.

Previous work was concerned with mechanical and mathematical simulation to produce stress-strain relation for different soft tissues [1]. Force-deformation relations for different layers of blood vessels were evaluated by studying the non- axial symmetric deformation of the vessel wall. Young's modulus is given for the intima media and the adventitial layers of the thoracic artery [1]. The circumferential and axial stretch of abdominal arteries are given for different pressure values for both normal and hypertensive patients [2] employing clinical data. Mechanical properties are obtained by reliable experimental setups especially for different arterial layers, collagen and body tissues [3]. The degree of nonlinearity and blood vessel elasticity was studied by J. Zhou and Y. C. Fung [4] introducing a pseudo strain energy function and fitting experimental data to produce its parameters. Furthermore the relationship between wall shear stress and intimal thickening for the abdominal aorta was determined [5], using the laser photochromic dye tracer technique.

Imaging the mechanical properties of thrombosis, plaques and arterial wall can aid in the characterizing and understanding of the pathogenesis of the cardiovascular disease. Intravascular ultrasound is a widely used method of imaging the coronary arteries. Strain and elasticity images are generated to determine mechanical properties of arterial tissues [6-8]. T. Shishido *et al.* developed a new technique by applying minute vibrations at various frequencies to evaluate regional myocardial elastance and Young's moduli at different regions [9]. Blood flow and pressure in the larger systemic arteries are modelled by structuring a tree attached to the terminal branches in which the root impedance is estimated using an approach based on a linearization of the viscous axisymmetric Navier-Stokes equations [10].

In the present work, periodic pressure pulses are assumed to act in a radial direction on the inner wall of an arterial transversal section. The periodic pulses are represented mathematically by a Fourier series. These pulses are considered to fluctuate between maximum values of a fraction of the mean arterial pressure. They approximately follow the well-known shape of the arterial pulse. Though the *in vivo* reported arterial pulses show faster rise than decay [11], we assume here that they are symmetric pulses. Biaxial symmetry of the artery is assumed as well. The radial pressure pulses are assumed to follow the heart beats with the same frequency. Some of the reported values concerning mechanical properties of arteries [1], [2], [11] are adopted. An R-L-C double loop circuit is proposed to simulate the double layer arterial wall. The flow of electric charge in the circuit is analogous to the strain fluctuations, the input voltage to the radial pressure pulses and the current is the pulse velocity. Parameters required to implement the mathematical model at hand are changed in appropriate ranges to show arterial stiffness, pulse damping, variation of the elasticity constant. Power dissipated and that stored are also computed.

The Model

A simplified model, introduced earlier [12], simulates the viscoelastic behaviour of a single layer blood vessel in response of the blood pulses acting in a radial direction on its inner wall. The proposed double layer arterial section is mechanically modelled as the spring-dash pot circuit shown in Fig.1

An arterial section of length, l, is assumed to be a double layer viscoelastic cylindrical tube. The inner layer is assumed to be the intima media layer with elasticity modulus, E_1, while the outer one is the adventitial layer with E_2 . The inner radii of the two layers are R_{01} and R_{02} .The elastic behaviour of the arterial segment is represented by that of a spring with spring constants, K_1`and $K_2$` while its viscous response is represented by that of a dash pot of friction coefficients B_1` and $B_2$` for the inner and outer layers respectively.

The second degree differential equations that govern the strain functions, ε_1(t) and ε_2(t) of both layers and the inertial forces on them are as follows:

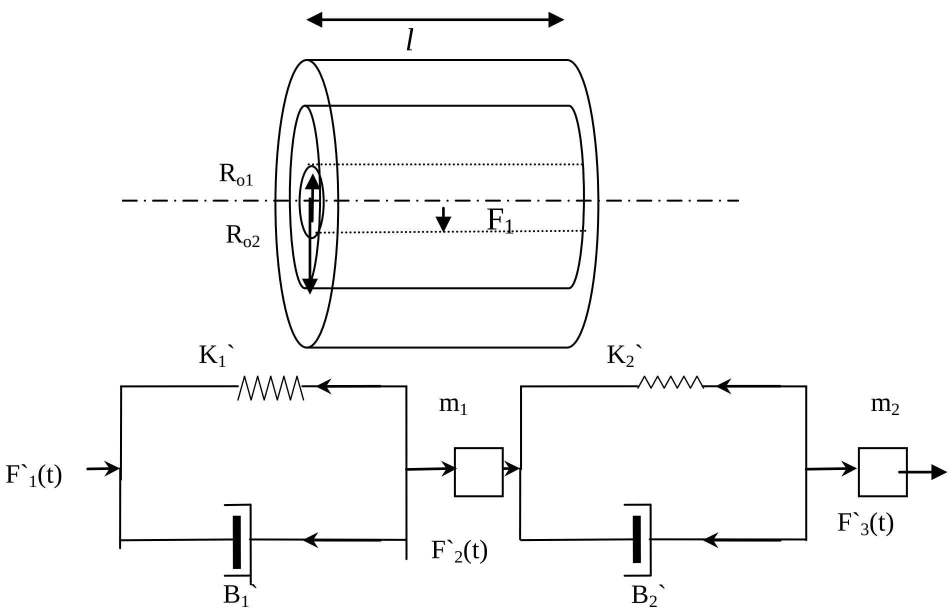
$$\frac{d^2\varepsilon_1(t)}{dt^2} = F_1(t) - B_1\frac{d\varepsilon_1(t)}{dt} - K_1\varepsilon_1(t)$$
$$\frac{d^2\varepsilon_2(t)}{dt^2} = F_2(t) - B_2\frac{d\varepsilon_2(t)}{dt} - K_2\varepsilon_2(t) \qquad (1)$$

where $B_1=B_1`/\rho_{\ell 1}$ , $K_1=K_1`/\rho_{\ell 1}$, $B_2=B_2`/\rho_{\ell 2}$ , $K_2=K_2`/\rho_{\ell 2}$, are normalized viscosity coefficients and elasticity constants, to the linear mass densities, $\rho_{\ell 1}$, $\rho_{\ell 2}$ for inner and outer layers respectively.
$F_1(t)$ and $F_2(t)$ are defined as the normalized radial components of the force acting on inner and outer layers respectively. These forces are generated due to radial blood pressure pulses, $P_1(t)$, pulsating on the inner layer producing successive pulses $P_2(t)$, on the interface between the layers. Hence:

$$F_1(t) = 2\pi R_{01} / \rho_{\ell 1} \times P_1(t)$$

$$F_2(t) = 2\pi R_{02} / \rho_{\ell 2} \times P_2(t) \qquad (2)$$

Fig.1. Double layer mechanical model



Mathematical Analysis

Starting with the mathematical representation of pressure, $P_1(t)$, assuming it takes the waveform of a biphasic exponential rise exponential decay periodic function, is given below:

$$P_1(t) = \begin{cases} P_o(1 - 2e^{-t\pi/T}) & 0 \le t \ge T \\ P_o(2e^{-\pi(t-T)/T} - 1) & T \le t \ge 2T \end{cases} \qquad (3)$$

The proposed model assumes the same time interval, T, for the rise and decay pulse components and hence equal decay constants. It is assumed that the pulses fluctuate between equal values $\pm P_o$.
A Fourier series is deduced to represent $P_1(t)$. The coefficients of the Fourier series, produced by integration on the two time intervals given in Eq.(3), give a final form of:

$$P_r(t) = \frac{2}{\pi} P_0 \sum_{n=1}^{\infty} \begin{matrix} (e^{-\pi}(\cos n\pi - 1) - (\cos n\pi + 1)) \cos \frac{n\pi t}{T} \\ + \frac{1}{n}(e^{-\pi}(\cos n\pi - 2) - (\cos n\pi + 1)) \sin \frac{n\pi t}{T} - \frac{1}{n} e^{-\pi} \cos n\pi \end{matrix} \quad (4)$$

Fig. (2) shows the pressure periodic waveform. The maximum value P_0, is assumed to be a 10% of the mean arterial pressure in adults.

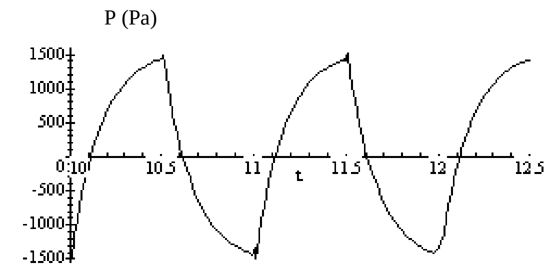


Fig. 2. Rise and fall of the radial pressure in Pa

An equivalent electric model, is introduced as a double loop R-L-C circuit to be analogous to the mechanical model above. Analogy between the mechanical system and the equivalent electrical circuit is shown in appendix 1. We should note that the stress on each layer is the difference in pressures on the bounding interfaces. Thus the stress on the inner layer, T_1, is difference in pressure $(P_1 - P_2)$ and the stress, T_2, on the outer layer is $(P_2 - P_3)$ where P_1 is the pressure on the inner wall, P_2 on the interface between the two layers and P_3 is that on the outer wall.

Consequently, the strain is calculated as:

$$\epsilon_1(t) = q_1 - q_2$$
$$\epsilon_2(t) = q_2 - q_3 \quad (5)$$

where q_1, q_2 and q_3 are the equivalent time dependent electric charges in analogous circuit.

Since P_1, P_2 and P_3 are analogous to V_1, V_2 and V_3, thus we can deduce the involved quantities in the *s*-domain as follows:

$$I_1(s) = Y_{eq}(s) V_1(s) \quad (6)$$

where Y_{eq} represents the total admittance of the two circuit section.

$$V_2(s) = V_1(s) - I_1(s)(R_1 + \frac{1}{sc_1})$$
$$V_3(s) = V_2(s) - I_3(s)(R_2 + \frac{1}{sc_2}) \quad (7)$$

Furthermore, the equivalent resistance, *Req*, can be introduced as the impeadance component responsible for the dissipated energy per pulse per meter, e_{diss}. Whereas the equivalent reactance, *Xeq*, is the impeadance component responsible for the stored energy per pulse per meter, e_{st} . These can be deduced as follows:

$$e_{diss}(t) = \frac{1}{T}\int_0^T i_1^2(t) R_{eq}\, dt, \quad e_{st}(t) = \frac{1}{T}\int_0^T i_1^2(t) X_{eq}\, dt \quad (8)$$

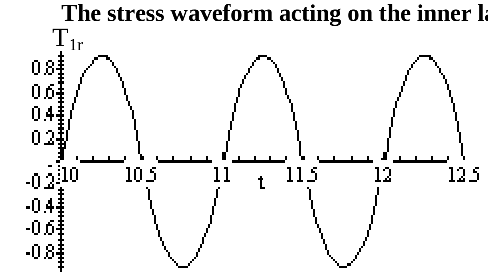
Results

The stress and the strain waveform for each layer is produced by Eqs.(5-7). The strain pulses follow those of the applied radial pressure having the same frequency with a slight time delay. They fluctuate between maximum minimum values of $\pm \epsilon_0$.

The strain response is calculated as the difference between strains of boundary interfaces. The computed values are represented graphically for normal viscoelastic wall performance. Table1 shows the results of three computer runs using three different sets of properties. The periodic time, T, for one pressure pulse is 1s while P_0 is 1.6 kPa. The first parameter set, set1, simulates normal performance with elasticity constants similar to those of the intima media and advetitia layers of a real arterial section.

Fig.3a **Fig.3b**

The stress waveform acting on the inner layer: a) real and b) imaginary components (kPa) vs. time (s)



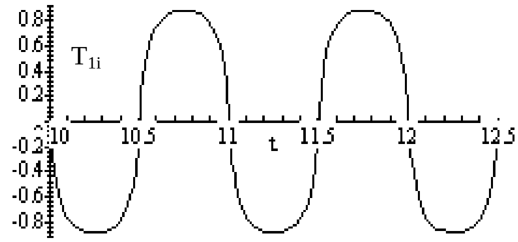
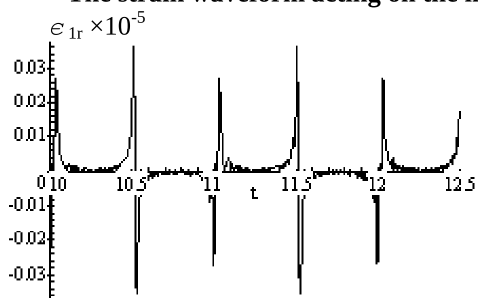


Fig.4a **Fig.4b**

The strain waveform acting on the inner layer: a) real and b) imaginary components vs. time



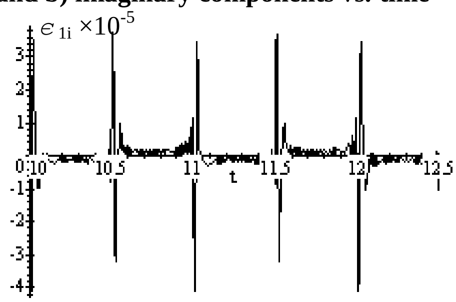
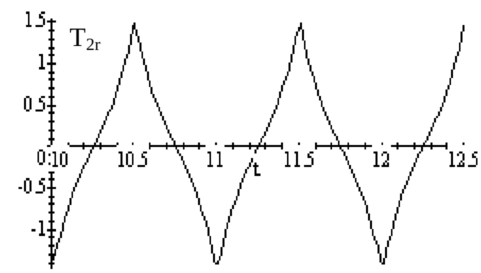


Fig.5a **Fig.5b**

The stress waveform acting on the second layer: a) real and b) imaginary components (kPa) vs. time (s)



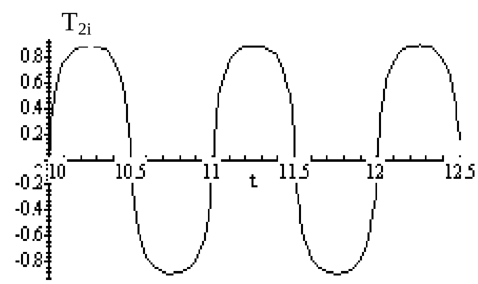
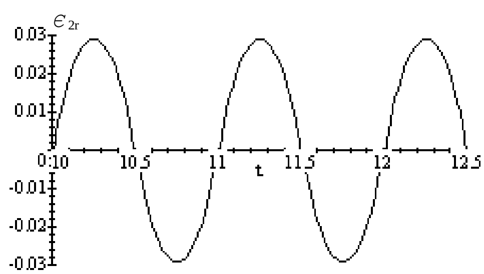
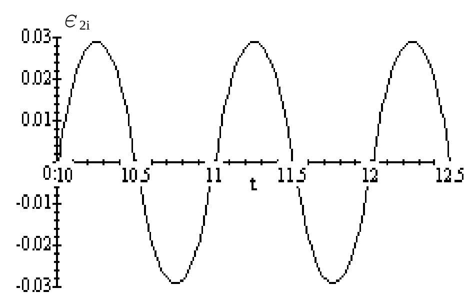


Fig.6a **Fig.6b**

The strain waveform acting on the second layer: a) real and b) imaginary components vs. time(s)





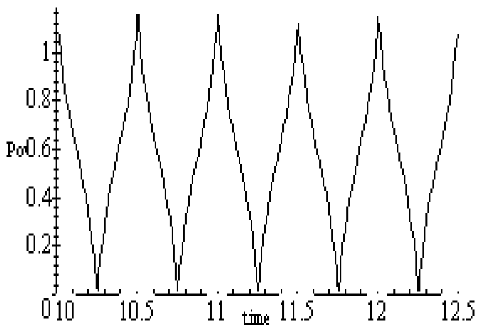
$\rho_{\ell 1}$, $\rho_{\ell 2}$ are assumed to be 25*g/m* and 12*g/m* respectively. Whereas B_1, B_2 are adjusted by computation to be $0.4s^{-1}$ and $0.417s^{-1}$ respectively.

Figs. (3a, 3b, 4a, 4b) show the complex stress-strain response for the inner layer, intima media. Whereas Figs. (5a, 5b, 6a, 6b) show the complex stress-strain response for the outer layer, adventitia. The latter show higher elastic response (about 3%) than the former layer.

Fig. (7a) represents the real component, P_{diss}, corresponding to the overall power loss. Fig. (7b) represents the imaginary component, P_{str}, the overall stored power calculated in *mw/mm*. The storage and loss moduli are computed, using assumed viscoelastic properties for each layer, given in Table 1.

Fig.7a. **Fig.7b**

The power density (*mw/mm*): a) dissipated and b) stored vs. time (s)



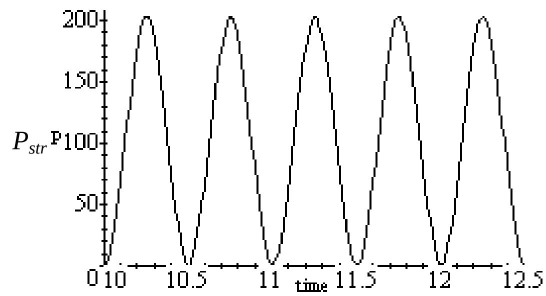


Table 1
Peak strain values, storage and loss moduli for different normalized

set	K_1 $10^6\ s^{-2}$	K_2 $10^6\ s^{-2}$	ϵ_{01r} 10^{-3}%	ϵ_{01i} 10^{-3}%	ϵ_{02r} %	ϵ_{02i} %	E_{loss1} 10^3 *kPa*	E_{str1} 10^3 *kPa*	E_{loss2} *kPa*	E_{str2} *kPa*
1	8	0.05	3	0.03	3	2	432.031	460.852	23.822	46.442
2	4	0.8	0.5	0.008	0.4	0.4	3744.905	4024.505	125.022	237.566
3	0.4	.02	0.1	8	6	6	10.3577	11.0498	0.9822	1.8067

DISCUSSION:

Though the proposed model assumes linear stress-strain relationship and biaxial isotropy of the blood vessel, it still offers a new perception towards the understanding of the mechanism of arterial pulsating nature. The results show non identical strain responses for each layer corresponding to its elasticity constant. I computed the elasticity modulus as the quotient of the stress and the strain complex functions for each layer with average value tabulated, Table1. The mathematical model constructed herein is tested to give strain response for a stiffer arterial section, set 2, and a highly elastic one, set 3. The elasticity constants produced in this work are non-linear complex functions of time. The real component represents the loss modulus and the imaginary represents the storage modulus.

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APPENDIX 1

The second order differential equations that govern the electrical charge *Q* with circuit components and input voltage V_0 is:

$$\frac{d^2Q(t)}{dt^2} = V_0(t)/L - \frac{R}{L}\frac{dQ(t)}{dt} - \frac{1}{LC}Q(t) \qquad (9)$$

By analogy between the two systems, mechanical and electrical, note that:

$V_0(t) \leftrightarrow 2\pi P_1(t)$,

$R/L \leftrightarrow B$

$1/LC \leftrightarrow K$, $L \leftrightarrow \rho_\ell$