Some Properties of Random Coefficients Regression Estimators

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Arabic Summary
## Glossary of Notation

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<th>Notation</th>
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<tbody>
<tr>
<td>ANCOVA</td>
<td>Analysis of Covariance</td>
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<tr>
<td>AR</td>
<td>Autoregressive</td>
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<tr>
<td>ARMA</td>
<td>Autoregressive Moving Average</td>
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<tr>
<td>BLUE</td>
<td>Best Linear Unbiased Estimator</td>
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<tr>
<td>CP</td>
<td>Classical Pooling</td>
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<tr>
<td>EM</td>
<td>Expectation Maximization</td>
</tr>
<tr>
<td>FGLS</td>
<td>Feasible Generalized Least Squares</td>
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<tr>
<td>GLM</td>
<td>General Linear Model</td>
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<tr>
<td>GLS</td>
<td>Generalized Least Squares</td>
</tr>
<tr>
<td>MA</td>
<td>Moving Average</td>
</tr>
<tr>
<td>MG</td>
<td>Mean Group</td>
</tr>
<tr>
<td>Mixed RCR</td>
<td>Mixed Random Coefficient Regression</td>
</tr>
<tr>
<td>MLE</td>
<td>Maximum Likelihood Estimator</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
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<tr>
<td>MVUE</td>
<td>Minimum Variance Unbiased Estimator</td>
</tr>
<tr>
<td>OLS</td>
<td>Ordinary Least Squares</td>
</tr>
<tr>
<td>RCR</td>
<td>Random Coefficient Regression</td>
</tr>
<tr>
<td>REML</td>
<td>Restricted Maximum Likelihood</td>
</tr>
<tr>
<td>SSE</td>
<td>Sum Square Error</td>
</tr>
<tr>
<td>SUR</td>
<td>Seemingly Unrelated Regression</td>
</tr>
<tr>
<td>TSCS</td>
<td>Time-Series-Cross-Section</td>
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Summary

An important assumption of the General Linear Model (GLM) is that the vector of regression coefficients is fixed vector, so the model will be called “Fixed Model”. But when we assumed that the regression coefficients are random variables, so the model will be called “Random Coefficient Regression (RCR) Model” examined by Swamy in several publications (Swamy 1970, 1971, 1973, and 1974). And if the regression coefficients in model contain both random and fixed coefficients, so the model will be called “Mixed Random Coefficient Regression (Mixed RCR) model”.

In this thesis, we studied the properties of RCR and Mixed RCR models. And also we studied the Swamy’s estimator (RCR estimator) for RCR model in panel data, and we proposed the alternative estimators for RCR model, such as unit by unit OLS, Mean Group (MG), Classical Pooling (CP), and Stein-rule estimators.

In this thesis, we used the Monte Carlo simulation to study the behavior of the Swamy’s estimator in small, medium and large samples in panel data. The parameters were set at several values, to allow the study of estimators under several situations, to know when the RCR model will be properly and improperly. This simulation provides some insight into how well the RCR estimator performs in different samples size. Also, we used the Mote Carlo simulation again for comparison between the behavior of RCR, CP, and MG estimators in three models (RCR, fixed, and Mixed RCR models). And we used the R language to conduct the Monte Carlo simulation study.
The thesis includes five chapters:

**Chapter 1: Definitions and Notations**

This chapter involved some definitions which be used in this thesis.

**Chapter 2: Introduction to Random Coefficient Regression Models**

This chapter presented an introduction to the general linear model estimators under the classical assumptions in section (2.1). While section (2.2) discussed RCR model for panel data, and we estimated the random coefficients by the generalized least square method. The literature review for RCR models in section (2.3). Finally, section (2.4) introduced the applications for RCR models.

**Chapter 3: Estimation of Random Coefficient Regression Model in Panel Data**

This chapter presented the RCR model in panel data when regression coefficients are viewed as invariant over time, but varying from one unit to another in section (3.1). While section (3.2) discussed the alternatives estimators for RCR model. Finally, in section (3.3) we proposed the Mixed RCR model as special case of the RCR model.

**Chapter 4: Estimation of Time and Cross-Sectionally Varying Parameter Models**

In this chapter, an introduction for the time and cross-sectionally varying parameter models and we proposed the Hsiao model in section (4.1). While in section (4.2), we discussed the estimation and tests of hypotheses for the random coefficients of Swamy and Mehta model. In section (4.3), we proposed other models with time varying coefficients. Finally, section (4.4)
introduced the applications for time and cross-sectionally varying parameter models.

**Chapter 5: Monte Carlo Simulation for Efficiency of Random Coefficient Regression Estimators**

In this chapter, we used the Monte Carlo simulation to study the behavior of the Swamy’s estimator in small, medium and large samples in panel data in section (5.1). While in section (5.2), we explained the results of simulation study. We used the Monte Carlo simulation again for comparison between the behavior of RCR, CP, and MG estimators in section (5.3). While in section (5.4), we explained the simulation results for the three estimators. Ending with section (5.5), we focused on displaying the concluding remarks of the Monte Carlo simulation studies.

Finally, the Monte Carlo simulation results suggest that the RCR estimators perform well in small samples if the coefficients are random but it does not in fixed or Mixed RCR models. But if the samples sizes are medium or large, the RCR estimators performs well for the three models. While CP estimators perform well in the fixed model only. But the MG estimators perform well if the coefficients are random or fixed. So, we can say that the MG method is the general estimation method for fixed, RCR, and Mixed RCR models.