CAIRO UNIVERSITY		ELECTRONICS AND COMMUNICATION DEPT.
FACULTY OF ENGINEERING		2^{nd} *Year*
Two-Port Networks	*ELECTRIC CIRUITS*	*Sheet (6)*

[1]Determine the O.C. Z – parameters for the network shown in Fig. 1 . Sketch its Z – parameters' equivalent circuits .

[2]Determine the S.C. Y – parameters for the network shown in Fig. 2 . Sketch the two forms of its equivalent circuit .

[3]Calculate the Hybrid H – parameters for the network given in Fig. 1 and sketch its equivalent circuit .

[4]For the Hybrid equivalent circuit shown in Fig. 3 , calculate the current gain A_i and the voltage gain A_v .

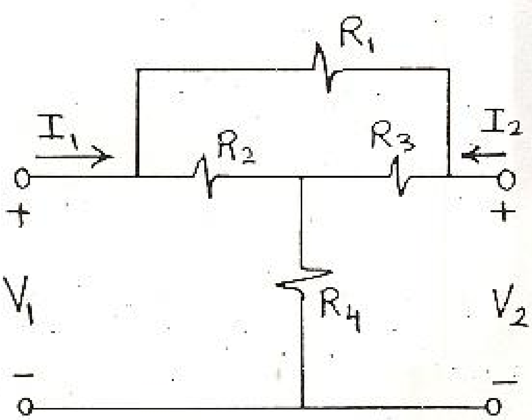


Fig.1

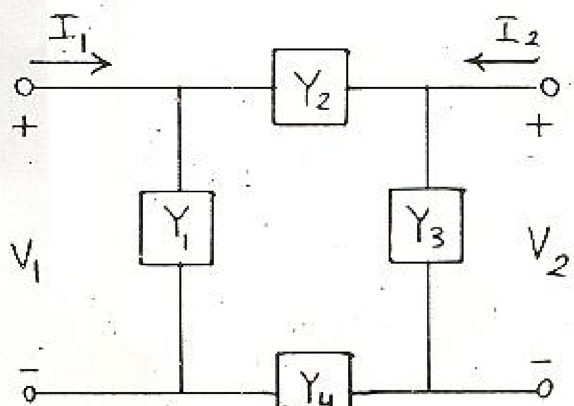


Fig.2

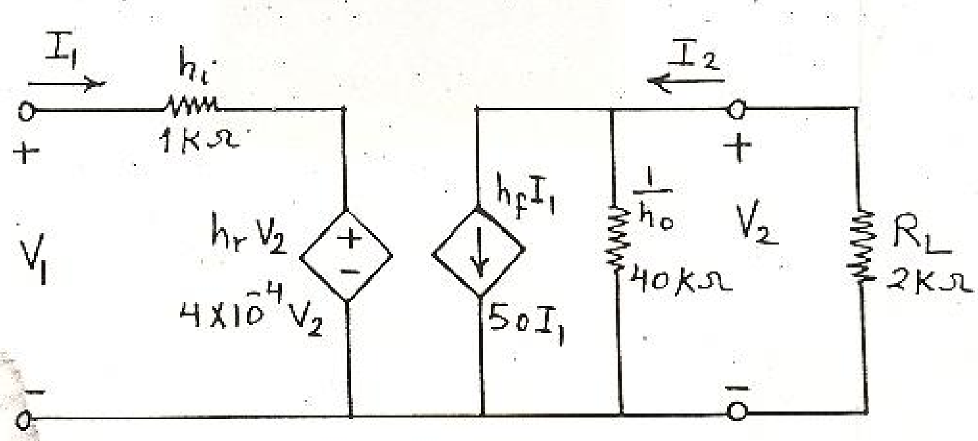


Fig.3

[5]For the Z – parameter equivalent circuit shown in Fig. 4, calculate the input and output impedances.

[6]The Hybrid H – parameters of a network are given by : $h_{11} = 1\ K\Omega$, $h_{12} = 0.0002$, $h_{21} = 100$ and $h_{22} = 20\ \mu$ mho. Calculate the H – parameters of two of such network connected in : a) series . b) parallel ...

[7]Determine the transmission – parameters for the network shown in Fig. 5 and write its 2 – port equations in the A – form .

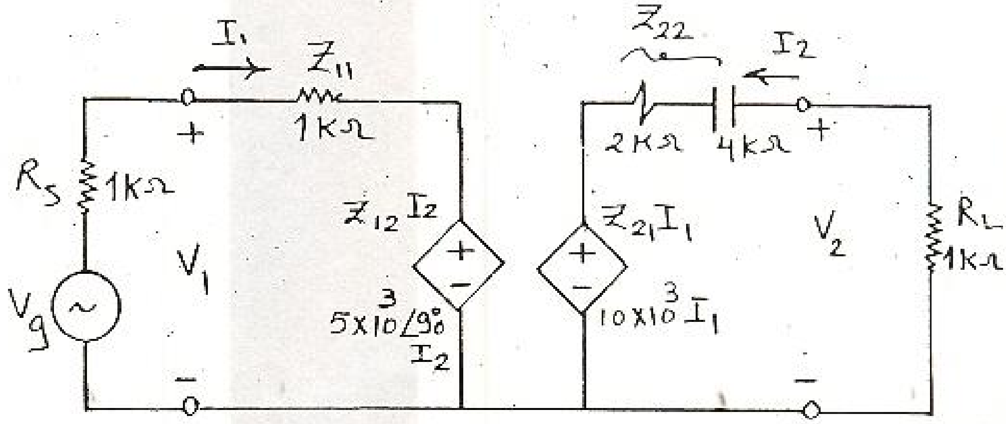


Fig.4

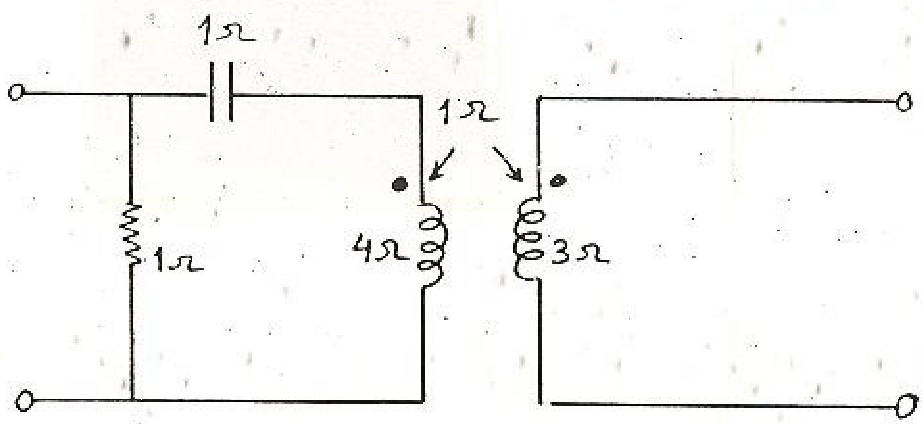


Fig.5

[8]Find the Z – parameters for the 2 – port network shown in Fig. 6 and calculate the voltage gain of the entire circuit with a 4 KΩ load attached to the output .

[9]Find the transmission – parameters for the 2 – port network shown in Fig. 7 .

[10]Simplify the network shown in Fig. 8 into two 2 – port networks connected in parallel Hence , calculate the Y – parameters ' matrix of the whole network .

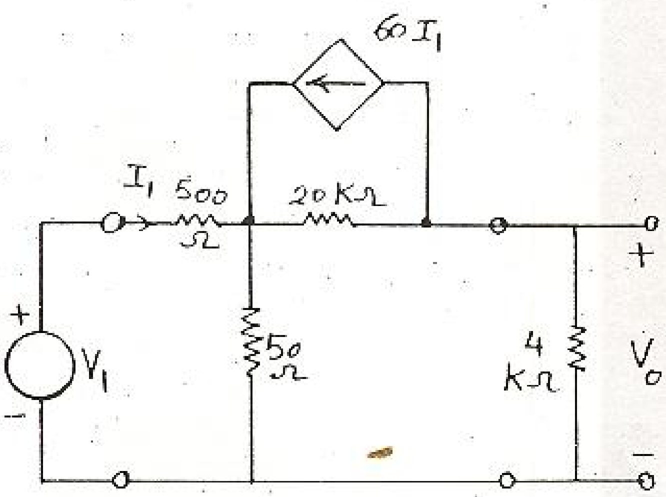


Fig.6

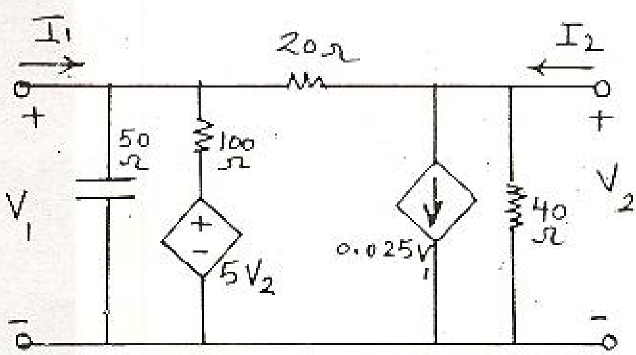


Fig.7

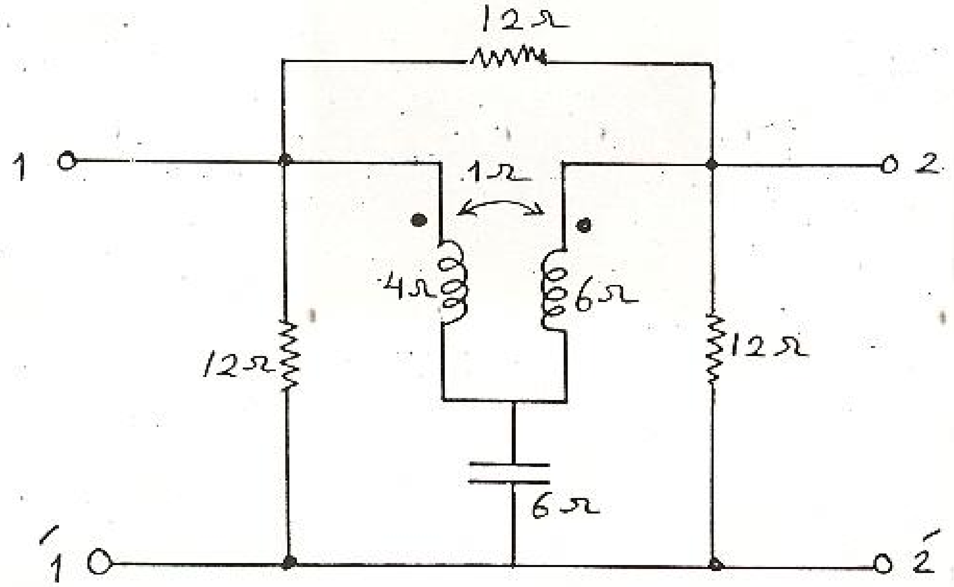


Fig.8

[11] Using the Z – parameters find Thevenin's equivalent circuit with respect to port 2 for the 2 – port network shown in Fig. 9 .

[12] The networks D and E in the circuit shown in Fig. 10 are reciprocal and symmetrical networks . D has $a_{11} = 5$ and $a_{12} = 24\ \Omega$. The impedance Z_o is adjusted for maximum average power transfer . Find Z_o if $Z_g = 5\ \Omega$.

[13] a) Derive an expression of the input and output impedances of the two-port network shown in Fig.11 in terms of the H-parameters of the network ,the internal impedance of the source Z_g and the load impedance Z_L .

b) If $h_{11} = 1500\ \Omega$, $h_{12} = 10^{-3}$, $h_{21} = 50$; $h_{22} = 50\mu$ mho , the internal impedance of the source is $Zg = 1500\Omega$ and the source voltage $V_g = 250\angle 0^{\circ}$, calculate :

i) The maximum average power delivered to the load .

ii) The average power delivered to the input port of the network at the condition of maximum power transfer to the load .

Fig.9

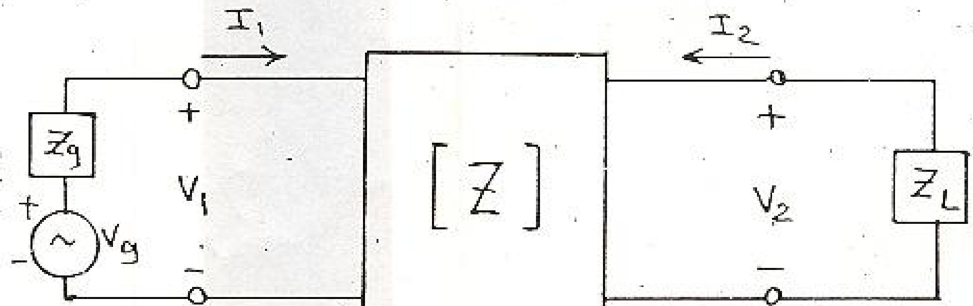


Fig.10

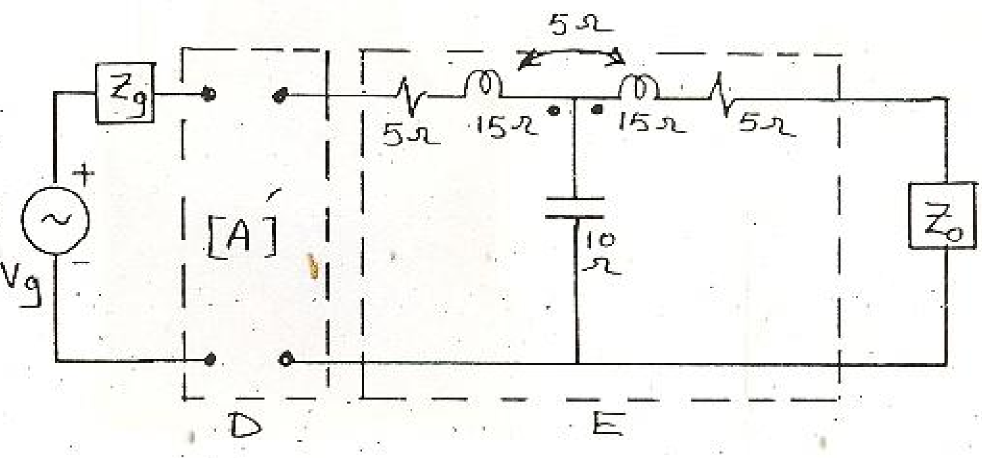
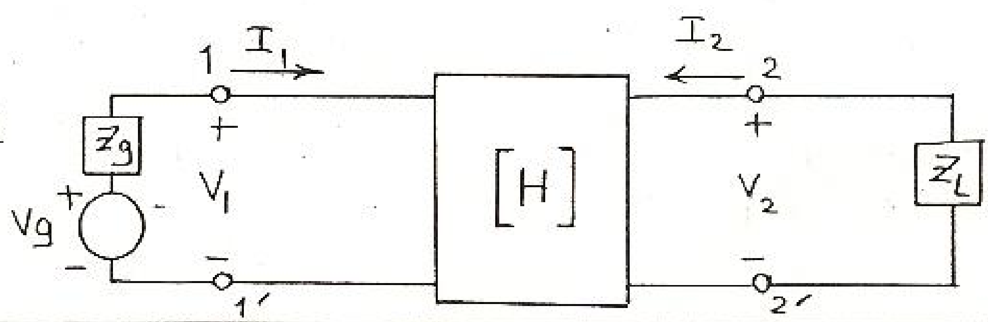


Fig.11



[14]The H-parameters of a two-port network are given by the matrix :

$$[H] = \begin{bmatrix} 2\ \Omega & 0.2 \\ -4 & 0.1\ mho \end{bmatrix}$$

a)*Check the conditions of symmetry and reciprocity for the network .*

b)*Sketch its equivalent circuit .*

c)*If two resistors of 2Ω and 5Ω are added to the network as shown in Fig.12 , calculate the H-parameters of the total network :*

*[15]****a)****Calculate the transmission A-parameters of a two-port network as a function of its Hybrid H-parameters .*

b)*The amplifier circuit shown in Fig.13 is formed of two identical stages each having the Hybrid H-parameters given by :*

$$[H] = \begin{bmatrix} 1\ K\Omega & 0.001 \\ 100 & 0.1\ m.mho \end{bmatrix}$$

What is the value of the input voltage V_s required to obtain an output voltage V_o of 25 volts ?

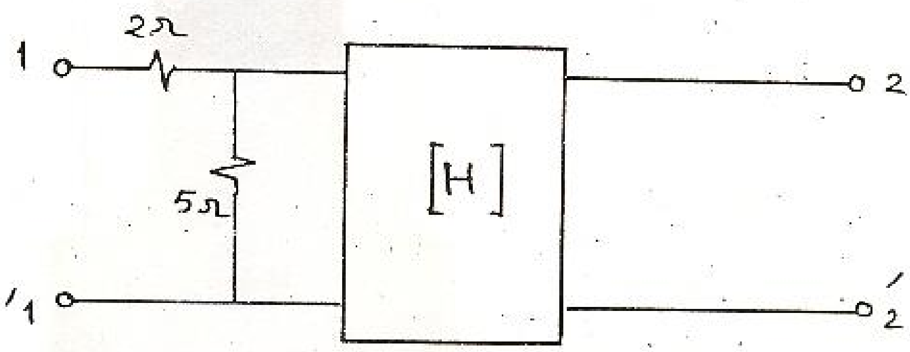


Fig.12

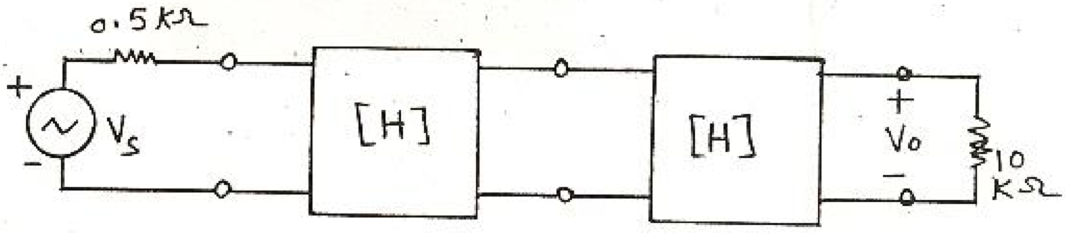


Fig.13

[16](a) Two sets of measurements were made on the two-port network N shown in Fig. 14. The results were as follows :

Measurement 1	Measurement 2
$V_1 = 20\ mV$	$V_1 = 4\ V$
$I_1 = 20\ \mu A$	$I_1 = 5mA$
$V_2 = 40\ V$	$V_2 = 0$
$I_2 = 0$	$I_2 = -200\ mA$

Find the H-parameters representing the two-port network.

b) Using the H-parameters, calculate the value of the variable resistance R_o when adjusted to receive maximum power and find the value of this power.

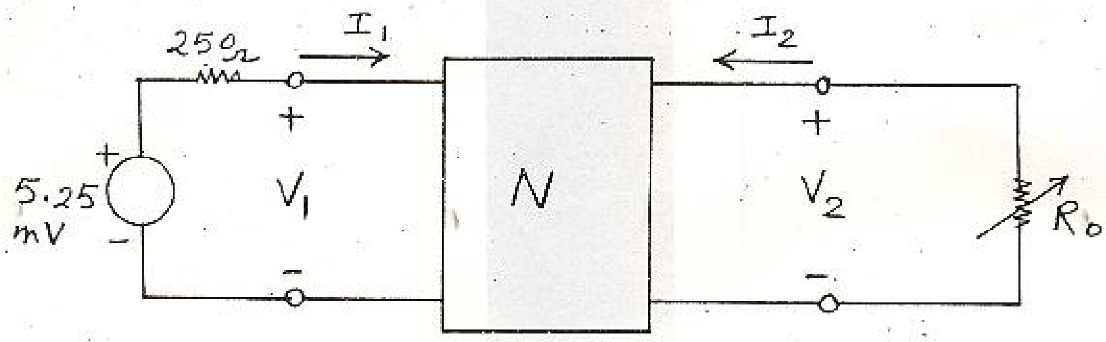


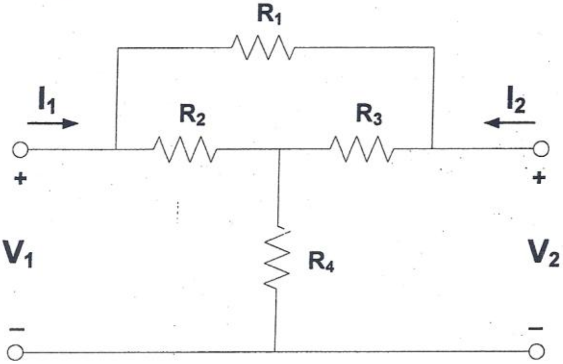
Fig.14

Question[1]:

a)Determine the O.C. Z-parameters for the network shown in figure and sketch its Z-parameters' equivalent circuits.

b)Calculate the Hybrid H-parameters of the same network and sketch its equivalent circuit.

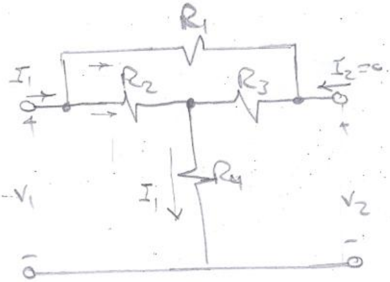
Insert your results in Table 1 .



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

@ $I_2 = 0$

$V_1 = Z_{11} I_1$, $V_2 = Z_{21} I_1$



$$V_1 = R_2 * I_1 * \frac{R_1 + R_3}{R_1 + R_2 + R_3} + I_1 R_4$$

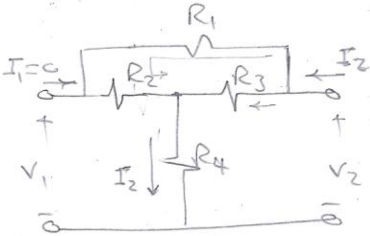
$$V_1 = \left[R_4 + \left(R_2 // (R_1 + R_3)\right)\right] I_1 \qquad \boxed{\therefore Z_{11} = R_4 + \left(R_2 // (R_1 + R_3)\right)}$$

$$V_2 = R_4 I_1 + R_3 * I_1 * \frac{R_2}{R_1+R_2+R_3}$$

$$\boxed{\therefore Z_{21} = R_4 + \frac{R_2 R_3}{R_1+R_2+R_3}}$$

the network is reciprocal so $\Rightarrow \boxed{Z_{12} = Z_{21} = R_4 + \frac{R_2 R_3}{R_1+R_2+R_3}}$

at $I_1 = 0$



$V_1 = Z_{12} I_2 \quad , \quad V_2 = Z_{22} I_2$

$$V_2 = I_2 R_4 + R_3 * I_2 * \frac{R_1+R_2}{R_1+R_2+R_3}$$

$$\therefore Z_{22} = \frac{V_2}{I_2} \Rightarrow \boxed{Z_{22} = R_4 + \left(R_3 // (R_1+R_2)\right)}$$

$$\therefore [Z] = \begin{bmatrix} R_4 + \left(R_2 // (R_1+R_3)\right) & R_4 + \frac{R_2 R_3}{R_1+R_2+R_3} \\ R_4 + \frac{R_2 R_3}{R_1+R_2+R_3} & R_4 + \left(R_3 // (R_1+R_2)\right) \end{bmatrix}$$

* For H-Parameters

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

* For Z-Parameters

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

at $I_1 = 0$

$V_1 = h_{12} V_2$ $\qquad$ $V_1 = Z_{12} I_2$

$I_2 = h_{22} V_2$ $\qquad$ $V_2 = Z_{22} I_2$

$$\boxed{h_{12} = \frac{V_1}{V_2} = \frac{Z_{12}}{Z_{22}}}$$

$$\boxed{h_{22} = \frac{I_2}{V_2} = \frac{1}{Z_{22}}}$$

$\because$ the network is reciprocal $\therefore h_{12} = -h_{21}$

$$\boxed{\therefore h_{21} = \frac{-Z_{12}}{Z_{22}}}$$

at $V_2 = 0$

$V_1 = h_{11} I_1$ $\qquad$ $V_1 = Z_{11} I_1 + Z_{12} I_2$

$I_2 = h_{21} I_1$ $\qquad$ $0 = Z_{21} I_1 + Z_{22} I_2 \Rightarrow -Z_{21} I_1 = Z_{22} I_2$

$$V_1 = Z_{11} I_1 + Z_{12}\left(\frac{-Z_{21}}{Z_{22}}\right) I_1$$

$$\therefore h_{11} = \frac{V_1}{I_1} = \frac{|Z|}{Z_{22}}$$

$$\therefore [H] = \begin{bmatrix} \frac{|Z|}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{-Z_{12}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$$

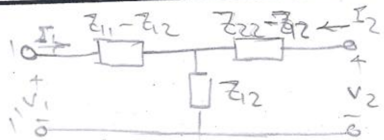
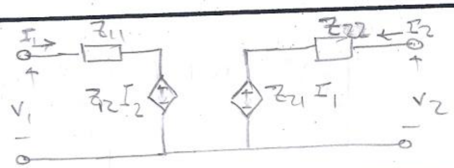
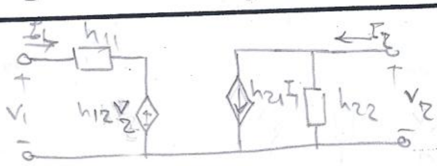
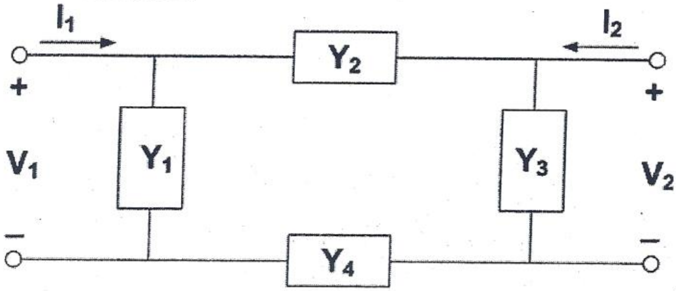
[Z]	$\begin{bmatrix} R_4 + (R_2 // (R_1+R_3)) & R_4 + \frac{R_2R_3}{R_1+R_2+R_3} \\ R_4 + \frac{R_2R_3}{R_1+R_2+R_3} & R_4 + (R_3 // (R_1+R_2)) \end{bmatrix}$
[illegible] Z-parameters	
[illegible] Z-parameters	
[illegible]	$\begin{bmatrix} \frac{\lvert Z \rvert}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{-Z_{12}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$
Equivalent Circuit H-parameters	

Table 1

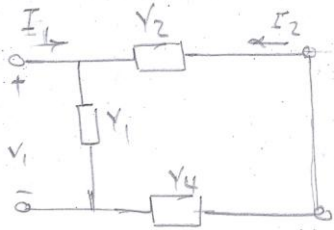
Question[2]:

Determine the S.C. Y-parameters for the network shown in figure and sketch the two forms of its equivalent circuit.
Insert your results in Table 2 .



$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

at $V_2 = 0$



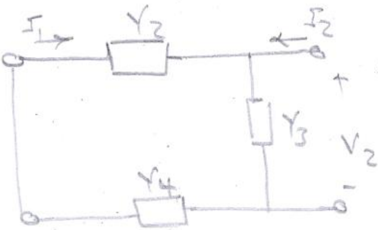
$$I_1 = Y_{11} V_1$$
$$I_2 = Y_{21} V_1$$

$$V_1 = \frac{1}{Y_1} * I_1 * \frac{Y_1}{Y_1 + (Y_2 // Y_4)}$$

$$V_1 = \frac{I_1}{Y_1 + (Y_2 // Y_4)}$$

$$\boxed{\therefore Y_{11} = Y_1 + (Y_2 // Y_4)}$$

$$V_1 = \frac{-I_2}{(Y_2 // Y_4)} \qquad \therefore Y_{21} = -(Y_2 // Y_4)$$



at $V_1 = 0$

$$I_1 = Y_{12} V_2$$

$$I_2 = Y_{22} V_2$$

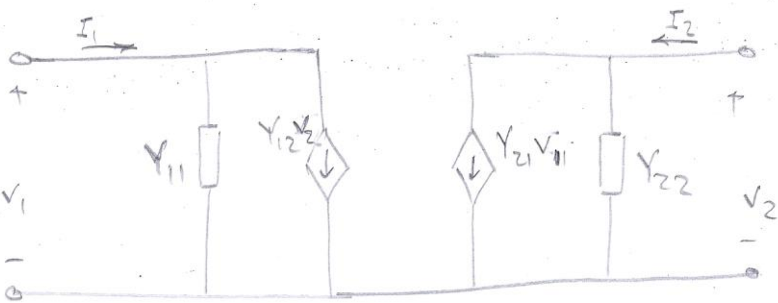
$$V_2 = \frac{1}{Y_3} * I_2 * \frac{Y_3}{Y_3 + (Y_2 // Y_4)} = \frac{I_2}{Y_3 + (Y_2 // Y_4)}$$

$$\therefore Y_{22} = Y_3 + (Y_2 // Y_4)$$

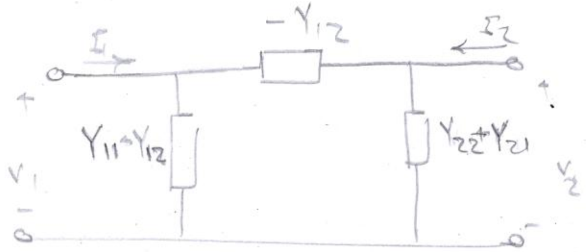
the network is reciprocal $\Rightarrow$ $Y_{12} = Y_{21} = -(Y_2 // Y_4)$

$$\therefore [Y] = \begin{bmatrix} Y_1 + (Y_2 // Y_4) & -(Y_2 // Y_4) \\ -(Y_2 // Y_4) & Y_3 + (Y_2 + Y_4) \end{bmatrix}$$

1st equivalent circuit



2nd equivalent circuit



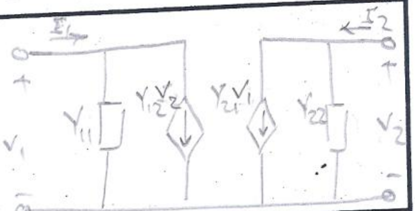
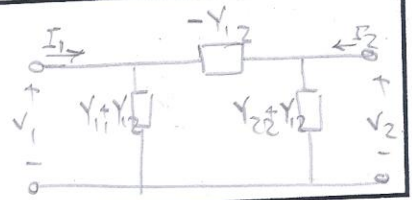
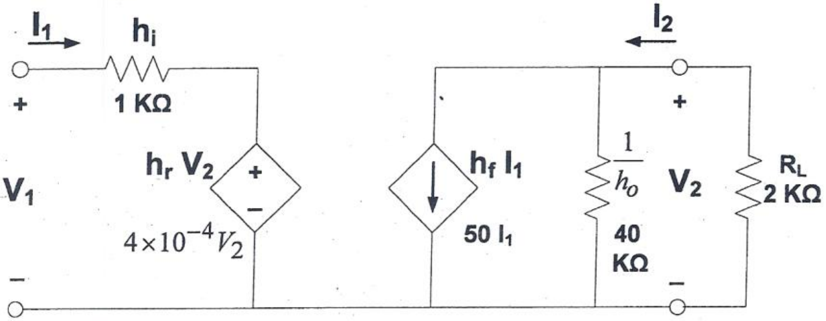
[Y]	$\begin{bmatrix} Y_1 + (Y_2 // Y_4) & -(Y_2 // Y_4) \\ -(Y_2 // Y_4) & Y_3 + (Y_2 // Y_4) \end{bmatrix}$
Equivalent Circuit for Y-parameters	
Equivalent Circuit for Y-parameters	

Table 2

Question[3]:

For the Hybrid equivalent circuit shown in figure, calculate the current gain A_i and the voltage gain A_v.
Insert your results in Table 3.



$V_1 = h_i I_1 + h_r V_2 \quad \rightarrow ①$

$I_2 = h_f I_1 + h_o V_2 \quad \rightarrow ②$

$V_2 = -I_2 R_L \quad \rightarrow ③$

Sub From ③ in ②

$I_2 = h_f I_1 + h_o(-I_2 R_L)$

$\therefore I_2(1 + h_o R_L) = h_f I_1$

$A_i = \frac{I_2}{I_1} = \frac{h_f}{1 + h_o R_L}$ $\boxed{\therefore A_i = 47.619}$

$$V_1 = h_i \left(\frac{1+h_o R_L}{h_f}\right) I_2 + h_r V_2$$

$$V_1 = h_i \left(\frac{1+h_o R_L}{h_f}\right)\left(\frac{-V_2}{R_L}\right) + h_r V_2$$

$$V_1 = V_2 \left[\frac{h_r h_f R_L - h_i(1+h_o R_L)}{h_f R_L}\right]$$

$$A_V = \frac{V_2}{V_1} = \frac{h_f R_L}{h_r h_f R_L - h_i(1+h_o R_L)}$$

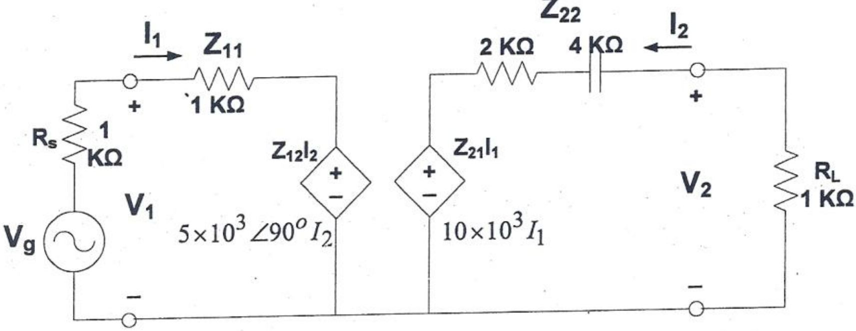
$$A_V \simeq -99$$

[redacted]	47.619
[redacted]	-99.0099

Table 3

Question[4]:

For the Z-parameters' equivalent circuit shown in figure, calculate the input and output impedances.
Insert your results in Table 4.



$Z_{11} = 1k\Omega$

$Z_{12} = 5jk\Omega$

$Z_{21} = 10k\Omega$

$Z_{22} = (2 - j4)\,k\Omega$

$V_1 = Z_{11} I_1 + Z_{12} I_2 \rightarrow ①$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow ②$

$V_1 = V_g - I_1 R_s \rightarrow ③$

$V_2 = - I_2 R_L \rightarrow ④$

From ④ → ②

$-I_2 R_L = Z_{21} I_1 + Z_{22} I_2$

$-I_2 (R_L + Z_{22}) = Z_{21} I_1 \qquad \therefore I_2 = \frac{-Z_{21}}{R_L + Z_{22}} I_1$

SuB. in ①

$V_1 = Z_{11} I_1 + Z_{12} * \frac{-Z_{21}}{R_L + Z_{22}} I_1$

$$Z_{in} = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{12} Z_{21}}{R_L + Z_{22}}$$

$$\boxed{Z_{in} = (9 - 6j)\,k\Omega}$$

$$Z_o = \left.\frac{V_2}{I_2}\right|_{V_g = 0}$$

at $V_g = 0$

$$\therefore V_1 = -I_1 R_S$$

$$\therefore -I_1 R_S = Z_{11} I_1 + Z_{12} I_2$$

$$-I_1 (R_S + Z_{11}) = Z_{12} I_2$$

$$V_2 = Z_{21} * \frac{-Z_{12}}{R_S + Z_{11}} I_2 + Z_{22} \cdot I_2$$

$$\therefore Z_o = Z_{22} - \frac{Z_{12} Z_{21}}{R_S + Z_{11}}$$

$$\boxed{Z_o = (2 - j29)\,k\Omega}$$

Z_{in}	(9 – j6) kΩ
Z_{out}	(12 – j29) kΩ

Table 4

Question[5]:

The Hybrid H-parameters of a network are given by the following matrix:

$$[H]=\begin{bmatrix} 1\,K\Omega & 0.0002 \\ 100 & 20\mu\, mho \end{bmatrix}$$

Calculate the H-parameters of two of such network connected in:

***a)** Series.*

***b)** Parallel.*

Insert your results in Table 5.

$|H| = 0$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \qquad V_1 = h_{11} I_1 + h_{12} V_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \qquad I_2 = h_{21} I_1 + h_{22} V_2$$

at $I_1 = 0$

$$Z_{12} = \frac{V_1}{I_2} = \frac{h_{12}}{h_{22}} = 10 \qquad V_1 = h_{12} V_2$$
$$Z_{22} = \frac{V_2}{I_2} = \frac{1}{h_{22}} = 50K\Omega \qquad I_2 = h_{22} V_2$$

at $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1} \qquad V_1 = h_{11} I_1 + h_{12} V_2$$
$$Z_{21} = \frac{V_2}{I_1} = \frac{-h_{21}}{h_{22}} = -5M\Omega \qquad -h_{21} I_1 = h_{22} V_2$$

$$\therefore V_1 = h_{11} I_1 + h_{12}\left(\frac{-h_{21}}{h_{22}}\right) I_1$$

$$\frac{V_1}{I_1} = Z_{11} = \frac{|H|}{h_{22}} = 0 \qquad \therefore [Z] = \begin{bmatrix} 0 & 10\Omega \\ -5M\Omega & 50K\Omega \end{bmatrix}$$

$I_1 = Y_{11} V_1 + Y_{12} V_2$ $\qquad$ $V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$ $\qquad$ $I_2 = h_{21} I_1 + h_{22} V_2$

at $V_2 = 0$

$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{h_{11}} = 1m\mho, \quad Y_{21} = \frac{I_2}{V_1} = \frac{h_{21}}{h_{11}} = 0.1\mho$$

at $V_1 = 0$

$$Y_{12} = \frac{I_1}{V_2}, \quad Y_{22} = \frac{I_2}{V_2}$$

$-h_{11} I_1 = h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$$\therefore Y_{12} = \frac{-h_{12}}{h_{11}} = -2\mu\mho$$

$$I_2 = \frac{-h_{12} h_{21}}{h_{11}} V_2 + h_{22} V_2 \quad \therefore Y_{22} = \frac{|H|}{h_{11}} = 0$$

$$\therefore [Y] = \begin{bmatrix} 1m\mho & -2\mu\mho \\ 0.1\mho & 0 \end{bmatrix}$$

(a) Series Connection

$$[Z_{eq}] = [Z_1] + [Z_2] = 2[Z] \quad \therefore [Z_{eq}] = \begin{bmatrix} 0 & 20\Omega \\ -10M\Omega & 100k\Omega \end{bmatrix}$$

(b) Parallel Connection

$$[Y_{eq}] = [Y_1] + [Y_2] = \begin{bmatrix} 2m\mho & -4\mu\mho \\ 0.2\mho & 0 \end{bmatrix}$$

$Z_{22} = \frac{1}{h_{22}} \quad \therefore h_{22} = \frac{1}{Z_{22}} = 10\mu ℧$

$Z_{12} = \frac{h_{12}}{h_{22}} \quad \therefore h_{12} = h_{22} Z_{12} = 0.0002$

$Z_{21} = \frac{-h_{21}}{h_{22}} \quad \therefore h_{21} = -h_{22} Z_{21} = 100$

$Z_{11} = \frac{|H|}{h_{22}} = 0 \quad h_{11} h_{22} = h_{12} h_{21} \quad \therefore h_{11} = 2k\Omega$

$\therefore$ in Series Connection

$$[H] = \begin{bmatrix} 2k\Omega & 0.0002 \\ 100 & 10\mu ℧ \end{bmatrix}$$

$Y_{11} = \frac{1}{h_{11}} \quad \therefore h_{11} = \frac{1}{Y_{11}} = 500\ \Omega$

$Y_{21} = \frac{h_{21}}{h_{11}} \quad \therefore h_{21} = 100$

$Y_{12} = \frac{-h_{12}}{h_{11}} \quad \therefore h_{12} = -h_{11} Y_{12} = 0.002$

$Y_{22} = \frac{|H|}{h_{11}} = 0 \quad \therefore h_{11} h_{22} = h_{12} h_{21} \quad \therefore h_{22} = 0.4 m℧$

$\therefore$ in Parallel Connection

$$[H] = \begin{bmatrix} 500\Omega & 0.002 \\ 100 & 0.4m℧ \end{bmatrix}$$

[illegible] Connection	$\begin{bmatrix} 2K\Omega & 0.0002 \\ 100 & 10\mu\mho \end{bmatrix}$
Parallel Connection	$\begin{bmatrix} 500\Omega & 0.002 \\ 100 & 0.4m\mho \end{bmatrix}$

Table 5

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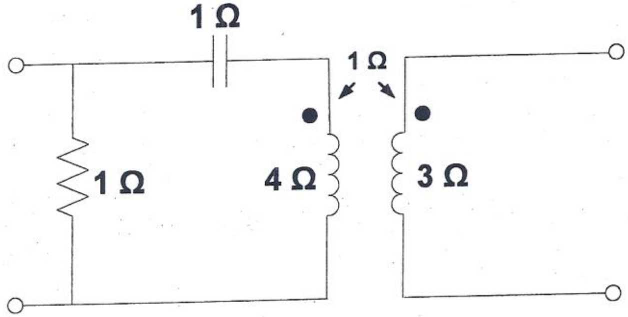
Department of Electronics & Electrical Communications

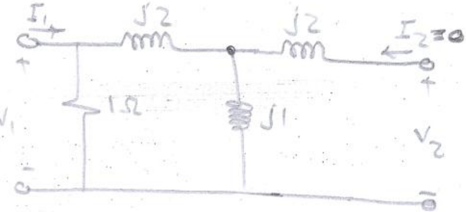
Question[6]:

Determine the transmission A-parameters for the network shown in figure and write its 2-port equations in the A-form.
Insert your results in Table 6.



$$V_1 = a_{11} V_2 - a_{12} I_2$$

$$I_1 = a_{21} V_2 - a_{22} I_2$$



ⓐ $I_2 = 0$

$V_1 = a_{11} V_2$, $I_1 = a_{21} V_2$

$$V_1 = I_1 * \frac{j3}{1+j3} \rightarrow (1)$$

$$V_1 = j2 * I_1 * \frac{1}{1+j3} + V_2$$

$$V_1 = V_1 * \frac{\cancel{(1+j3)}}{j3} * \frac{j2}{\cancel{(1+j3)}} + V_2 \qquad \therefore \frac{V_1}{3} = V_2$$

$$\boxed{\therefore a_{11} = \frac{V_1}{V_2} = 3}$$

in eqn ①

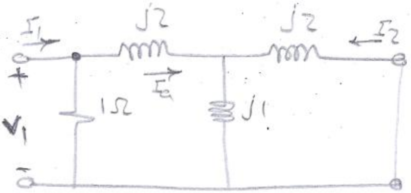
$$V_1 = I_1 * \frac{j3}{1+j3}$$

$$3V_2 = I_1 * \frac{j3}{1+j3}$$

$$a_{21} = \frac{I_1}{V_2} = \frac{1+j3}{j}$$

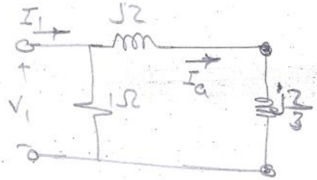
$$\boxed{\therefore a_{21} = (3-j)\ \text{mho}}$$

@ $V_2 = 0$



$$V_1 = -a_{12} I_2$$

$$I_1 = -a_{22} I_2$$



$$I_a = I_1 * \frac{1}{1 + j\frac{8}{3}}$$

$$I_a = \frac{3I_1}{3+j8}$$

$$I_2 = -I_a * \frac{j1}{j3} = \frac{-I_a}{3}$$

$$\therefore I_2 = \frac{-I_1}{3+j8}$$

$$a_{22} = \frac{-I_1}{I_2} \qquad \boxed{\therefore a_{22} = 3+j8}$$

$$V_1 = j\frac{8}{3} I_a = -j8 I_2$$

$$a_{12} = \frac{-V_1}{I_2} = j8 \qquad \boxed{\therefore a_{12} = j8\ \Omega}$$

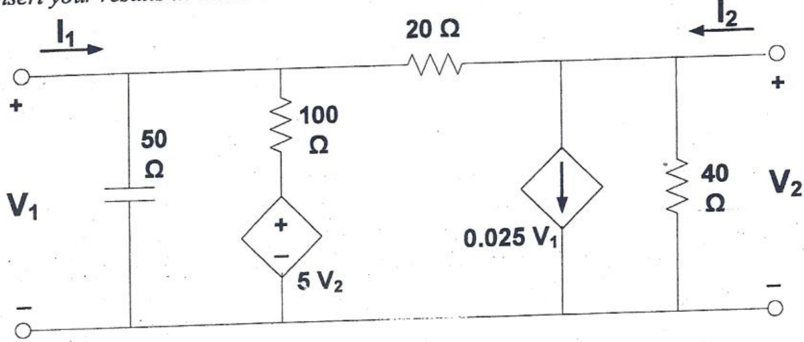
$$\therefore [A] = \begin{bmatrix} 3 & j8 \\ 3-j & 3+j8 \end{bmatrix}$$

[A]	$\begin{bmatrix} 3 & j8 \\ 3-j & 3+j8 \end{bmatrix}$
Transmission Equations	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 3 & j8 \\ 3-j & 3+j8 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

Table 6

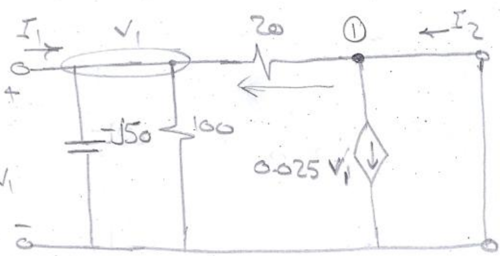
Question[7]:

For the 2-port network shown in figure, find the transmission A-parameters. Insert your results in Table 7.



$$V_1 = a_{11} V_2 - a_{12} I_2$$

$$I_1 = a_{21} V_2 - a_{22} I_2$$



at $V_2 = 0$

$$V_1 = -a_{12} I_2$$

$$I_1 = -a_{22} I_2$$

kcl at 1

$$I_2 = 0.025 V_1 + \frac{0 - V_1}{20} \Rightarrow I_2 = -0.025 V_1$$

$$\frac{-V_1}{I_2} = a_{12} \qquad \boxed{\therefore a_{12} = 40\,\Omega} \qquad \Rightarrow V_1 = -40 I_2$$

$$V_1 = I_1 (20 // 100 // -j50) = I_1 (15 - j5)$$

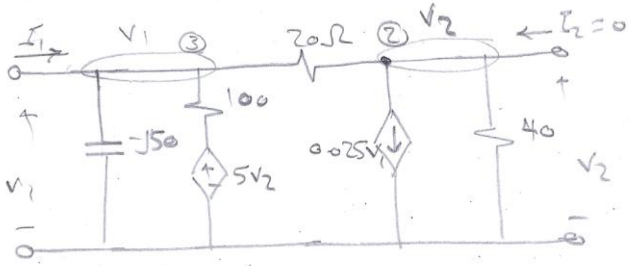
$$-40 I_2 = I_1 (15 - j5)$$

$$a_{22} = \frac{-I_1}{I_2} = \frac{40}{15 - j5} \Rightarrow \boxed{a_{22} = (2.4 + j0.8)}$$

at $I_2 = 0$

$V_1 = a_{11} V_2$

$I_1 = a_{21} V_2$



KCL at point ②

$$\frac{V_2}{40} + 0.025V_1 + \frac{V_2 - V_1}{20} = 0$$

$$\frac{3V_2}{40} = \frac{V_1}{40} \qquad \therefore V_1 = 3V_2$$

$$\boxed{\therefore a_{11} = 3}$$

KCL at ③

$$I_1 = \frac{V_1}{-j50} + \frac{V_1 - 5V_2}{100} + \frac{V_1 - V_2}{20}$$

$$I_1 = V_1\left(\frac{3}{50} + \frac{j}{50}\right) - 0.1V_2$$

$$I_1 = V_2\left(\frac{9}{50} + \frac{j3}{50}\right) - 0.1V_2$$

$$I_1 = V_2\,(0.08 + j0.06)$$

$$\boxed{\therefore a_{21} = (0.08 + j0.06)\ \mho}$$

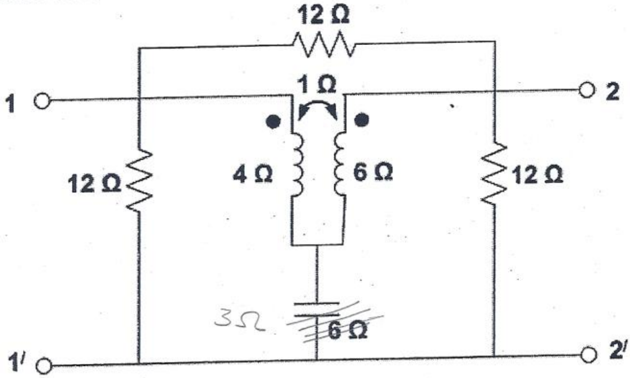
$$\therefore [A] = \begin{bmatrix} 3 & 40 \\ 0.08 + j0.06 & 2.4 + j0.8 \end{bmatrix}.$$

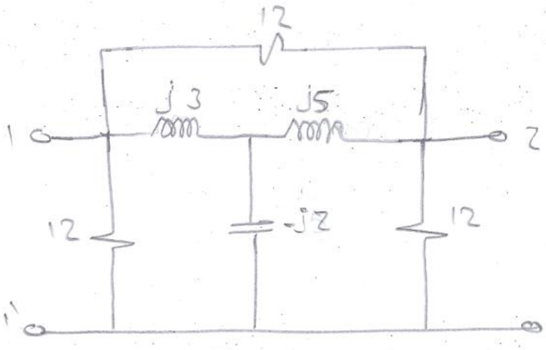
[redacted]	$\begin{bmatrix} 3 & 40 \\ 0.08+j0.06 & 2.4+j0.8 \end{bmatrix}$

Table 7

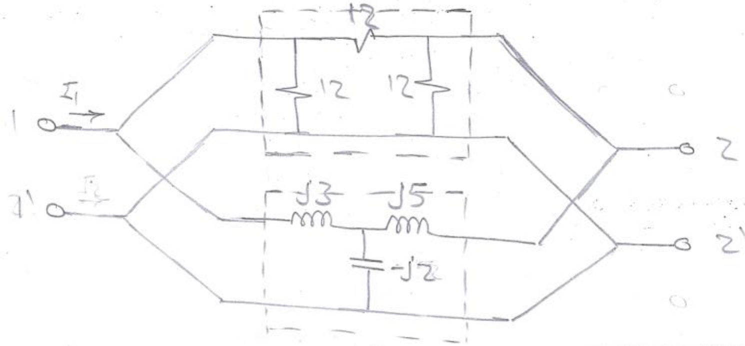
Question[8]:

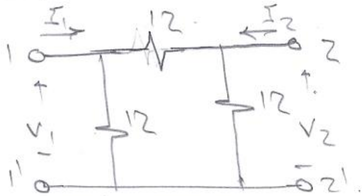
Simplify the network shown in figure into two 2-port networks connected in parallel. Hence, calculate the Y-parameters' matrix of the whole network.
Insert your results in Table 8.





$$[Y_{eq}] = [Y_1] + [Y_2]$$

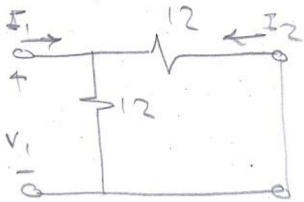




$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

<u>at $V_2 = 0$</u>

$$I_1 = Y_{11} V_1$$
$$I_2 = Y_{21} V_1$$



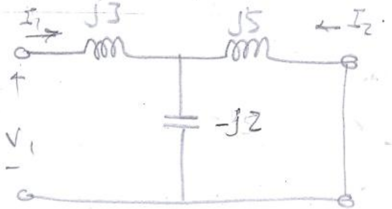
$$V_1 = 12 * I_1 * \frac{12}{24}$$
$$V_1 = 6 I_1 \qquad \boxed{\therefore Y_{11} = \frac{1}{6}\ \mho}$$

$$V_1 = -12 I_2 \qquad \boxed{\therefore Y_{21} = \frac{-1}{12}\ \mho}$$

the network is symmetrical and reciprocal

$$\therefore Y_{11} = Y_{22} = \frac{1}{6}\ \mho$$
$$Y_{12} = Y_{21} = \frac{-1}{12}\ \mho$$

$$\therefore [Y_1] = \begin{bmatrix} \frac{1}{6} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{1}{6} \end{bmatrix}$$



$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

@ $V_2 = 0$

$I_1 = Y_{11} V_1$

$I_2 = Y_{21} V_1$

$V_1 = j3 * I_1 - j2 * I_1 * \frac{j5}{j5 - j2} = \frac{1}{3j} I_1$

$\boxed{\therefore Y_{11} = 3j}$

$I_2 = -I_1 * \frac{-j2}{j5 - j2}$

$I_2 = \frac{2 I_1}{3}$

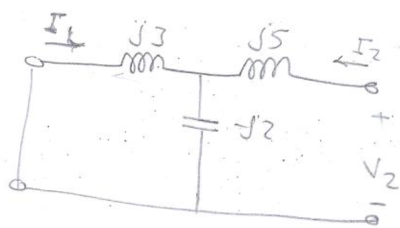
$\therefore I_1 = \frac{3 I_2}{2}$

$V_1 = (j3 * I_1) + (j5 * (-I_2))$

$V_1 = \left(\frac{9j}{2} - j5\right) I_2$ $\boxed{Y_{21} = 2j}$

the network is reciprocal $\rightarrow Y_{12} = Y_{21} = 2j$

at $V_1 = 0$



$I_1 = Y_{12} V_2$

$I_2 = Y_{22} V_2$

$V_2 = j5 I_2 - j2 * I_2 * \frac{j3}{j}$

$V_2 = -j I_2$

$\boxed{\therefore Y_{22} = j \ \mho}$ $\therefore [Y_2] = \begin{bmatrix} 3j & 2j \\ 2j & j \end{bmatrix}$

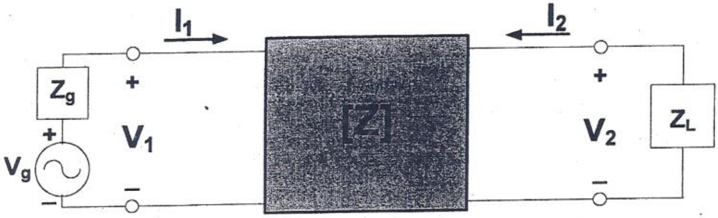
$$\therefore [Y_{eq}] = \begin{bmatrix} \frac{1}{6}+3j & \frac{-1}{12}+2j \\ \frac{-1}{12}+2j & \frac{1}{6}+j \end{bmatrix}$$

[Y]	$\begin{bmatrix} (\frac{1}{6}+3j) & (\frac{-1}{12}+2j) \\ (\frac{-1}{12}+2j) & (\frac{1}{6}+j) \end{bmatrix}$

Table 8

Question[9]:

Using the Z-parameters, find Thevenin's equivalent circuit with respect to port 2 for the 2-port network shown in figure.
Insert your results in Table 9.



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$
$$V_1 = V_g - I_1 Z_g$$
$$V_2 = - I_2 Z_L$$

$$V_{th} = V_2 \Big|_{I_2 = 0} \Rightarrow \therefore \text{at } I_2 = 0 \Rightarrow \quad V_1 = Z_{11} I_1, \quad V_2 = Z_{21} I_1, \quad V_1 = V_g - I_1 Z_g$$

$$\therefore V_g - I_1 Z_g = Z_{11} I_1$$
$$V_g = I_1 (Z_{11} + Z_g)$$
$$\therefore I_1 = \frac{V_g}{Z_{11} + Z_g}$$
$$V_2 = Z_{21} I_1 = \frac{Z_{21} V_g}{Z_{11} + Z_g}$$

$$\boxed{\therefore V_{th} = \left(\frac{Z_{21}}{Z_{11} + Z_g} \right) V_g}$$

$$Z_{th} = \left.\frac{V_2}{I_2}\right|_{V_g=0}$$

$$V_1 = -I_1 Z_g$$

$$\therefore \quad -I_1 Z_g = Z_{11} I_1 + Z_{12} I_2$$

$$-I_1 (Z_{11} + Z_g) = Z_{12} I_2$$

$$\therefore I_1 = \frac{-Z_{12}}{Z_{11} + Z_g} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$= \left(\frac{-Z_{12} Z_{21}}{Z_{11} + Z_g} + Z_{22} \right) I_2$$

$$\therefore Z_{th} = Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11} + Z_g}$$

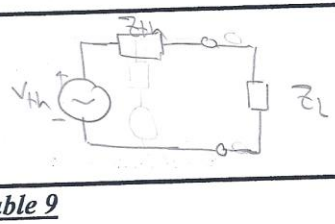
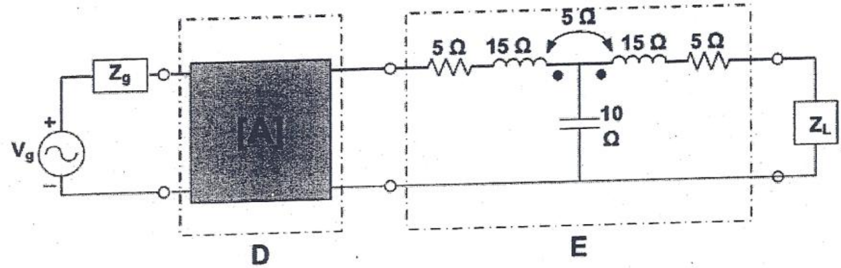
V_{th}	$\left(\dfrac{Z_{21}}{Z_{11}+Z_g}\right) V_g$
Z_{th}	$Z_{22} - \dfrac{Z_{12} Z_{21}}{Z_{11}+Z_g}$
Equiv. Cct.	

Table 9

Question[10]:

*The networks **D** and **E** in the circuit shown in figure are reciprocal and symmetrical networks. **D** has $a_{11}=5$ and $a_{12}=24\ \Omega$. The impedance Z_L is adjusted for maximum average power transfer. Find Z_L if $Z_g = 5\ \Omega$.*
Insert your results in Table 10.



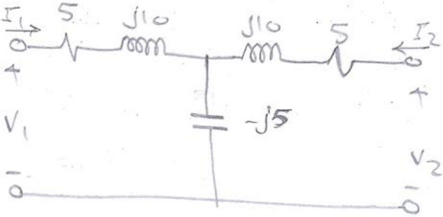
For network D

Symm. $\Rightarrow a_{11} = a_{22} = 5$

reciprocal $\Rightarrow a_{11} a_{22} - a_{12} a_{21} = 1$ $\quad \therefore a_{21} = 1\ \mho$

$$\therefore [A_D] = \begin{bmatrix} 5 & 24\Omega \\ 1\mho & 5 \end{bmatrix}$$

For network E



$V_1 = a_{11} V_2 - a_{12} I_2$
$I_1 = a_{21} V_2 - a_{22} I_2$

@ $I_2 = 0$

$V_1 = a_{11} V_2$
$I_1 = a_{21} V_2$

$$V_2 = V_1 * \frac{-j5}{5+j5}$$

$$\boxed{\therefore a_{11} = -1+j}$$

$V_1 = (5+j10)I_1 + V_2$

$(-1+j)V_2 = (5+j10)I_1 + V_2$

$\therefore (-2+j)V_2 = (5+j10)I_1$ $\quad \boxed{\therefore a_{21} = 0.2j \ \mho}$

the network is symm. $\Rightarrow \boxed{a_{11} = a_{22} = (-1+j)}$

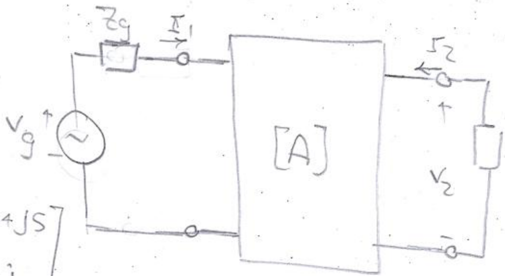
reciprocal $\Rightarrow a_{11}a_{22} - a_{12}a_{21} = 1$

$$\boxed{\therefore a_{12} = (-10+j5)\ \Omega}$$

$$\therefore [A_E] = \begin{bmatrix} -1+j & -10+j5 \\ 0.2j & -1+j \end{bmatrix}$$

$[A] = [A_D][A_E]$

$$= \begin{bmatrix} 5 & 24 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} -1+j & -10+j5 \\ 0.2j & -1+j \end{bmatrix}$$



$$[A] = \begin{bmatrix} (-5+9.8j) & (-74+49j) \\ (-1+2j) & (-15+10j) \end{bmatrix}$$

$$V_1 = a_{11} V_2 - a_{12} I_2$$

$$I_1 = a_{21} V_2 - a_{22} I_2$$

$$V_1 = V_g - I_1 Z_g$$

$$V_2 = - I_2 Z_L$$

$$Z_{th} = \left. \frac{V_2}{I_2} \right|_{V_g = 0}$$

at $V_g = 0$ $\quad \therefore V_1 = - I_1 Z_g$

$$\therefore - I_1 Z_g = a_{11} V_2 - a_{12} I_2$$

$$\therefore -Z_g (a_{21} V_2 - a_{22} I_2) = a_{11} V_2 - a_{12} I_2$$

$$-(Z_g a_{21} + a_{11}) V_2 = -(Z_g a_{22} + a_{12}) I_2$$

$$\frac{V_2}{I_2} = \frac{Z_g a_{22} + a_{12}}{Z_g a_{21} + a_{11}}$$

$$\therefore Z_{th} = \frac{Z_g a_{22} + a_{12}}{Z_g a_{21} + a_{11}}$$

to receive max. Power $\quad Z_L = Z_{th}^*$

$$Z_{th} = 8.06 \underline{/29.6^\circ} \ \Omega$$

$$\therefore Z_L = 8.06 \underline{/-29.6^\circ} \ \Omega$$

Question[1]:

***a)** Derive an expression of the input and output impedances of the two-port network shown in figure in terms of the H-parameters of the network, the internal impedance of the source Z_g and the load impedance Z_L.*

***b)** If the network parameters are given by :*

$$[H]=\begin{bmatrix} 1500\Omega & 10^{-3} \\ 50 & 50\mu\, mho \end{bmatrix}$$

$$Z_g = 1500\,\Omega$$

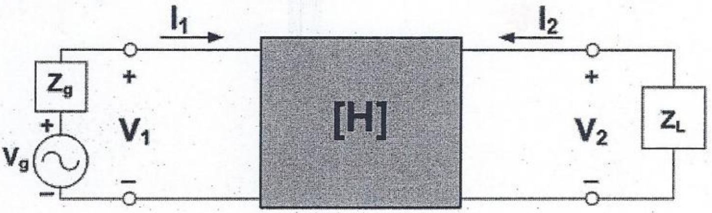
$$V_g = 250\angle 0^o\ V$$

Calculate:

***i)** The maximum average power delivered to the load.*

***ii)** The average power delivered to the input port of the network at the condition of maximum power transfer to the load.*

Insert your results in Table 1.



$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$

$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$

$V_1 = V_g - I_1 Z_g \rightarrow (3)$

$V_2 = -I_2 Z_L \rightarrow (4)$

From (4) → (2)

$I_2 = h_{21} I_1 - h_{22} Z_L I_2$

$I_2 = \dfrac{h_{21} I_1}{1 + h_{22} Z_L}$

$\therefore V_2 = \dfrac{-h_{21} Z_L I_1}{1 + h_{22} Z_L} \rightarrow (5)$

in eq. (1) $V_1 = h_{11} I_1 - \dfrac{h_{12} h_{21} Z_L}{1 + h_{22} Z_L} I_1$

$$\boxed{\therefore Z_{in} = \frac{V_1}{I_1} = h_{11} - \frac{h_{12} h_{21} Z_L}{1 + h_{22} Z_L}}$$

Z_L		$\therefore Z_{in} = \dfrac{h_{11} + Z_L \lvert H \rvert}{1 + h_{22} Z_L}$

Table 10

$$Z_o = \left.\frac{V_2}{I_2}\right|_{V_g=0} \qquad \text{at } V_g = 0 \Rightarrow V_1 = -I_1 Z_g$$

$\therefore$ in equation ① $\quad -I_1 Z_g = h_{11} I_1 + h_{12} V_2$

$$\therefore I_1 = \frac{-h_{12}}{h_{11}+Z_g} V_2$$

in equation ② $\quad I_2 = \dfrac{-h_{12} h_{21}}{h_{11}+Z_g} V_2 + h_{22} V_2$

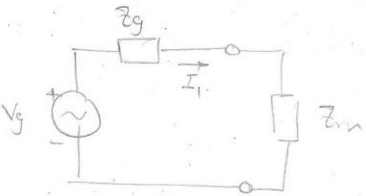
$$I_2 = \left(\frac{|H| + h_{22} Z_g}{h_{11}+Z_g}\right) V_2$$

$$\therefore Z_o = \frac{h_{11} + Z_g}{|H| + h_{22} Z_g}$$

$$Z_o = Z_L \Big|_{\text{at max Power transfer}}$$

$\therefore Z_L = 30 k\Omega$

$\therefore Z_{in}\Big|_{\text{max Power}} = 900 \Omega$



$$I_1 = \frac{V_g}{Z_g + Z_{in}} = 0.104 \text{ A}$$

From equation ⑤

$$V_2 = \frac{-h_{21} Z_L I_1}{1 + h_{22} Z_L} \qquad \therefore V_2 = -62400 \text{ Volts}$$

$$P_{L\,max} = \frac{|V_2|^2}{Z_L} = 130 \text{ K watts}$$

$$P_{in} = |I_1|^2 Z_{in}$$

$$= 9.7344 \text{ watts}$$

Z_{in}	$\frac{h_{11} + Z_L \lvert H \rvert}{1 + h_{22} Z_L}$
Z_{out}	$\frac{h_{11} + Z_g}{\lvert H \rvert + h_{22} Z_g}$
P_L	130 k watts
P_{in}	9.7344 watts

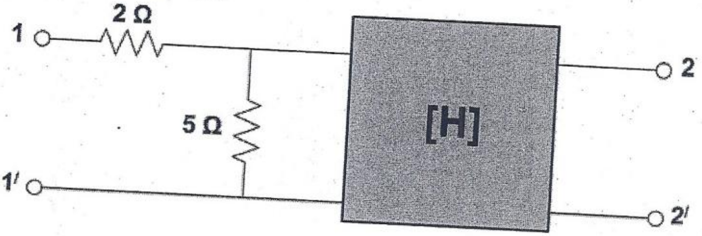
Table 1

Question[2]:

The H-parameters of a two-port network are given by the matrix :

$$[H]=\begin{bmatrix}2\,\Omega & 0.2\\ -4 & 0.1\,mho\end{bmatrix}$$

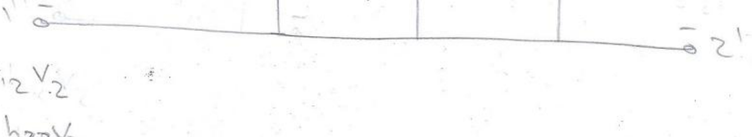
a) *Check the conditions of symmetry and reciprocity of the network.*
b) *Sketch its equivalent circuit.*
c) *If two resistors of 2 Ω and 5 Ω are added to the network as shown in figure, calculate the H-parameters of the total network.*
Insert your results in Table 2.



a) For reciprocity $h_{12} = -h_{21}$ ⇒ not satisfied
∴ the network is not reciprocal.

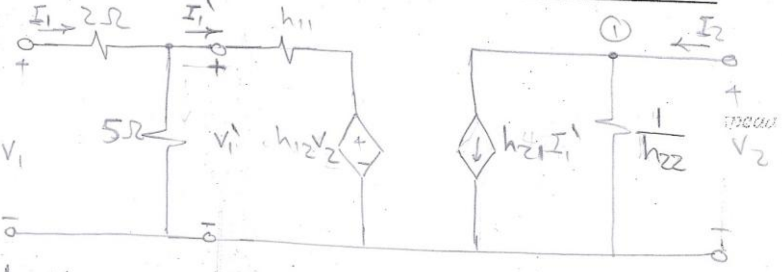
For Symmetry $|H| = 1$ ⇒ $2 * 0.1 + 0.2 * 4 = 1$ Satisfied
∴ the network is symmetric

b)



$V_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{21} I_1 + h_{22} V_2$

C



$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

① at $I_1 = 0$ $\qquad V_1 = h_{12} V_2$

$V_1 = V_1' \qquad I_2 = h_{22} V_2$

$$V_1 = h_{12} V_2 * \frac{5}{5+h_{11}} \Rightarrow h_{12_{eq}} = \frac{V_1}{V_2} = \frac{5\, h_{12}}{5+h_{11}}$$

$$\boxed{h_{12_{eq}} = \frac{1}{7}}$$

KCL at node ①

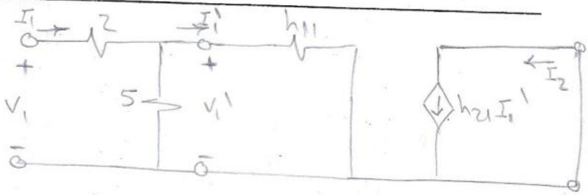
$$I_2 = h_{22} V_2 + h_{21} I_1'$$

$$I_1' = \frac{-h_{12} V_2}{5+h_{11}}$$

$$\therefore I_2 = \left(h_{22} - \frac{h_{12} h_{21}}{5+h_{11}}\right) V_2 \Rightarrow h_{22} = \frac{I_2}{V_2}$$

$$\therefore h_{22_{eq}} = h_{22} - \frac{h_{12} h_{21}}{5+h_{11}} \Rightarrow \boxed{h_{22_{eq}} = \frac{3}{14}\ \mho}$$

at $V_2 = 0$



$V_1 = h_{11} I_1$

$I_2 = h_{21} I_1$

$$V_1 = I_1 * \left[2 + (5 // h_{11}) \right] \qquad h_{11} = \frac{V_1}{I_1}$$

$$\therefore h_{11_{eq}} = 2 + (5 // h_{11}) \Rightarrow \boxed{h_{11_{eq}} = \frac{24}{7}\ \Omega}$$

$$I_2 = h_{21} I_1'$$

$$I_1' = I_1 * \frac{5}{5 + h_{11}}$$

$$\therefore I_2 = I_1 * \frac{5 h_{21}}{5 + h_{11}} \qquad h_{21} = \frac{I_2}{I_1}$$

$$\therefore h_{21_{eq}} = \frac{5 h_{21}}{5 + h_{11}} \Rightarrow \boxed{h_{21_{eq}} = \frac{-20}{7}}$$

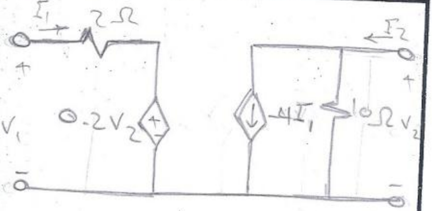
Is the net. sym. ?	yes
Is the net. recip. ?	no
Equivalent Circuit	
[H]	$\begin{bmatrix} \frac{24}{7} & \frac{1}{7} \\ \frac{-20}{7} & \frac{3}{14} \end{bmatrix}$

Table 2

Question[3]:

a) *Calculate the transmission A-parameters of a two-port network as a function of its Hybrid H-parameters.*

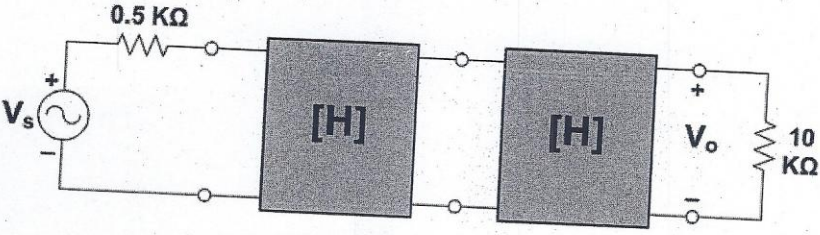
b) *The amplifier circuit shown in figure is formed of two identical stages each having the Hybrid H-parameters given by :*

$$[H]=\begin{bmatrix} 1\,K\Omega & 0.001 \\ 100 & 0.1\,m.mho \end{bmatrix}$$

$|H| = 0$

What is the value of the input voltage V_s required to obtain an output voltage V_o of 25 volts ?

Insert your results in Table 3.



$V_1 = a_{11} V_2 - a_{12} I_2$ | $V_1 = h_{11} I_1 + h_{12} V_2$

$I_1 = a_{21} V_2 - a_{22} I_2$ | $I_2 = h_{21} I_1 + h_{22} V_2$

at $V_2 = 0$

$V_1 = h_{11} I_1$

$I_2 = h_{21} I_1$

$$a_{12} = -\frac{V_1}{I_2} = \frac{-h_{11} I_1}{h_{21} I_1} = \frac{-h_{11}}{h_{21}}$$

$$a_{22} = \frac{-I_1}{I_2} = \frac{-1}{h_{21}}$$

at $I_2 = 0$

$V_1 = h_{11} I_1 + h_{12} V_2$

$h_{21} I_1 = -h_{22} V_2$

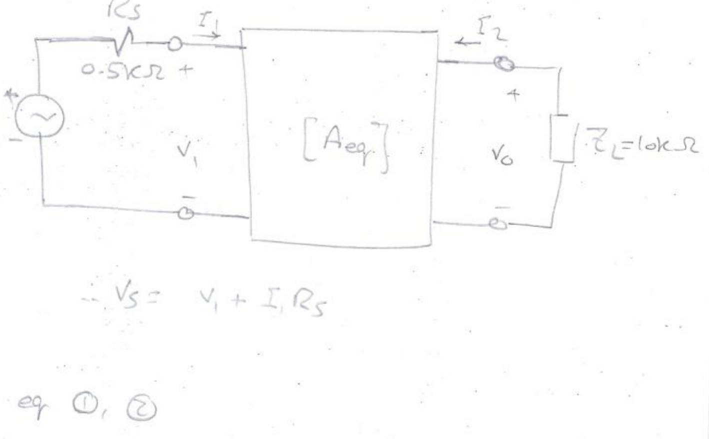
$$a_{21} = \frac{I_1}{V_2} = \frac{-h_{22}}{h_{21}}$$

$$V_1 = \frac{1 - h_{11}h_{22}}{h_{21}} V_2 + h_{12}V_2 = \frac{-|H|}{h_{21}} V_2$$

$$a_{11} = \frac{V_1}{V_2} = \frac{-|H|}{h_{21}}$$

$$\therefore [A] = \begin{bmatrix} 0 & -10 \\ -10^{-6} & -0.01 \end{bmatrix}$$

$$[A_{eq}] = [A][A] = \begin{bmatrix} 10^{-5} & 0.1 \\ 10^{-8} & 1.1 * 10^{-4} \end{bmatrix}$$



$V_1 = a_{11}V_2 - a_{12}I_2 \rightarrow ①$

$I_1 = a_{21}V_2 - a_{22}I_2 \rightarrow ②$

$V_2 = -I_2 Z_L \rightarrow ③$

$V_1 = V_S - I_1 R_S \rightarrow ④ \qquad \therefore V_S = V_1 + I_1 R_S$

sub. from eq ③ in eq ①, ②

$$V_1 = a_{11} V_2 + \frac{a_{12}}{Z_L} V_2 \qquad \therefore V_1 = \left(a_{11} + \frac{a_{12}}{Z_L}\right) V_2$$

$$V_1 = 5 * 10^{-4} \text{ volts}$$

$I_1 = a_{21} V_2 + \frac{a_{22}}{Z_L} V_2 \qquad \therefore I_1 = \left(a_{21} + \frac{a_{22}}{Z_L}\right) V_2$

$\therefore I_1 = 5.25 \times 10^{-7} A$

$V_S = V_1 + I_1 R_S$

$V_S = 7.625 \times 10^{-4}$ Volts

[A]	$\begin{bmatrix} \frac{-\lvert H \rvert}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
V_s	7.625×10^{-4} Volts

Table 3

Question[4]:

a) *Two sets of measurements were made on the two-port network **N** shown in figure .The results were as follows :*

Measurement 1

V_1=20 mV
I_1=20 μA
V_2=40 V
I_2=0

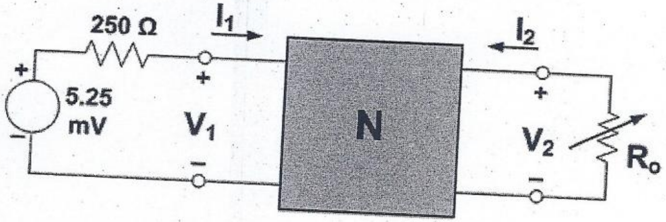
Measurement 2

V_1=4 V
I_1=5 mA
V_2=0
I_2=-200 mA

Find the H-parameters representing the two-port network.

b) *Using the H-parameters, calculate the value of the variable resistance* R_o *when adjusted to receive maximum power and find the value of this power.*

Insert Your results in Table 4.



$V_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{21} I_1 + h_{22} V_2$

For Measurement 2

$V_2 = 0$ $\quad \therefore V_1 = h_{11} I_1$
$I_2 = h_{21} I_1$

$\therefore h_{11} = \frac{V_1}{I_1} = 800\,\Omega$
$h_{21} = \frac{I_2}{I_1} = \frac{-200}{5} = -40$

For Measurement 1

$I_2 = 0$

$V_1 = h_{11} I_1 + h_{12} V_2$
$h_{21} I_1 = -h_{22} V_2$

$h_{12} = \frac{V_1 - h_{11} I_1}{V_2} = 10^{-4}$
$h_{22} = \frac{-h_{21} I_1}{V_2} = 2 \times 10^{-5}$

$$\therefore [H] = \begin{bmatrix} 800\,\Omega & 10^{-4} \\ -40 & 2\times 10^{-5}\,\mho \end{bmatrix}$$

$\boxed{b}$ $V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow (1)$

$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow (2)$

$V_2 = -I_2 Z_L \longrightarrow (3)$

$V_1 = V_g - I_1 Z_g \longrightarrow (4)$

to receive Maximum power $\quad Z_L = Z_{th}^*$

$Z_{th} = \left.\dfrac{V_2}{I_2}\right|_{V_g=0} \qquad$ at $V_g = 0 \Rightarrow V_1 = -I_1 Z_g$

at equation (1) $\quad -I_1 Z_g = h_{11} I_1 + h_{12} V_2$

$$-I_1 (Z_g + h_{11}) = h_{12} V_2$$

$$\therefore I_1 = \frac{-h_{12} V_2}{h_{11} + Z_g}$$

at equation (2)

$$I_2 = \frac{-h_{12} h_{21} V_2}{h_{11} + Z_g} + h_{22} V_2$$

$$I_2 = \left(\frac{h_{11} h_{22} - h_{12} h_{21} + h_{22} Z_g}{h_{11} + Z_g}\right) V_2 = \left(\frac{|H| + h_{22} Z_g}{h_{11} + Z_g}\right) V_2$$

$$Z_{th} = \frac{V_2}{I_2} = \frac{h_{11} + Z_g}{|H| + h_{22} Z_g} = 42\,k\Omega$$

$$\boxed{\therefore R_o = 42\ k\Omega}$$

$$V_{th} = V_2 \Big|_{I_2=0} \qquad \therefore h_{21} I_1 = -h_{22} V_2$$

$$\therefore I_1 = \frac{-h_{22}}{h_{21}} V_2$$

in equation ① $\quad V_1 = h_{11} I_1 + h_{12} V_2$

$$V_g - I_1 Z_g = h_{11} I_1 + h_{12} V_2$$

$$V_g = I_1 (h_{11} + Z_g) + h_{12} V_2$$

$$V_g = V_2 \left[h_{12} - \frac{h_{22}(h_{11} + Z_g)}{h_{21}} \right]$$

$$V_g = V_2 \left[\frac{h_{12} h_{21} - h_{22} h_{11} - h_{22} Z_g}{h_{21}} \right]$$

$$V_g = -V_2 \left[\frac{|H| + h_{22} Z_g}{h_{21}} \right]$$

$$\therefore V_2 = \frac{-h_{21} V_g}{|H| + h_{22} Z_g} = 8.4 \text{ volts}$$

$$\therefore V_{th} = 8.4 \text{ volts}$$

$$P_{max} = \frac{|V_{th}|^2}{4 R_{th}} = \frac{(8.4)^2}{4 * 42 * 10^3} = 4.2 * 10^{-4} \text{ watts}$$

[H]	$\begin{bmatrix} 800\,\Omega & 10^{-4} \\ -40 & 2*10^{-5}\,\mho \end{bmatrix}$
R_o	$42\,k\Omega$
$P_{max.}$	$4.2*10^{-4}$ watts

Table 4