CAIRO UNIVERSITY FACULTY OF ENGINEERING	 ELECTRIC CIRUITS	ELECTRONICS AND COMMUNICATION DEPT. 2^{nd} Year SHEET (8)

[1]For the circuit shown in Fig.1 , find the value of the current i(t) after 0.4m sec. and 1.0m sec. respectively.

[2]For the two circuits shown in Fig.2a,b , derive an expression for the current through the resistance and the voltage across it as a function of time .

[3]Derive the expression of the output voltage of the network shown in Fig.3 , v(t) . What would be its value at $t = 0$ and as t tends to ∞ .

[4]For the network shown in Fig.4 , find $v_o(t)$ for $t > 0$ using loop analysis . Verify your answer using the superposition theorem .

[5]Find $v_o(t)$, $t > 0$, for the network shown inFig.5 using node analysis .

[6]Derive an expression for the current $i_o(t)$, $t > 0$, in the network shown in Fig.6 . What would be the value of this current as t tends to ∞ .

[7]In the network shown in Fig.7 , use Thevenin's Theorem to derive an expression for $v_o(t)$, $t > 0$.

[8]In the circuit shown in Fig.8 ,find the branch currents $i_1(t)$, $i_2(t)$, $i_3(t)$, $t > 0$. Apply the initial and final value theorems to the S-domain expressions .

[9]Using the Laplace Transform Technique , find an expression for the current i(t) shown in Fig.9 , $t > 0$, hence calculate its initial and final values .

[10]For the circuit shown in Fig.10 , calculate the branch currents which will result when the switch is closed at t=0 .

[11]When the switch in the circuit shown in Fig.11 is closed , the condenser was charged such that $v_c(0) = 20$ V and the current in the coil was $i_L(0) = 2$ A . Derive expressions for the instantanuous currents $i_1(t)$, $i_2(t)$ and $i_3(t)$ for $t > 0$.

[12]In the circuit shown in Fig.12 , the capacitor is initially charged such that $v_C(0)=50V$. Calculate $v_C(t)$, $t > 0$. Using current sources only , derive an expression for the Capacitor current in the time domain $i_C(t)$, $t >$ 1 msec. Calculate also $i_C(\infty)$ and verify your result using the final value theorem .

[13]The circuit shown in Fig.13 is fed from a source v(t) given by :

$v(t) = 100\,e^{-100t}$ V

Calculate i(t) , t > 0 .

[14]For the circuit shown in Fig.14 , calculate the potentials at nodes 1 , 2 $v_1(t)$ and $v_2(t)$. Using the initial and final value theorems , calculate $v_1(0)$, $v_2(0)$, $v_1(\infty)$ and $v_2(\infty)$.

[15]Using the Laplace Transform , find $v_0(t)$ for t > 0 , in the circuit shown in Fig.15. Assume that the circuit has reached steady state before switching the source off .

[16]For the circuit shown in Fig.16 , find i(t) for t > 0 .

[17]Find theZ-Parameters of the two-port network shown in Fig.17a . If the same network is used in Fig.17b , calculate $i_2(t)$ for t > 0 .

[18]Find $v_0(t)$ for t > 0 in the circuit shown in Fig. 18 .

[19]Derive the expression of the output voltage $v_o(t)$ for the circuit shown in Fig.19a if the input is v(t) shown in :

i) Fig.19b .

ii) Fig.19c .

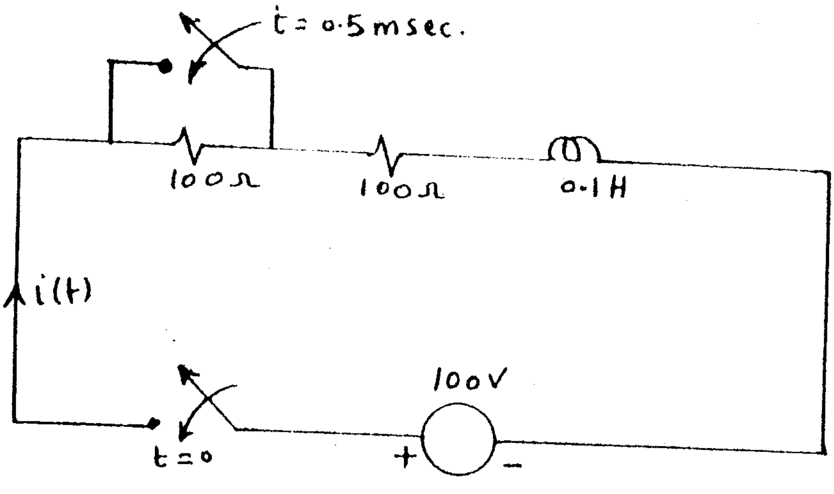


Fig. 1

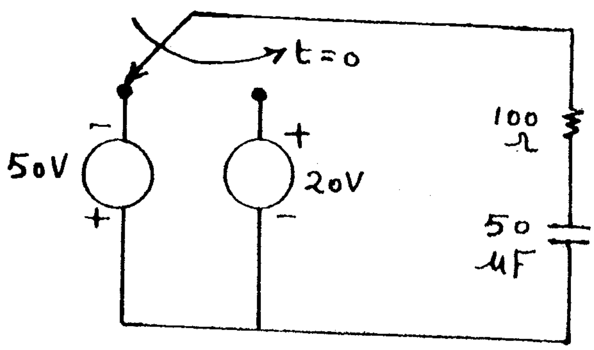


Fig. 2a

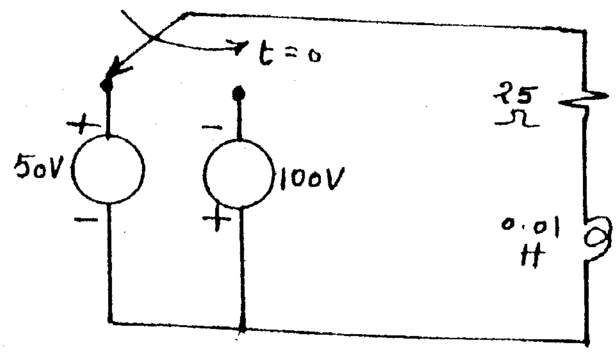


Fig. 2b

Fig. 3

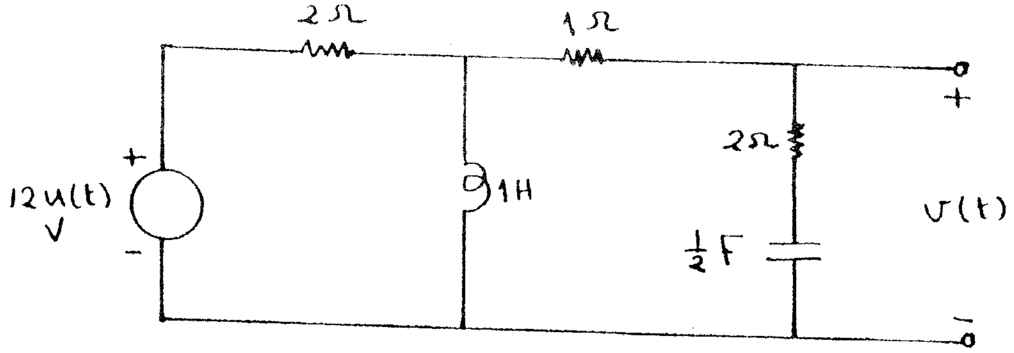


Fig. 4

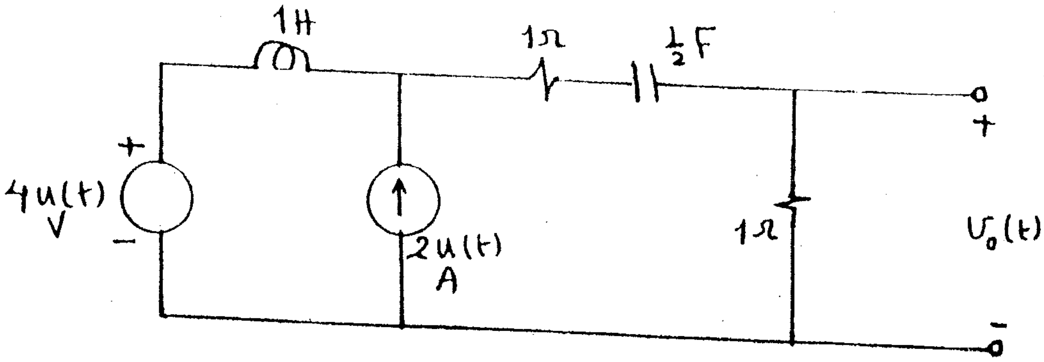


Fig. 5

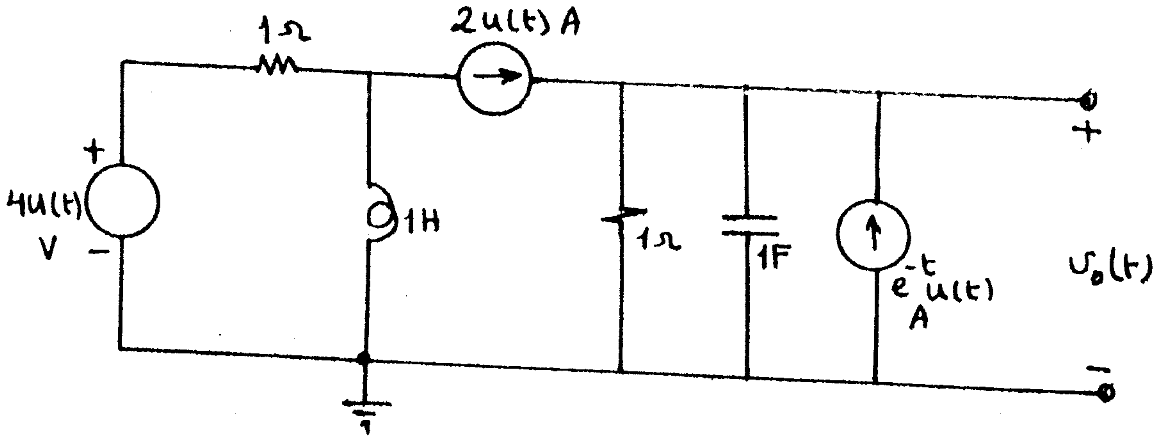
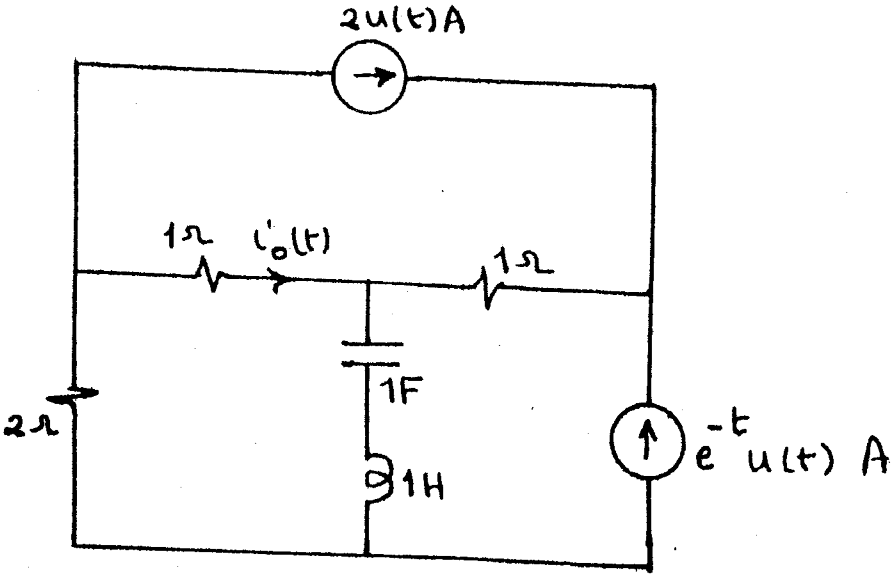


Fig. 6



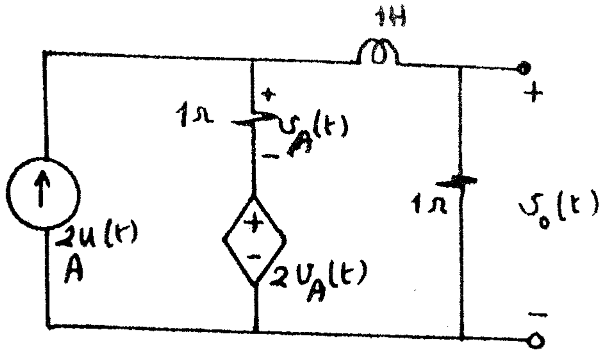


Fig.7

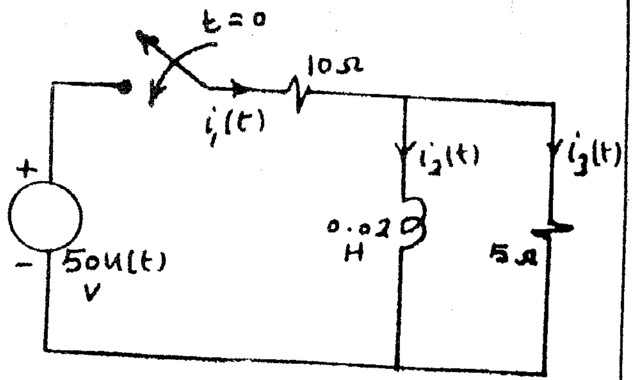


Fig.8

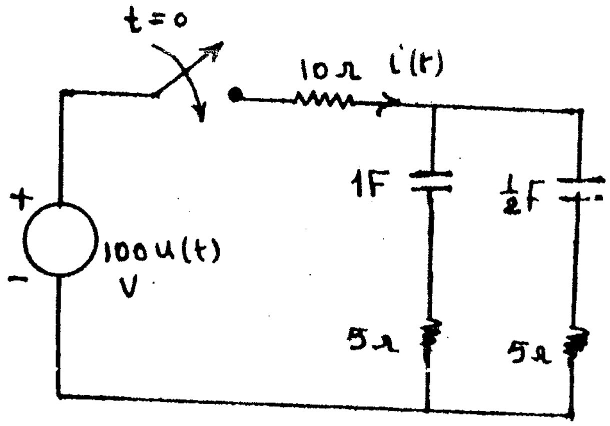


Fig.9

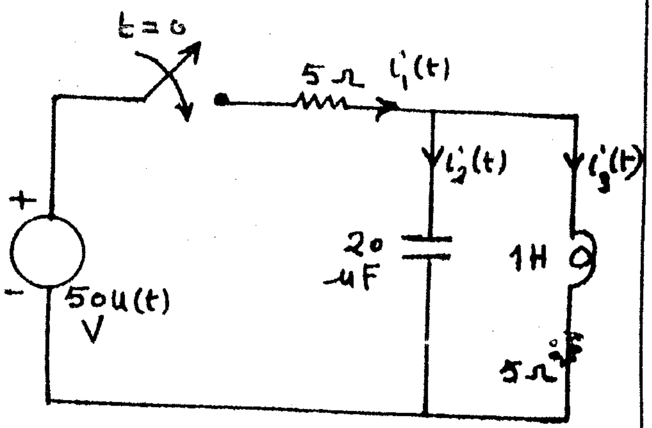


Fig.10

Fig.11

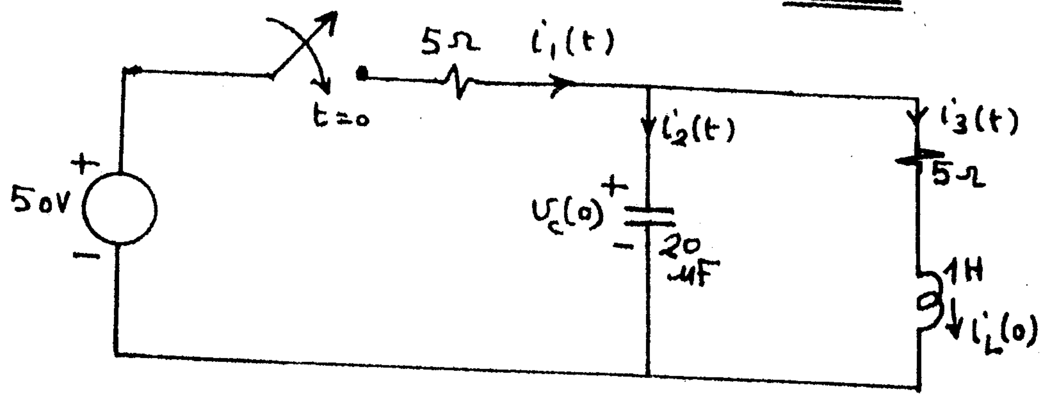


Fig.12

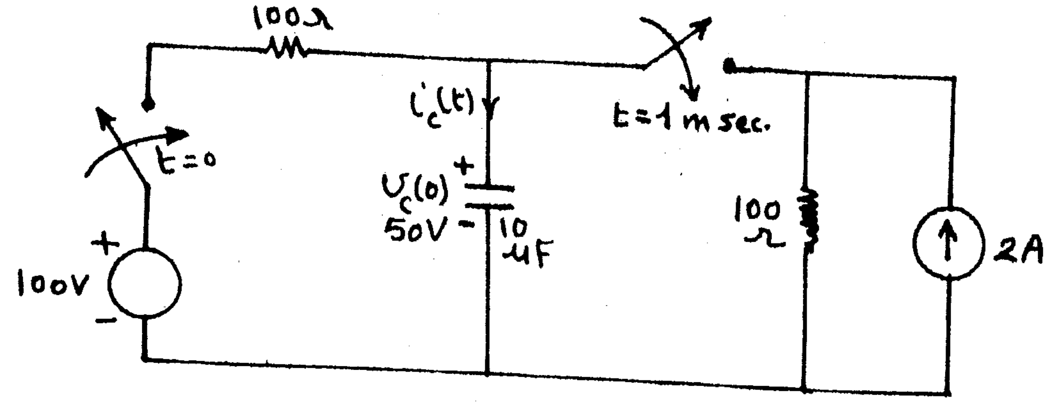


Fig. 13

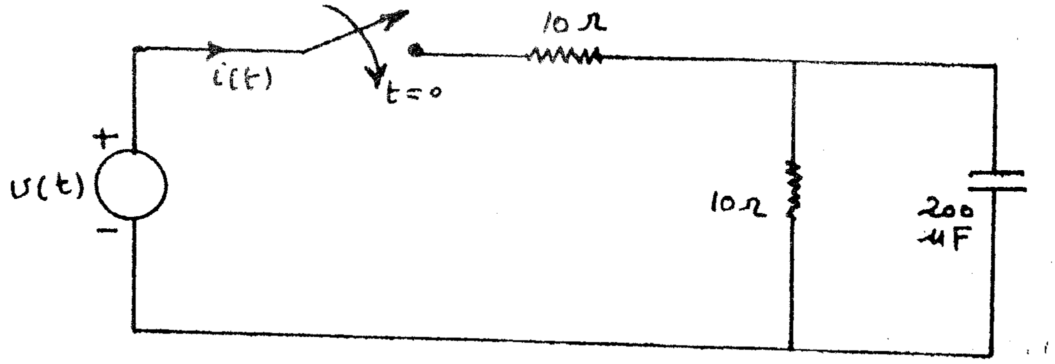


Fig. 14

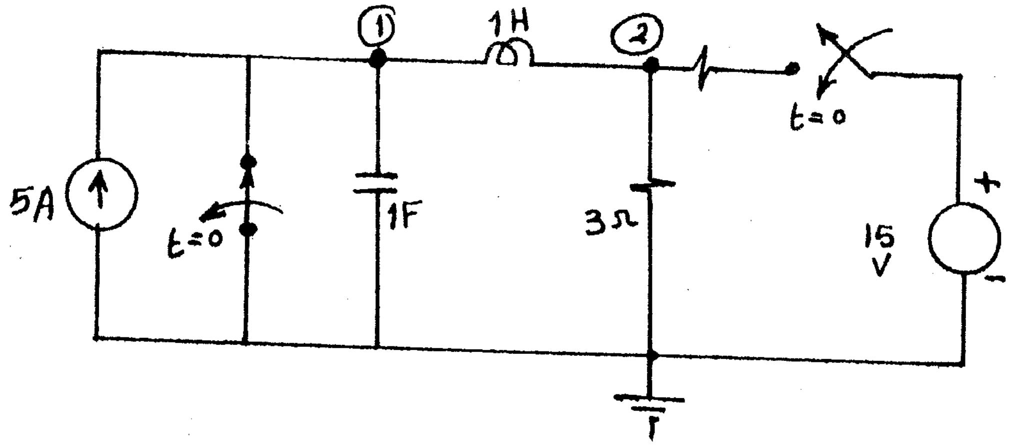


Fig. 15

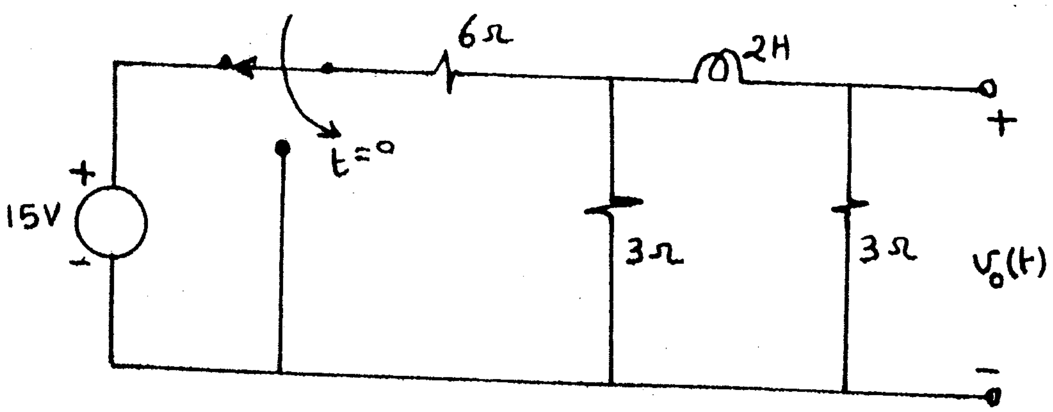
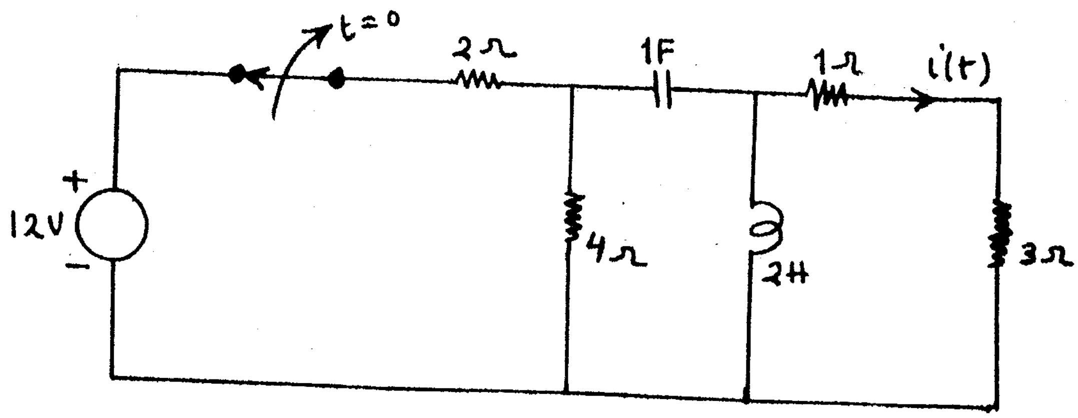


Fig. 16



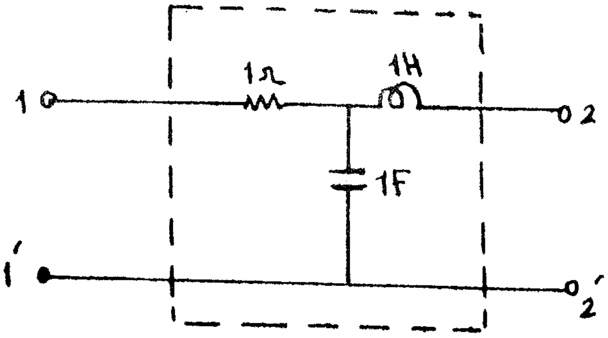


Fig. 17a

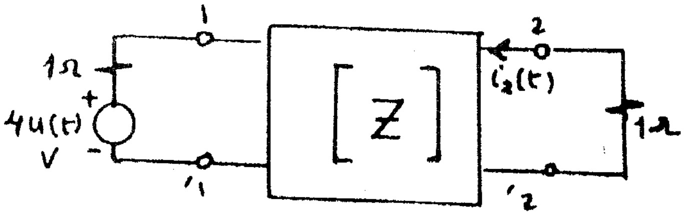


Fig. 17b

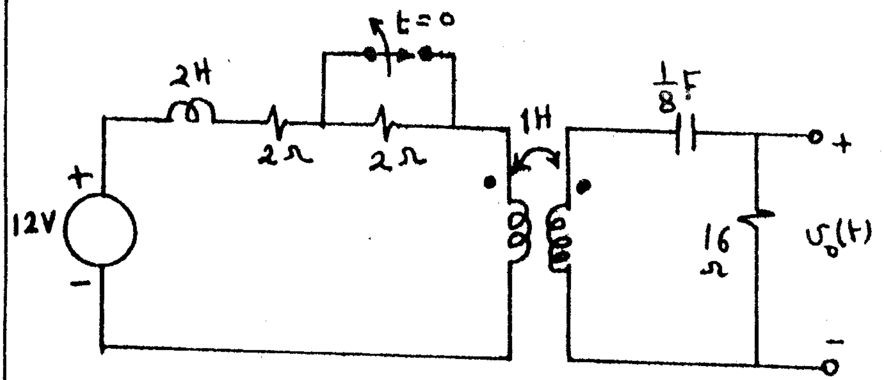


Fig. 18

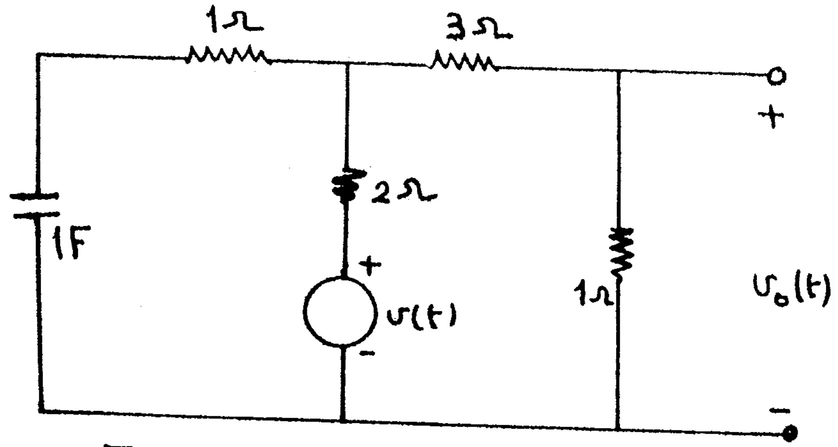


Fig. 19a

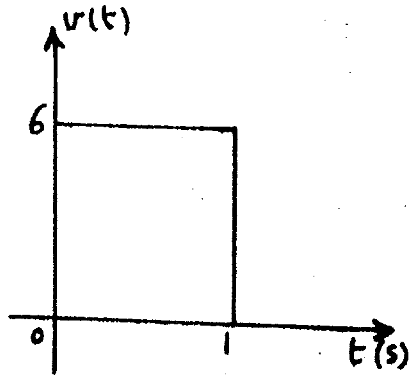


Fig. 19b

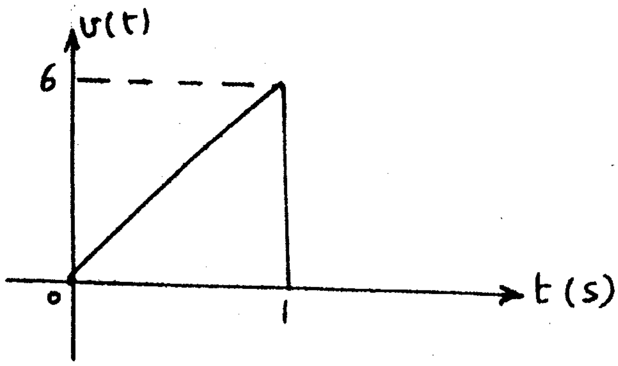


Fig. 19c

Prof. Afaf Abdel-Fattah.

Question[1]: Answer of Question 2

For the two circuits shown in Figs.a,b, derive an expression for the current through the resistance and the voltage across it as a function of time.
Insert your results in Table 1.

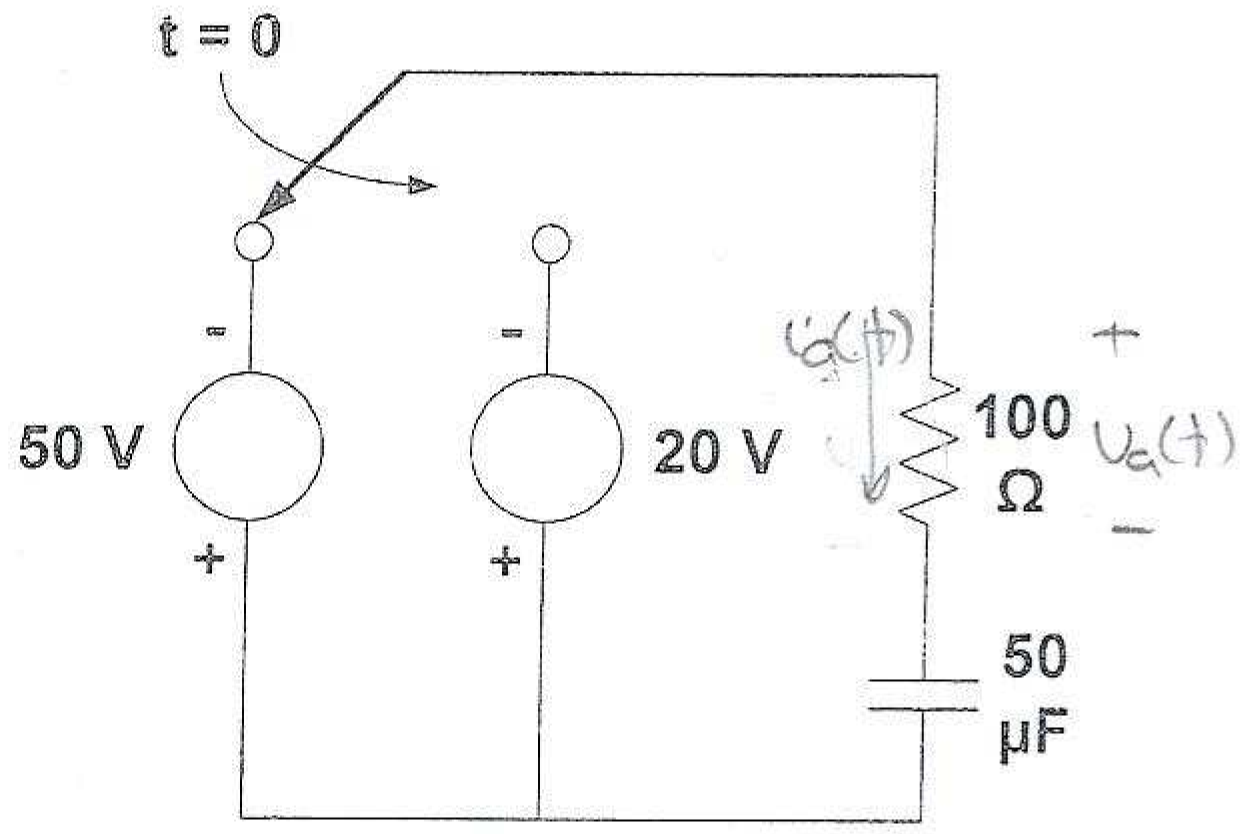


Fig.a

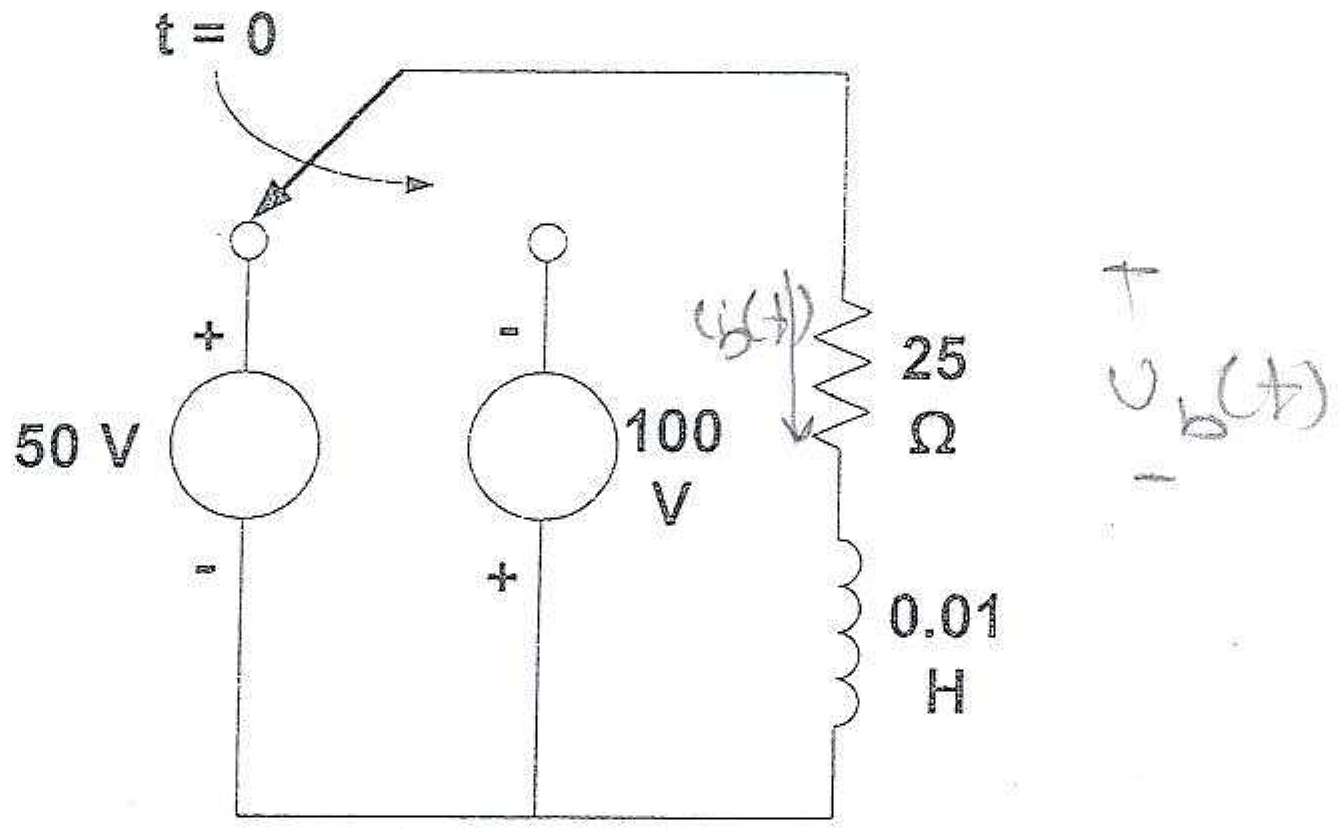
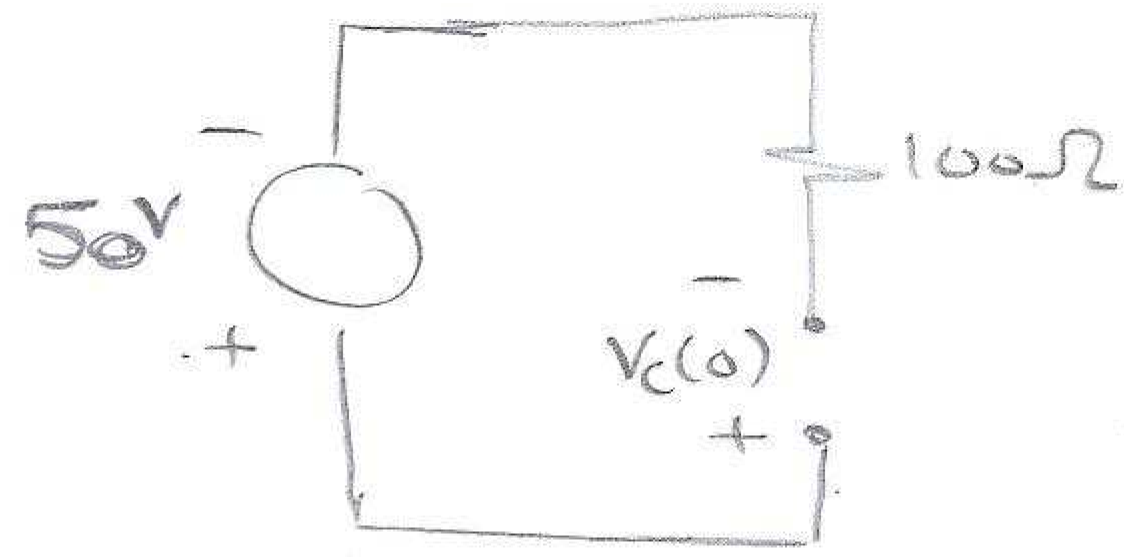


Fig.b

For Fig. a

→ Initial Condition :

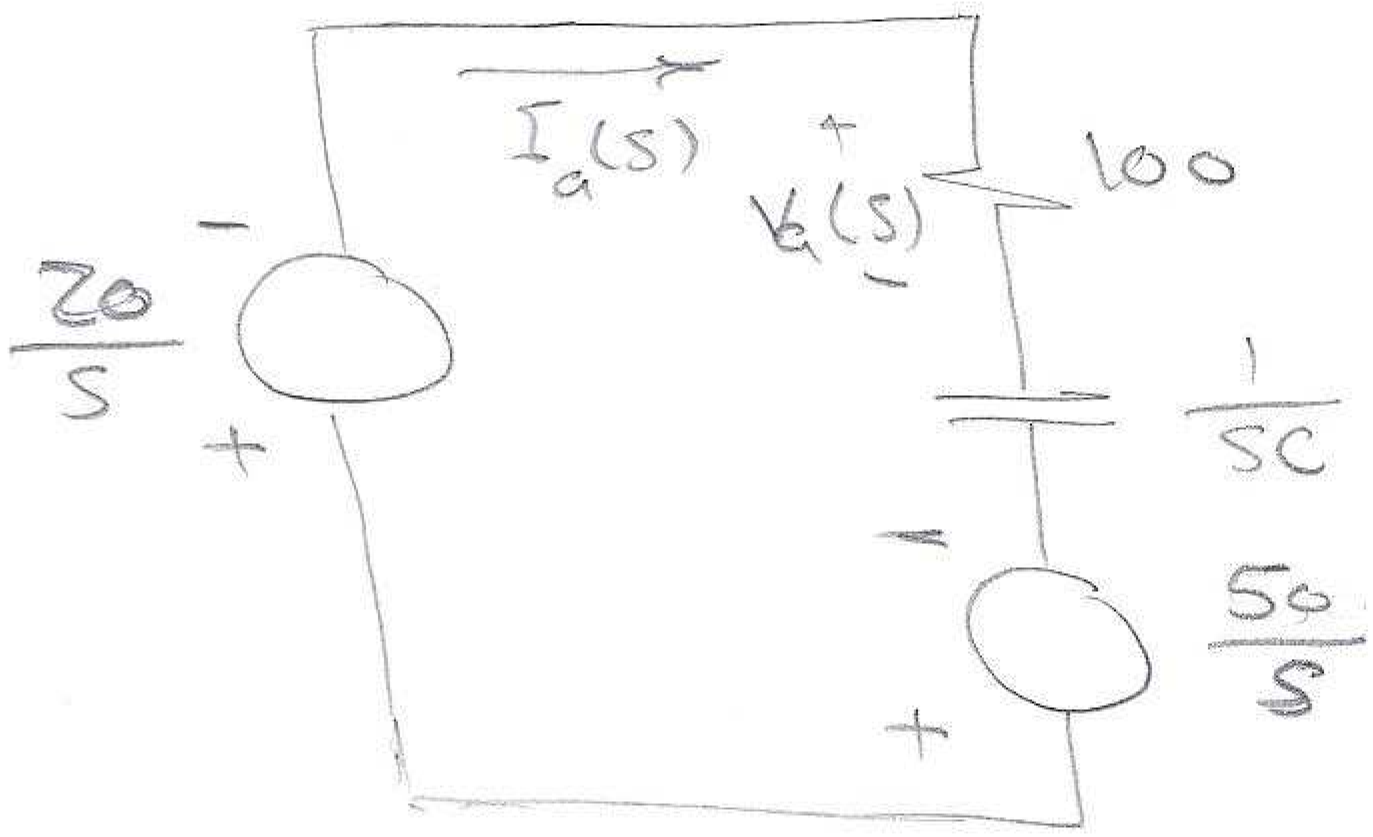
$V_c(0) = 50^V$



$$I_a(s) = \frac{\frac{50}{s} - \frac{20}{s}}{100 + \frac{1}{sc}}$$

$$= \frac{30}{100s + \frac{1}{c}}$$

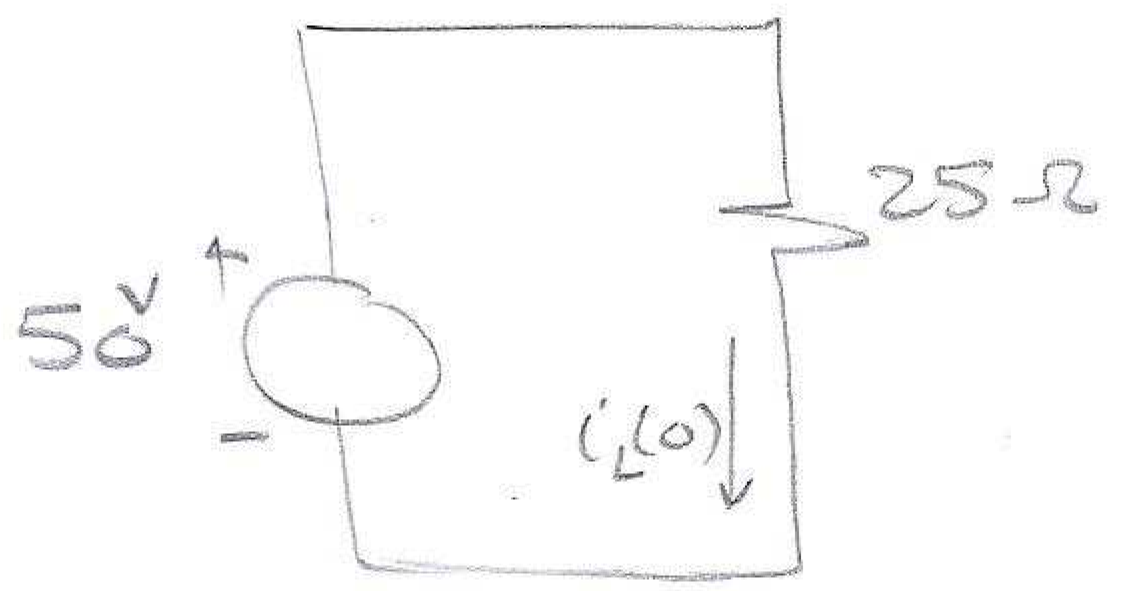
$$= \frac{0.3}{s + 200}$$



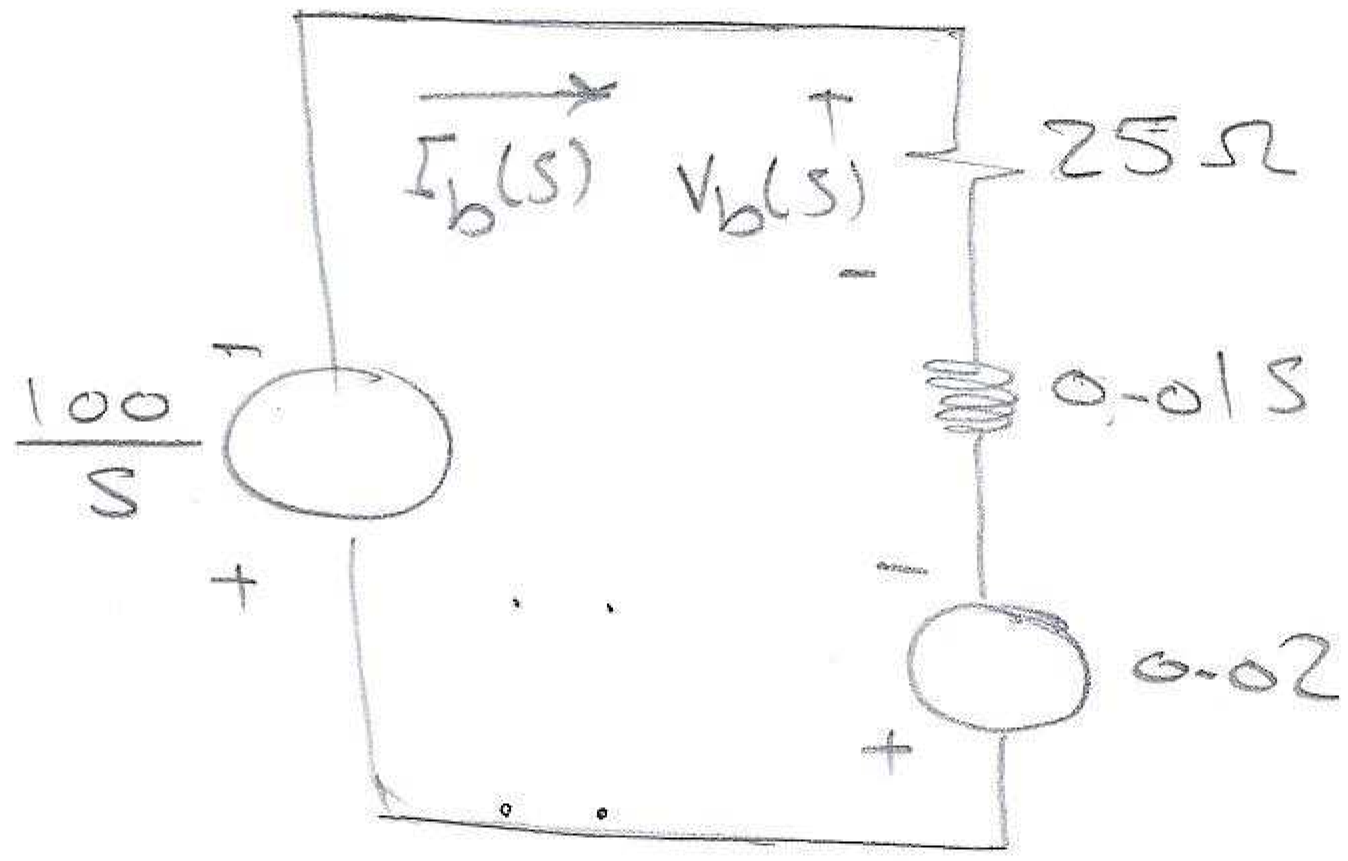
$\therefore i_a(t) = 0.3e^{-200t}$ Amp , $V_a(t) = 30e^{-200t}$ Volts

For Fig. b.

$$i_L(0) = \frac{50}{25} = 2A$$



$$I_b(s) = \frac{0.02 - \frac{100}{s}}{0.01s + 25}$$



$$= \frac{2s - 10^4}{s(s + 2500)}$$

$$= \frac{-4}{s} + \frac{6}{s + 2500}$$

$$\therefore i_b(t) = -4 + 6e^{-2500t} \quad \text{Amp}$$

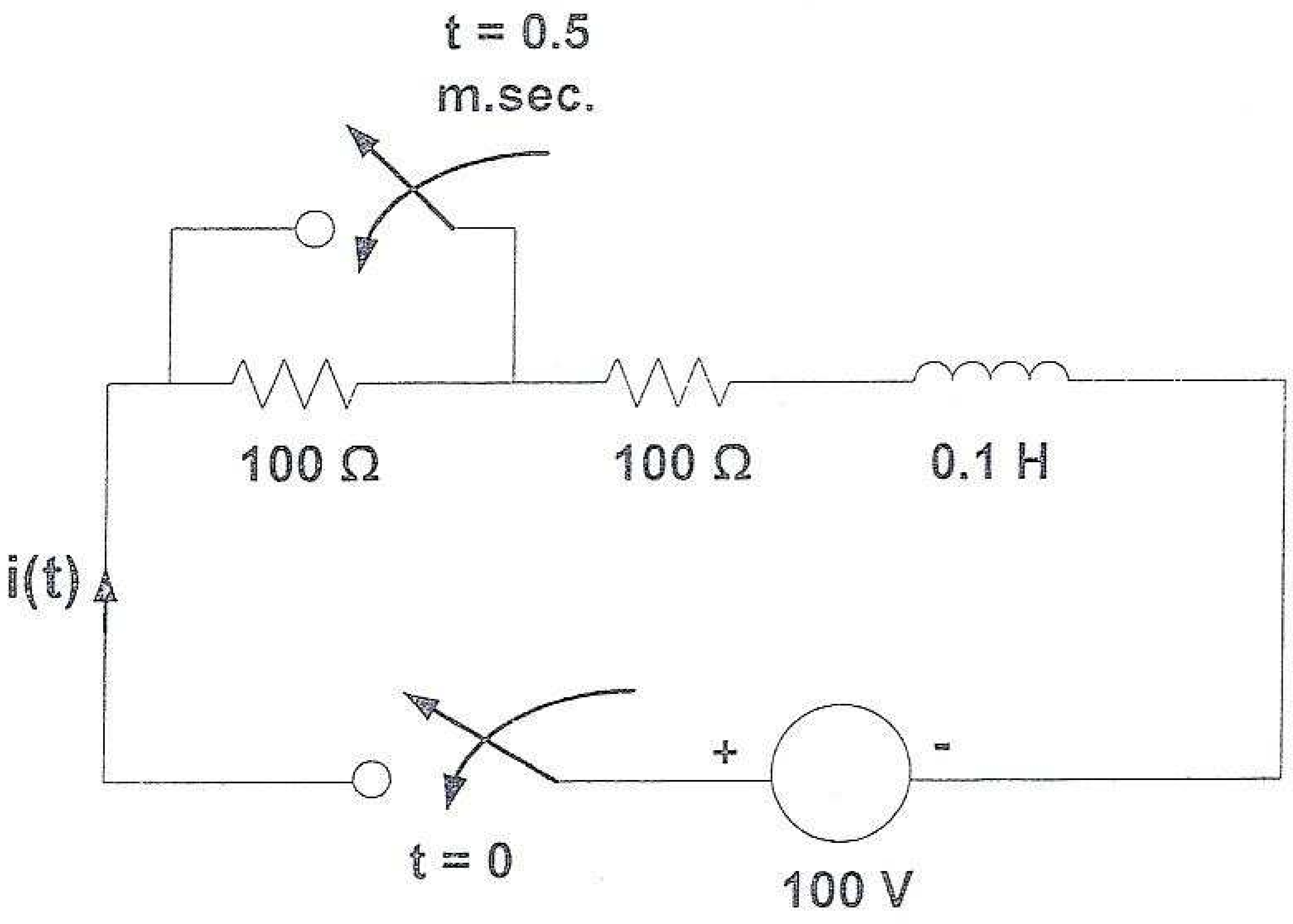
$$v_b(t) = -100 + 150\, e^{-2500t} \quad \text{Volts}$$

$i_a(t)$	$0.3\, e^{-200t}$ Amp
$v_a(t)$	$30\, e^{-200t}$ Volts
$i_b(t)$	$-4 + 6\, e^{-2500t}$ Amp
$v_b(t)$	$-100 + 150\, e^{-2500t}$ Volts.

Table 1

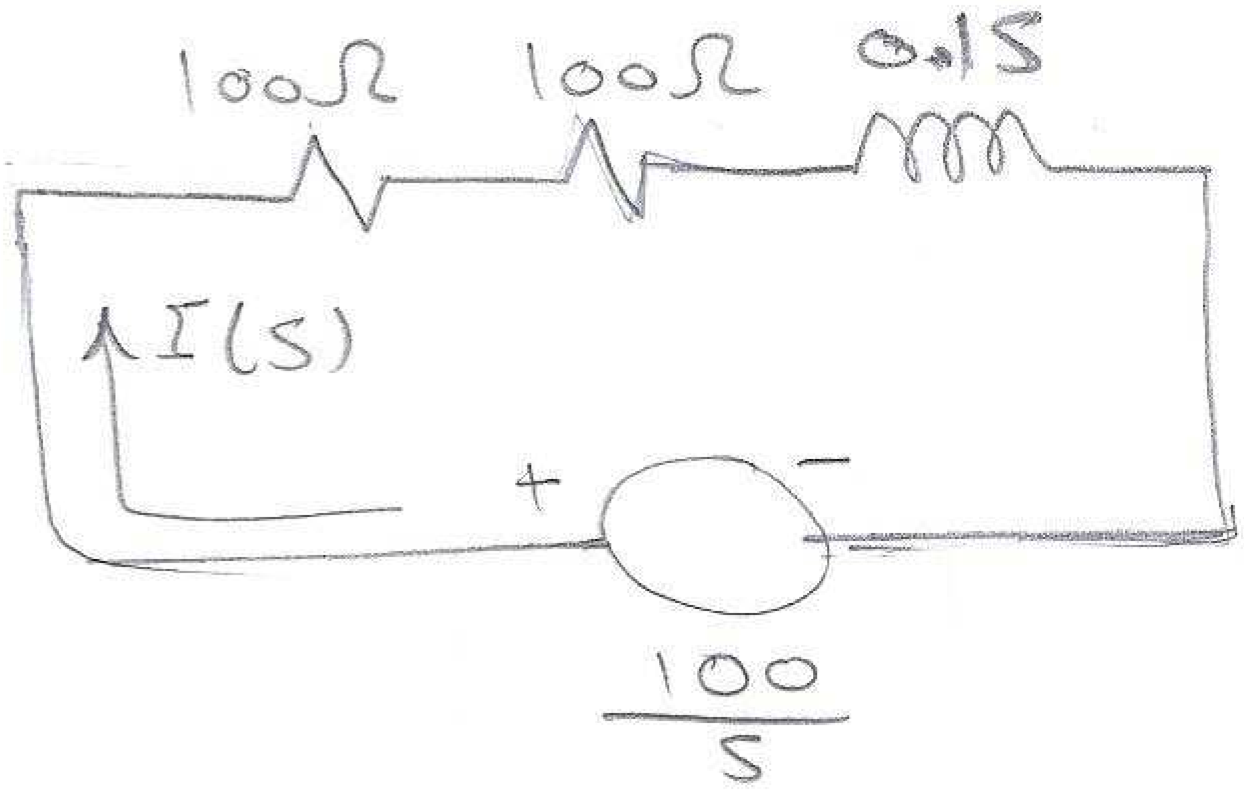
Question[2]: Answer of Question 1

For the circuit shown in figure, find the value of the current i(t) at 0.4 m.sec. and 1.0 m.sec. respectively .
Insert your results in Table 2 .



* at $0 < t < 0.5\,msec$

→ no-initial conditions



$$I(S) = \frac{\frac{100}{S}}{0.1S + 200}$$

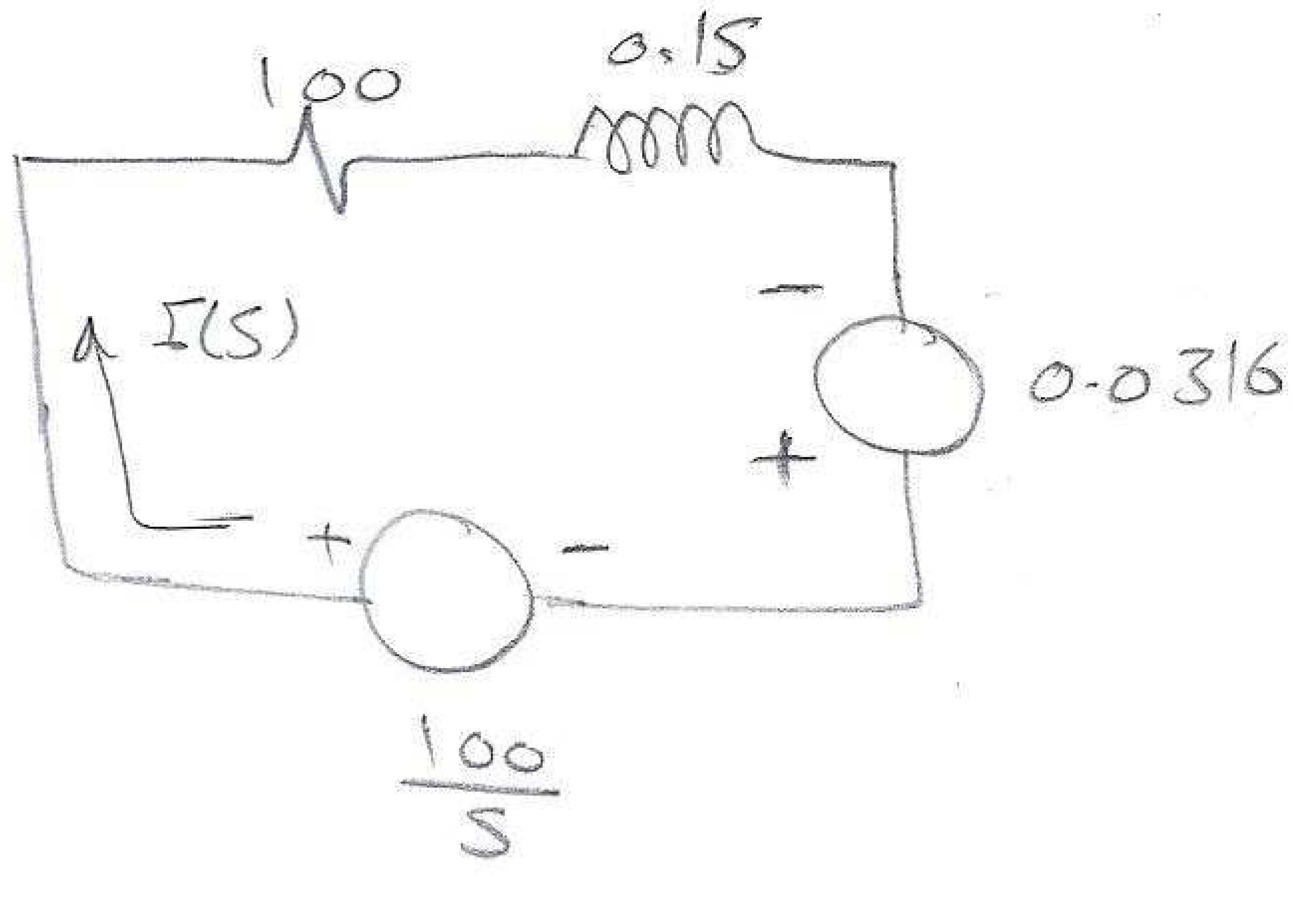
$$= \frac{1000}{S(S+2000)}$$

$$= \frac{1/2}{S} - \frac{1/2}{S+2000}$$

$$\therefore i(t) = \frac{1}{2}(1 - e^{-2000t})\{u(t) - u(t-0.5msec)\}.\ Amp.$$

$i_L(0.5msec) = 0.316\ Amp.$

$$I(S) = \frac{\frac{100}{S} + 0.0316}{0.1S + 100}$$



$$= \frac{0.316S + 1000}{S(S+1000)}$$

$$= \frac{1}{S} - \frac{0.684}{S+1000}$$

$$\therefore i(t) = (1 - 0.684\, e^{-1000t})\, u(t)$$

$$\therefore i(t - 0.5msec) = (1 - 0.684\, e^{-1000(t-0.5msec)})\, u(t-0.5msec) A$$

$$i(0.4msec) = \frac{1}{2}(1 - e^{-2000 * 0.4 * 10^{-3}})$$

$$\boxed{i(0.4msec) = 0.275\ Amp}$$

$$i(1msec) = 1 - 0.684\, e^{-1000 * 0.5 * 10^{-3}}$$

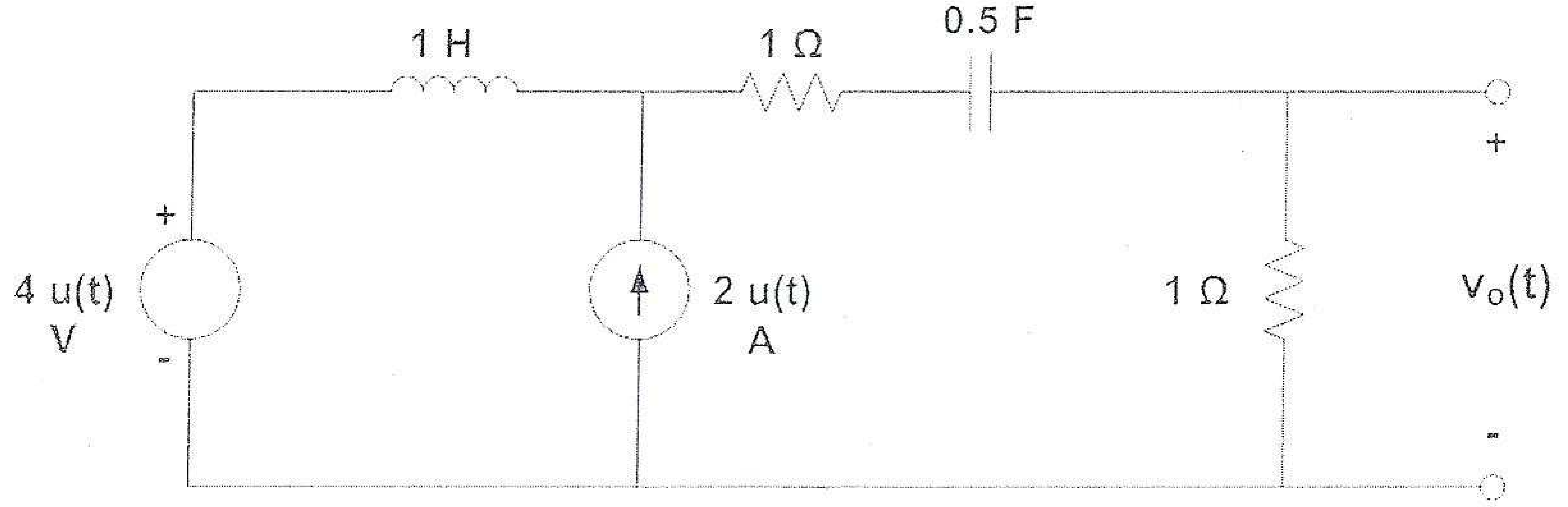
$$\boxed{\therefore i(1msec) = 0.585\ A}$$

i(t) (*at 0.4 msec.*)	0.275 Amp
i(t) (*at 1.0 msec.*)	0.585 Amp.

Table 2

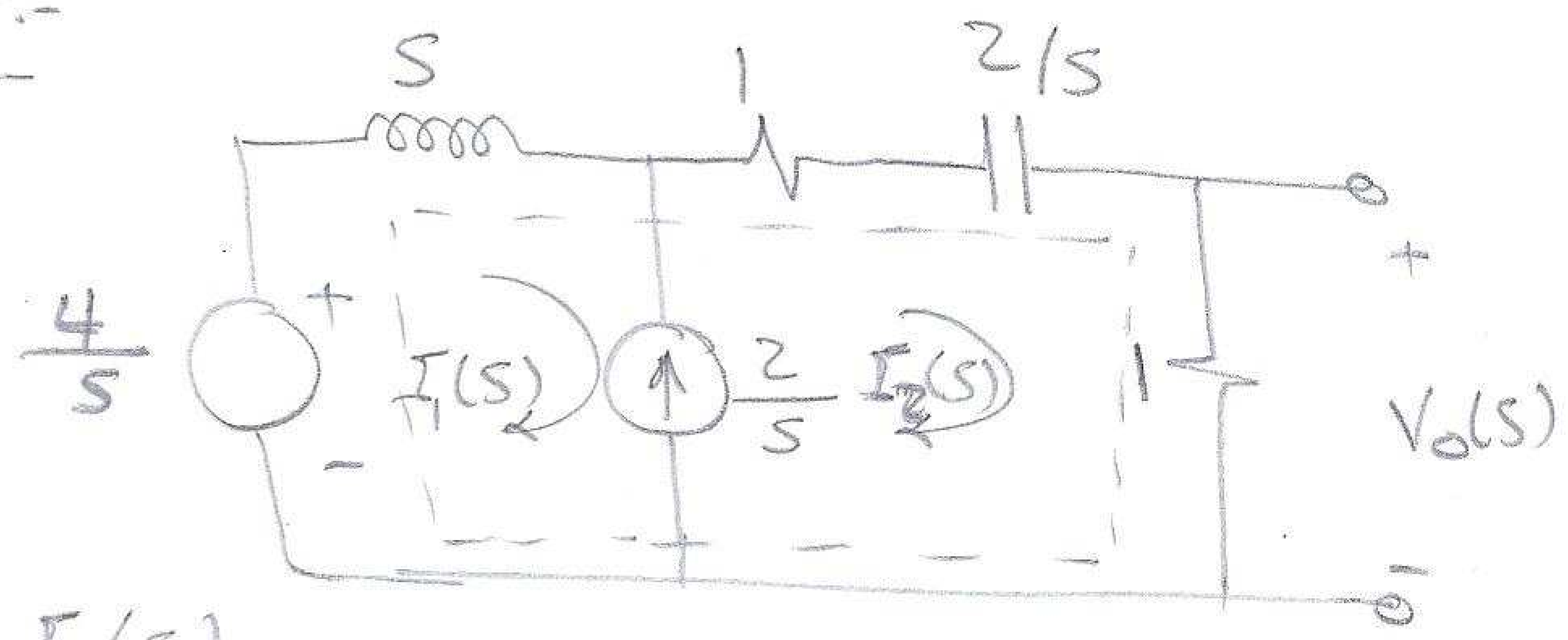
Question[3]: Answer of Question 4

For the network shown in figure, find $v_o(t)$ for $t > 0$ using loop analysis . Verify your answer using the superposition theorem .
Insert your result in Table 3 .



→ no – Initial Conditions

∗ using loop Analysis :-



$$I_2(s) - I_1(s) = \frac{2}{s} \leftarrow ①$$

$$\frac{4}{s} = s I_1(s) + \left(2 + \frac{2}{s}\right) I_2(s)$$

$$\therefore \begin{bmatrix} -1 & 1 \\ s & 2 + \frac{2}{s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{2}{s} \\ \frac{4}{s} \end{bmatrix}$$

$$\Delta = -2 - \frac{2}{s} - s = \frac{-(s^2 + 2s + 2)}{s}$$

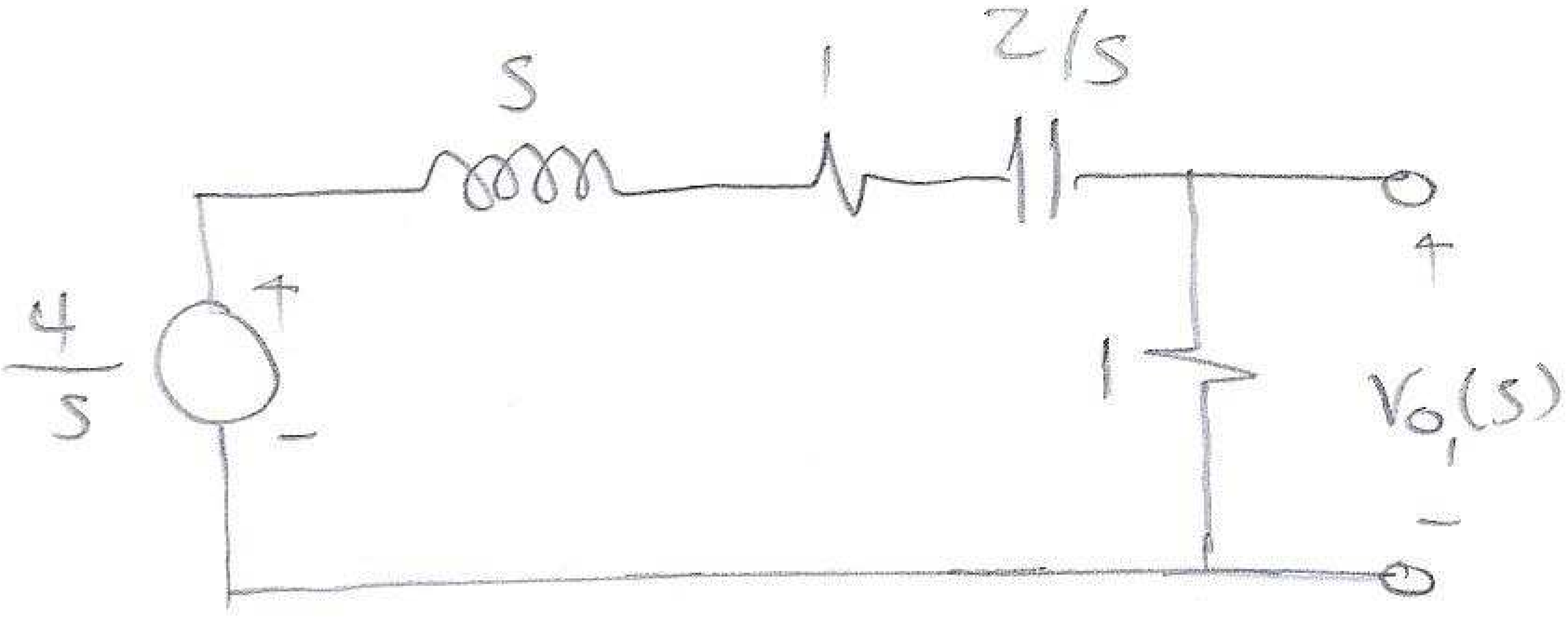
$$\Delta_2 = \begin{vmatrix} -1 & \frac{2}{S} \\ S & \frac{4}{S} \end{vmatrix} = \frac{-4}{S} - 2 = \frac{-(2S+4)}{S}$$

$$I_2(S) = \frac{\Delta_2}{\Delta} = \frac{-(2S+4)}{S} * \frac{-S}{S^2+2S+2} = \frac{2S+4}{S^2+2S+2}$$

$$V_o(S) = 1 * I_2(S) \qquad \therefore V_o(S) = \frac{2S+4}{S^2+2S+2} \leftarrow ①$$

→ using super position

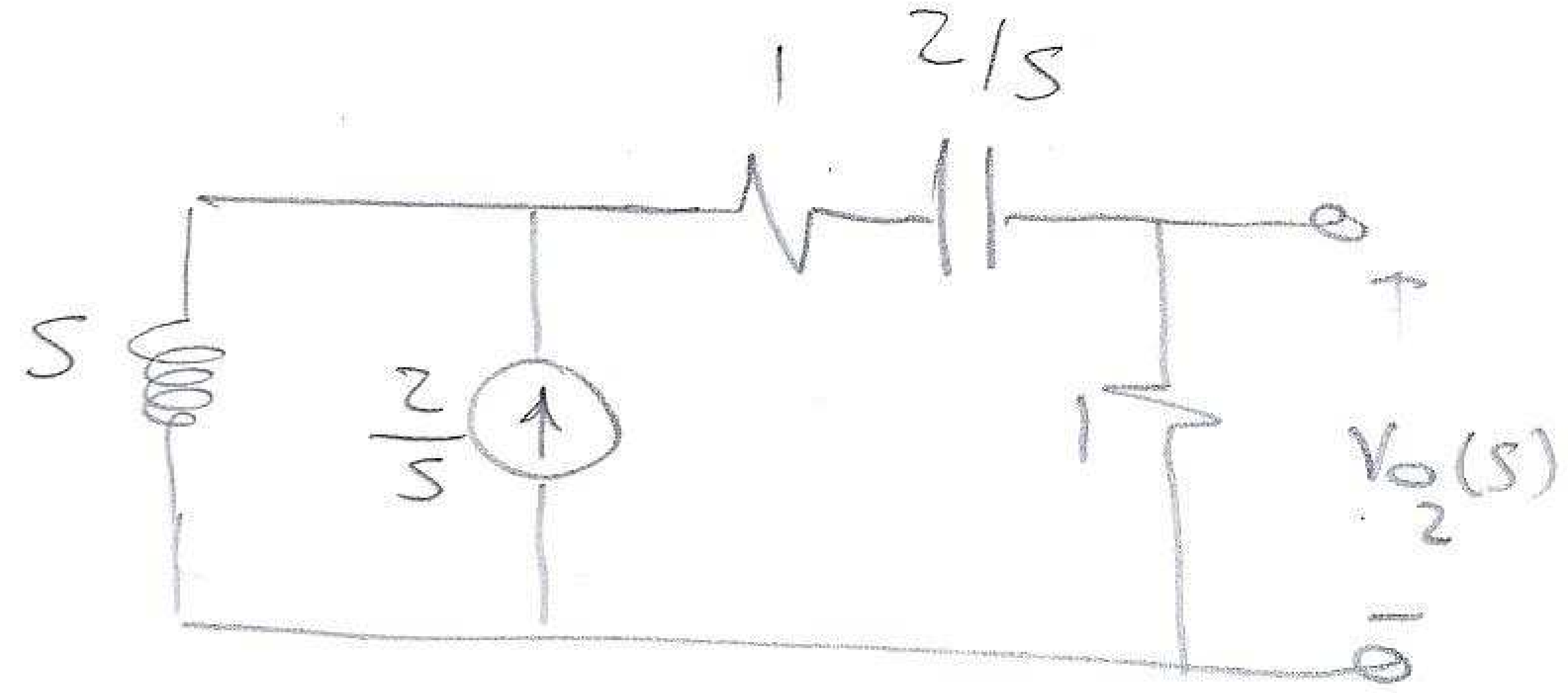
[1] due to $(\frac{4}{S} V)$ Source



$$V_{o_1}(S) = \frac{4}{S} * \frac{1}{2+S+\frac{2}{S}}$$

$$\therefore V_{o_1}(S) = \frac{4}{S^2+2S+2}$$

[2] due to $(\frac{2}{S} A)$ Source



$$V_{o_2}(S) = 1 * \frac{2}{S} * \frac{S}{2+S+\frac{2}{S}}$$

$$= \frac{2S}{S^2+2S+2}$$

$$V_o(S) = V_{o_1}(S) + V_{o_2}(S) \qquad \therefore V_o(S) = \frac{2S+4}{S^2+2S+4} \leftarrow ②$$

From ①, ② $V_o(S)\Big|_{\text{Loop Analysis}} = V_o(S)\Big|_{\text{superposition}} = \dfrac{2S+4}{S^2+2S+2}$

Poles at $S = -1 \pm j$

$$V_o(S) = \frac{2S+4}{(S+1-j)(S+1+j)}$$

$$= \frac{K}{S+1-j} + \frac{K^*}{S+1+j}$$

$$K = \sqrt{2}\,\angle{-45}$$

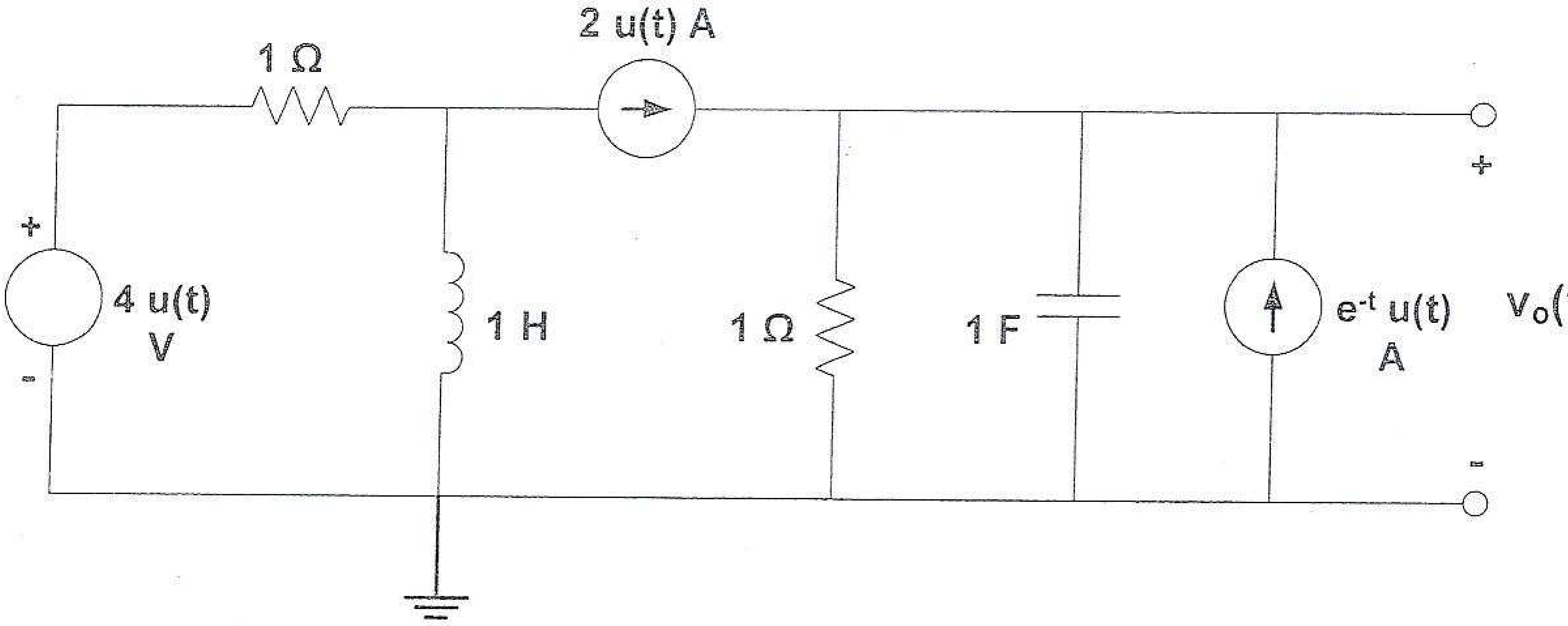
$$\therefore v_o(t) = \left(2\sqrt{2}\, e^{-t} \cos(t-45^\circ)\right) u(t) \text{ volts}$$

$v_o(t)$	$\left(2\sqrt{2}\, e^{-t} \cos(t-45^\circ)\right) u(t)$

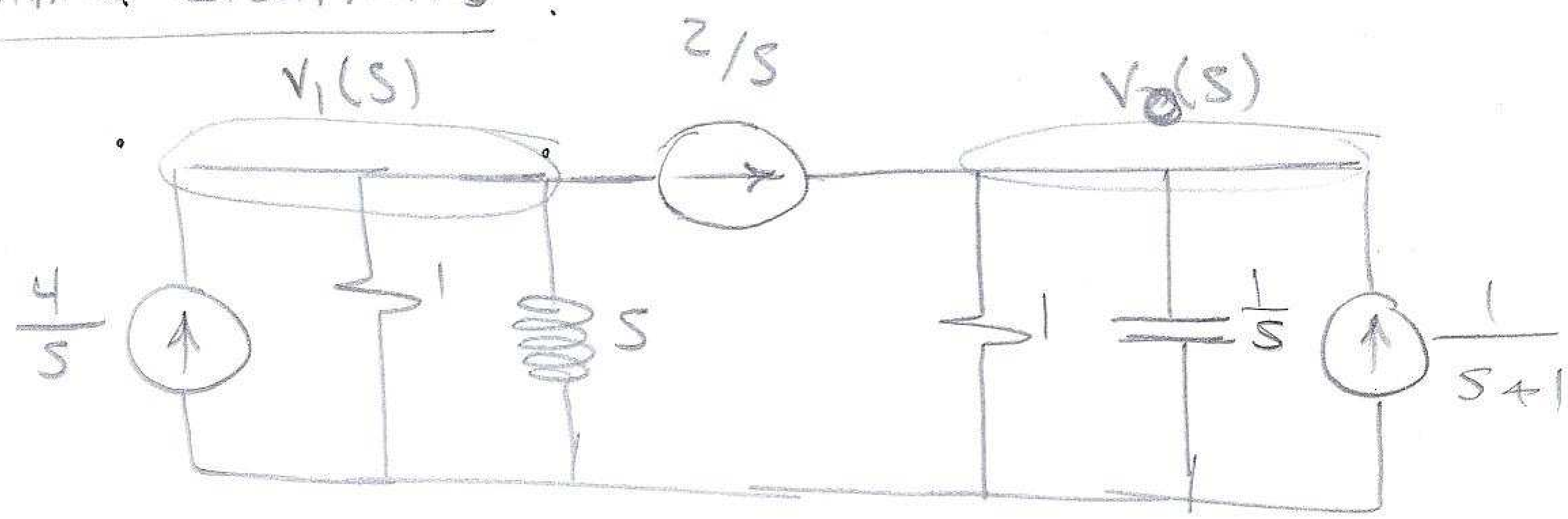
Table 3

Question[4]: Answer of Question 5

Find $v_o(t)$, $t > 0$, for the network shown in figure using node analysis.
Insert your result in Table 4.



⇒ no-initial conditions.



$$\begin{bmatrix}1+\frac{1}{s} & 0\\ 0 & 1+s\end{bmatrix}\begin{bmatrix}V_1(s)\\ V_o(s)\end{bmatrix}=\begin{bmatrix}\frac{4}{s}-\frac{2}{s}\\ \frac{2}{s}+\frac{1}{s+1}\end{bmatrix}$$

$$\therefore \begin{bmatrix}\frac{s+1}{s} & 0\\ 0 & s+1\end{bmatrix}\begin{bmatrix}V_1(s)\\ V_o(s)\end{bmatrix}=\begin{bmatrix}\frac{2}{s}\\ \frac{3s+2}{s(s+1)}\end{bmatrix}$$

$$\Delta = \frac{(S+1)}{S} * (S+1) - 0 = \frac{(S+1)^2}{S}$$

$$\Delta_2 = \begin{vmatrix} \frac{S+1}{S} & \frac{2}{S} \\ 0 & \frac{3S+2}{S(S+1)} \end{vmatrix} = \frac{3S+2}{S^2}$$

$$V_o(S) = \frac{\Delta_2}{\Delta} = \frac{3S+2}{S^2} * \frac{S}{(S+1)^2} = \frac{3S+2}{S(S+1)^2}$$

$$= \frac{K_0}{S} + \frac{K_1}{S+1} + \frac{K_2}{(S+1)^2}$$

$$3S+2 = K_0(S+1)^2 + K_1 S(S+1) + K_2 S$$

$K_0 = 2$

$K_2 = 1$

$K_1 + K_0 = 0 \quad \therefore K_1 = -2$

$$\therefore V_o(S) = \frac{2}{S} - \frac{2}{S+1} + \frac{1}{(S+1)^2}$$

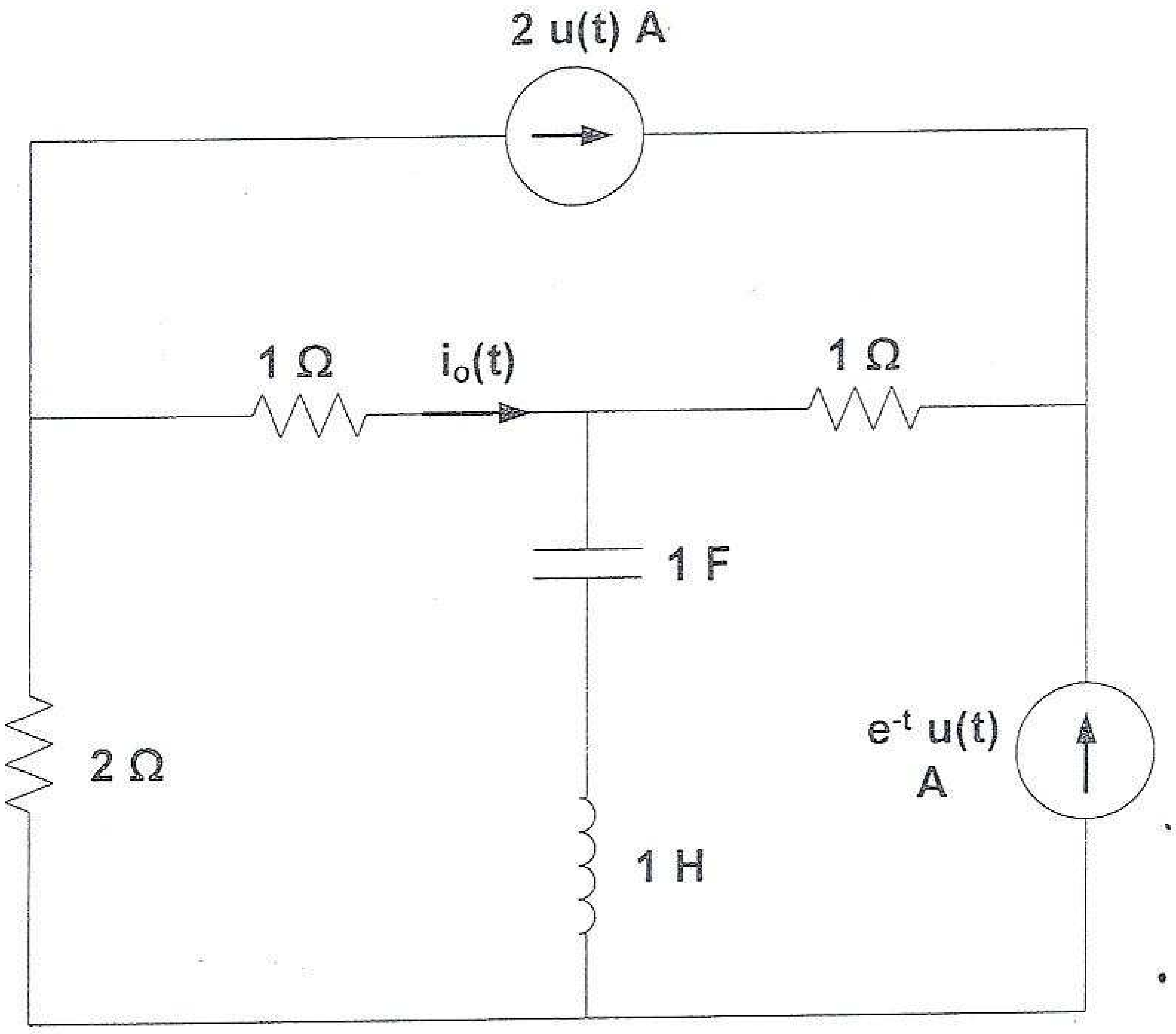
$$\boxed{\therefore v_o(t) = \left(2 - 2e^{-t} + te^{-t}\right) u(t)}$$

$v_o(t)$	$(2 - 2e^{-t} + te^{-t})\,u(t)$

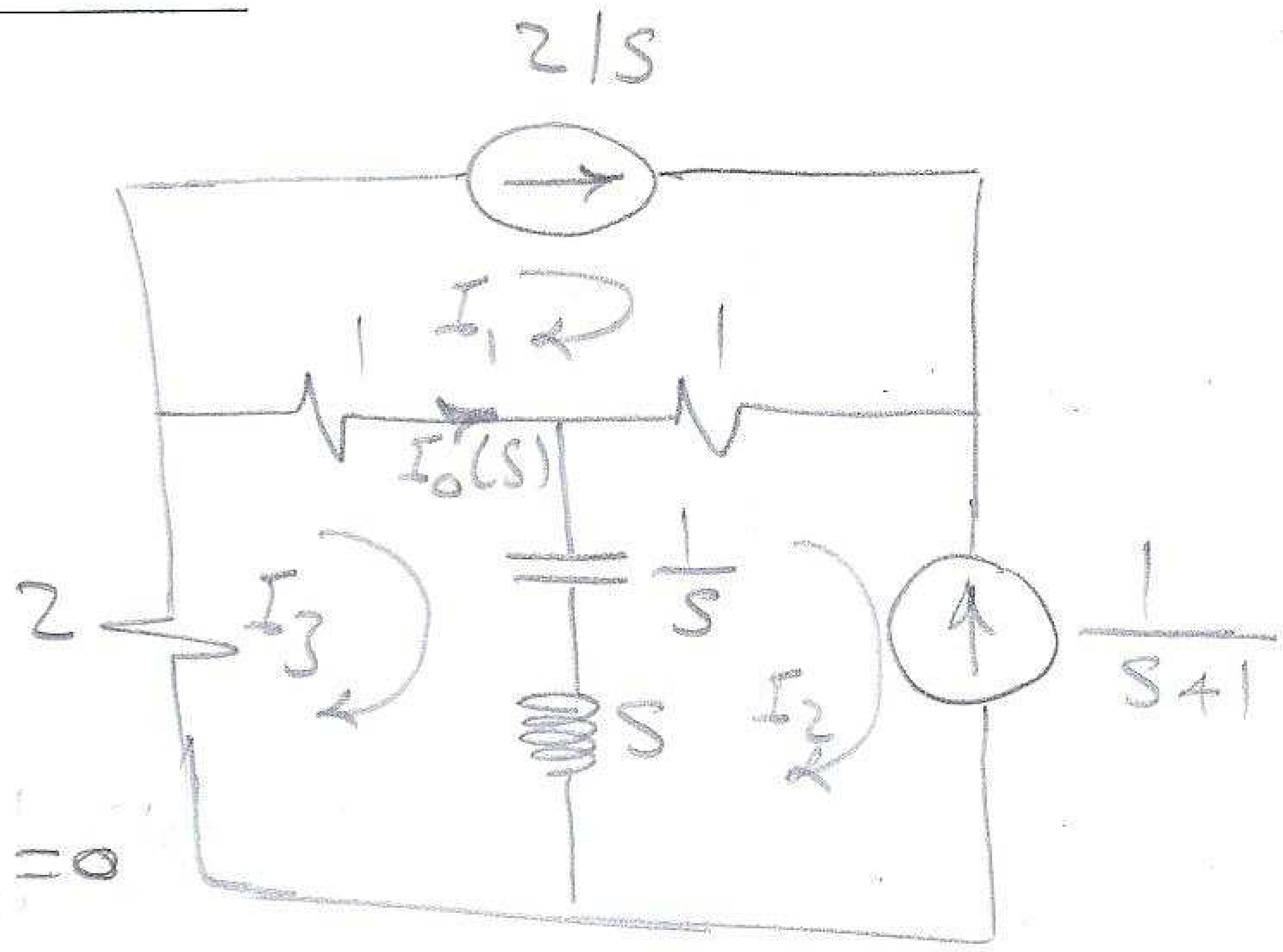
Table 5

Question[5]: Answer of Question 6

Derive an expression for the current $i_o(t)$, $t > 0$, in the network shown in figure . What would be the steady state value of this current ?
Insert your results in Table 5 .



⇒ no-initial conditions



$$I_1 = \frac{2}{S}$$

$$I_2 = \frac{-1}{S+1}$$

$$-I_1 - \left(S + \frac{1}{S}\right) I_2 + \left(3 + S + \frac{1}{S}\right) I_3 = 0$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & \frac{-(S^2+1)}{S} & \frac{S^2+3S+1}{S} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{S} \\ \frac{-1}{S+1} \\ 0 \end{bmatrix}$$

$$\Delta = \frac{S^2+3S+1}{S}$$

$$\Delta_1 = \begin{vmatrix} \frac{2}{S} & 0 & 0 \\ \frac{-1}{S+1} & 1 & 0 \\ 0 & \frac{-(S^2+1)}{S} & \frac{S^2+3S+1}{S} \end{vmatrix} = \frac{2}{S} * \frac{S^2+3S+1}{S}$$

$$\Delta_3 = \begin{vmatrix} 1 & 0 & \frac{2}{S} \\ 0 & 1 & \frac{-1}{S+1} \\ -1 & \frac{-(S^2+1)}{S} & 0 \end{vmatrix} = \frac{-(S^2+1)}{S(S+1)} + \frac{2}{S}$$

$$= \frac{-S^2-1+2S+2}{S(S+1)}$$

$$\Delta_3 = \frac{-S^2+2S+1}{S(S+1)}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{2}{S}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-S^2+2S+1}{\cancel{S}(S+1)} * \frac{\cancel{S}}{(S^2+3S+1)}$$

$$\therefore I_3 = \frac{-S^2+2S+1}{(S+1)(S^2+3S+1)}$$

$$I_o = I_3 - I_1 = \frac{-S^2+2S+1}{(S+1)(S^2+3S+1)} - \frac{2}{S}$$

$$\therefore I_o(S) = \frac{-S^2+2S+1}{(S+1)(S+0.382)(S+2.618)} - \frac{2}{S}$$

$$= \frac{2}{S+1} + \frac{0.065}{S+0.382} - \frac{3.065}{S+2.618} - \frac{2}{S}$$

$$\therefore i_o(t) = \left(2e^{-t} + 0.065\,e^{-0.382t} - 3.065\,e^{-2.618t} - 2\right)u(t)$$

$$\boxed{i_o(\infty) = -2\,\text{Amp}}$$

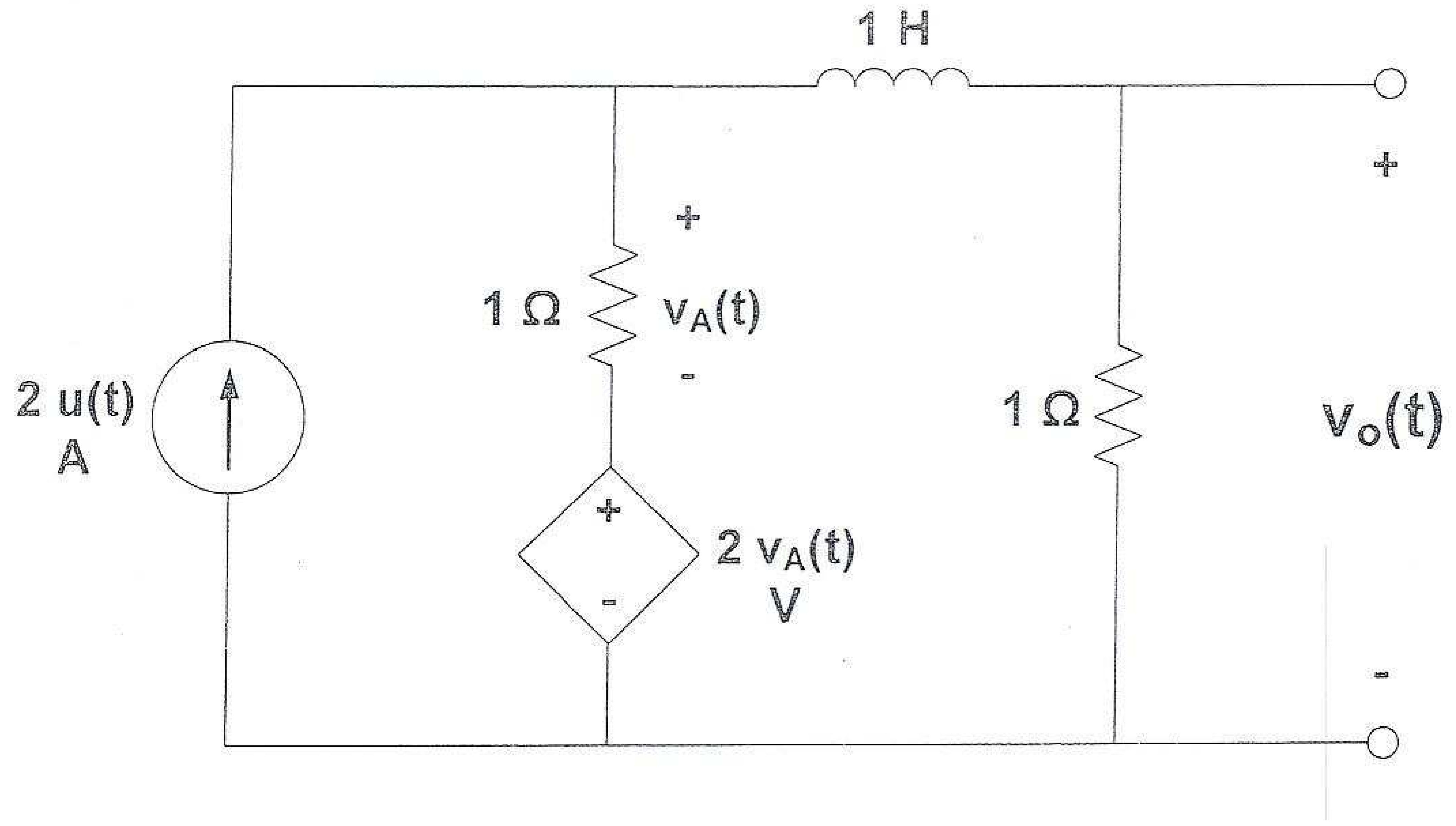
$v_o(t)$ *for t > 0*	$\left(2e^{-t} + 0.065\,e^{-0.382t} - 3.065e^{-2.618t} - 2\right)u(t)$
v_o *(Steady State)*	-2 Amp.

Table 5

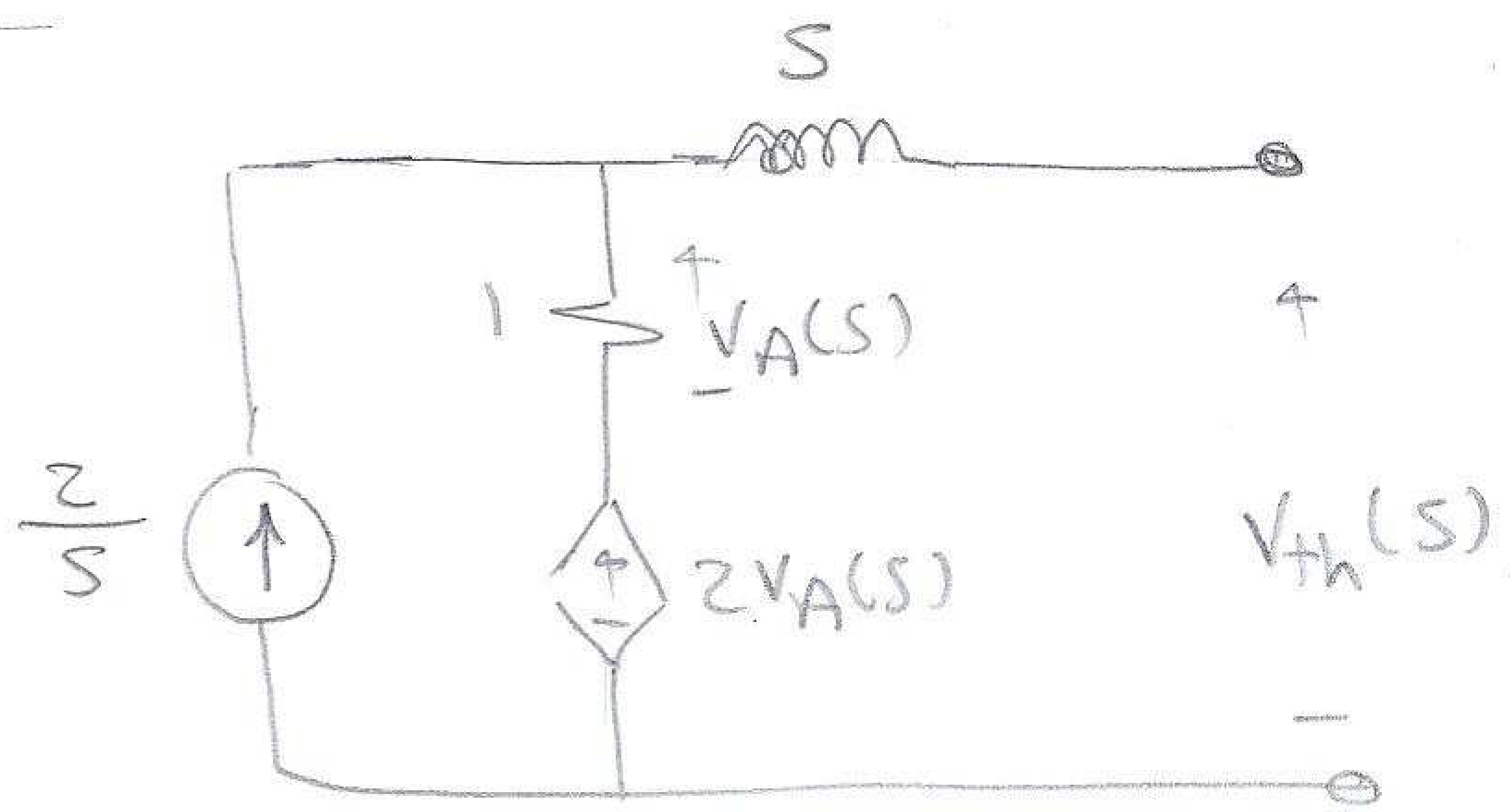
Question[6]: Answer of Question 7

In the network shown in figure, use Thevenin's Theorem to derive an expression for $v_o(t)$, $t > 0$.

Insert your result in Table 6 .



→ no - initial Conditions



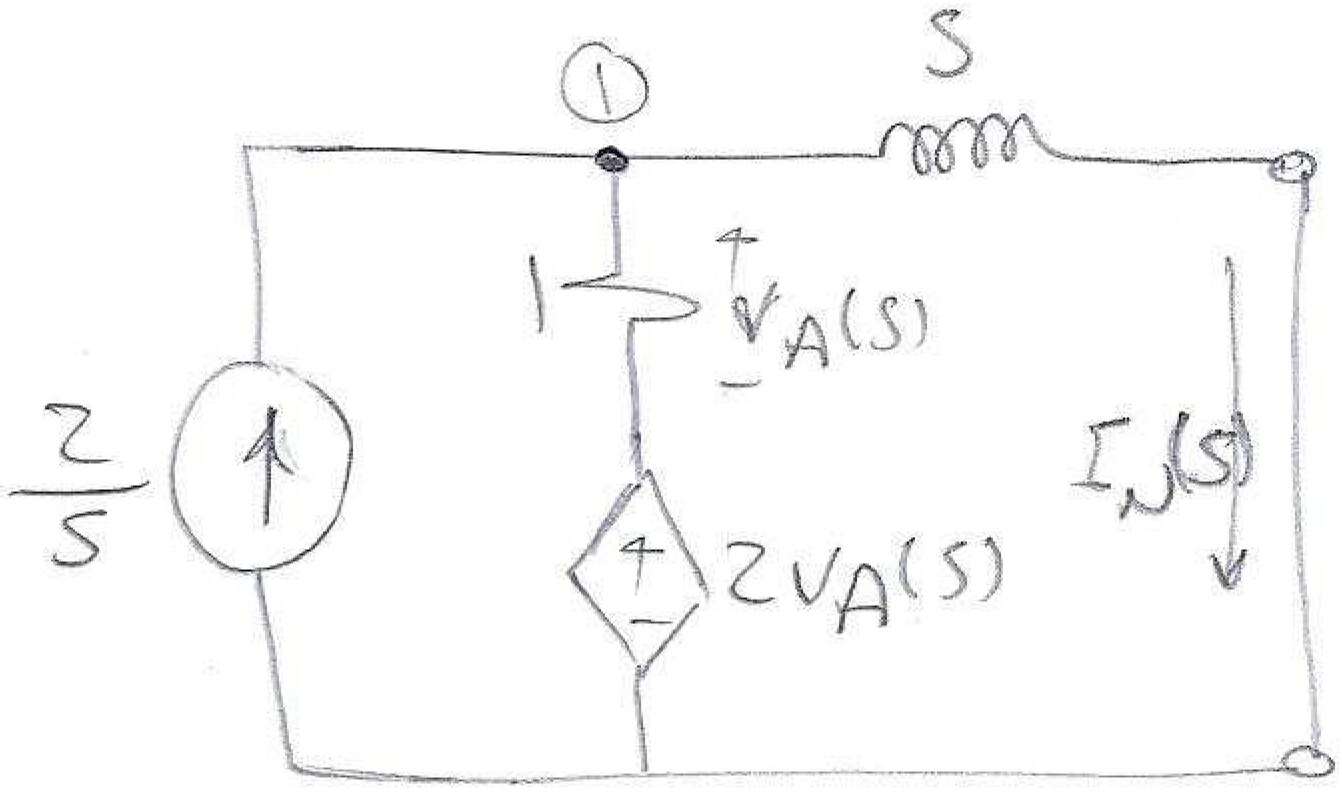
$V_{th} = V_A + 2V_A$

$= 3V_A$

$V_A = \frac{2}{s} \times 1 = \frac{2}{s}$

$\therefore V_{th} = \frac{6}{s}$

KCl at ①



$$\frac{2}{S} = \frac{V_A}{1} + I_N$$

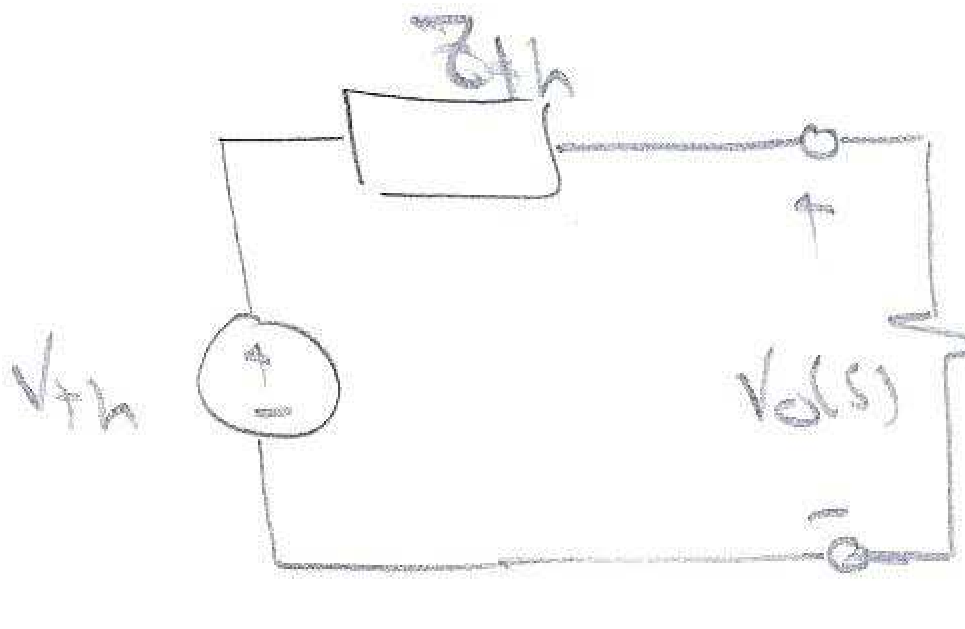
$$\therefore I_N + V_A = \frac{2}{S}$$

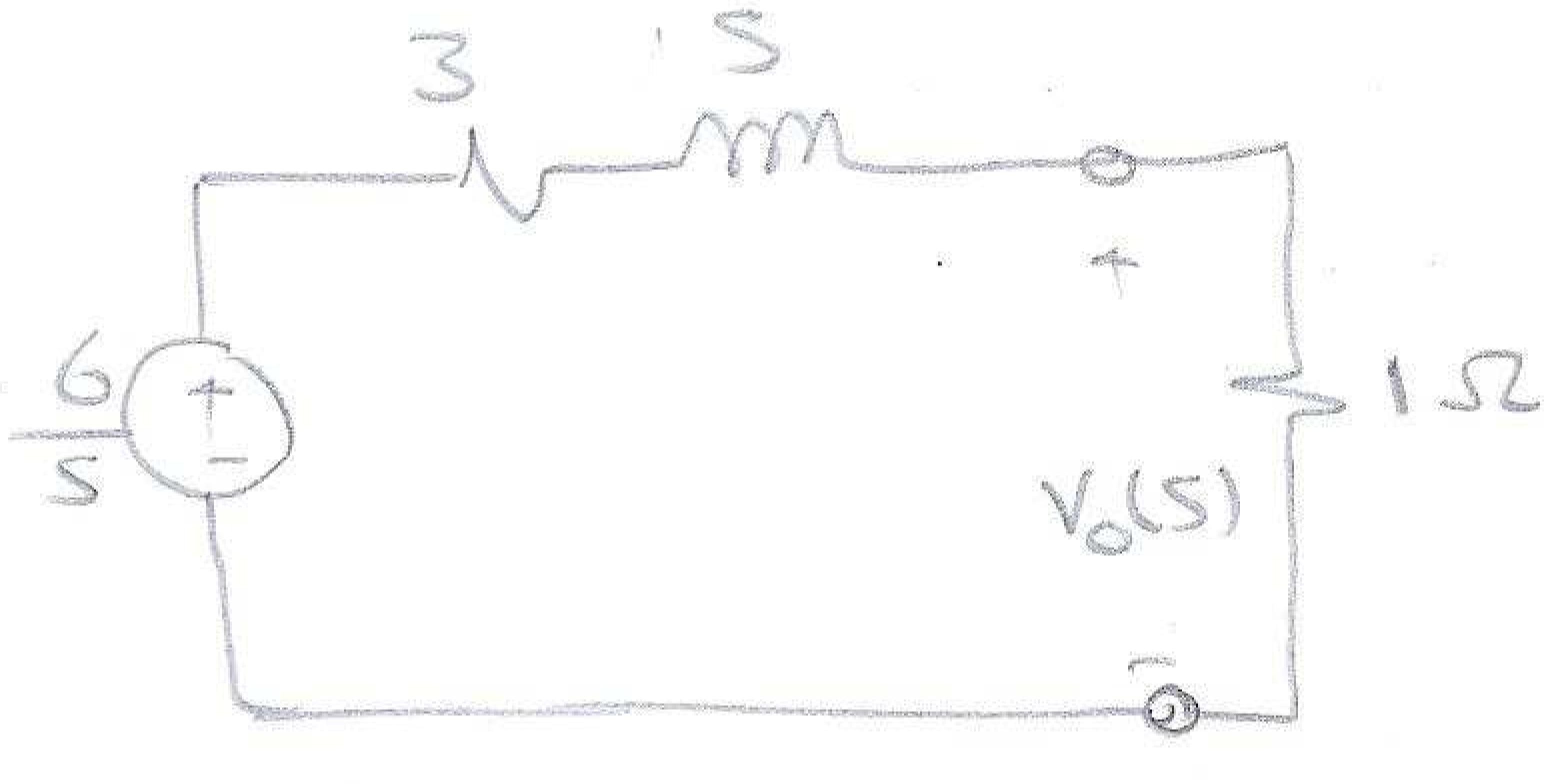
$$I_N = \frac{V_A + 2V_A}{S} = \frac{3V_A}{S} \qquad \therefore V_A = \frac{S}{3} I_N$$

$$\therefore I_N + \frac{S}{3} I_N = \frac{2}{S} \Rightarrow I_N\left(\frac{S+3}{3}\right) = \frac{2}{S}$$

$$\therefore I_N = \frac{6}{S(S+3)}$$

$$Z_{th} = \frac{V_{th}}{I_N} = S+3$$





$$V_o(S) = \frac{6}{S} * \frac{1}{S+4} = \frac{6}{S(S+4)}$$

$$= \frac{1.5}{S} - \frac{1.5}{S+4}$$

$$\therefore v_o(t) = 1.5\left(1 - e^{-4t}\right) u(t) \text{ volt}$$

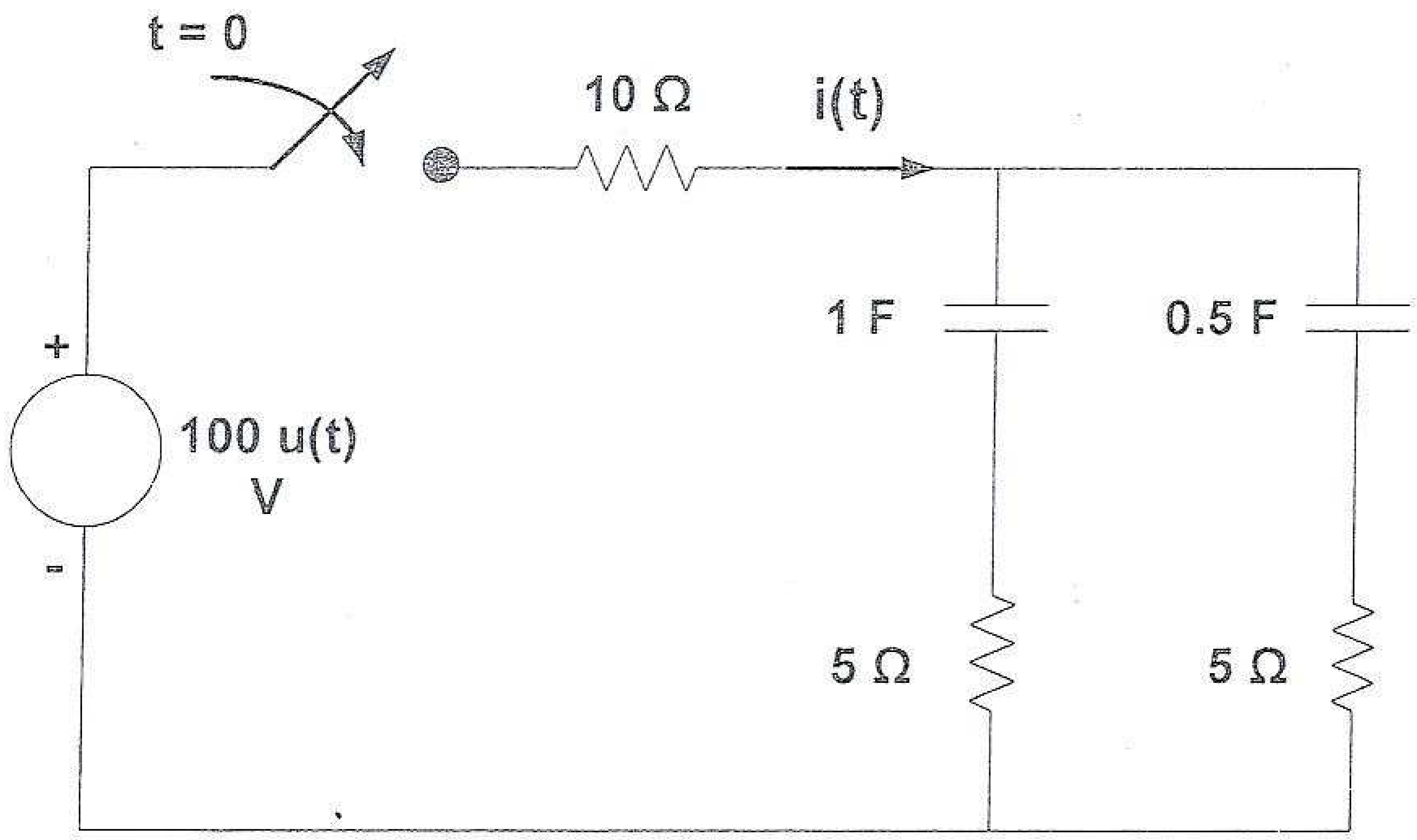
$v_o(t)$	$1.5\left(1-e^{-4t}\right)u(t)$ volt

Table 6

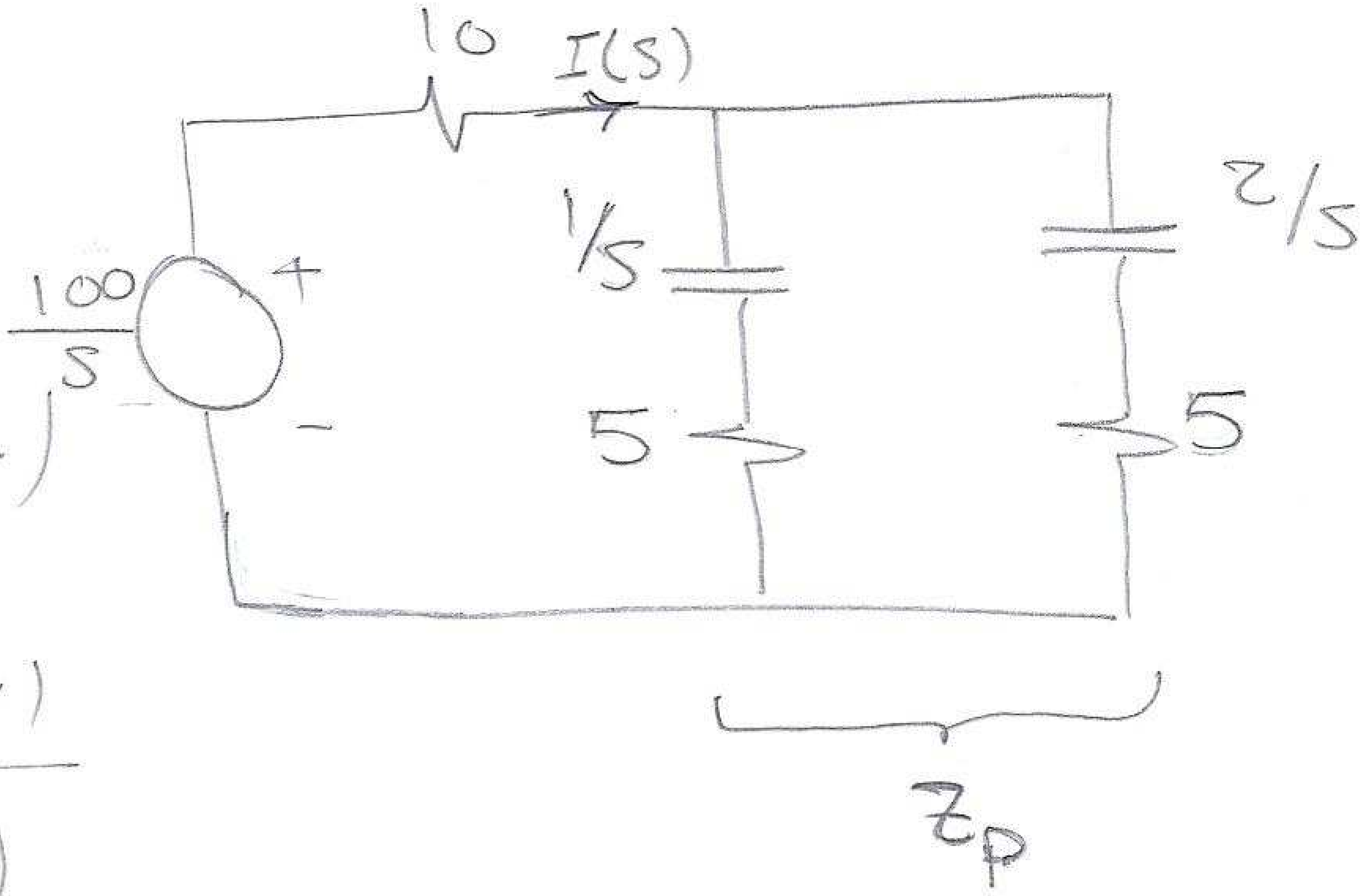
Question[7]: Answer of Question 9

Using the Laplace Transform Technique, derive an expression for the current i(t),t > 0, shown in figure, hence, calculate its initial and steady state values . Verify your results using the Initial and Final Value Theorems .

Insert your results in Table 7 .



⇒ no - initial Conditions



$$Z_p = \left(5 + \frac{2}{s}\right) // \left(5 + \frac{1}{s}\right)$$

$$= \left(\frac{5s+2}{s}\right) // \left(\frac{5s+1}{s}\right)$$

$$= \frac{(5s+2)(5s+1)}{s^2\left(\frac{10s+3}{s}\right)}$$

$$Z_p = \frac{(5s+2)(5s+1)}{s(10s+3)}$$

$$Z = 10 + Z_P$$

$$I(S) = \frac{\frac{100}{S}}{10 + Z_P} = \frac{\frac{100}{S}}{10 + \frac{(5S+2)(5S+1)}{S(10S+3)}}$$

$$\therefore I(S) = \frac{100(10S+3)}{10S(10S+3) + (5S+2)(5S+1)}$$

$$= \frac{100(10S+3)}{125S^2 + 45S + 2} = \frac{0.8(10S+3)}{S^2 + 0.36S + 0.016}$$

$$= \frac{0.8(10S+3)}{(S+0.052)(S+0.308)} = \frac{7.75}{S+0.052} + \frac{0.25}{S+0.308}$$

$$\therefore i(t) = \left(7.75\, e^{-0.052t} + 0.25\, e^{-0.308t}\right) u(t)$$

$$i(0) = 8 \text{ Amp}$$

$$i(\infty) = 0$$

$$i(0) = \lim_{S \to \infty} \frac{0.8(10S^2 + 3S)}{S^2 + 0.36S + 0.016} = 8 \text{ Amp}$$

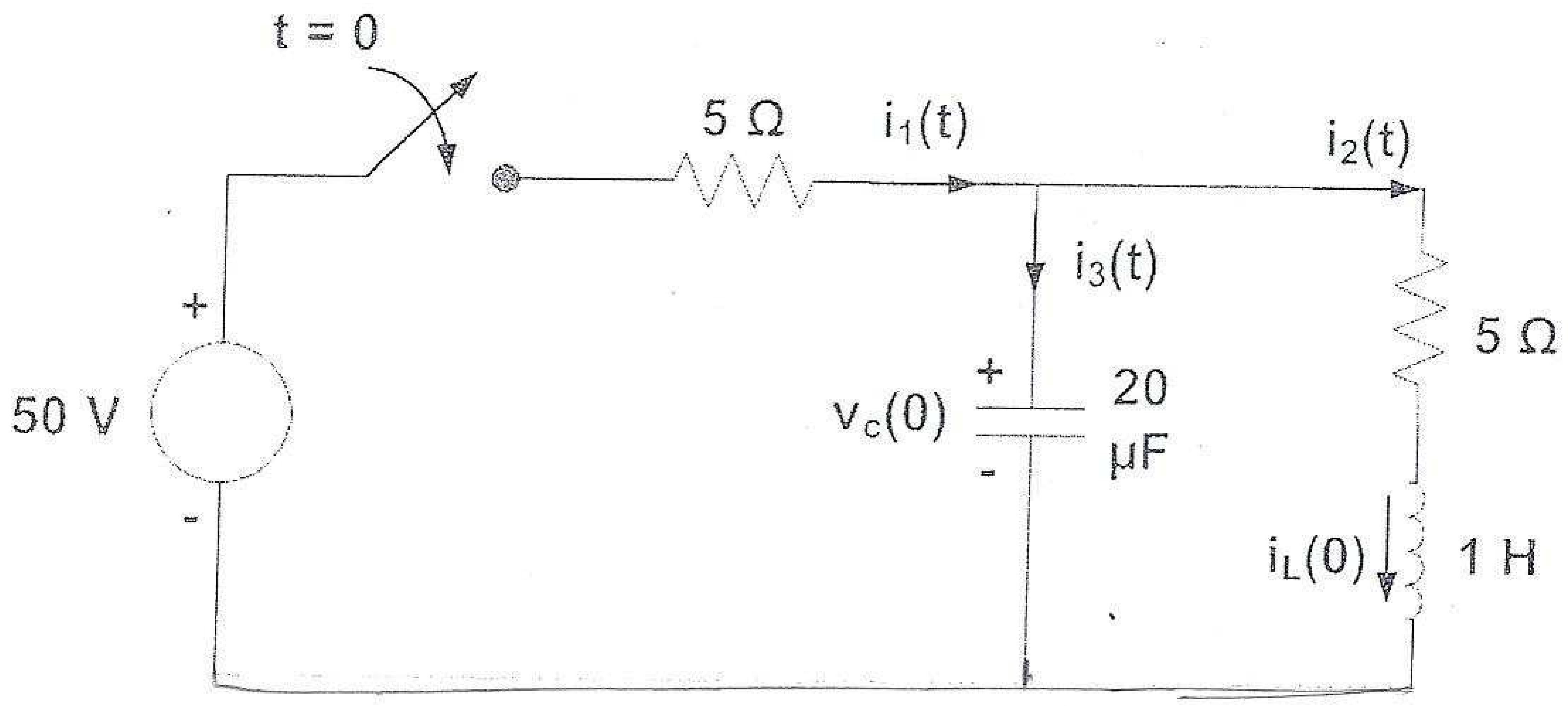
$$i(\infty) = \lim_{S \to 0} \frac{0.8(10S^2 + 3S)}{S^2 + 0.36S + 0.016} = 0$$

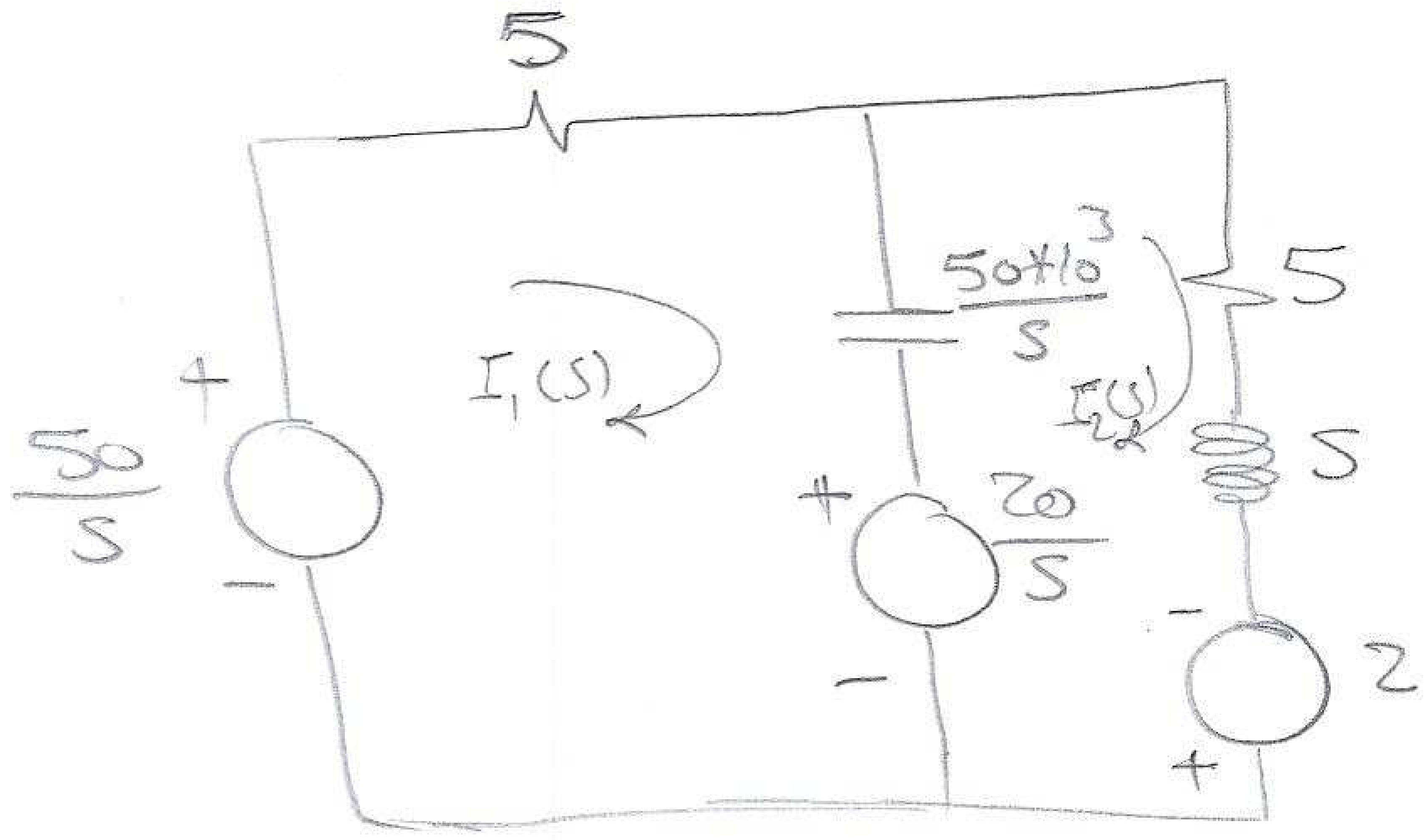
i(t)	$\left(7.75\, e^{-0.052t} + 0.25\, e^{-0.308t}\right)u(t)$
Initial Value	8 Amp
Final Value	0

Table 7

Question[8]: Answer of Question 10

When the switch in the circuit shown in figure is closed, the condenser was charged such that $v_c(0)=20$ volts and the current in the coil was $i_L(0)=2A$. Derive expressions for the instantaneous currents $i_1(t)$, $i_2(t)$ and $i_3(t)$ for $t > 0$.
Insert your results in Table 8.





$$\begin{bmatrix} 5+\frac{5*10^4}{s} & \frac{-5*10^4}{s} \\ \frac{-5*10^4}{s} & 5+\frac{5*10^4}{s}+s \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{30}{s} \\ \frac{2(s+10)}{s} \end{bmatrix}$$

$$\Delta = \left(5 + \frac{5*10^4}{S}\right)^2 + 5S + 5*10^4 - \left(\frac{5*10^4}{S}\right)^2$$

$$= 25 + \frac{5*10^5}{S} + 5S + 5*10^4$$

$$= \frac{5(S^2 + 10005S + 10^5)}{S}$$

$$\Delta_1 = \begin{vmatrix} \frac{30}{S} & \frac{-5*10^4}{S} \\ \frac{2(S+10)}{S} & 5+S+\frac{5*10^4}{S} \end{vmatrix}$$

$$= \frac{150}{S} + 30 + \frac{15*10^5}{S^2} + \frac{10^5(S+10)}{S^2}$$

$$= \frac{5(6S^2 + 20030S + 5*10^5)}{S^2}$$

$$\Delta_2 = \begin{vmatrix} 5+\frac{5*10^4}{S} & \frac{30}{S} \\ \frac{-5*10^4}{S} & \frac{2(S+10)}{S} \end{vmatrix} = \frac{10(S+10)}{S} + \frac{10^5(S+10)}{S^2} + \frac{15*10^5}{S^2}$$

$$= \frac{10S^2 + 100S + 10^5 S + 25*10^5}{S^2}$$

$$= \frac{5(2S^2 + 20020 + 5*10^5)}{S^2}$$

$$I_1(s) = \frac{\Delta_1}{\Delta} = \frac{6s^2 + 20030s + 5*10^5}{s(s^2 + 10025s + 10^5)}$$

$$= \frac{6s^2 + 20030s + 5*10^5}{s(s^2 + 10005s + 10^5)} = \frac{6s^2 + 20030s + 5*10^5}{s(s+10)(s+9995)}$$

$$= \frac{5}{s} - \frac{3}{s+10} + \frac{4}{s+9995}$$

$$\therefore i_1(t) = \left(5 - 3e^{-10t} + 4e^{-9995t}\right)u(t)$$

$$I_2(s) = \frac{\Delta_2}{\Delta} = \frac{2s^2 + 20020s + 5*10^5}{s(s+10)(s+9995)}$$

$$= \frac{5}{s} - \frac{3}{(s+10)} + \frac{2*10^{-3}}{(s+9995)}$$

$$\therefore i_2(t) = \left(5 - 3e^{-10t} + 2*10^{-3}e^{-9995t}\right)u(t)$$

$$I_3(s) = I_1(s) - I_2(s) = \frac{3.998}{s+9995}$$

$$\therefore i_3(t) = \left(3.998\,e^{-9995t}\right)u(t)$$

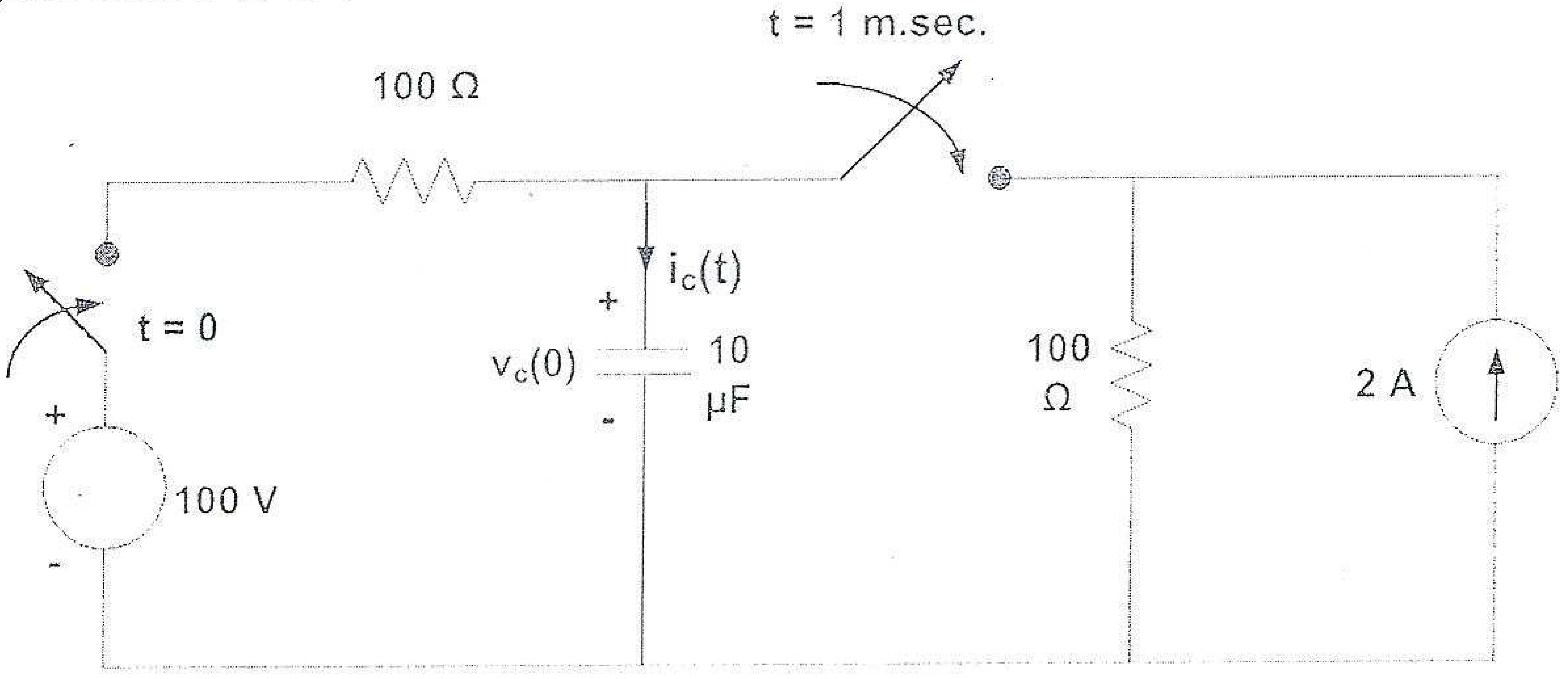
$i_1(t)$	$(5-3e^{-10t}+4e^{-9995t})u(t)$
$i_2(t)$	$(5-3e^{-10t}+2*10^{-3}\,e^{-9995t})u(t)$
$i_3(t)$	$(3.998\,e^{-9995t})u(t)$.

Table 8

Question[9]: Answer of Question 12

In the circuit shown in figure, the capacitor is initially charged such that $v_c(0) = 50$ volts. Calculate $v_c(t)$, $t > 0$. Derive an expression for the capacitor current in the time domain $i_c(t)$, $t > 1$ m.sec. and calculate its steady state value $i_c(\infty)$. Verify your result using the final value theorem.

Insert your results in Table 9.



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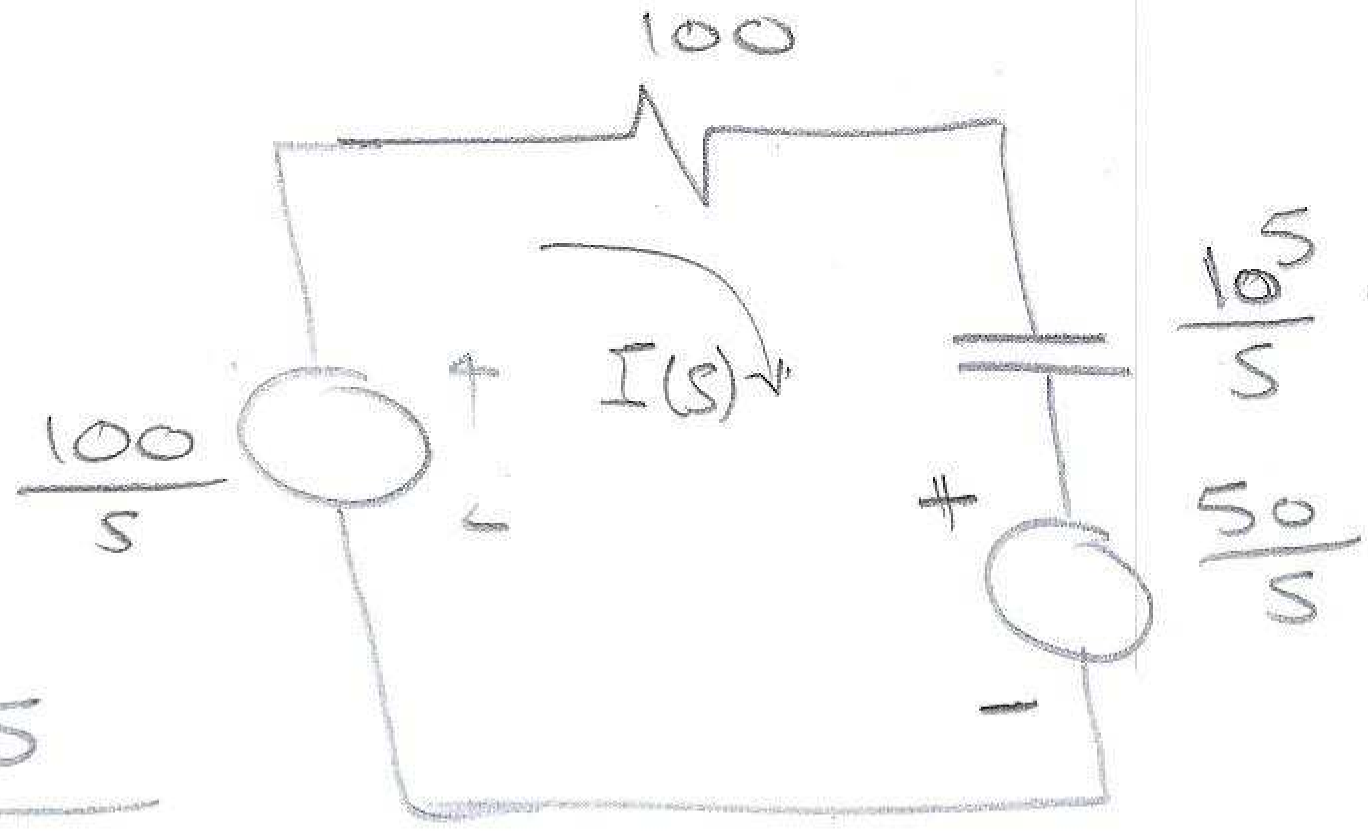
* at $0 \leq t \leq 1\,\text{msec}$



$$I(s) = \frac{\frac{100}{s} - \frac{50}{s}}{100 + \frac{10^5}{s}}$$

$$= \frac{50}{100s + 10^5} = \frac{0.5}{s + 1000}$$

$$\therefore I_c(s) = \frac{0.5}{s+1000}$$

$$\therefore i_c(t) = 0.5\, e^{-1000t} \{u(t) - u(t - 1\,\text{msec})\}$$

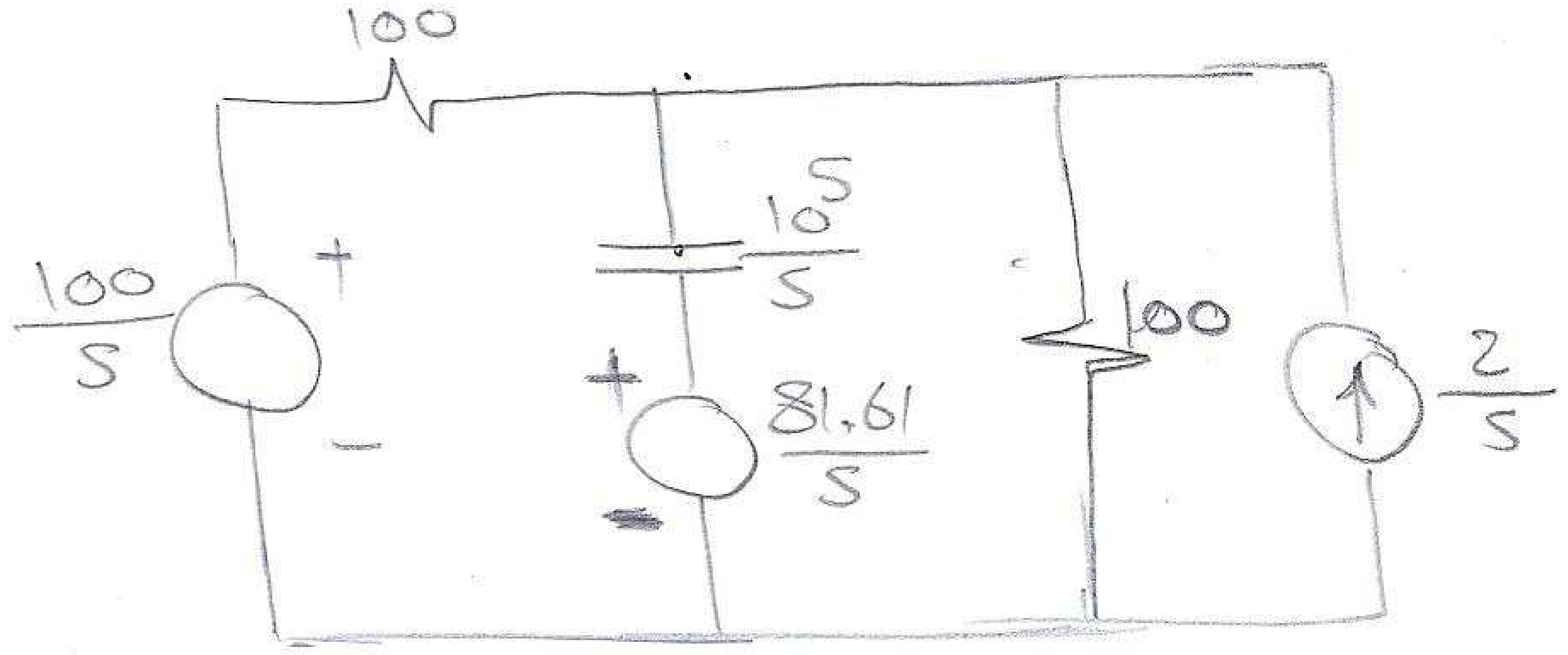
$$V_C(s) = \frac{10^5}{s} * I_C(s) + \frac{50}{s}$$

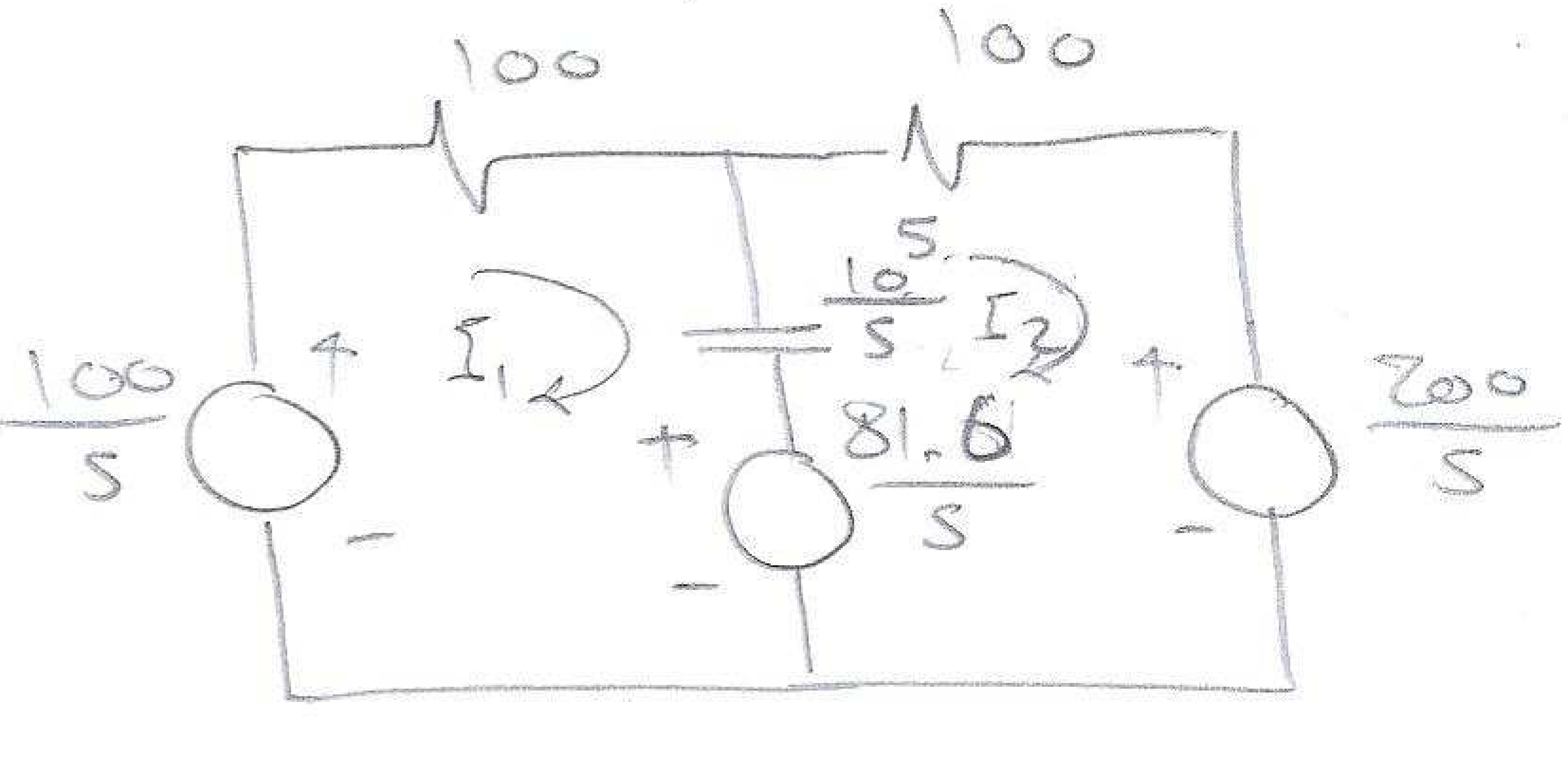
$$= \frac{5*10^4}{s(s+1000)} + \frac{50}{s}$$

$$= \frac{100}{s} - \frac{50}{s+1000}$$

$$\therefore v_C(t) = 50\left(2 - e^{-1000t}\right)\left\{u(t) - u(t - 1\,\text{msec})\right\}$$

$$v_C(1\,\text{msec}) = 50(2 - e^{-1}) = 81.61 \text{ volts.}$$





$$\begin{bmatrix} 100 + \frac{10^5}{s} & \frac{-10^5}{s} \\ \frac{-10^5}{s} & 100 + \frac{10^5}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{18.4}{s} \\ \frac{-118.4}{s} \end{bmatrix}$$

$$\Delta = \left(100 + \frac{10^5}{s}\right)^2 - \left(\frac{10^5}{s}\right)^2 = 10^4 + \frac{2*10^7}{s} = \frac{10^4(s+2000)}{s}$$

$$\Delta_1 = \begin{vmatrix} \frac{18.4}{s} & \frac{-10^5}{s} \\ \frac{-118.4}{s} & 100 + \frac{10^5}{s} \end{vmatrix} = \frac{1840}{s} + \frac{18.4*10^5}{s^2} - \frac{118.4*10^5}{s^2}$$

$$= \frac{1840s - 100*10^5}{s^2}.$$

$$\Delta_2 = \begin{vmatrix} 100 + \frac{10^5}{s} & \frac{18.4}{s} \\ \frac{-10^5}{s} & \frac{-118.4}{s} \end{vmatrix} = \frac{-11840}{s} - \frac{118.4*10^5}{s^2} + \frac{18.4*10^5}{s^2}$$

$$= \frac{-11840s - 100*10^5}{s^2}$$

$$I_1(s) = \frac{\Delta_1}{\Delta} = \frac{1840s - 100*10^5}{s^2} * \frac{s}{10^4(s+2000)}$$

$$= \frac{0.184s - 1000}{s(s+2000)} = \frac{-1/2}{s} + \frac{0.684}{s+2000}$$

$$I_2(s) = \frac{\Delta_2}{\Delta} = \frac{-11840s - 100*10^5}{s^2} * \frac{s}{10^4(s+2000)}$$

$$= \frac{-1.184s - 1000}{s(s+2000)} = \frac{-1/2}{s} - \frac{0.684}{s+2000}$$

$$I_c(s) = I_1(s) - I_2(s) = \frac{1.368}{s+2000}$$

$$\therefore i_c(t) = \left(1.368\, e^{-2000t}\right)u(t).$$

$$i_c(t-1msec) = \left(1.368\, e^{-2000(t-1msec)}\right) u(t-1msec)$$

$$V_c(s) = \frac{1.368 * 10^5}{s(s+2000)} + \frac{81.61}{s}$$

$$= \frac{150}{s} - \frac{68.4}{s+2000}$$

$$\therefore v_c(t) = \left(150 - 68.4\, e^{-2000t}\right) u(t).$$

$$v_c(t-1msec) = \left\{150 - 68.4\, e^{-2000(t-1msec)}\right\} u(t-1msec)$$

$$\boxed{i_c(\infty) = 0}$$

$$i_c(\infty) = \lim_{s \to 0} \frac{1.368\, s}{s+2000} = 0$$

$v_c(t), t > 0$	$50(2 - e^{-1000t})\{u(t) - u(t-1msec)\}$ $+ \left(150 - 68.4\, e^{-2000(t-1msec)}\right) u(t-1msec)$
$i_c(t), t > 1$ m.sec.	$\left(1.368\, e^{-2000(t-1msec)}\right) u(t-1msec)$
$i_c(\infty)$	0

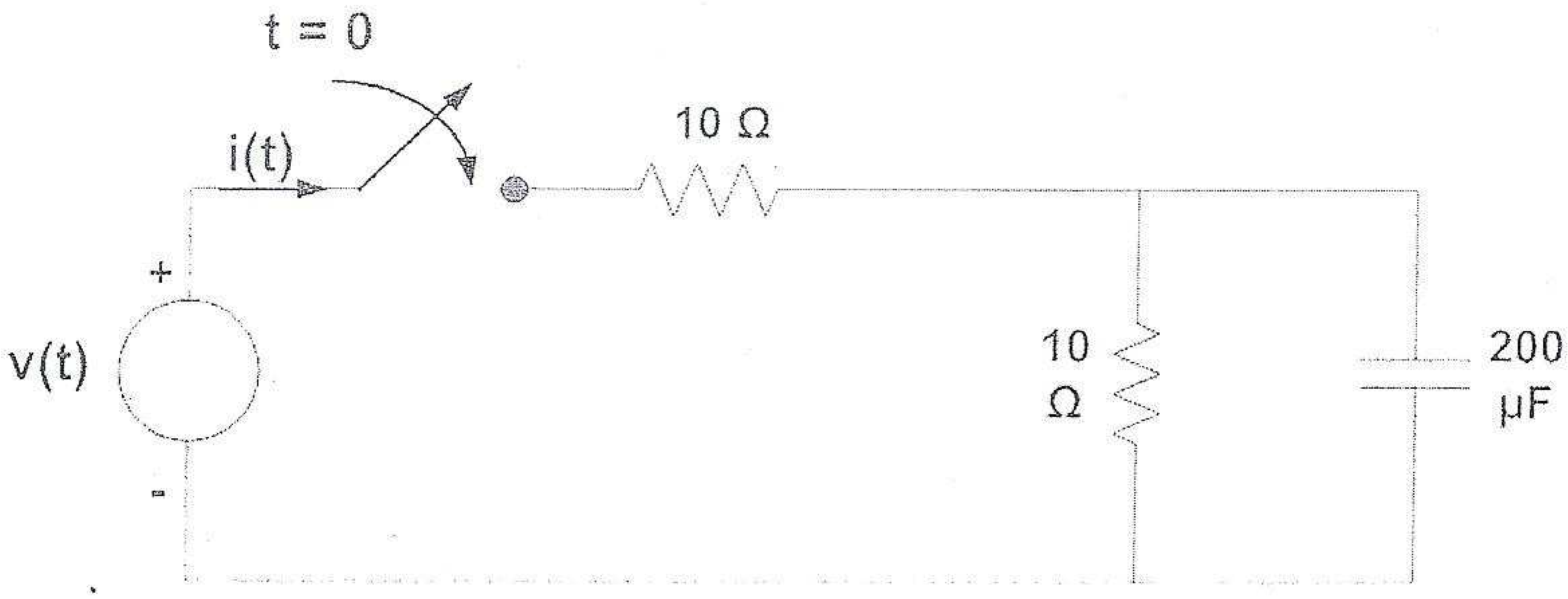
Table 9

Question[10]: Answer of Question 13

The circuit shown in figure is fed from a source v(t) given by :

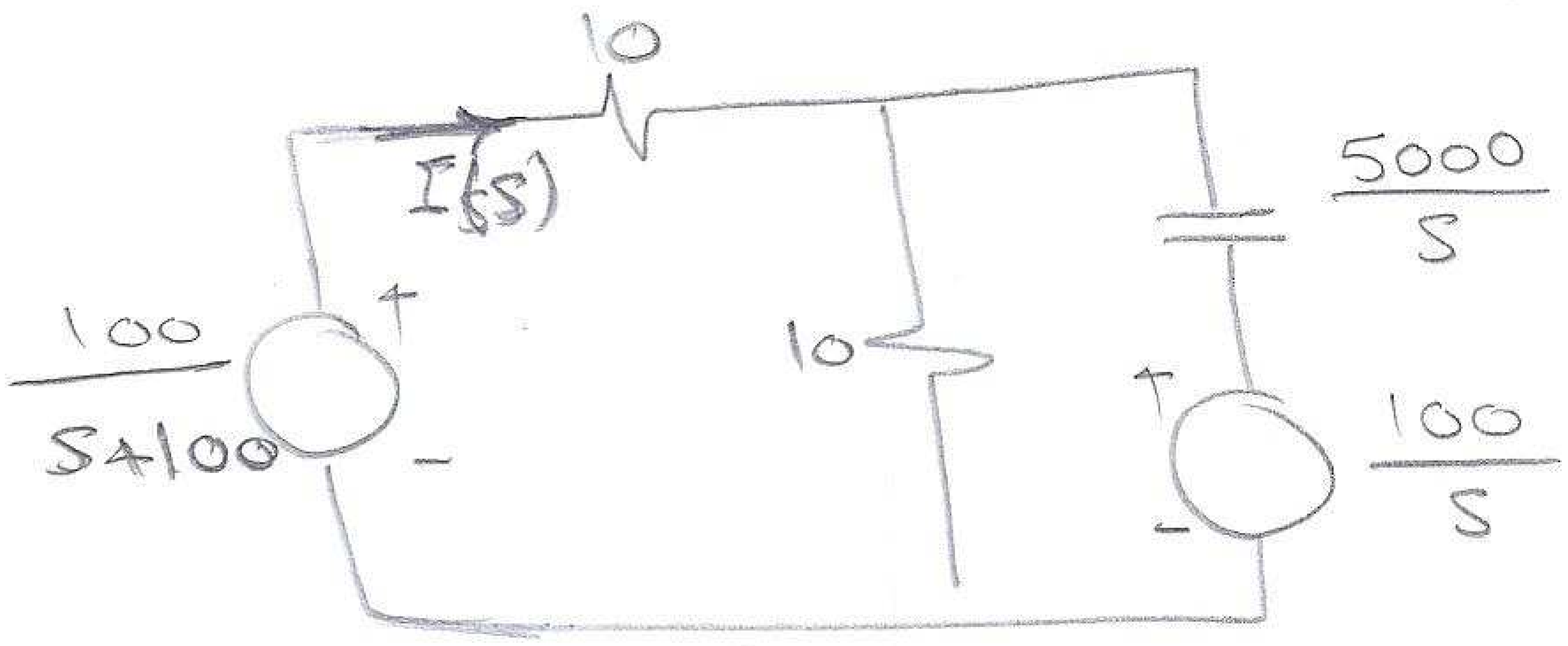
$$v(t) = 100\, e^{-100t} \quad Volts$$

Calculate i(t), t > 0 .
Insert your result in Table 10 .

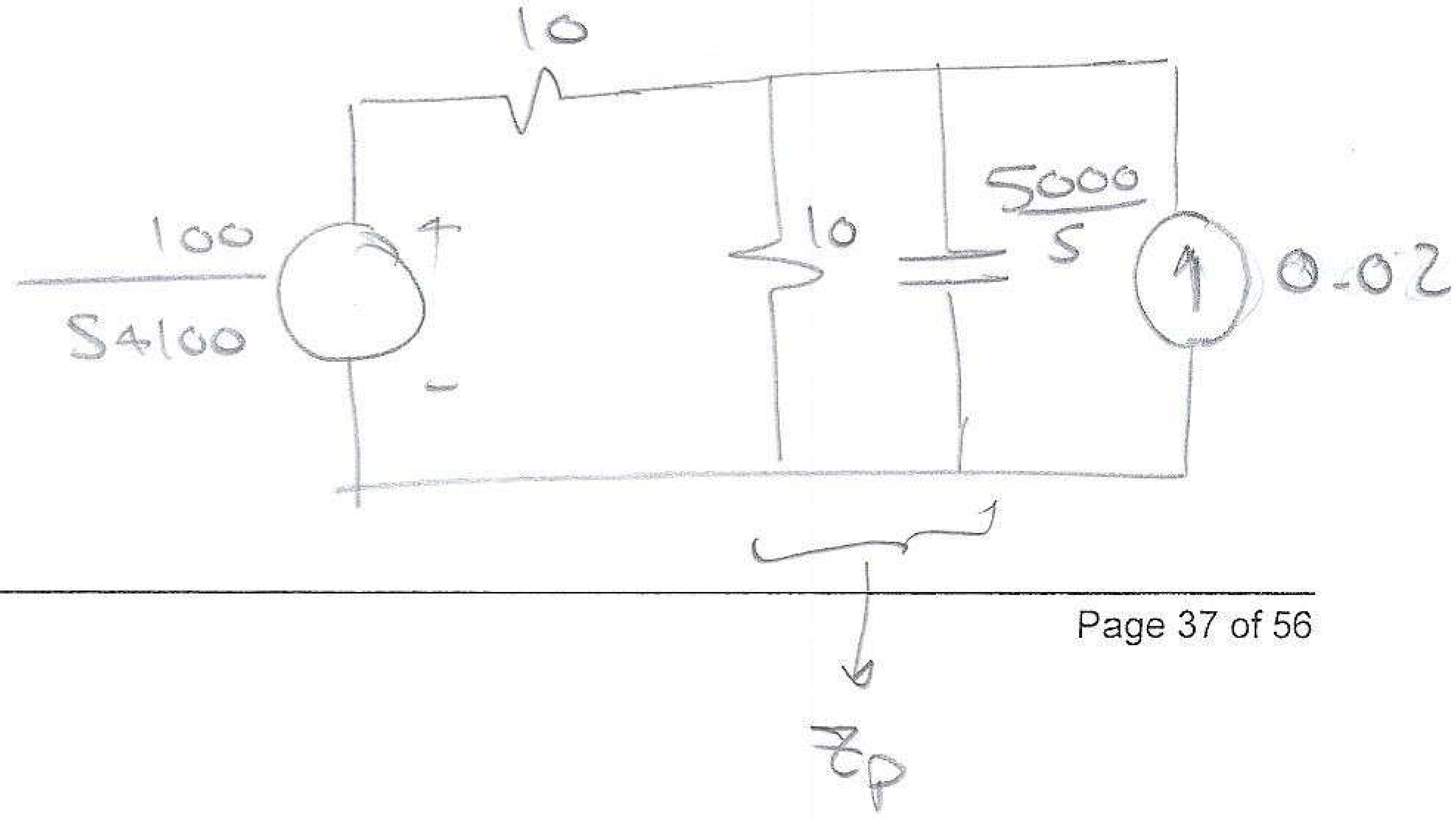


$\Rightarrow$ at $t=0$ $v(t) = 100\, e^{0} = 100$ Volts.

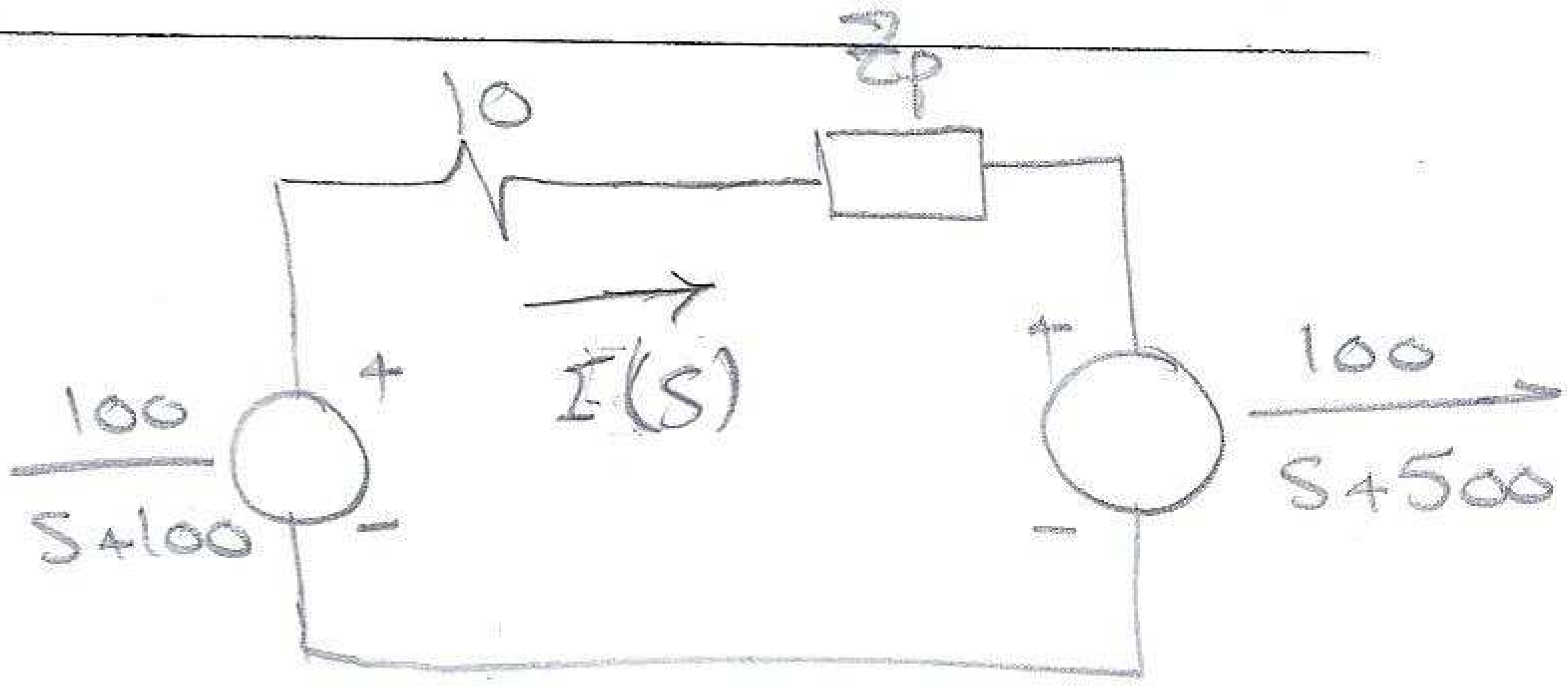
$\Rightarrow$ $V_C(0) = 100$ Volts



$$Z_p = \frac{\frac{5 * 10^4}{S}}{10 + \frac{5000}{S}} = \frac{5 * 10^4}{10S + 5000} = \frac{5000}{S + 500}$$



$$I(s) = \frac{\frac{100}{s+100} - \frac{100}{s+500}}{10 + \frac{5000}{s+500}}$$



$$= \frac{10(s+500) - 10(s+100)}{(s+100)(s+500) + 500(s+100)}$$

$$= \frac{4000}{s^2 + 1100s + 10^5} = \frac{4000}{(s+100)(s+1000)}$$

$$= \frac{40/9}{s+100} - \frac{40/9}{s+1000}$$

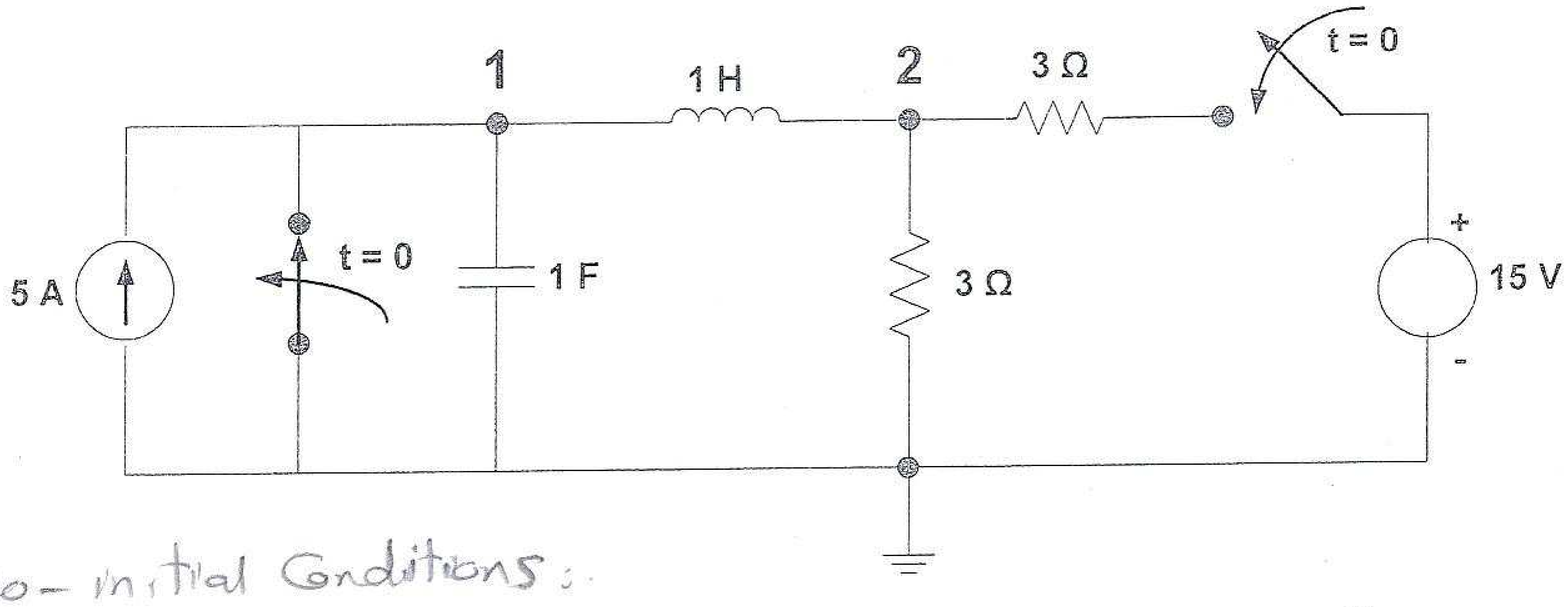
$$\therefore i(t) = \frac{40}{9}\left(e^{-100t} - e^{-1000t}\right) u(t)$$

i(t), t > 0	$\frac{40}{9}\left(e^{-100t} - e^{-1000t}\right)u(t)$

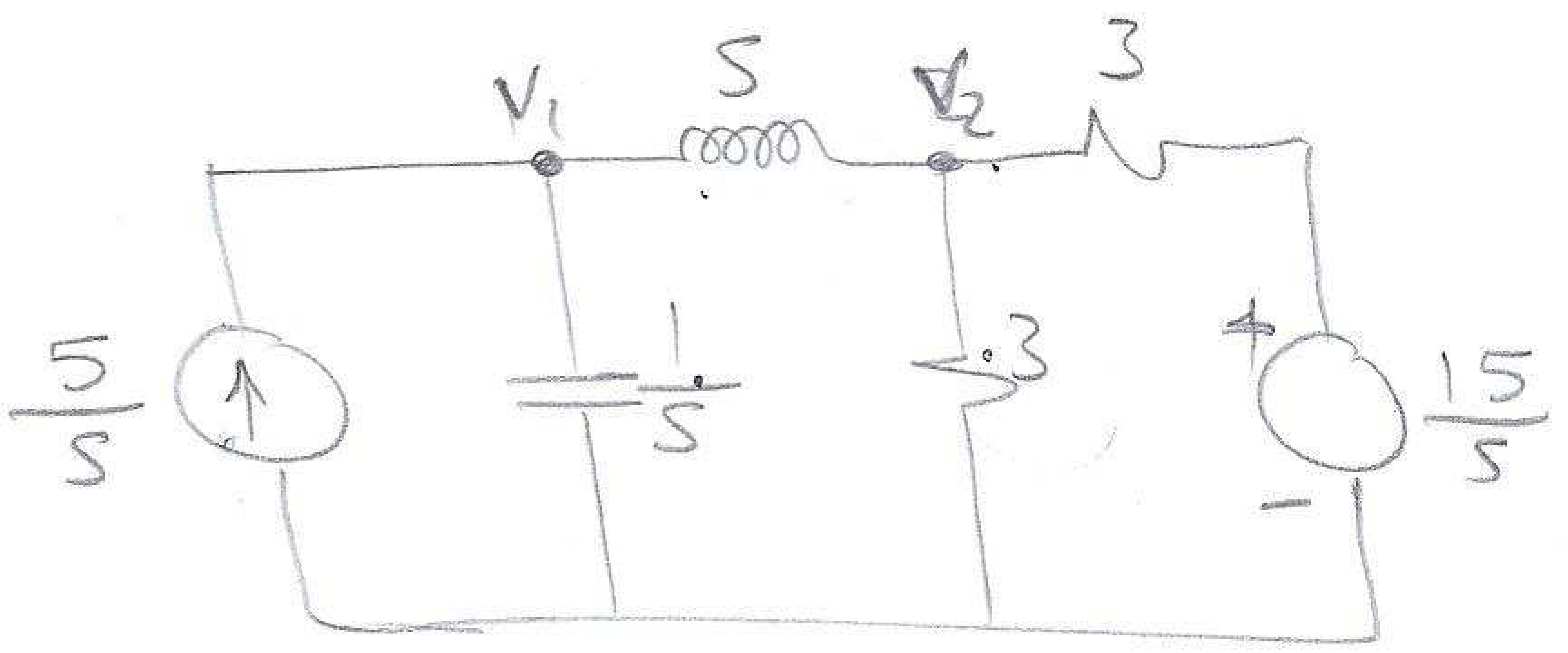
Table (10)

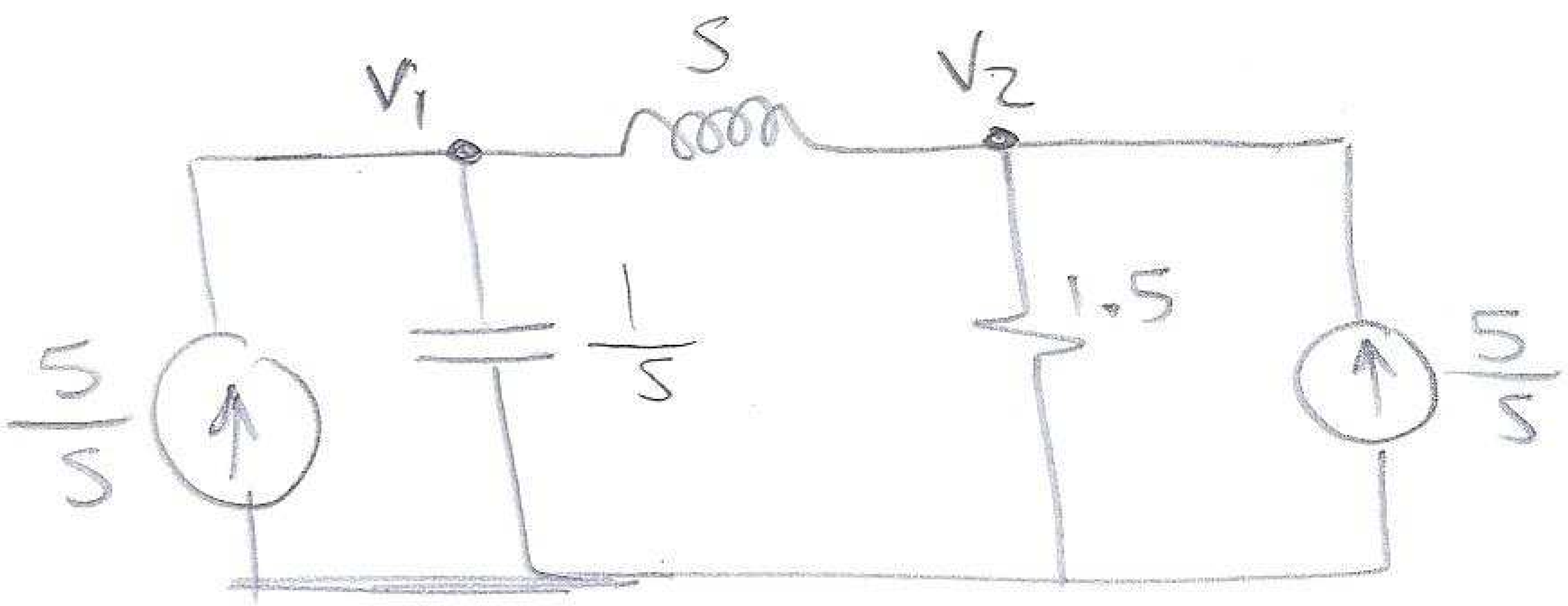
Question[11]: Answer of Question 14

For the circuit shown in figure, calculate the node potentials $v_1(t)$ and $v_2(t)$. Using the initial and final value theorems, calculate $v_1(0)$, $v_2(0)$, $v_1(\infty)$ and $v_2(\infty)$.
Insert your results in Table 11 .



→ no-initial Conditions :





$$\begin{bmatrix} s+\frac{1}{s} & \frac{-1}{s} \\ \frac{-1}{s} & \frac{1}{s}+\frac{2}{3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{s} \\ \frac{5}{s} \end{bmatrix}$$

$$\begin{bmatrix} \frac{s^2+1}{s} & \frac{-1}{s} \\ \frac{-1}{s} & \frac{2s+3}{3s} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{s} \\ \frac{5}{s} \end{bmatrix}$$

$$\Delta = \frac{(2s+3)(s^2+1)}{3s^2} - \frac{1}{s^2} = \frac{2s^3+3s^2+2s+\cancel{3}-\cancel{3}}{3s^2}$$

$$\therefore \Delta = \frac{2s^2+3s+2}{3s}$$

$$\Delta_1 = \begin{vmatrix} \frac{5}{s} & \frac{-1}{s} \\ \frac{5}{s} & \frac{2s+3}{3s} \end{vmatrix} = \frac{10s+15}{3s^2} + \frac{5}{s^2}$$

$$= \frac{10s+30}{3s^2} = \frac{10(s+3)}{3s^2}$$

$$\Delta_2 = \begin{vmatrix} \frac{s^2+1}{s} & \frac{5}{s} \\ \frac{-1}{s} & \frac{5}{s} \end{vmatrix} = \frac{5s^2+5+5}{s^2} = \frac{5(s^2+2)}{s^2}$$

$$V_1(s) = \frac{\Delta_1}{\Delta} = \frac{10(s+3)}{\cancel{3}s^2} \times \frac{\cancel{3}s}{2s^2+3s+2} = \frac{5(s+3)}{s(s^2+1.5s+1)}$$

$$= \frac{5(s+3)}{s\left(s+\frac{3}{4}-j\frac{\sqrt{7}}{4}\right)\left(s+\frac{3}{4}+j\frac{\sqrt{7}}{4}\right)}$$

$$= \frac{15}{s} + \frac{k_1}{s+\frac{3}{4}-j\frac{\sqrt{7}}{4}} + \frac{k_1^*}{s+\frac{3}{4}+j\frac{\sqrt{7}}{4}}$$

$$k_1 = 8.86 \angle 147.8^\circ$$

$$\therefore v_1(t) = 15 + 17.72\, e^{-\frac{3}{4}t} \cos\left(\frac{\sqrt{7}}{4}t + 147.8^\circ\right) \text{ Volts.}$$

$$V_2(s) = \frac{5(s^2+2)}{s^2} * \frac{3s}{2s^2+3s+2}$$

$$= \frac{7.5(s^2+2)}{s\left(s+\frac{3}{4}-j\frac{\sqrt{7}}{4}\right)\left(s+\frac{3}{4}+j\frac{\sqrt{7}}{4}\right)}$$

$$= \frac{15}{s} + \frac{k_2}{s+\frac{3}{4}-j\frac{\sqrt{7}}{4}} + \frac{k_2^*}{s+\frac{3}{4}+j\frac{\sqrt{7}}{4}}$$

$$k_2 = 13.3 \angle 106.38^\circ$$

$$\therefore v_2(t) = 15 + 26.6\, e^{-\frac{3}{4}t} \cos\left(\frac{\sqrt{7}}{4}t + 106.38\right)$$

$$v_1(0) = \lim_{s \to \infty} \frac{5(s+3)}{s^2+\frac{3}{2}s+1} = 0$$

$$v_1(\infty) = \lim_{s \to 0} \frac{5(s+3)}{s^2+\frac{3}{2}s+1} = 15 \text{ Volts.}$$

$$v_2(0) = \lim_{s \to \infty} \frac{7.5(s^2+2)}{s^2+\frac{3}{2}s+1} = 7.5 \text{ Volts.}$$

$$v_2(\infty) = \lim_{s \to 0} \frac{7.5(s^2+2)}{s^2+\frac{3}{2}s+1} = 15 \text{ Volts.}$$

$v_1(t)$	$15 + 17.72\, e^{-\frac{3}{4}t} \cos(\frac{\sqrt{7}}{4}t + 147.8^\circ)$
$v_2(t)$	$15 + 26.6\, e^{-\frac{3}{4}t} \cos(\frac{\sqrt{7}}{4}t + 106.38^\circ)$
$v_1(0)$	0
$v_2(0)$	7.5 Volts.
$v_1(\infty)$	15 volts.
$v_2(\infty)$	15 volts.

Table 11

Question[12]: Answer of Question 17

Find the Z-parameters of the two-port network shown in Fig.a . If the same network is used in Fig.b, calculate $i_2(t)$ for $t > 0$.
Insert your results in Table 12 .

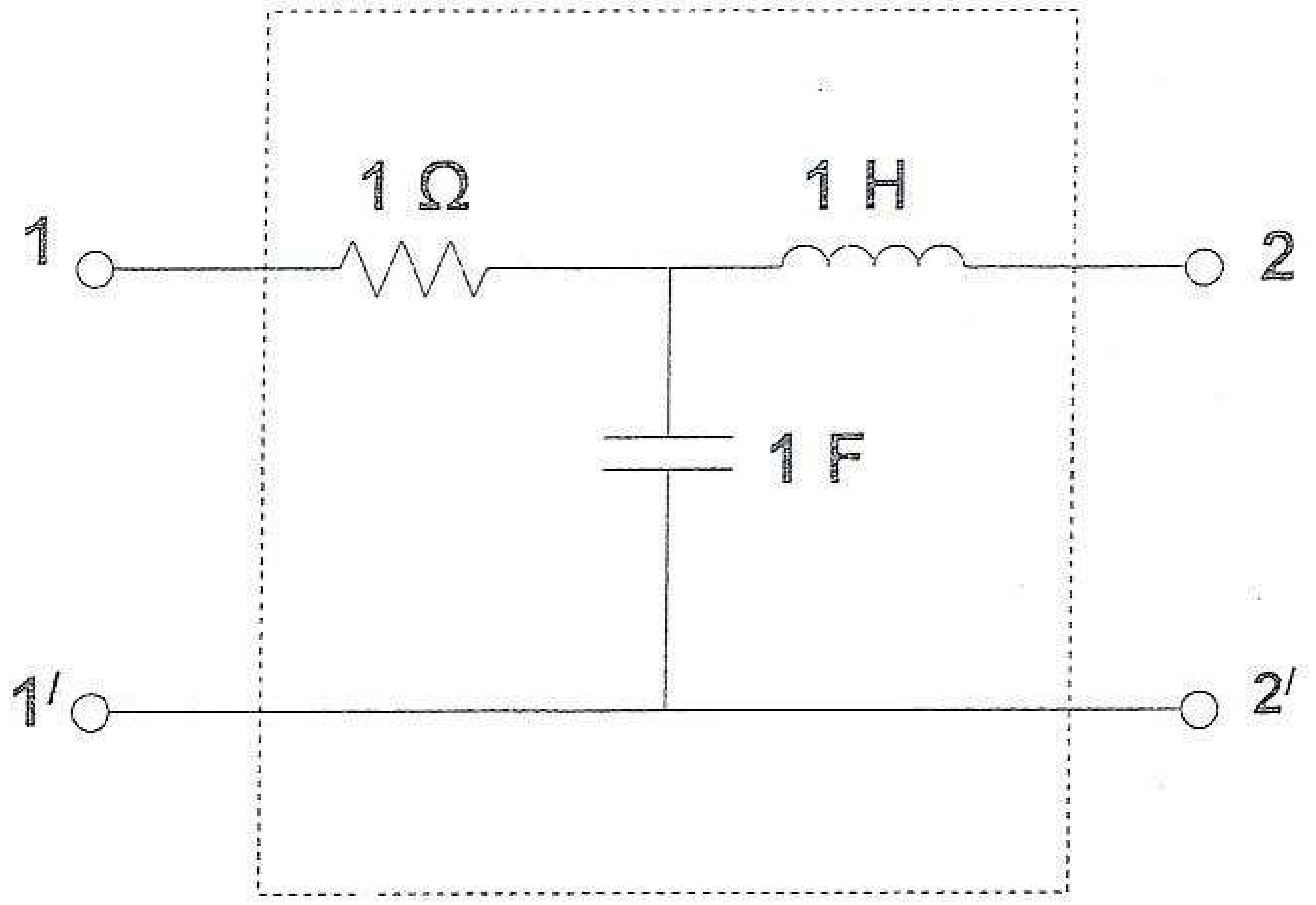


Fig.a

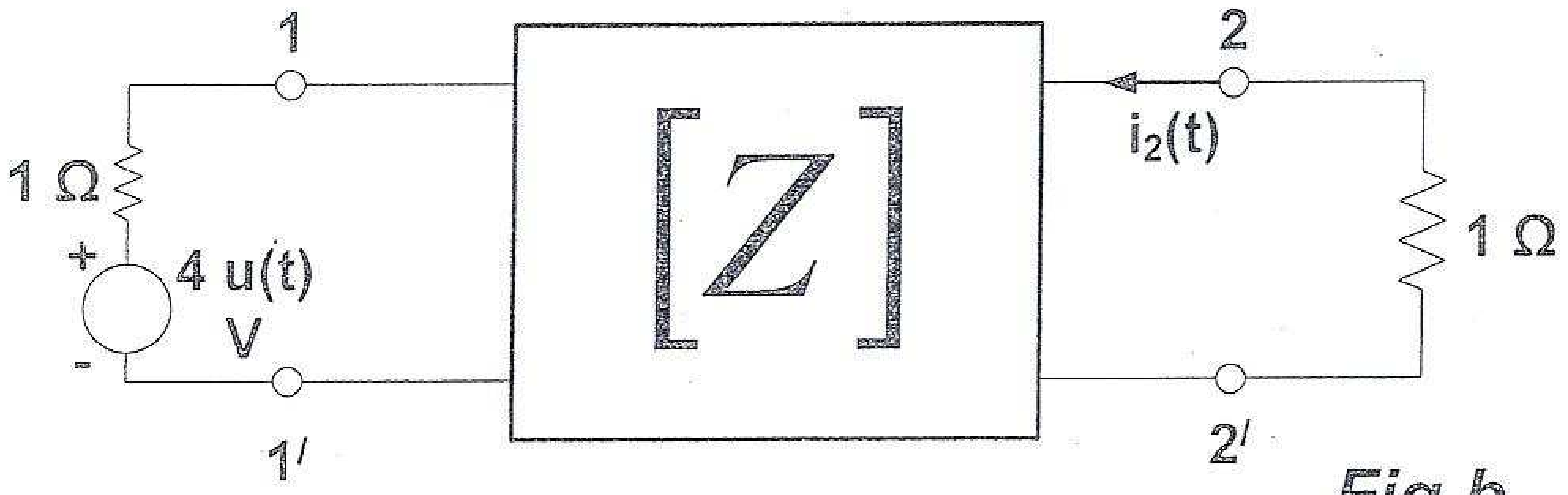
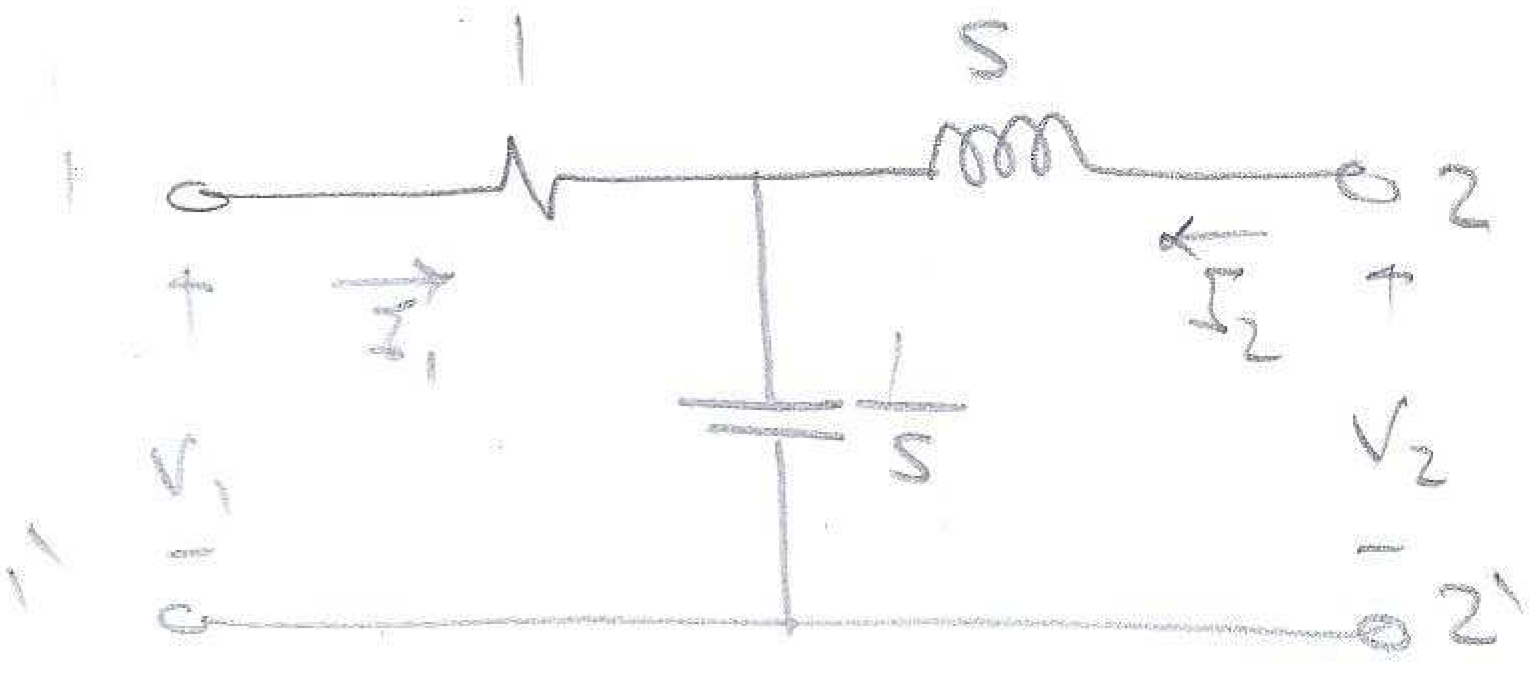


Fig.b

in Fig-a

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$



$\Rightarrow$ let $I_2 = 0$ $\quad \therefore V_1 = Z_{11} I_1 \quad , \quad V_2 = Z_{21} I_1$

$$Z_{11} = \frac{V_1}{I_1} = \frac{(1+\frac{1}{S}) I_1}{I_1} \Rightarrow \boxed{\therefore Z_{11} = \frac{S+1}{S}}$$

$V_2 = \frac{1}{s} I_1$ $\quad \therefore Z_{21} = \frac{1}{s}$

$\therefore$ the system is reciprocal $\quad \boxed{\therefore Z_{21} = Z_{12} = \frac{1}{s}}$

② let $I_1 = 0$

$\therefore V_2 = Z_{22} I_2 \qquad V_2 = \left(s + \frac{1}{s}\right) I_2$

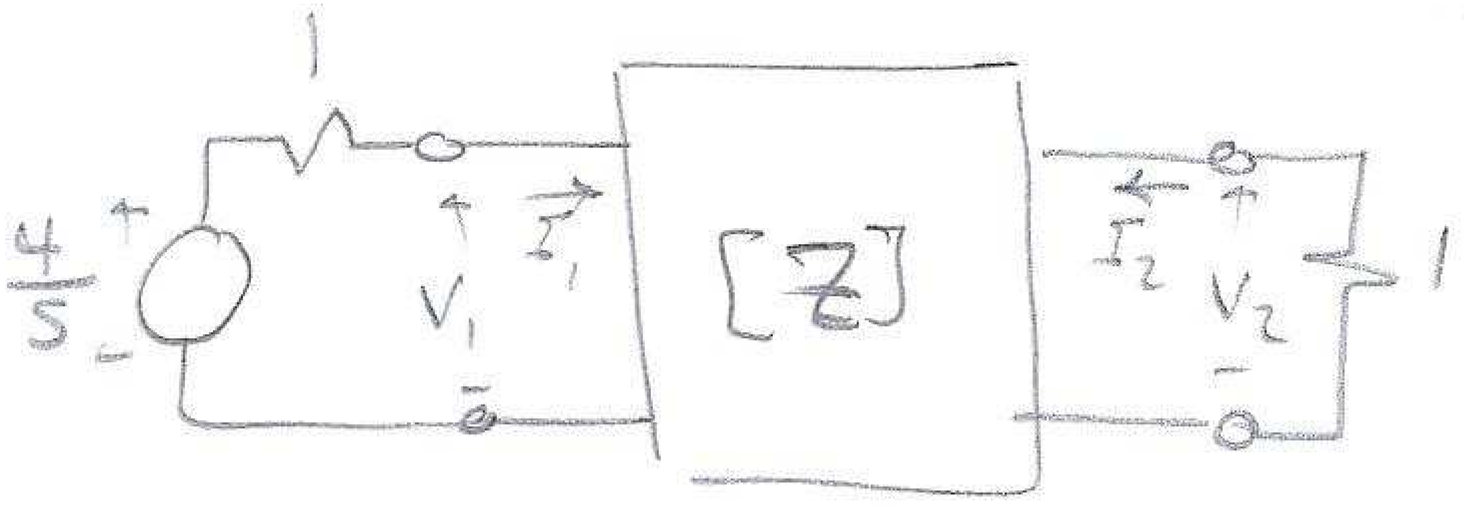
$$\boxed{\therefore Z_{22} = \frac{s^2+1}{s}}$$

$$\therefore [Z] = \begin{bmatrix} \frac{s+1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{s^2+1}{s} \end{bmatrix}.$$

<u>in Fig. b</u>

$V_1 = Z_{11} I_1 + Z_{12} I_2 \leftarrow ①$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \leftarrow ②$



$V_1 = -I_1 + \frac{4}{s} \leftarrow ③$

$V_2 = -I_2 \leftarrow ④$

sub. ④ in ②

$-I_2 = Z_{21} I_1 + Z_{22} I_2 \qquad \therefore -(1 + Z_{22}) I_2 = Z_{21} I_1$

$$\therefore I_1 = \frac{-(1 + Z_{22}) I_2}{Z_{21}}.$$

$-I_1 + \frac{4}{s} = Z_{11} I_1 + Z_{12} I_2$

$\frac{4}{s} = (Z_{11} + 1) I_1 + Z_{12} I_2$

$$\therefore \left(\frac{-(1+Z_{11})(1+Z_{22})}{Z_{21}} + Z_{12} \right) I_2 = \frac{4}{s}$$

$$I_2 = \frac{4 Z_{21}}{Z_{12} Z_{21} - (1+Z_{11})(1+Z_{22})}$$

$$I_2 = \frac{4/s}{\frac{1}{s^2} - \left(1 + \frac{s+1}{s}\right)\left(1 + \frac{s^2+1}{s}\right)}$$

$$\frac{-5 \pm \sqrt{25-32}}{4} = \frac{-5}{4} \pm j \frac{\sqrt{7}}{4}$$

$$= \frac{4/s}{\frac{1}{s^2} - \frac{2s^3+5s^2+4s+1}{s^2}} = \frac{4s}{-(2s^3+5s^2+4s)}$$

$$I_2 = \frac{-2}{s^2 + 2.5s + 2} = \frac{-2}{\left(s + \frac{5}{4} - j\frac{\sqrt{7}}{4}\right)\left(s + \frac{5}{4} + j\frac{\sqrt{7}}{4}\right)}$$

$$= \frac{k}{s + \frac{5}{4} - j\frac{\sqrt{7}}{4}} + \frac{k^*}{s + \frac{5}{4} + j\frac{\sqrt{7}}{4}}$$

$$k = 1.512 \angle 90^\circ$$

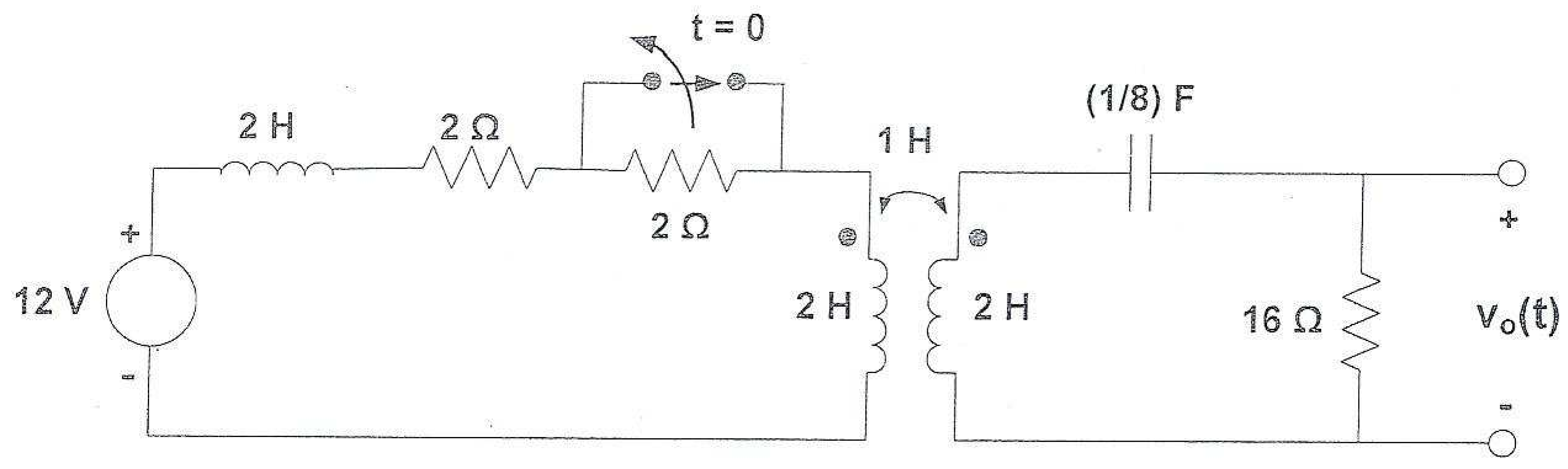
$$\therefore i_2(t) = \left\{ 3.024\, e^{-\frac{5}{4}t} \cos\left(\frac{\sqrt{7}}{4} t + 90\right) \right\} u(t)$$

[Z]	$\begin{bmatrix} 1+\frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & s+\frac{1}{s} \end{bmatrix}$
$i_2(t)$	$\left\{3.024\, e^{-\frac{5}{4}t} \cos\left(\frac{\sqrt{7}}{4}t + 90\right)\right\} u(t)$.

Table 12

Question[13]: Answer of Question 18

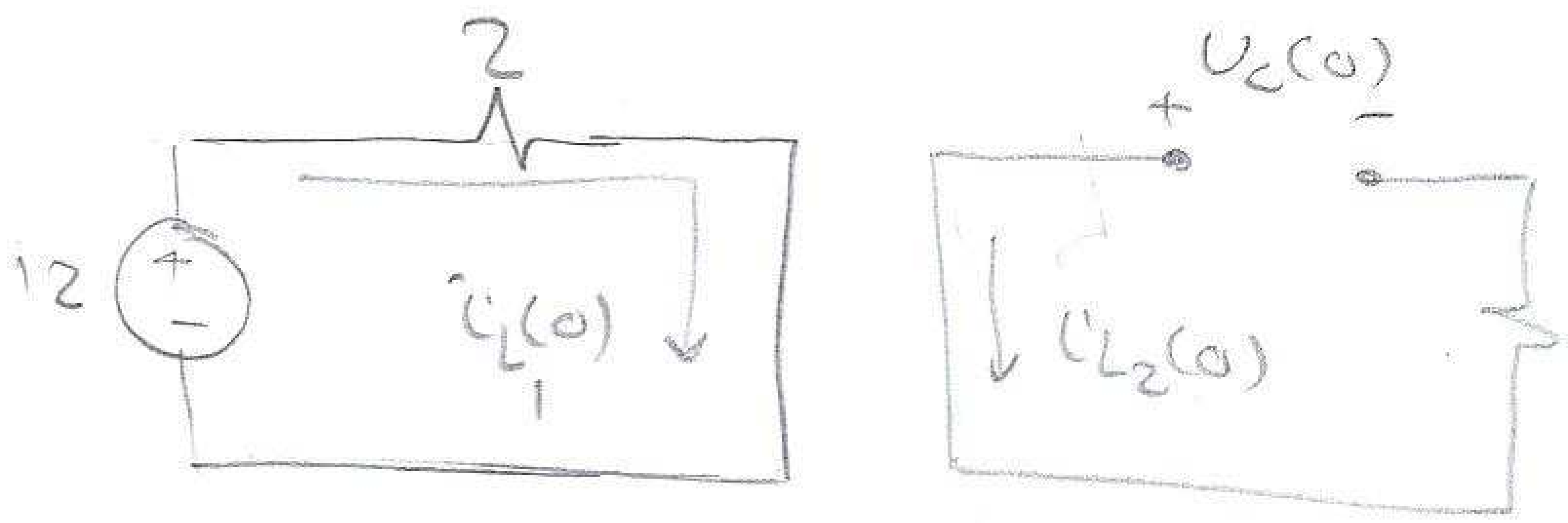
In the circuit shown in figure, derive an expression for the voltage $v_o(t)$ for $t > o$ and sketch its value in volts against time in seconds .
Insert your results in Table 13 and sketch a .

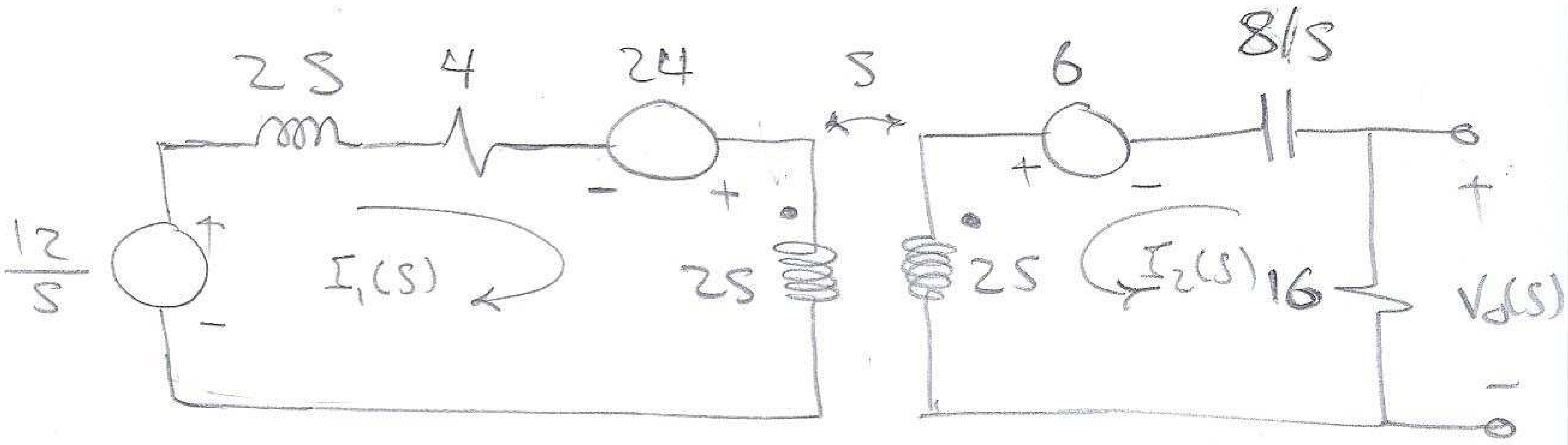


$i_{L_1}(0) = 6A$

$i_{L_2}(0) = 0$

$v_c(0) = 0$





$$\frac{12}{S} + 24 = (4S+4)\,I_1(S) + S\,I_2(S) \longrightarrow (1)$$

$$6 = S\,I_1(S) + \left(\frac{8}{S} + 2S + 16\right) I_2(S) \longrightarrow (2)$$

$$\begin{bmatrix} 4S+4 & S \\ S & \frac{2S^2+16S+8}{S} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{24S+12}{S} \\ 6 \end{bmatrix}$$

$$\Delta = \frac{8(s+1)(s^2+8s+4)}{s} - s^2$$

$$= \frac{7s^3+72s^2+96s+32}{s}$$

$$\Delta_2 = \begin{vmatrix} 4(s+1) & \frac{12(2s+1)}{s} \\ s & 6 \end{vmatrix} = 24(\cancel{s}+1) - 12(2\cancel{s}+1)$$

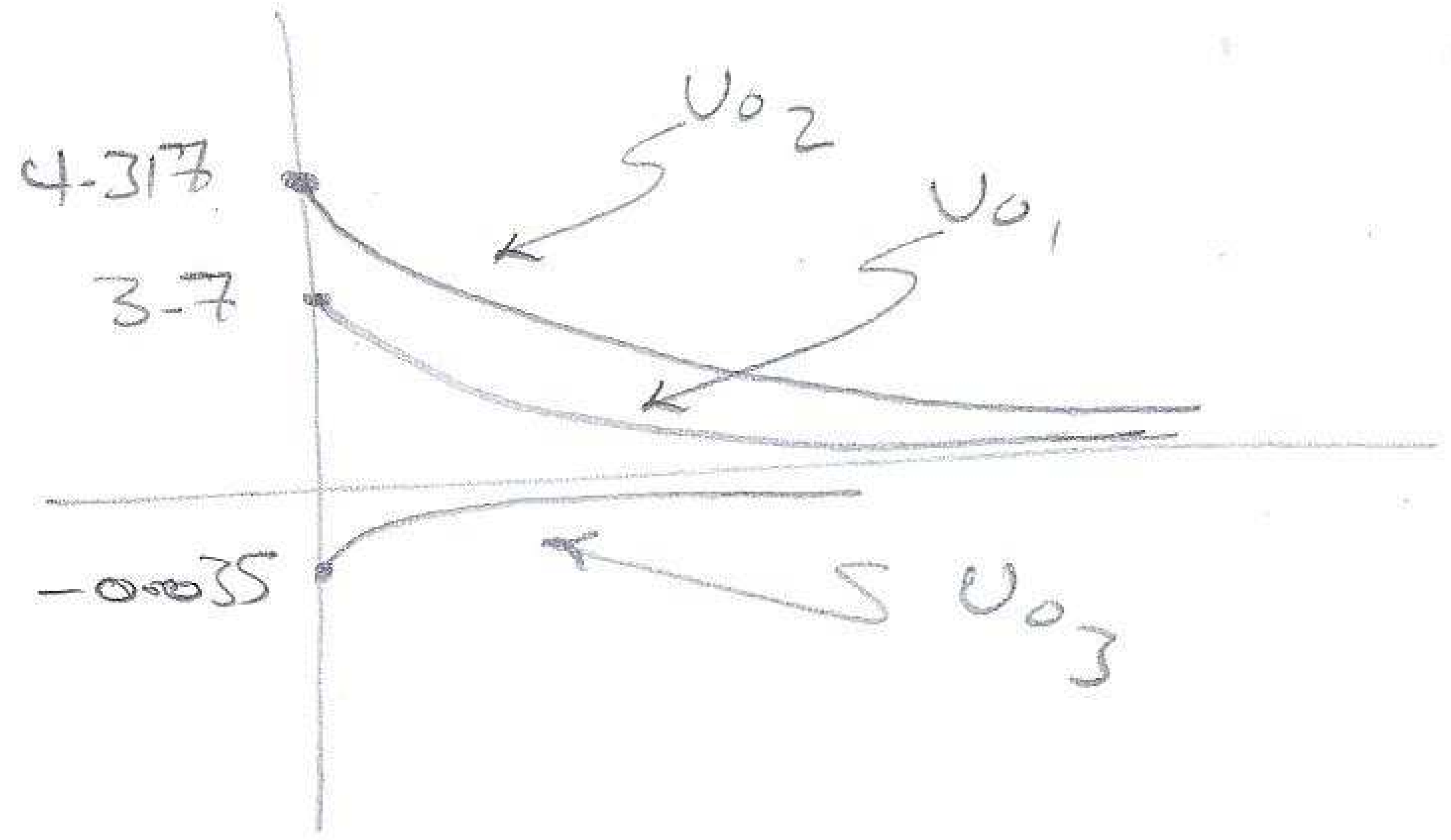
$$= 12$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{12s}{7s^3+72s^2+96s+32}$$

$$V_o(s) = -16\, I_2 = \frac{-192s}{7(s+8.78)(s+0.542)(s+0.96)}$$

$$= \frac{3.7}{s+8.78} + \frac{4.317}{s+0.542} - \frac{0.035}{s+0.96}$$

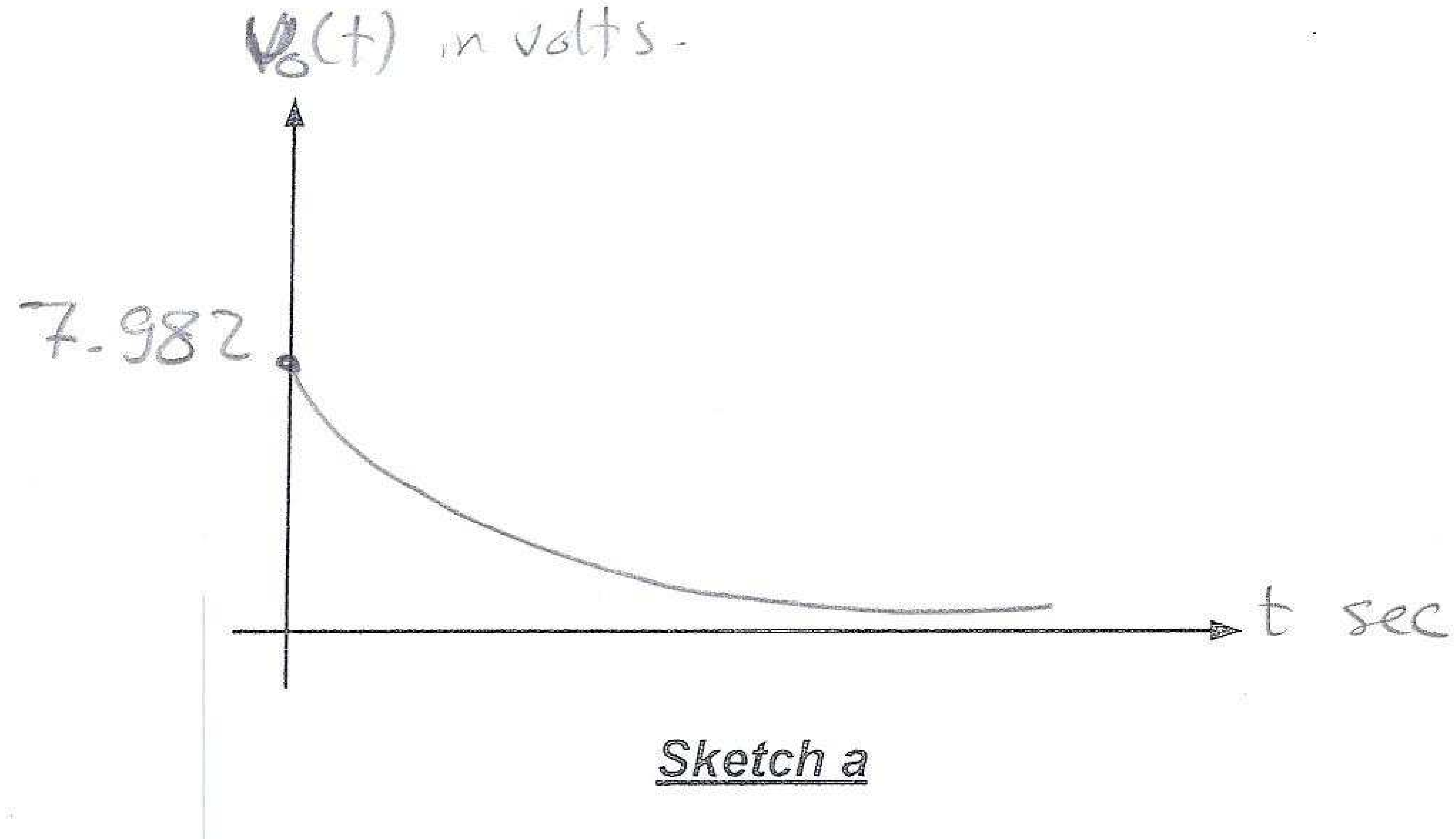
$$\therefore v_o(t) = \left(3.7e^{-8.78t} + 4.317e^{-0.542t} - 0.035e^{-0.96t}\right)u(t)$$



$$v_o(t) = \big(\overset{v_{o1}}{3.7e^{-8.78t}} + \overset{v_{o2}}{4.317e^{-0.542t}} \overset{v_{o3}}{-0.035e^{-0.96t}}\big)u(t).$$

$v_o(t)$	

Table 13



Sketch a

Question[14]: Answer of Question 19

Derive an expression of the output voltage $v_o(t)$ for the circuit shown in Fig.a if the input is the voltage $v(t)$ shown in :
i)Fig.b .
ii)Fig.c .
Insert your result in Table 14 .

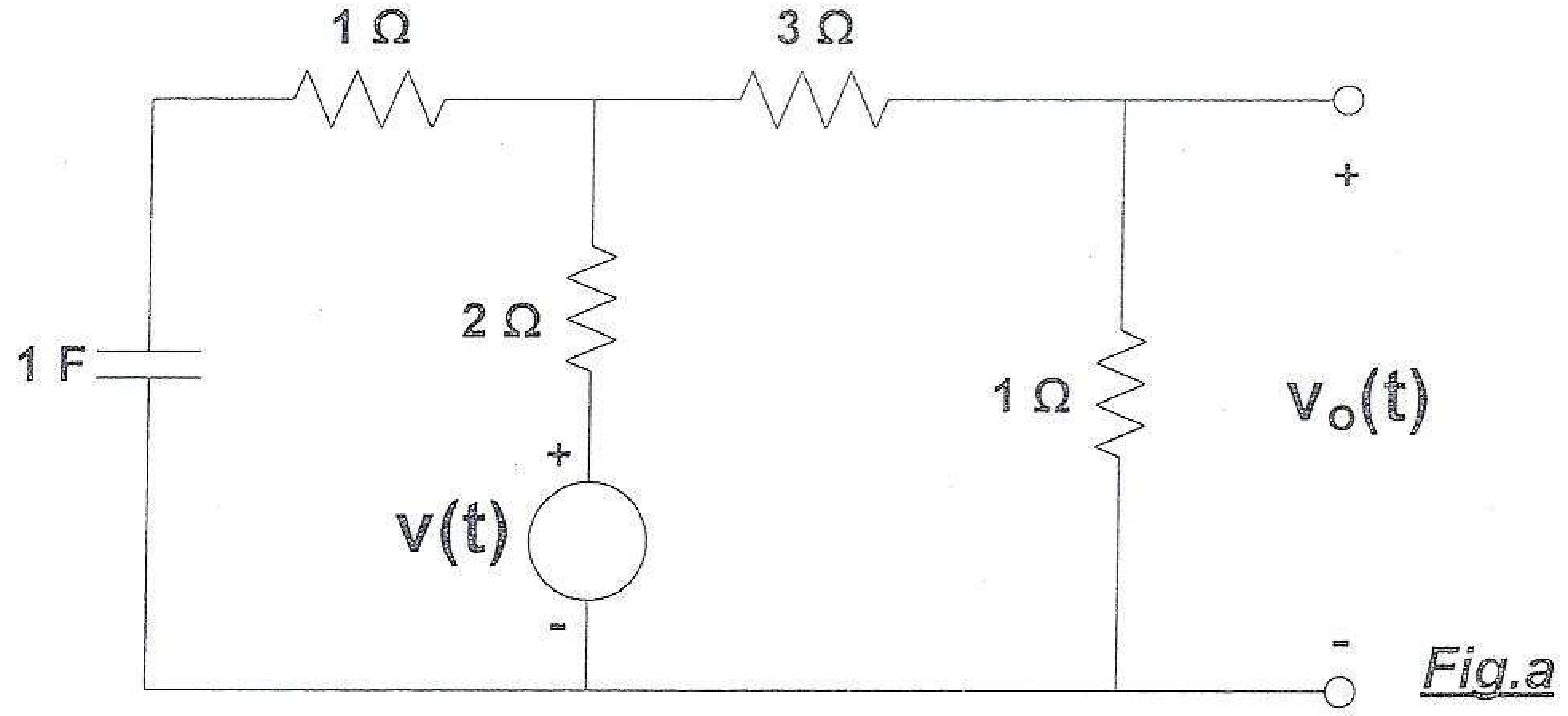


Fig.a

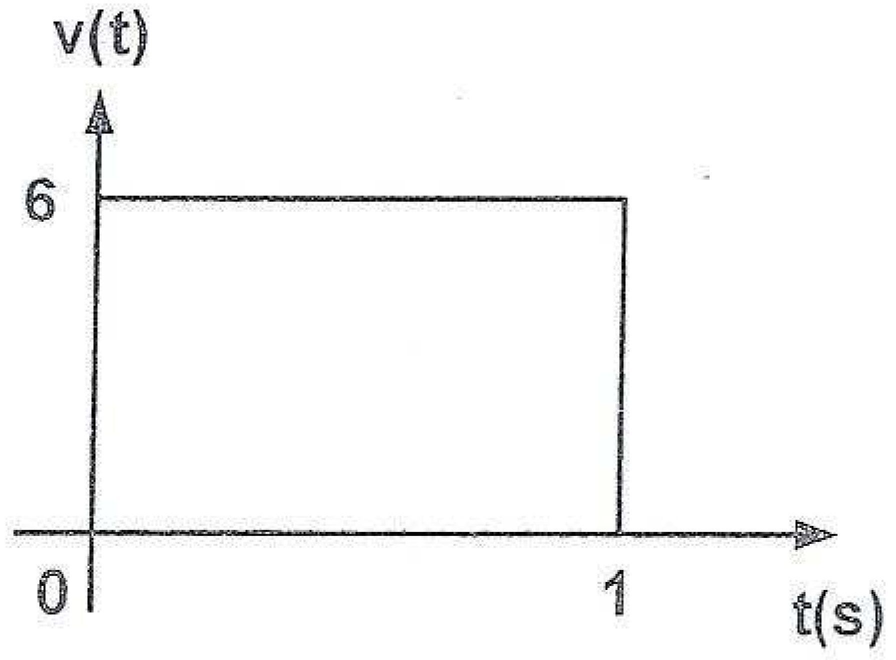


Fig.b

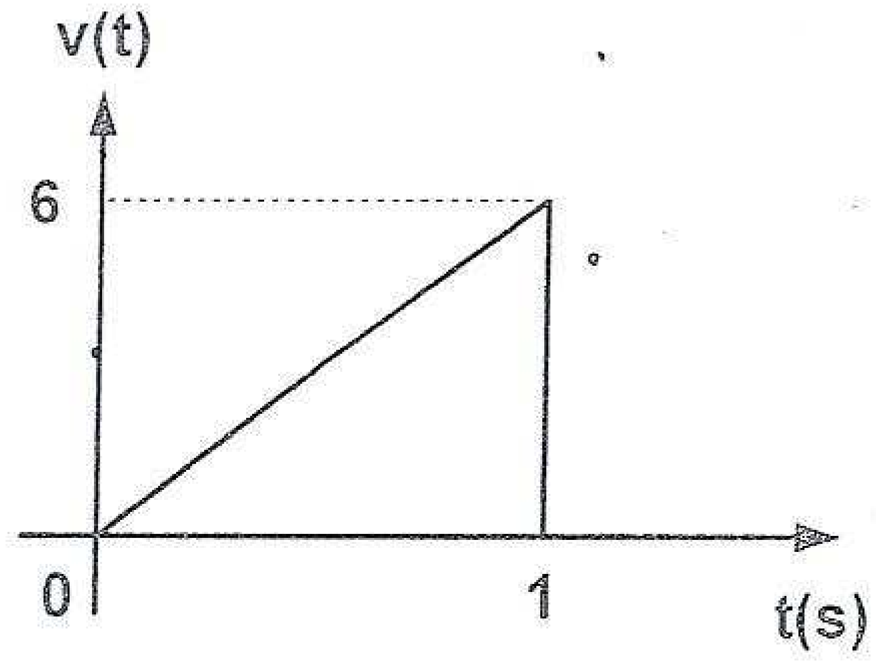


Fig.c

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* For Fig. a

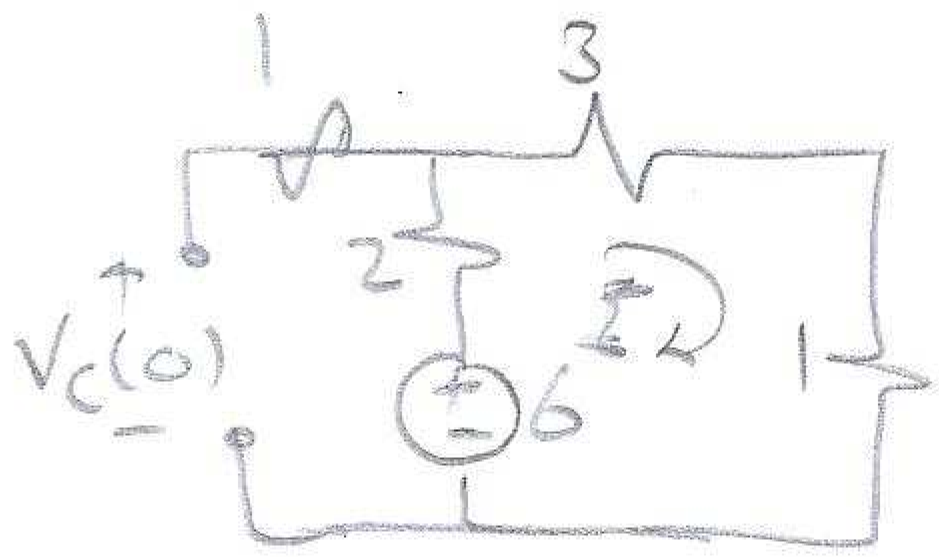
$v(t) = 6\left(u(t) - u(t-1)\right)$

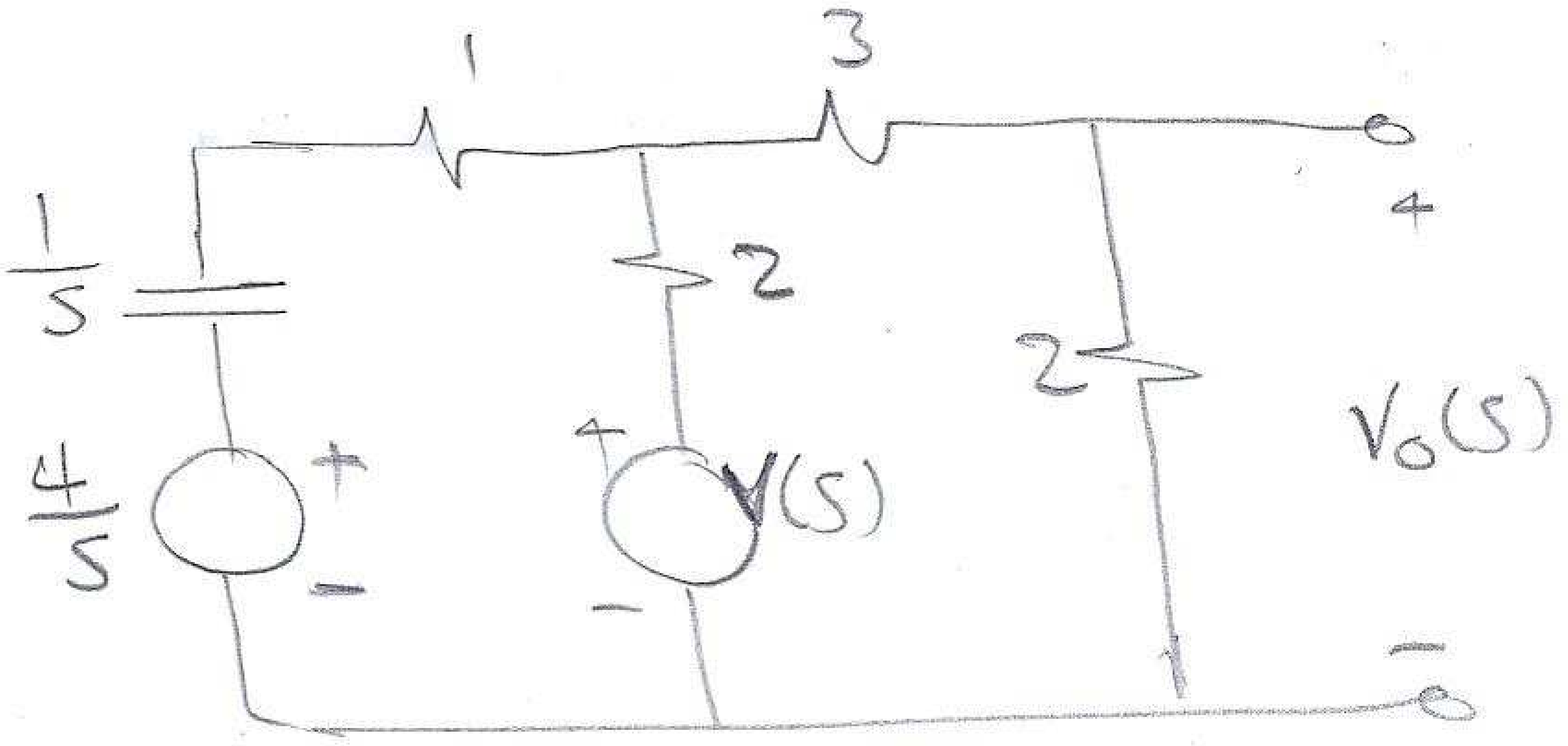
at $t=0$ $\quad v(t) = 6$ volts.

$v_c(0) = 6 - 2I$

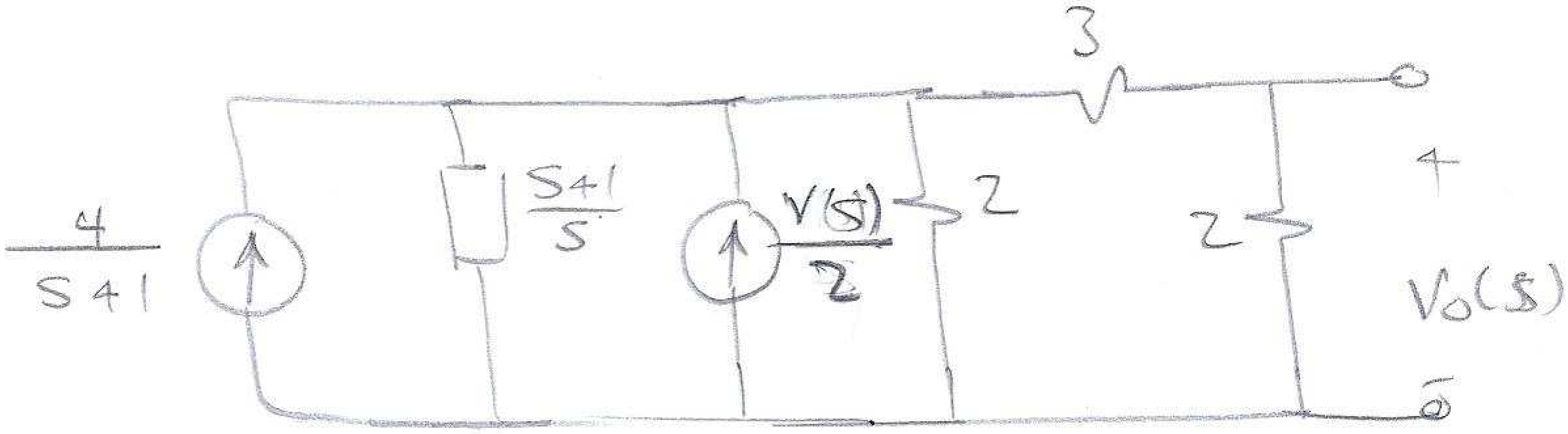
$= 6 - 2 * \frac{6}{6}$

$\boxed{\therefore v_c(0) = 4 \text{ volts.}}$

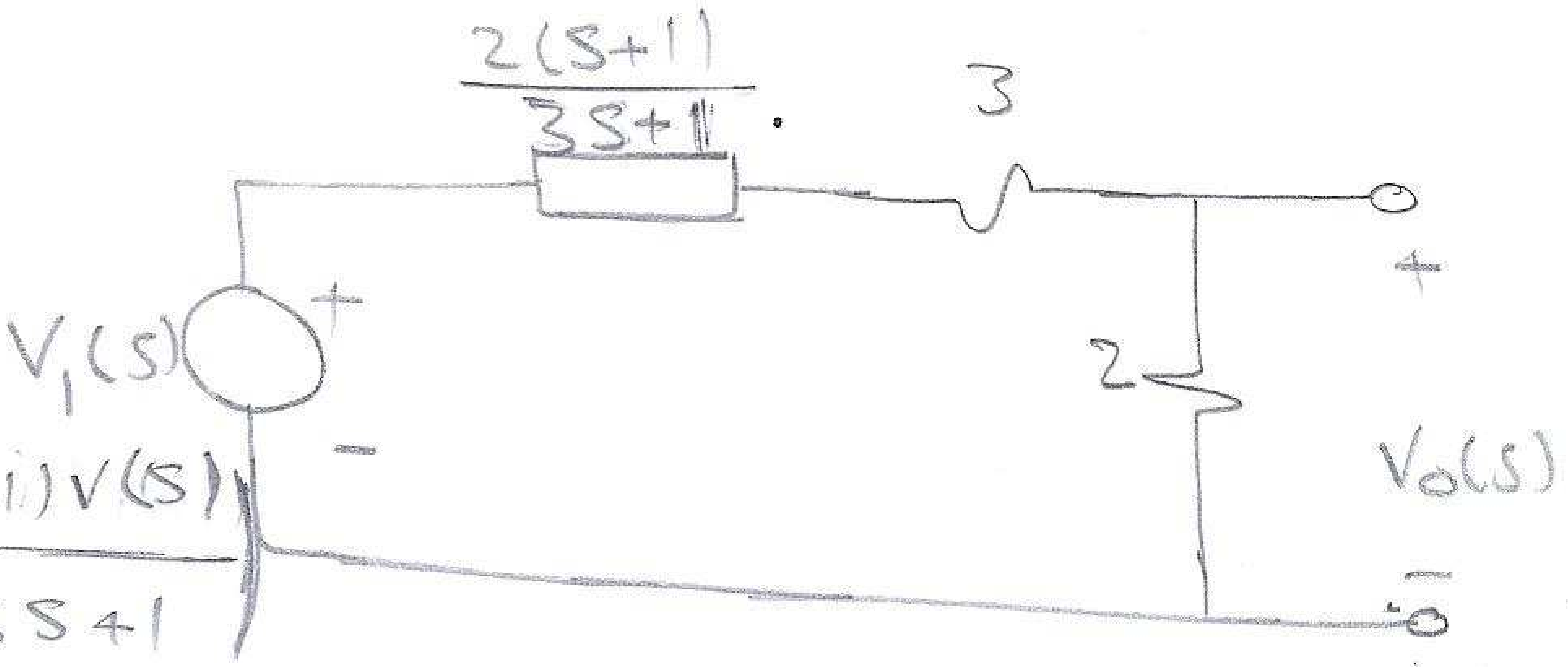




$$Z_p = \frac{2 * \frac{S+1}{S}}{2 + \frac{S+1}{S}} = \frac{2(S+1)}{3S+1}$$



$$V(s) = \frac{6}{s} - \frac{6e^{-s}}{s}$$



$$V_1(S) = \left(\frac{8}{3S+1} + \frac{(S+1)V(S)}{3S+1} \right)$$

$$V_o(S) = V_1(S) * \frac{2}{5 + \frac{2(S+1)}{3S+1}} = \frac{8+(S+1)V(S)}{3S+1} * \frac{2}{5+\frac{2(S+1)}{3S+1}}$$

$$V_o(S) = \frac{16+2(S+1)V(S)}{17S+7} = \frac{12\left(8+(S+1)(1-e^{-S})\right)}{S(17S+7)}$$

$$= \frac{96}{17S(S+\frac{7}{17})} + \frac{12(1-e^{-S})}{17(S+\frac{7}{17})} + \frac{12(1-e^{-S})}{17S(S+\frac{7}{17})}$$

$$V_o(S) = \frac{96/7}{S} - \frac{96/7}{S+\frac{7}{17}} + \frac{12/17}{S+\frac{7}{17}} - \frac{12/17\, e^{-S}}{(S+\frac{7}{17})}$$

$$+ \frac{12/7}{S} - \frac{12/7}{S+\frac{7}{17}} - \frac{12/7\, e^{-S}}{S} + \frac{12/7\, e^{-S}}{S+\frac{7}{17}}$$

$$\therefore V_o(t) = \left(\frac{96}{7} - \frac{96}{7} e^{-\frac{7}{17}t} + \frac{12}{17} e^{-\frac{7}{17}t} + \frac{12}{7} - \frac{12}{7} e^{-\frac{7}{17}t}\right) u(t)$$

$$+ \left(\frac{12}{17} e^{-\frac{7}{17}(t-1)} - \frac{12}{7} + \frac{12}{7} e^{-\frac{7}{17}(t-1)}\right) u(t-1)$$

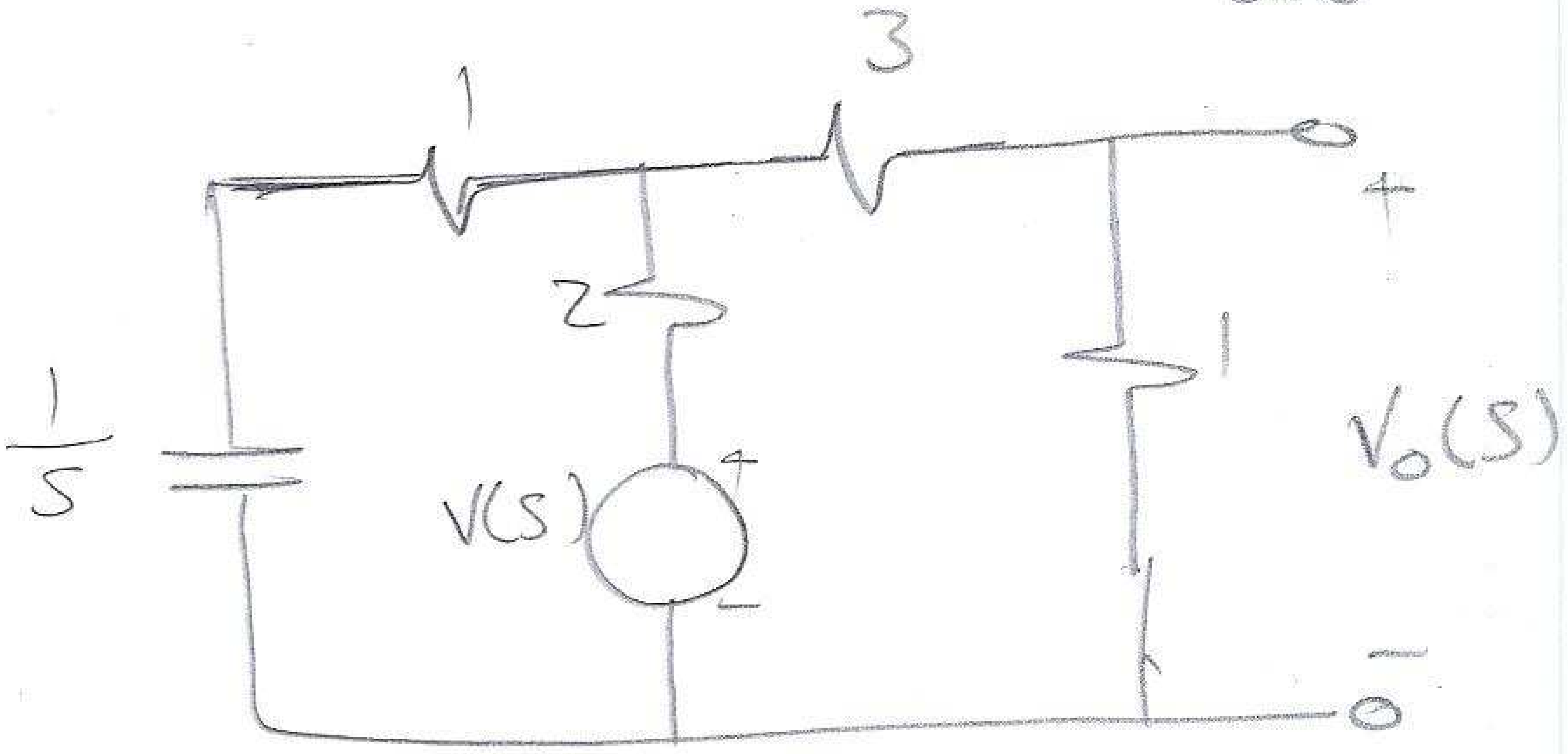
$$V_o(t) = \left(\frac{108}{7} - 14.722\, e^{-\frac{7}{17}t}\right) u(t)$$

$$+ \left(\frac{-12}{7} + 2.42\, e^{-\frac{7}{17}(t-1)}\right) u(t-1)$$

in Fig.b

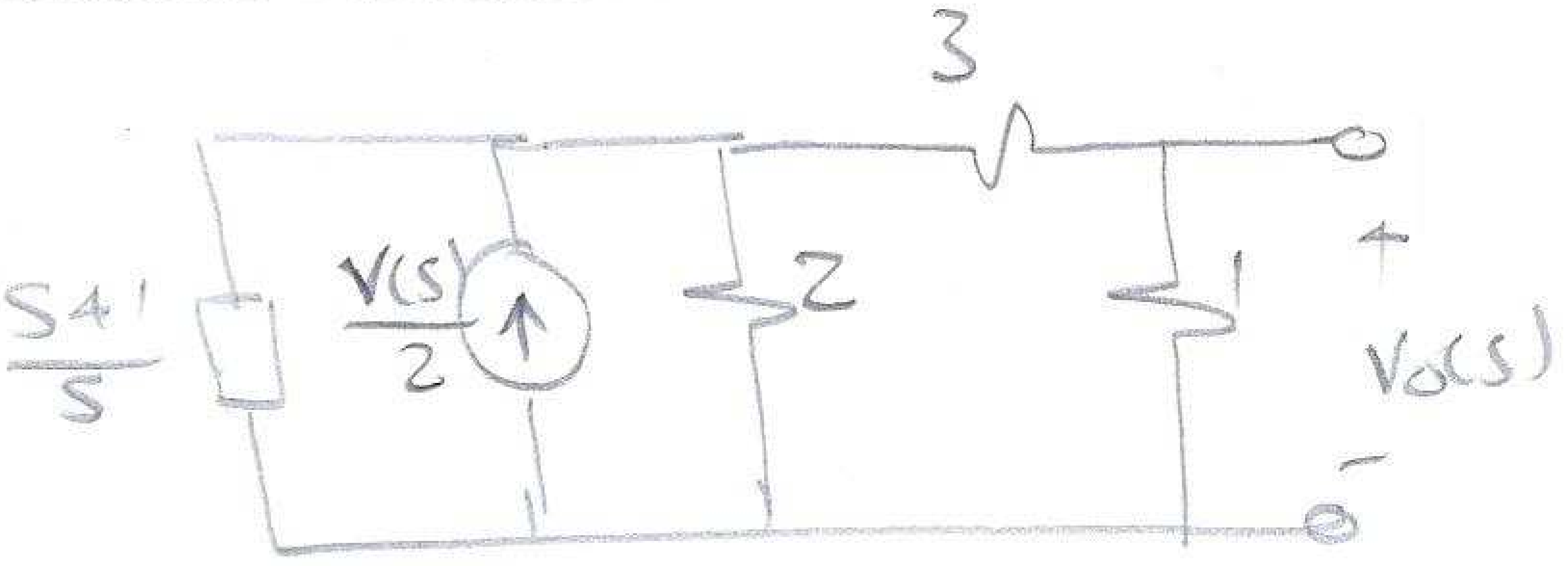
no-initial condital as $V(t) = 6t\{u(t) - u(t-1)\} = 0\Big|_{t=0}$

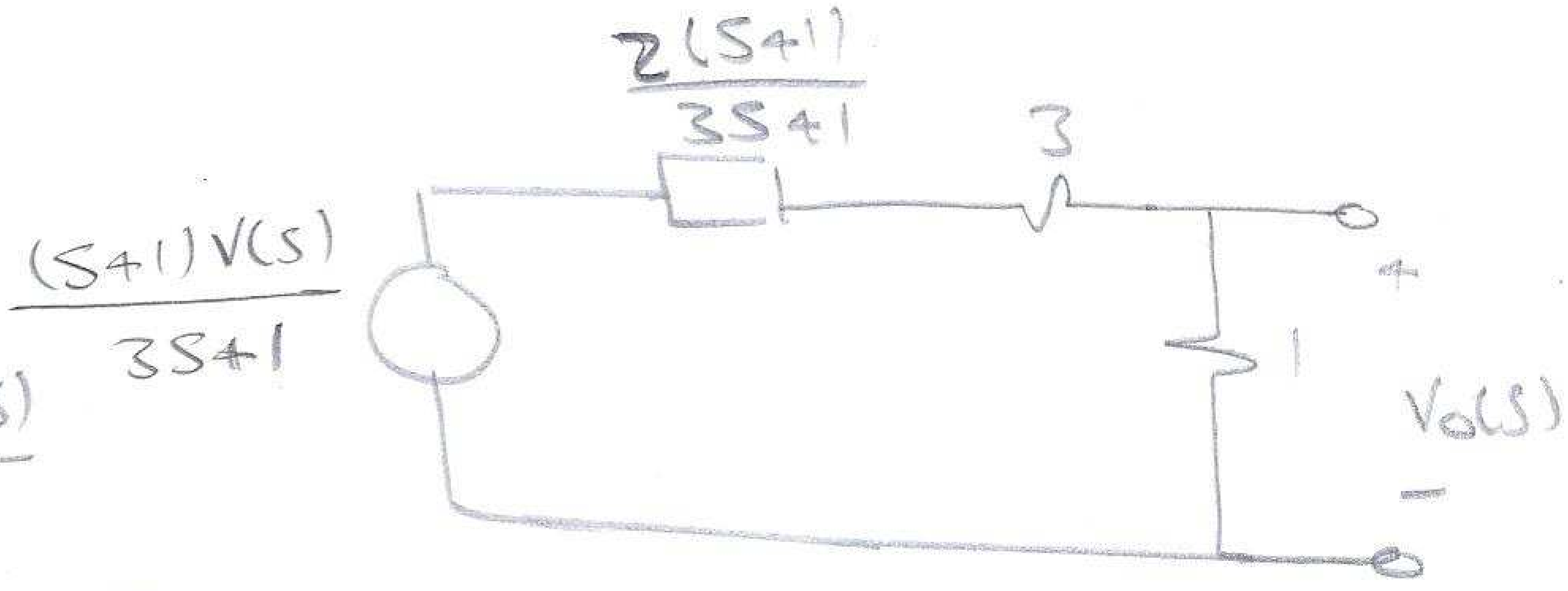
$$V(t) = 6t\,u(t) - 6(t-1)\,u(t-1) - 6\,u(t-1)$$



$$V(S) = \frac{6}{S^2} - \frac{6e^{-S}}{S^2} - \frac{6e^{-S}}{S}$$

$$V(S) = \frac{6(1 - e^{-S} - Se^{-S})}{S^2}$$





$$V_o(S) = \frac{\dfrac{(S+1)\,V(S)}{3S+1}}{4 + \dfrac{2(S+1)}{3S+1}}$$

$$V_o(S) = \frac{(S+1)\,V(S)}{14S+6}$$

a) $v_o(t)$	
b) $v_o(t)$	

Table 14