



- [1] Using nodal analysis for the circuit shown in Fig.1, calculate the current  $I_o$  and the power dissipated in the resistance  $R$ .
- [2] Find  $V_o$  in the circuit shown in Fig.2 using node analysis.
- [3] Using loop equations, calculate the voltage across the resistance  $R$  in the circuit shown in Fig.3.
- [4] Calculate the power dissipated in the  $3\Omega$  resistor in the circuit shown in Fig.4.

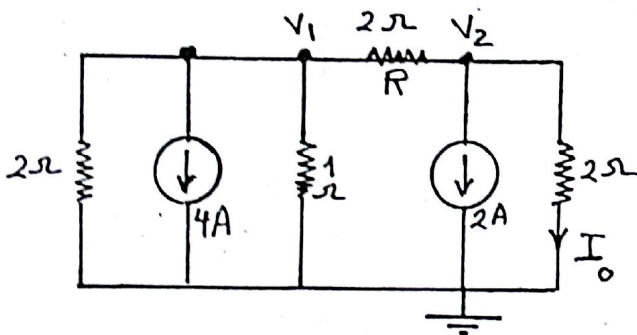


Fig.1

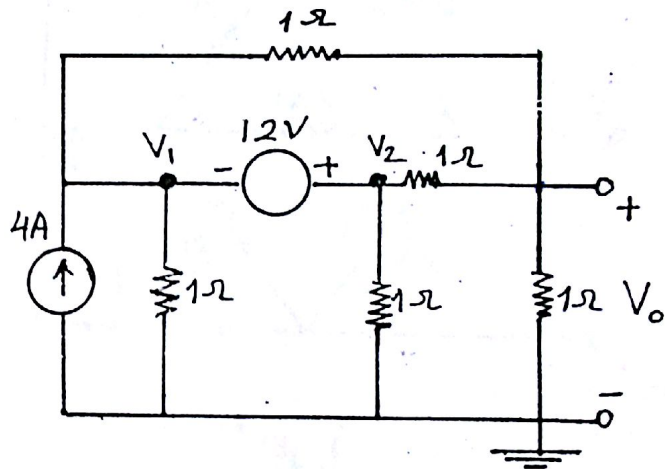


Fig.2

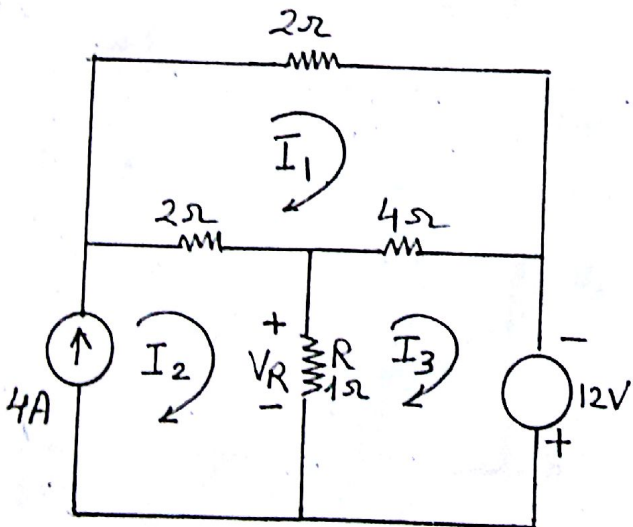


Fig.3

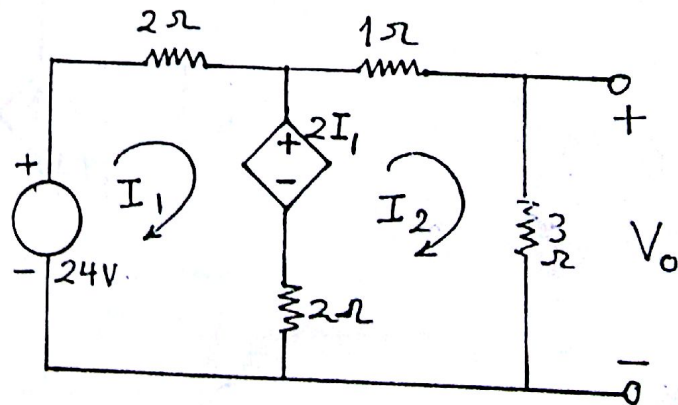


Fig.4

[5] Use loop equations to calculate the output voltage  $V_o$  of the circuit shown in Fig. 5.

[6] Calculate the voltage  $V_o$  in the circuit shown in Fig. 6 using the superposition theorem.

[7] Apply Thevenin's theorem to calculate the voltage  $V_o$  in the circuit shown in Fig. 7.

[8] Use Thevenin's theorem to calculate the output voltage  $V_o$  of the circuit shown in Fig. 8.

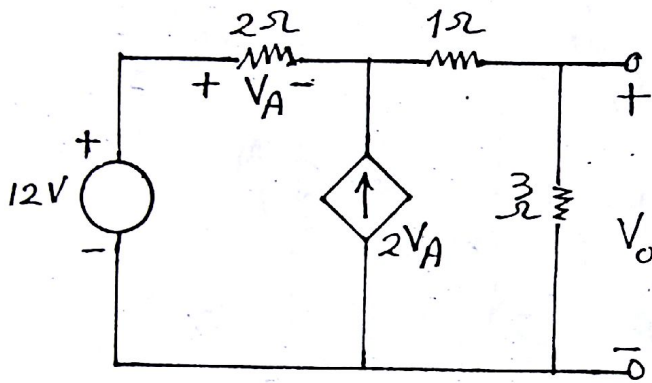


Fig. 5

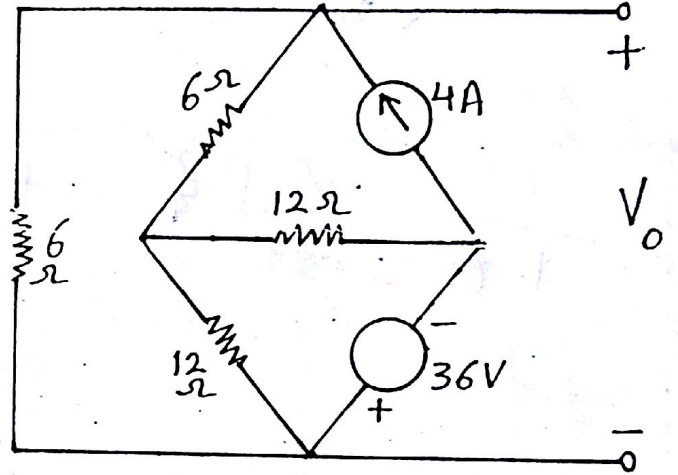


Fig. 6

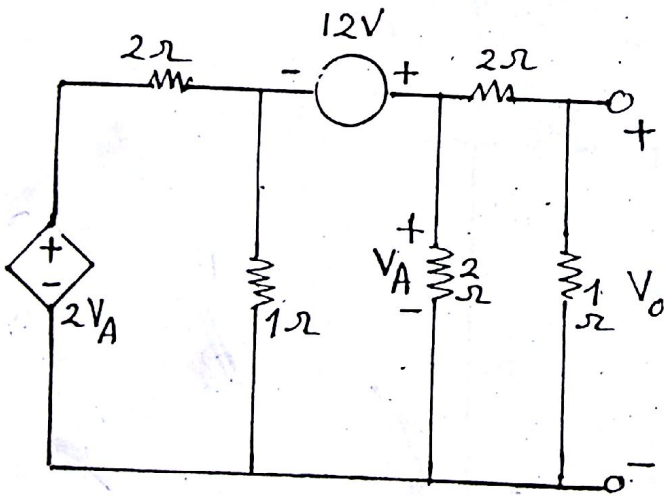


Fig. 7

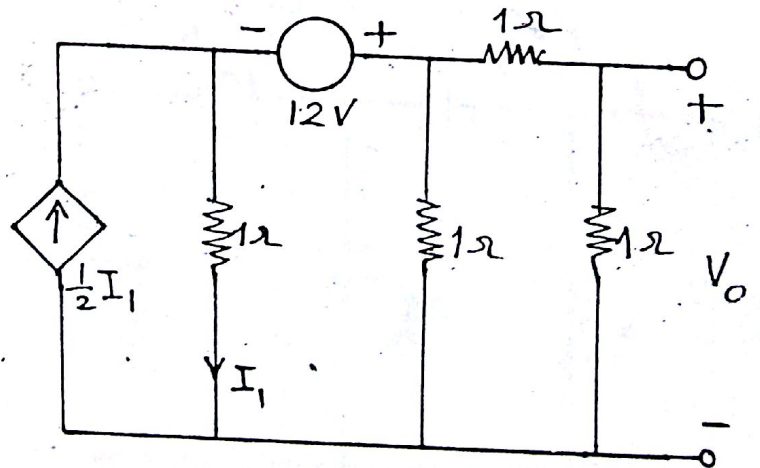


Fig. 8

[9] Apply Norton's theorem to find the current  $I_o$  in the circuit shown in Fig.9.

[10] Calculate the value of the resistance  $R_L$  in the circuit shown in Fig.10 to receive maximum power and the value of this power.

[11] Calculate the input impedance  $Z_{in}$  to the circuit shown in Fig.11.

[12] Calculate the input admittance  $Y_{in}$  to the circuit shown in Fig.12.

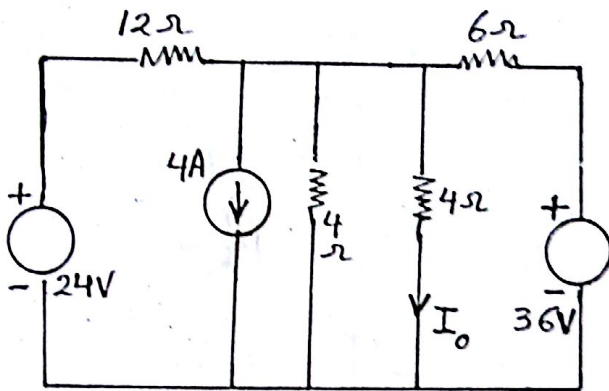


Fig.9

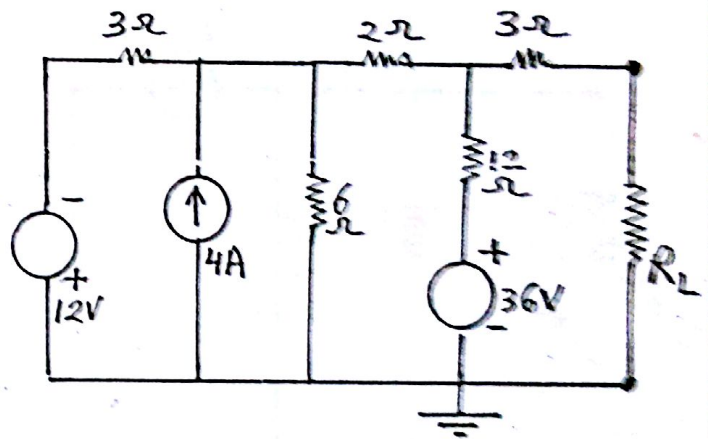


Fig.10

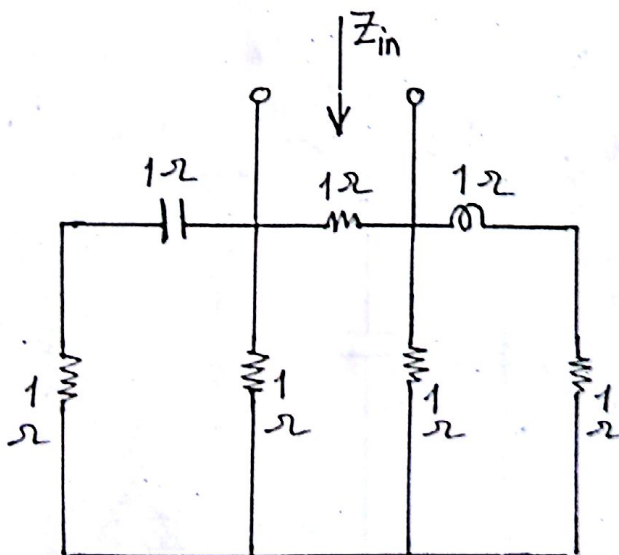


Fig.11

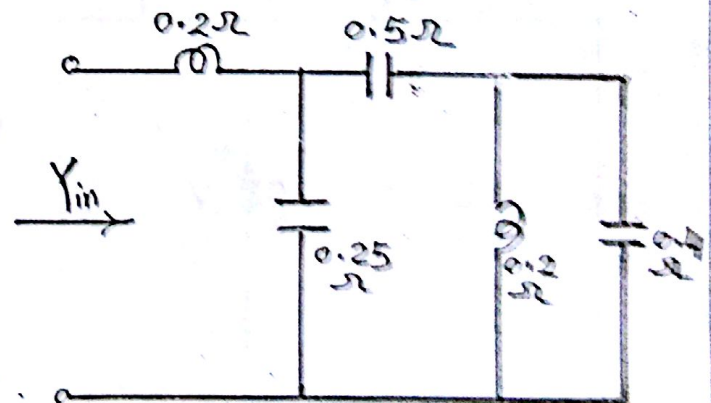


Fig.12

[13] Make the necessary analysis to calculate the branch currents of the circuit shown in Fig.13.

[14] Find the source current  $I_s$  in the circuit shown in Fig.14 if the output voltage  $V_o$  is  $12 \angle 0^\circ$  volts.

[15] For the circuit shown in Fig.15, calculate the output voltage  $V_o$ .

[16] Calculate the source voltage  $V_s$  in the circuit shown in Fig.16 if  $V_1 = 4 \angle 0^\circ$  volts.

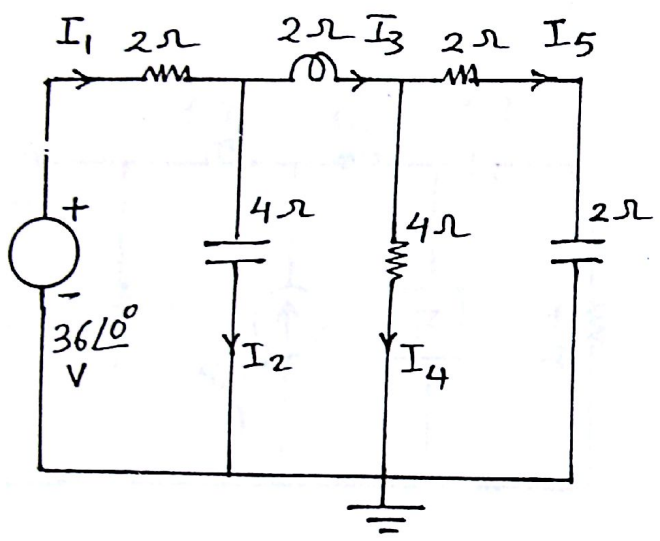


Fig.13

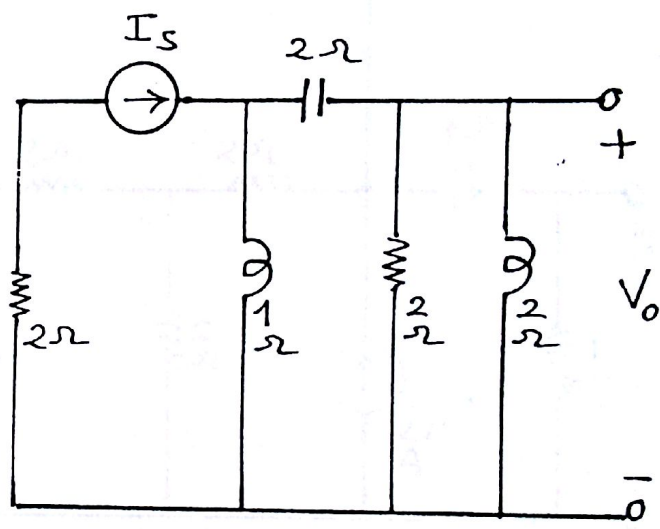


Fig.14

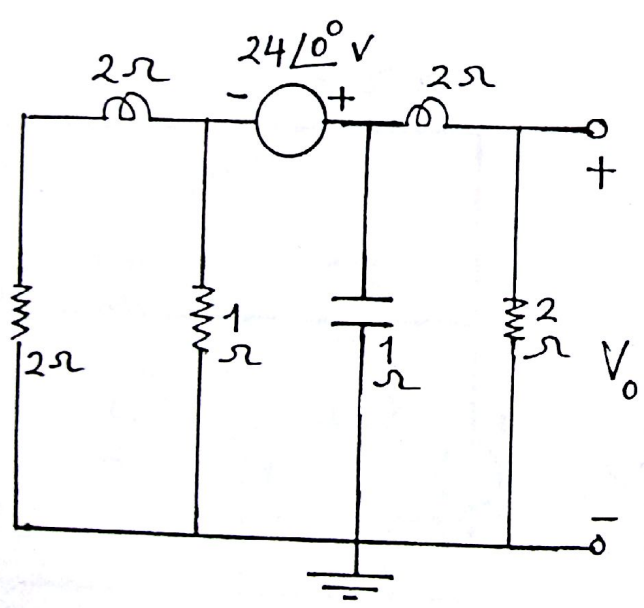


Fig.15

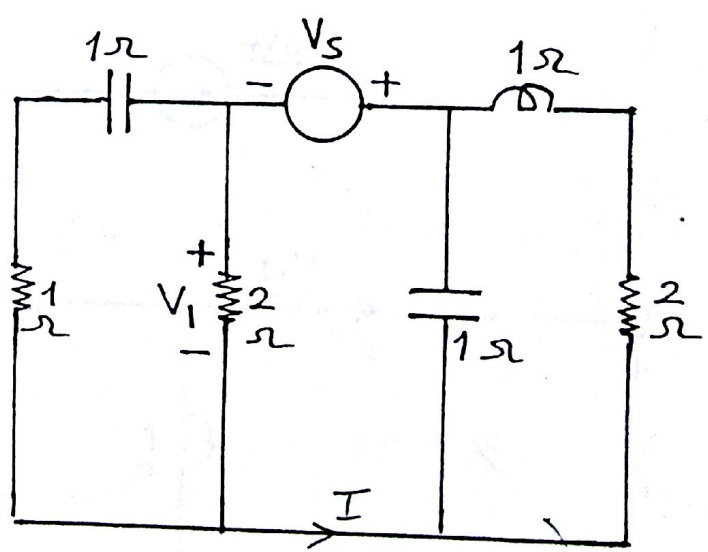


Fig.16

[17] In the circuit shown in Fig.17,  $V_1 = 2 \angle 45^\circ$  volts. Calculate the impedance  $Z$ .

[18] Calculate the output voltage  $V_o$  in the circuit shown in Fig.18.

[19] Given the circuit shown in Fig.19, write the loop equations to calculate the currents  $I_1$  and  $I_2$ .

[20] In the circuit shown in Fig.20, calculate the voltage  $V_R$ .

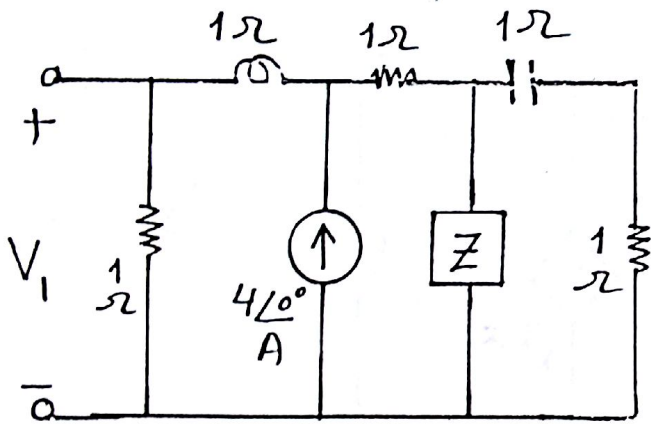


Fig.17

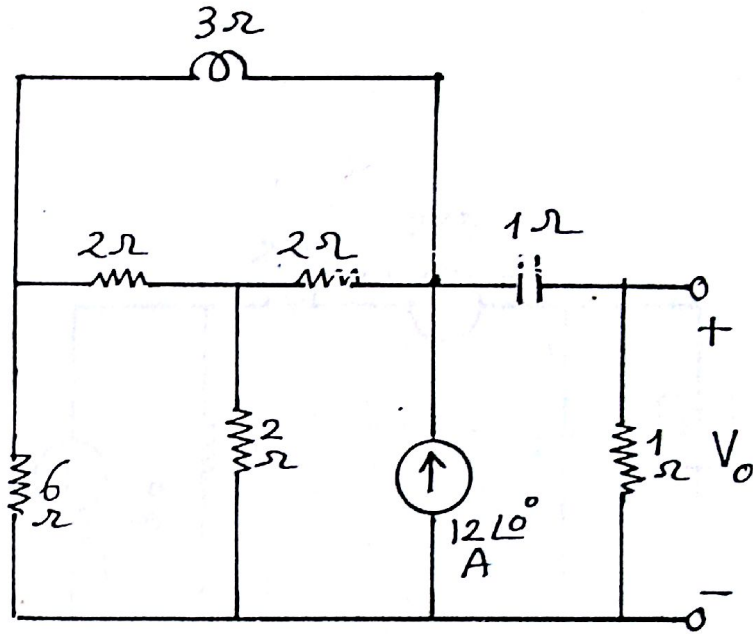


Fig.18

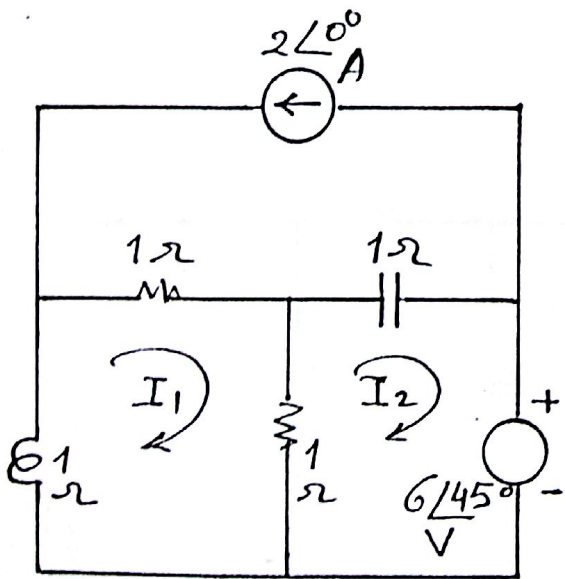


Fig.19

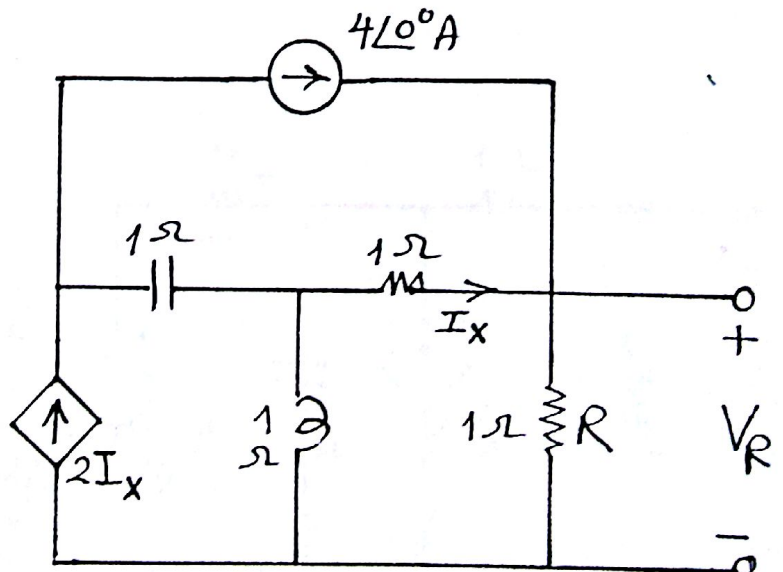


Fig.20

[21] Use Thevenin's theorem to calculate the voltage  $V_o$  in the circuit shown in Fig.21 .

[22] Determine the value of the load impedance  $Z_L$ , shown in Fig.22 , to receive maximum average power and the value of this power .

[23] Calculate the average power absorbed by the  $4 \Omega$  resistor shown in Fig.23 .

[24] Calculate the value of the load impedance  $Z_L$ , shown in Fig.24 , to receive maximum average power and the value of this power .

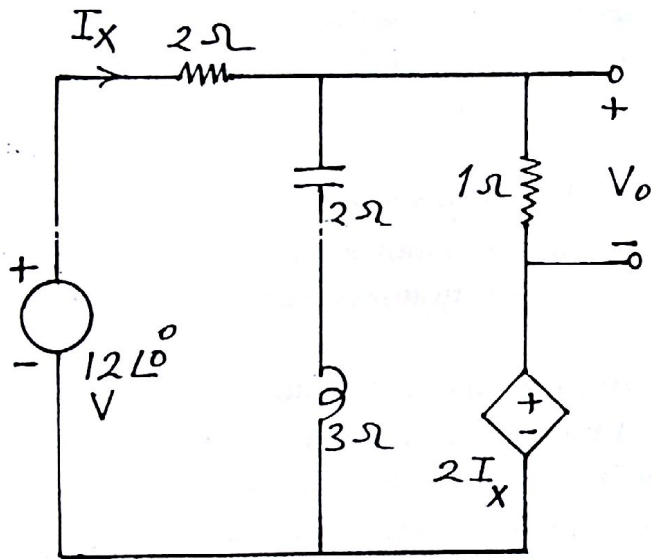


Fig.21

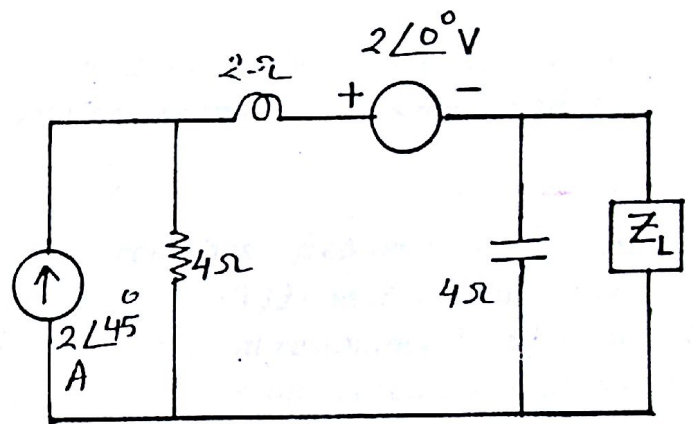


Fig.22

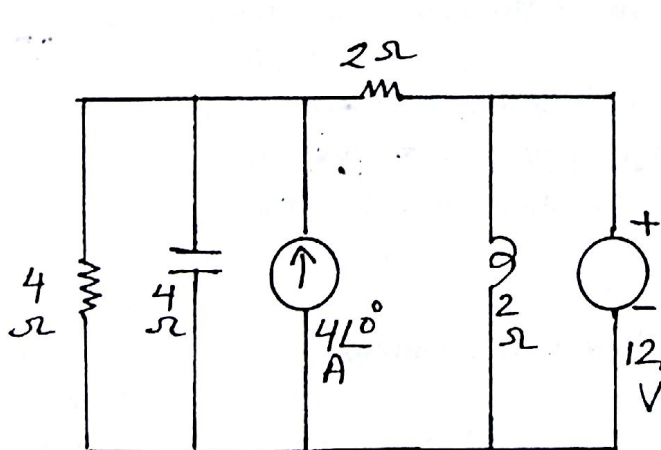


Fig.23

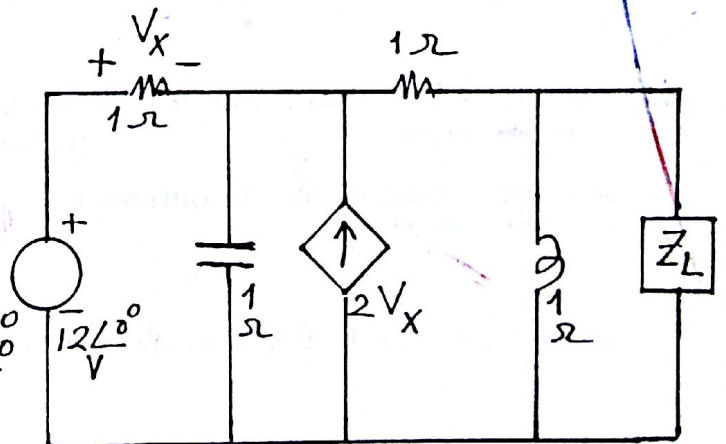
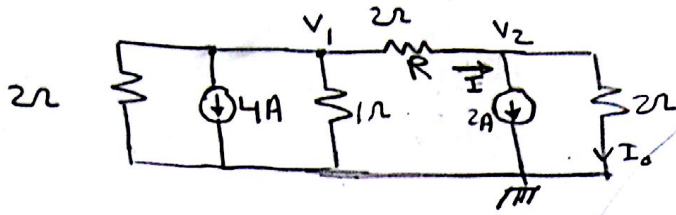


Fig.24

I



Apply Nodal Analysis

$$\begin{bmatrix} 2 & -0.5 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

$$\text{Solve to get } \begin{aligned} v_1 &= -20/7 \text{ V} \\ v_2 &= -24/7 \text{ V} \end{aligned}$$

to get  $I_0$ :  $I_0 = I - 2$ 

$$I_0 = \frac{v_1 - v_2}{2} - 2 = \frac{-12}{7} \text{ A} \quad \text{or} \quad I_0 = \frac{v_2}{2}$$

$$P_R = I^2 * R = \left(\frac{2}{7}\right)^2 * 2 = \frac{28}{49} \text{ watt} \quad \text{or} \quad \frac{(v_1 - v_2)^2}{R}$$

checking Power balance (Not Required)

$$P_{2\Omega} = \frac{v_1^2}{R} = \frac{200}{49} \text{ watt}$$

$$P_{1\Omega} = \frac{v_1^2}{R} = \frac{400}{49} \text{ watt}$$

$$P_{2\Omega} = I_0^2 * R = \frac{288}{49} = \frac{v_2^2}{R} = \frac{288}{49} \text{ watt}$$

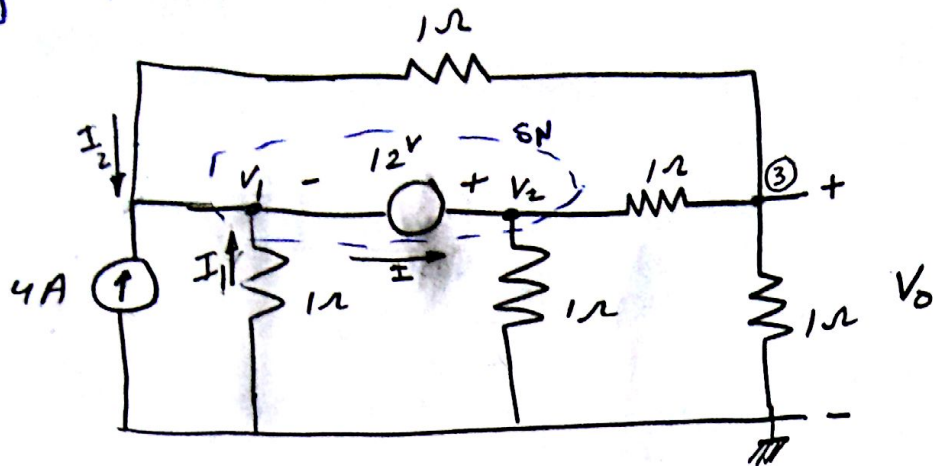
$$\sum P_{\text{diss.}} = \frac{8}{49} + \frac{200}{49} + \frac{400}{49} + \frac{288}{49} = \boxed{\frac{128}{7}} \text{ watt}$$

$$P_{4A} = (0 - v_1) * 4 = \frac{20}{7} * 4 = \frac{80}{7} \text{ watt}$$

$$P_{2A} = (0 - v_2) * 2 = \frac{24}{7} * 2 = \frac{48}{7} \text{ watt}$$

$$\sum P_{\text{supplied}} = \frac{80 + 48}{7} = \boxed{\frac{128}{7}} \text{ watt} \quad \checkmark$$

②



$$\begin{matrix} \text{SN}(1,2) \\ \text{V eqn} \\ \text{Node } \textcircled{3} \end{matrix} \begin{bmatrix} 2 & 2 & -2 \\ -1 & 1 & 0 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 0 \end{bmatrix}$$

Solve to get

$$\begin{aligned} V_1 &= \frac{-19}{2} \text{ V} \\ V_2 &= \frac{15}{2} \text{ V} \\ V_3 &= 1 \text{ V} \end{aligned}$$

$$\therefore V_0 = V_3 = \underline{1 \text{ V}}$$

Power (Not Required)

$$\sum P_{\text{diss}} = \frac{V_1^2}{1} + \frac{V_2^2}{1} + \frac{V_3^2}{1} + \frac{(V_2 - V_3)^2}{1} + \frac{(V_1 - V_3)^2}{1} = \underline{150 \text{ watt}}$$

$$P_{4A} = V_1 * 4 = -18 \text{ watt}$$

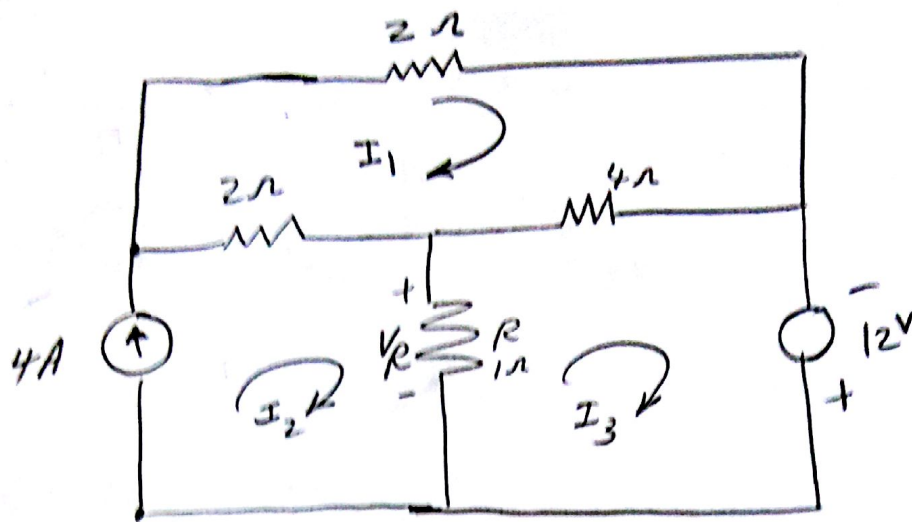
$$P_{12V} = 12 * I$$

$$I = 4 + I_1 + I_2 = 4 + \left(\frac{9/2}{1}\right) + \left(\frac{1 + 9/2}{1}\right) = \underline{14 \text{ A}}$$

$$\therefore P_{12V} = 12 * 14 = 168 \text{ watt}$$

$$\sum P_{\text{supp.}} = 168 - 18 = \underline{150 \text{ watt}}$$

③



$$\begin{array}{l} \text{loop 1} \\ \text{loop 3} \\ \text{current equation} \end{array} \begin{bmatrix} 8 & -2 & -4 \\ -4 & -1 & 5 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 4 \end{bmatrix}$$

$$I_1 = \frac{13}{3} \text{ A}$$

$$I_2 = 4 \text{ A}$$

$$I_3 = \frac{20}{3} \text{ A}$$

$$V_R = (I_2 - I_3) R = \left( \frac{-8}{3} \right) \text{ V}$$

Power

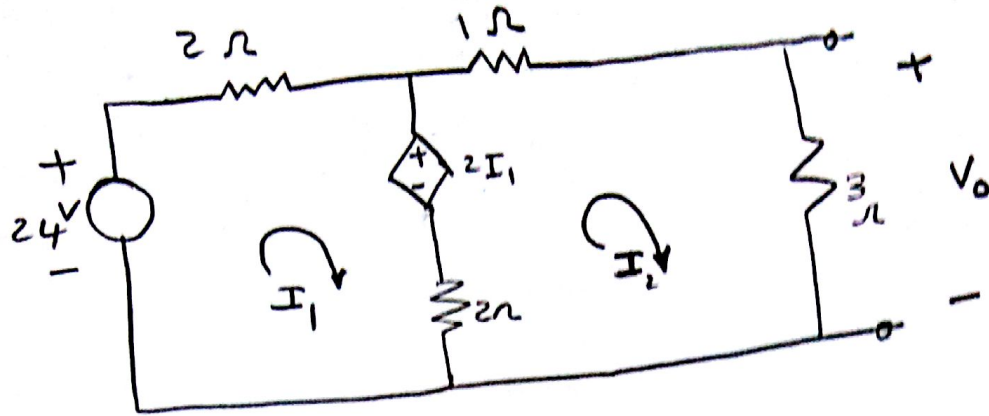
$$\begin{aligned} \sum P_{\text{diss}} &= 2(I_1 - I_2)^2 + 1(I_2 - I_3)^2 + 4(I_1 - I_3)^2 + 2I_1^2 \\ &= \frac{2}{9} + \frac{64}{9} + \frac{196}{9} + \frac{338}{9} = \boxed{\frac{200}{3}} \text{ watt} \end{aligned}$$

$$P_{12V} = 12 I_3 = \boxed{80} \text{ watt}$$

$$P_{4A} = 4 \times [2(I_2 - I_1) + 1(I_2 - I_3)] = \boxed{\frac{-40}{3}} \text{ watt}$$

$$\sum P_{\text{supp}} = 80 - \frac{40}{3} = \left( \frac{200}{3} \right) \text{ watt} \quad \checkmark$$

7



Apply loop Analysis

$$\begin{bmatrix} 6 & -2 \\ -4 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \end{bmatrix}$$

$$4I_1 - 2I_2 = 24 + 2I_1$$

$$6I_1 - 2I_2 = 24$$

$$6I_2 - 2I_1 = 2I_1$$

$$6I_2 - 4I_1 = 0$$

Solve to get:  $I_1 = \frac{36}{7} \text{ A}$

$I_2 = \frac{24}{7} \text{ A}$

$$P_{3\Omega} = I_2^2 * R = \frac{1728}{49} = 35.265 \text{ watt}$$

Power

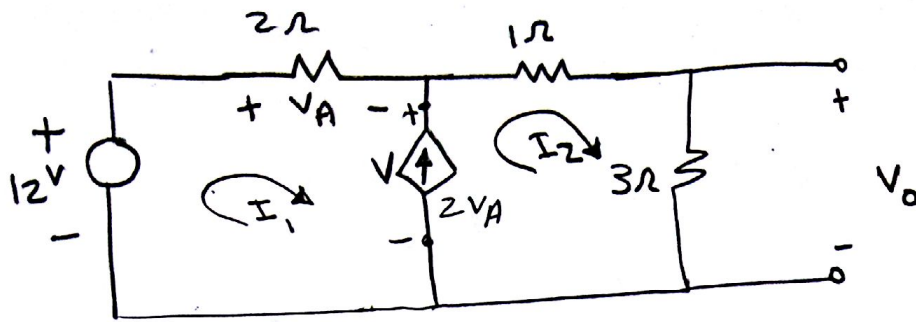
$$\sum P_{diss} = 2I_1^2 + 4I_2^2 + 2(I_1 - I_2)^2 = \frac{5184}{49} \text{ watt}$$

$$P_{24V} = 24I_1 = \frac{864}{7} \text{ watt}$$

$$P_{2\Omega} = 2I_1 * (I_2 - I_1) = \frac{-864}{49} \text{ watt}$$

$$\sum P_{supp} = \frac{864}{7} - \frac{864}{49} = \frac{5184}{49} \text{ watt} \quad \checkmark$$

⑤



$$I_2 - I_1 = 2VA$$

$$\therefore I_2 - I_1 = 2 * 2I_1$$

$$\therefore -5I_1 + I_2 = 0 \rightarrow \textcircled{1}$$

$$12 = 2I_1 + 4I_2 \rightarrow \textcircled{2}$$

Solve to get

$$I_1 = \frac{6}{11} \text{ A}$$

$$I_2 = \frac{30}{11} \text{ A}$$

$$V_0 = 3I_2 = 3 * \frac{30}{11} = \frac{90}{11} \text{ V}$$

Power

$$\sum P_{\text{diss}} = 2I_1^2 + 4I_2^2 = \boxed{\frac{3672}{121}} \text{ watt}$$

$$P_{12V} = 12I_1 = \frac{72}{11} \text{ watt}$$

$$P_{2VA} = 2VA * V$$

$$2VA = 4I_1 = \frac{24}{11}$$

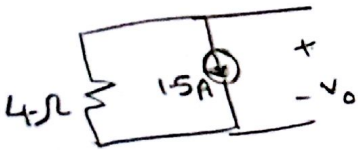
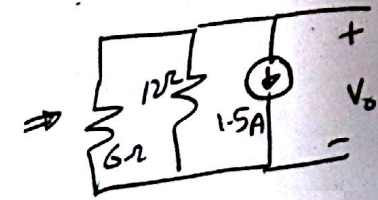
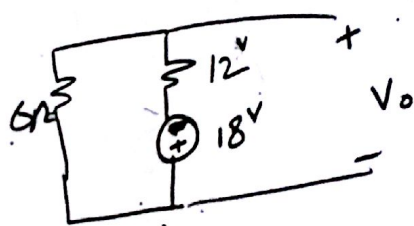
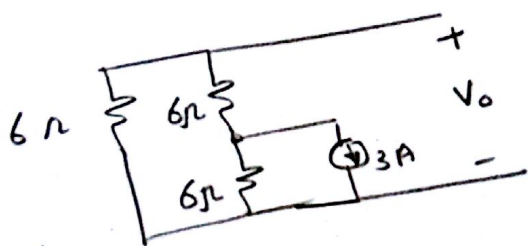
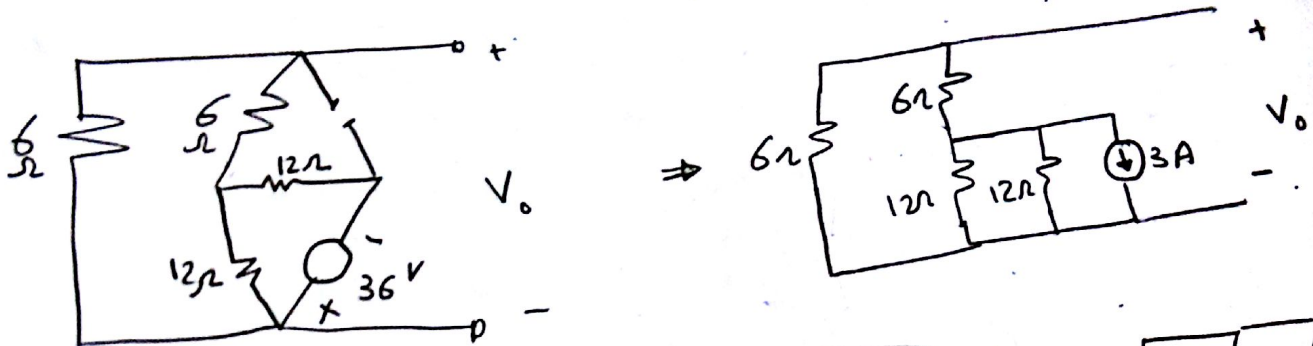
$$V = -2I_1 + 12 = \frac{120}{11}$$

$$\therefore P_{2VA} = \frac{2880}{121} \text{ watt}$$

$$\sum P_{\text{supp.}} = \frac{72}{11} + \frac{2880}{121} = \boxed{\frac{3672}{121}} \text{ watt} \quad \checkmark$$

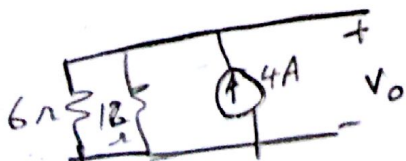
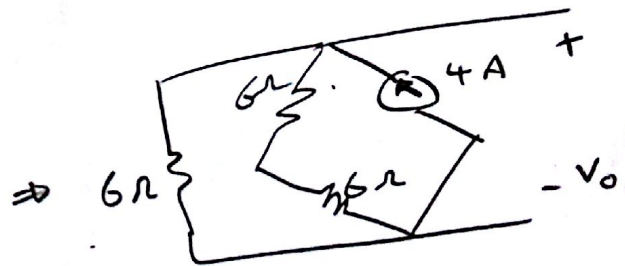
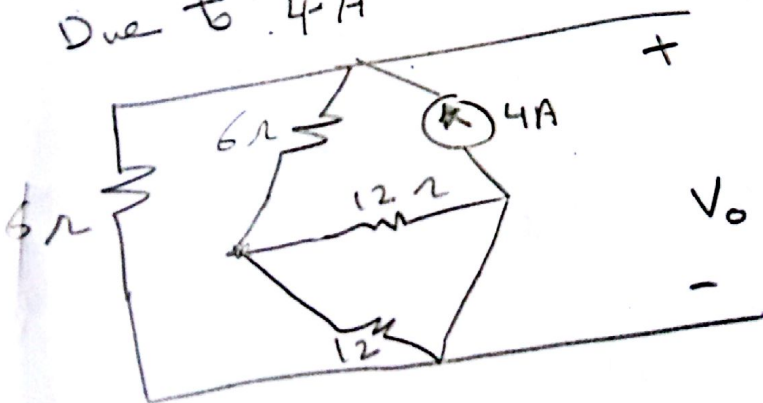
# 6 Super Position Theorem

□ Due to 36V



$$\therefore V_0 = 6V$$

Due to 4A

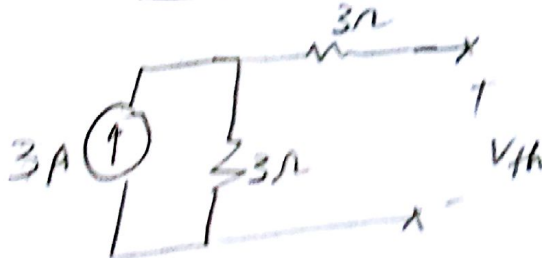
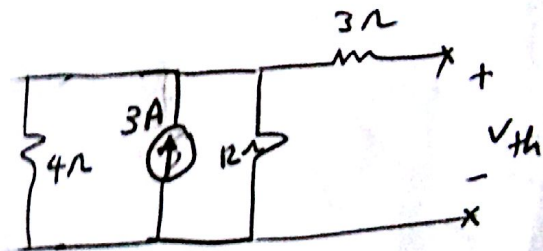
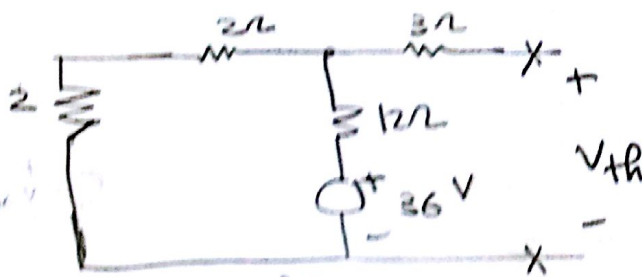
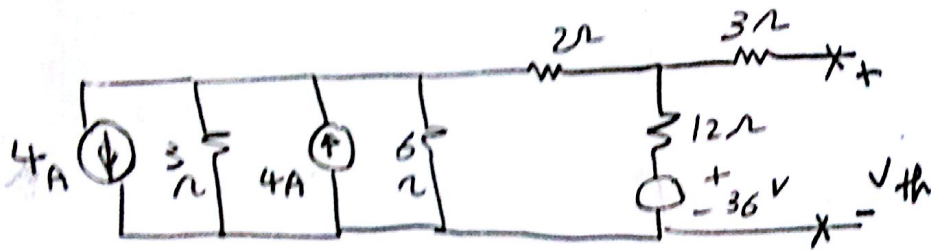
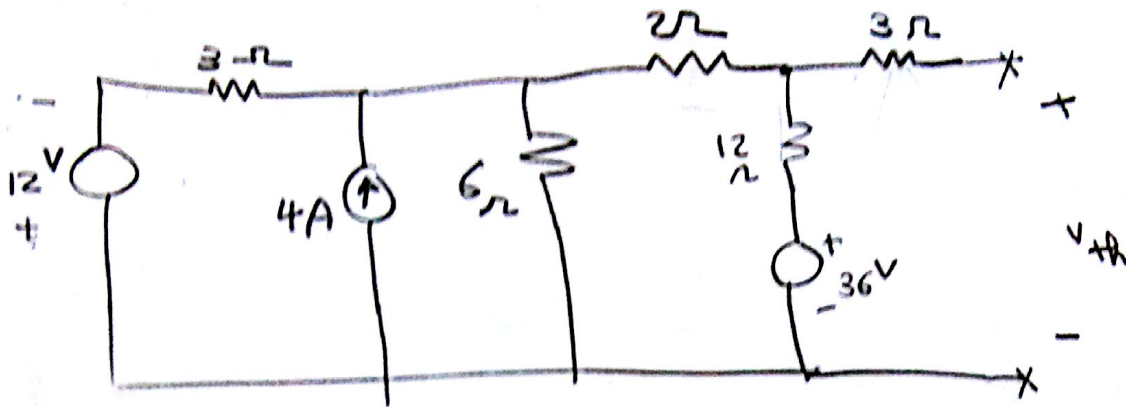


$$V_0 = 16V$$

$$\therefore V_0 = 16 - 6 = 10V \quad \square$$

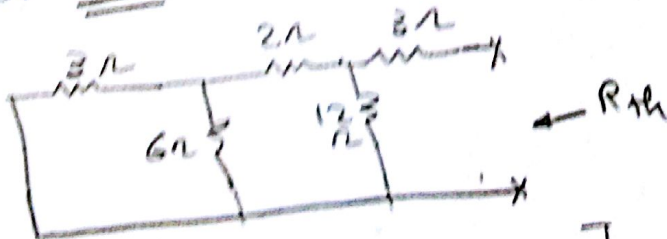
10

get  $V_R$



$V_R = \underline{\underline{9V}}$

get  $R_{th}$

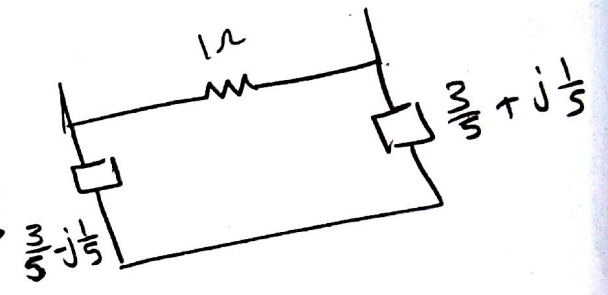
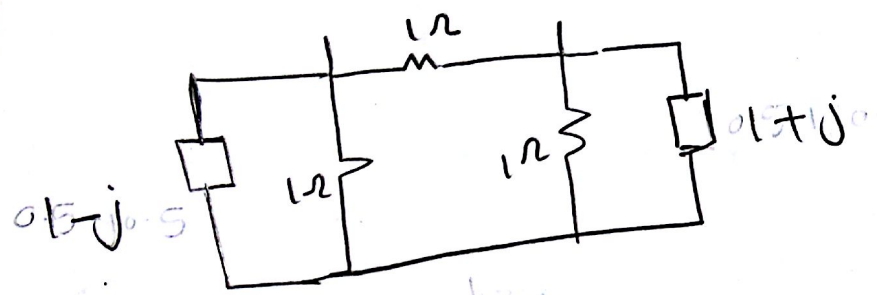
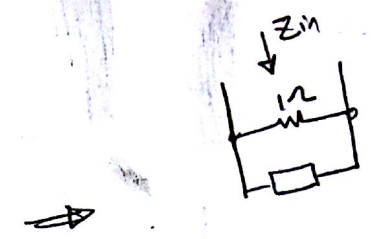
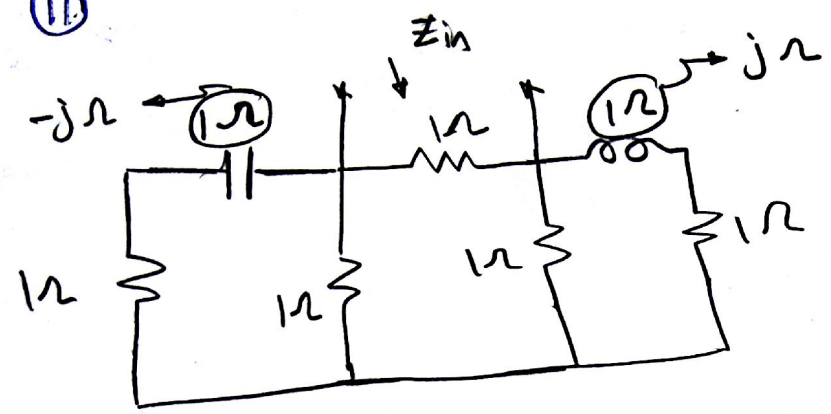


$$R_{th} = \left[ \left[ \left( 3 \parallel 6 \right) + 2 \right] \parallel 12 \right] + 2 = \underline{\underline{6 \Omega}}$$

$$R_L = R_{th} = \underline{\underline{6 \Omega}} = R_L$$

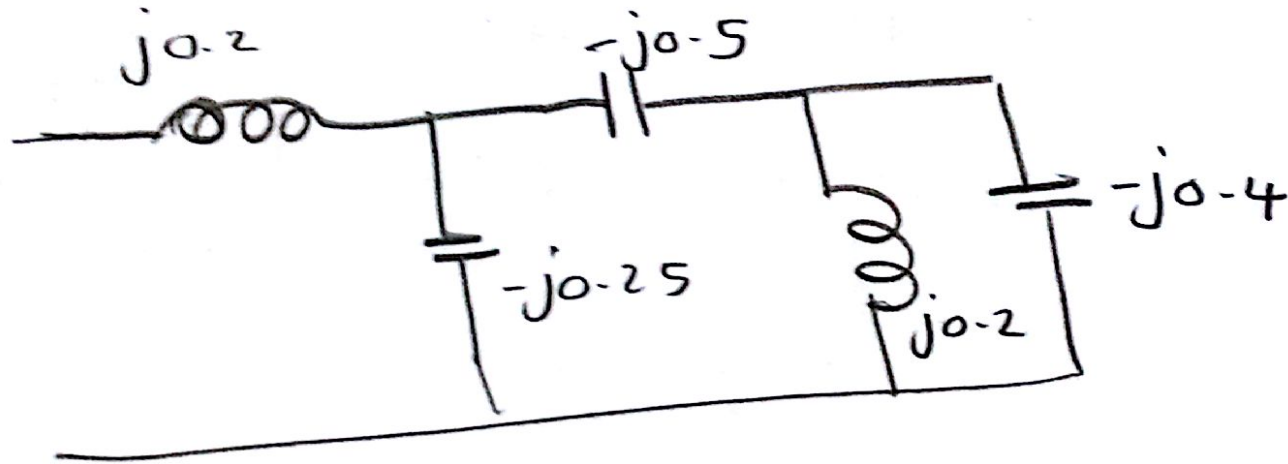
$$P_{max} = \frac{V_{th}^2}{4 R_{th}} = \frac{(9)^2}{24} = \frac{81}{24} = \underline{\underline{3.375 \text{ Watt}}}$$

11



$$\begin{aligned}
 \text{in } Z_{in} &= \left( \frac{3}{5} - j\frac{1}{5} + \frac{3}{5} + j\frac{1}{5} \right) \parallel 1 \\
 &= \left( \frac{6}{5} \right) \parallel 1 = \frac{6/5}{\frac{11}{5}} = \frac{6}{11} = 0.545 \Omega
 \end{aligned}$$

12

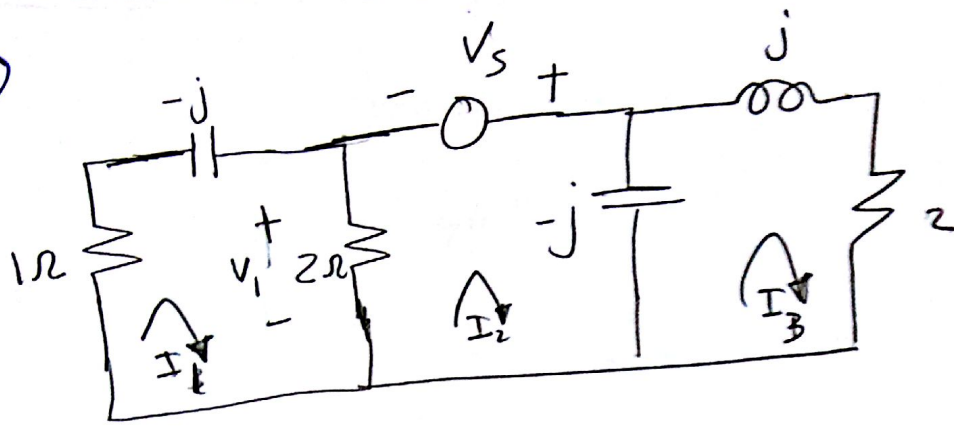


$$Z_{in} = \left[ \left[ (-j0.4 \parallel j0.2) + (-j0.5) \right] \parallel (-j0.25) \right] + j0.2$$

$$Z_{in} = j \frac{9}{70}$$

$$Y_{in} = -j \frac{70}{9} \approx -j7.78 \Omega^{-1} \quad (2)$$

16



$$V_1 = (I_1 - I_2) \cdot 2$$

$$\therefore I_1 - I_2 = 2 \rightarrow \textcircled{1}$$

$$-2I_1 + (2 - j)I_2 + jI_3 = V_s \rightarrow \textcircled{2}$$

$$(3 - j)I_1 - 2I_2 = 0 \rightarrow \textcircled{3}$$

$$2I_3 + jI_2 = 0 \rightarrow \textcircled{4}$$

Solve  $\textcircled{1}$  &  $\textcircled{3}$  to get

$$I_1 = -2 - j2 \text{ A} = 2\sqrt{2} \angle 135^\circ$$
$$I_2 = -4 - j2 \text{ A} = 2\sqrt{5} \angle -153.43^\circ$$

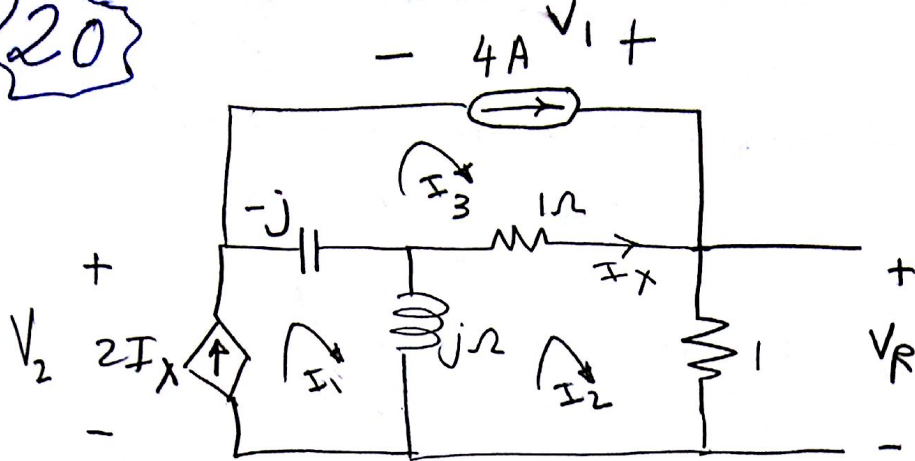
$$\therefore I_3 = -1 + j2 \text{ A}$$

$$V_s = \underline{-18 + 3j \text{ V}} = \sqrt{73} \angle 159.4439^\circ$$

check :  $P_{\text{supp}} = \sqrt{73} \times 2\sqrt{5} \times \cos(-159.443^\circ + 153.434^\circ)$   
 $\cong \textcircled{26} \text{ watt}$

$$P_{\text{diss}} = |I_1|^2 + |I_1 - I_2|^2 \times 2 + |I_3|^2 \times 2$$
$$= 8 + 8 + 10 = \underline{\underline{\textcircled{26} \text{ watt}}}$$

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$$I_3 = 4 \text{ A}$$

$$I_1 = 2I_x = 2(I_2 - I_3)$$

$$I_1 - 2I_2 + 2I_3 = 0$$

$$\therefore I_1 - 2I_2 = -8 \rightarrow I_1 = 2I_2 - 8 \rightarrow \textcircled{1}$$

$$(2+j)I_2 - jI_1 - I_3 = 0$$

$$-jI_1 + (2+j)I_2 = 4 \rightarrow \textcircled{2}$$

Substitute from ① into ②

$$-j(2I_2 - 8) + (2+j)I_2 = 4$$

$$(-j2 + 2 + j)I_2 + j8 = 4$$

$$(2 - j)I_2 = 4 - j8 \rightarrow I_2 = \frac{16}{5} - j\frac{12}{5} \text{ A}$$

$$I_1 = \frac{-8}{5} - j\frac{24}{5} \text{ A}$$

$$V_R = I_2 = \frac{16}{5} - j\frac{12}{5} = 4 \angle -36.869^\circ \text{ V}$$

Check  $P_{4A} = 4 |V_1| \cos \theta$

$$V_1 = (I_3 - I_2) - j(I_3 - I_1)$$

$$V_1 = 2.5298 \angle 71.565^\circ + 7.3756 \angle -49.398^\circ$$

$$= 6.44981275 \angle -29.745^\circ$$