Analysis of composite plates using moving least squares differential quadrature method

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\begin{abstract}
In this work, the moving least squares differential quadrature method (MLSDQM) is employed to analyze bending problems of composite plates. Based on a transverse shear theory, the governing equations of the problem are derived. The transverse deflection and two rotations of the plate are independently approximated with MLS approximations. The weighting coefficients used in the MLSDQ approximation are obtained through the fast computation of the MLS shape functions and their partial derivatives. The obtained results are compared with the previous analytical and numerical ones. Further a parametric study is introduced to investigate the effects of elastic and geometric characteristics on the values of transverse deflection of the plate.
\end{abstract}

1. Introduction

Owing to their superior material properties, composite plates have been extensively used in mechanical, civil, nuclear and aerospace structures. So, analysis of such plates is one of the most important problems in structural analysis. Due to the mathematical complexity of such problems, only limited cases can be solved analytically \cite{1–3}. Finite difference, finite element, boundary element, and discrete singular convolution methods have been widely applied to solve such plate problems \cite{4–10}. The main disadvantage of such techniques is to require a large number of grid points as well as a large computer capacity to attain a considerable accuracy \cite{11–16}.

In seeking a more efficient technique that requires fewer grid points and achieves acceptable accuracy, the differential quadrature (DQ) method is recently introduced. As stated by Civan and Sliepcevich \cite{17}, the DQM approximates the partial derivative of a variable at a given discrete point as a weighted linear sum of the function values at all the discrete points in the domain of interest. For the discontinuity problems, classical version of DQM leads to in-accurate results. This drawback can be overcome using domain decomposition technique with DQM \cite{18–20}. The second drawback of the classical DQM is the requirement of regular (rectangular) computational domain. Geometric mapping may be employed to transform the irregular domain of the problem to a regular one. Geometric mapping may also complicate the governing equations of the problem \cite{21,22}. For several applications, harmonic version of DQM leads to more accurate results than that obtained by the classical version \cite{23,24}. Ritz method or finite element method can also be combined with DQM to solve irregular plate problems \cite{25–27}.
Liew et al. [28–32] introduced a new version of DQ based on a moving least squares approximations which is termed by moving least squares differential quadrature method (MLSDQM). The main advantage of MLSDQM is its capability to deal with discontinuity plate problems as well as irregular plate ones. The proposed technique is employed for solving several engineering problems, such as vibration and buckling problems [33–37].

The present work extends the applications of MLSDQM to analyse material discontinuity, (composite plate), problems. The shape functions and their derivatives are derived through MLS approximations for regular and irregular node patterns. The obtained results are compared with the previous analytical and numerical ones. Further a parametric study is introduced to investigate the effects of radius of support domain, order of the basis functions, sensitivity of the discrete irregular pattern, elastic and geometric characteristics of the plate on the values of the obtained results.

2. Formulation of the problem

Consider a non-homogeneous composite consisting of an isotropic plate bonded, (along x-axis), to another one made of a functionally graded material (FGM). The elastic characteristics of the composite vary such that:

\[ G' = Ge^{\partial}, \quad E' = Ee^{\partial}, \quad \nu' = \nu, \]  

(1)

where \( G, E \) and \( \nu \) are shear modulus, Young’s modulus and Poisson’s ratio of the isotropic plate. \( G', E' \) and \( \nu' \) are shear modulus, Young’s modulus and Poisson’s ratio of the FG plate. \( \gamma \) is a constant characterizing the composite gradation.

Assume that the composite is subjected to a pure bending due to a laterally distributed load \( q \). Based on a first-order shear deformation theory, the equilibrium equations for such plate can be written, (in tensorial notations), as [38]:

\[ M_{ij} = Q_i, \quad Q_{ij} = -q, \quad (i,j = x,y) \]  

(2)

where \( M_{ij}, (i,j = x,y) \), are the bending and twisting moment resultants.

\( Q_i, (i = x,y) \), are the shearing force resultants.

The transverse deflection \( w(x,y) \) and the normal rotations \( \varphi_x(x,y), \varphi_y(x,y) \) are related to the moment and shear resultants through the following constitutive relations [39].

\[ M_{ij} = -D \left[ \delta_{ij}(\delta_{kk}\phi_{kk} + \nu(1 - \delta_{kk})\phi_{kk}) + \frac{(1 - \nu)(1 - \delta_{kk})}{2}(\phi_{ij} + \phi_{ji}) \right], \quad (i,j,k = x,y) \]  

(3)

\[ Q_i = kGh(w_i - \varphi_x), \quad (i = x,y), \]  

(4)

where \( \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \) and the flexural rigidity of the plate \( D = \frac{Eh^3}{12(1 - \nu^2)} \).

\( k \) is the shear correction factor [39,40], which is to be taken 5/6. \( h \) is thickness of the plate.

On suitable substitution from Eqs. (3) and (4) into (2), the equilibrium equations can be written as:

\[ D\left( \frac{\partial^2 \phi_x}{\partial x^2} + \frac{1 - \nu}{2} \frac{\partial^2 \phi_x}{\partial y^2} + \frac{1 + \nu}{2} \frac{\partial^2 \phi_y}{\partial x \partial y} + \frac{(1 - \nu)}{2}\gamma \left( \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right) \right) + kGh\left( \frac{\partial w}{\partial x} - \varphi_x \right) = 0, \]  

(5-a)

\[ D\left( \frac{\partial^2 \phi_y}{\partial y^2} + \frac{1 - \nu}{2} \frac{\partial^2 \phi_x}{\partial x^2} + \frac{1 + \nu}{2} \frac{\partial^2 \phi_x}{\partial x \partial y} + \gamma \left( \frac{\partial \phi_x}{\partial y} + \nu \frac{\partial \phi_y}{\partial x} \right) \right) + kGh\left( \frac{\partial w}{\partial y} - \varphi_y \right) = 0, \]  

(5-b)

\[ kGh\left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \gamma \left( \frac{\partial w}{\partial y} - \frac{\partial \varphi_x}{\partial x} \right) - \gamma \varphi_y \right) + qe^{-\partial x} = 0, \]  

(5-c)

where \( \gamma \neq 0 \) for FG plate while \( \gamma = 0 \) for isotropic one.

The boundary conditions can be described (according to the case of supporting) as follows:

(a) Simply supported:

\[ \begin{align*}
\text{SS1} & \quad w = 0, \quad M_n = 0, \quad M_m = 0, \\
\text{SS2} & \quad w = 0, \quad \varphi_x = 0, \quad M_n = 0,
\end{align*} \]  

(6)

(7)

(b) Clamped :

\[ \begin{align*}
\text{clamped} & \quad w = 0, \quad \varphi_x = 0, \quad \varphi_y = 0,
\end{align*} \]  

(8)

(c) Free :

\[ \begin{align*}
\text{free} & \quad Q_n = 0, \quad M_n = 0, \quad M_m = 0,
\end{align*} \]  

(9)

where the subscripts \( n \) and \( s \) represent the normal and tangent directions to the boundary edge, respectively; \( M_n, M_m \) and \( Q_n \) denote the normal bending moment, twisting moment and shear force on the plate edge; \( \varphi_x \) and \( \varphi_y \) are the normal and tangent rotations about the plate edge.
The continuity conditions, (along the interface), must be also satisfied such that:

\[ w(x, 0^-) = w^f(x, 0^+), \quad \phi_x(x, 0^-) = \phi_x^f(x, 0^+), \quad \phi_y(x, 0^-) = \phi_y^f(x, 0^+), \]  

(10)

which means that the deflection and rotations, (along the interface), of isotropic and FG plates must be equaled. Further, the force resultants and the rotations on the edge can be expressed in terms of the basic unknowns \( \phi_x \) and \( \phi_y \) as follows [39,40]:

\[
M_n = n_x^2 M_{xx} + 2n_xn_y M_{xy} + n_y^2 M_{yy},
\]

(11-a)

\[
M_m = (n_x^2 - n_y^2) M_{xy} + n_xn_y(M_{xy} - M_{xx})
\]

(11-b)

\[
Q_n = n_xQ_x + n_yQ_y,
\]

(11-c)

\[
\phi_n = n_x\phi_x + n_y\phi_y.
\]

(11-d)

\[
\phi_s = n_x\phi_y - n_y\phi_x
\]

(11-e)

where \( n_x \) and \( n_y \) are the direction cosines at a point on the boundary edge.

3. Solution of the problem

Moving least squares DQM can be applied to solve the problem as follows:

- Discretize the domain of the problem, \( \Omega \), into a finite number of nodes: \( \{X_i = (x_i, y_i), i = 1, N\} \). Each node is associated with three nodal unknowns \( (w, \phi_x, \phi_y) \). The influence domain, for each node, is determined as shown in Fig. 1. Over each influence domain, \( (\Omega, i = 1, N) \), the nodal unknowns can be approximated as [28–31]:

\[
\rho(x) = \rho(x_i, y_i) \cong \phi^h(x_i, y_i) = \sum_{j=1}^{n} \phi_j(x_i, y_i) \rho^j, \quad (\rho = w, \phi_x, \phi_y), \quad (i = 1, N),
\]

(12)

where \( n \) is the number of nodes within the influence domain, \( \Omega_i \). \( \rho^h = w^h, \phi^h_x, \phi^h_y \) are approximate values for nodal unknowns \( w, \phi_x \) and \( \phi_y \), respectively. \( \phi_j(x_i, y_i) \) is defined as the shape function of MLS approximation over the influence domain, \( (\Omega_i, i = 1, N) \). The nodal parameters: \( \{w, \phi^h_x, \phi^h_y\} \) are always not equal to the physical values \( \{w(x_i, y_i), \phi_x(x_i, y_i), \phi_y(x_i, y_i)\} \), since the MLS shape functions \( \phi_j(x_i, y_i) \) do not satisfy the Kronecker delta condition generally.

- Apply the MLS technique to approximate \( u^h(x) \) to \( u(x) \), for any \( x \in \Omega \), such as [41,42]:

\[
u^h(x) = \sum_{i=1}^{m} P_i(x)a_i(x) = P^T(x)a(x),
\]

(13)

where \( a(x) = \{a_1(x), a_2(x), \ldots, a_m(x)\}^T \) is a vector of unknown coefficients. \( P^T(x) = \{p_1(x), p_2(x), \ldots, p_m(x)\} \) is a complete set of monomial basis. \( m \) is the number of basis terms. The coefficients \( a_j(x) (j = 1, m) \), can be obtained at any point \( x \) by minimizing the following weighted quadratic form:

\[
\Pi(a) = \sum_{i=1}^{n} \omega(x - x_i)(u^h(x_i) - u_i)^2 = \sum_{i=1}^{n} \omega(x - x_i)(P^T(x_i)a(x) - u_i)^2,
\]

(14)

Fig. 1. Domain descretization for moving least squares differential quadrature method.
where \( n \) is the number of nodes in the neighborhood of \( x \) and \( u_i \) is the nodal parameter of \( u(x) \) at point \( x_i \). \( \omega_i(x) = \omega(x - x_i) \) is a positive weight function which decreases as \( ||x - x_i|| \) increases. It always takes unit value at the sampling point \( x \) and vanishes outside the domain of \( x \).

The stationary value of \( \Pi(a) \) with respect to \( a(x) \) leads to a linear relation between the coefficient vector \( a(x) \) and the vector of fictitious nodal values \( u \), such as:

\[
\mathbf{A}(x)a(x) = \mathbf{B}(x)u \tag{15}
\]

from which \( a(x) = \mathbf{A}^{-1}(x)\mathbf{B}(x)u \). \( \mathbf{A}(x) = \mathbf{P}(x)\mathbf{\omega}_n(x)\mathbf{P}_T(x) = \sum_{i=1}^{n} \mathbf{P}(x)\mathbf{P}_T(x_i) \), \( \mathbf{B}(x) = \mathbf{P}(x)\mathbf{\omega}(x) = [\mathbf{\omega}_1(x)\mathbf{P}(x_1) \ldots \mathbf{\omega}_n(x)\mathbf{P}(x_n)] \).

On suitable substitution from Eq. (16) into (13), \( u^h(x) \) can then be expressed in terms of the shape functions as:

\[
u^h(x) = \mathbf{P}^T(x)a(x) = \mathbf{P}^T(x)\mathbf{A}^{-1}(x)\mathbf{B}(x)u = \sum_{i=1}^{n} \phi_i(x)u_i \tag{17}
\]

Where the nodal shape function: \( \phi_i(x) = \mathbf{P}^T(x)\mathbf{A}^{-1}(x)\mathbf{B}(x_i) \) \( \tag{18} \)

It should be noted that the MLS shape function and its derivatives are dependent on the weight function and the radius of influence domain. It's also required that \( n \geq m \) in the domain of influence so that the matrix \( \mathbf{A}(x) \) in Eq. (16) can be inverted [28–31].

- Determine the MLS shape functions \( \phi_i(x_i, y_i) \) and its partial derivatives using the technique proposed by Belytschko [43], to reduce the computational effort as follows:

Eq. (18) can be rewritten as:

\[
\phi_i(x) = \mathbf{P}^T(x)\mathbf{A}^{-1}(x)\mathbf{B}_i(x) = \mathbf{z}^T(x)\mathbf{B}_i(x) \tag{19}
\]

Since \( \mathbf{A}(x) \) is a symmetric matrix, then Eq. (19) yields

\[
\mathbf{A}(x)\mathbf{z}(x) = \mathbf{P}(x) \tag{20}
\]

Therefore, determination of the shape function and its partial derivatives can be reduced to solution of Eq. (20). This equation can be solved using LU decomposition and back-substitution. Moreover, one can compute the partial derivatives of \( \mathbf{z}(x) \) in any order using the same matrix after LU decomposition of \( \mathbf{A}(x) \) as in Eq. (20). For example, we have:

\[
\mathbf{A}_I(x)\mathbf{z}(x) + \mathbf{A}(x)\mathbf{z}_J(x) = \mathbf{P}_I(x), \quad (I = x, y) \tag{21}
\]

Or \( \mathbf{A}(x)\mathbf{z}_J(x) = \mathbf{P}_I(x) - \mathbf{A}_I(x)\mathbf{z}(x), \quad (I = x, y) \tag{22} \)

where only one back-substitution is required to obtain \( \mathbf{z}_J(x) \). Similarly, higher order derivatives can be computed in the same way. The second order derivatives of \( \mathbf{z}(x) \) can be computed with a back-substitution using the new right hand vector as shown in the following equations:

\[
\mathbf{A}_{IJ}(x)\mathbf{z}(x) + \mathbf{A}_I(x)\mathbf{z}_J(x) + \mathbf{A}_J(x)\mathbf{z}_I(x) + \mathbf{A}(x)\mathbf{z}_{IJ}(x) = \mathbf{P}_{IJ}(x), \quad (I, J = x, y) \tag{23}
\]

Or \( \mathbf{A}(x)\mathbf{z}_{IJ}(x) = \mathbf{P}_{IJ}(x) - \mathbf{A}_I(x)\mathbf{z}(x) - \mathbf{A}_J(x)\mathbf{z}_I(x) - \mathbf{A}_J(x)\mathbf{z}_J(x), \quad (I, J = x, y) \tag{24} \)

Defining the first and second order partial derivatives of the shape function as:

\[
\phi_{ijL}(x) = \mathbf{c}_{ij}^L(x) = \mathbf{z}_{ijL}(x)\mathbf{B}_i(x) + \mathbf{z}_{ijL}(x)\mathbf{B}_j(x), \quad (L = x, y) \tag{25}
\]

\[
\phi_{ijkl}(x) = \mathbf{c}_{ijkl}^K(x) = \mathbf{z}_{ijkl}(x)\mathbf{B}_i(x) + \mathbf{z}_{ijkl}(x)\mathbf{B}_j(x) + \mathbf{z}_{ijkl}(x)\mathbf{B}_k(x) + \mathbf{z}_{ijkl}(x)\mathbf{B}_l(x), \quad (L, K = x, y) \tag{26} \)

Therefore, one can consider the following expressions to be substituted in the governing equations (6), the problem can be reduced to the following system of linear algebraic equations:

\[
\sum_{j=1}^{n} [k\mathbf{G}c_{ijw} + D \left( c_{ijy} + \frac{1}{2} - \mathbf{c}_{ijy} \right) + \gamma c_{ijy} - k\mathbf{G}\phi_{ij} \mathbf{p}_i + D \left( \frac{1}{2} - \mathbf{c}_{ijy} \right) \mathbf{p}_j] = 0, \quad (i = 1, N) \tag{27} \]

On suitable substitution from Eq. (27) into the governing equations (6), the problem can be reduced to the following system of linear algebraic equations:

\[
\sum_{j=1}^{n} [k\mathbf{G}c_{ijw} + D \left( c_{ijy} + \frac{1}{2} - \mathbf{c}_{ijy} \right) + \gamma c_{ijy} - k\mathbf{G}\phi_{ij} \mathbf{p}_i + D \left( \frac{1}{2} - \mathbf{c}_{ijy} \right) \mathbf{p}_j] = 0, \quad (i = 1, N) \tag{28} \]
\[
\sum_{j=1}^{n} \left[ k_{ij} \right] w_{x}^{i} + D \left( 1 + \frac{\gamma}{2} c_{ij}^{x} \right) \phi_{x}^{i} + D \left( c_{ij}^{xy} + 1 - \frac{\gamma}{2} c_{ij}^{x} + \gamma c_{ij}^{y} - k_{ij} \phi_{y}^{i} \right) \phi_{y}^{i} = 0, \quad (i = 1, N)
\]
(29)

\[
\sum_{j=1}^{n} k_{ij} \left[ c_{ij}^{x} + c_{ij}^{y} + \gamma c_{ij}^{y} \right] w_{x}^{i} - c_{ij}^{x} \phi_{x}^{i} - (c_{ij}^{y} + \gamma c_{ij}^{y}) \phi_{y}^{i} = -q \cdot e^{-\gamma t}, \quad (i = 1, N)
\]
(30)

The boundary conditions can also be reduced to the following linear algebraic equations:
Fig. 3. Variation of the results with the radius of support domain, completeness order and the number of grid points for a regular discretized clamped plate.

Fig. 4. Variation of the results with the radius of support domain and completeness order for an irregular discretization: (a) simply supported plate, (b) clamped plate.

(a) Simply supported:

\[ w' = 0, \quad \sum_{j=1}^{n} \left[ \left( n_{x}^2 + n_{y}^2 \right) c_{x}^j + (1 - \nu) n_{x} n_{y} c_{y}^j \right] \phi_{x}^{j} + \left( (\nu n_{x}^2 + n_{y}^2) c_{y}^j + (1 - \nu) n_{x} n_{y} c_{x}^j \right) \phi_{y}^{j} = 0, \]

\[ -\frac{1}{2} D \sum_{j=1}^{n} \left[ \left( n_{x}^2 - n_{y}^2 \right) c_{x}^j - 2n_{x} n_{y} c_{y}^j \right] \phi_{x}^{j} + \left( (n_{x}^2 - n_{y}^2) c_{y}^j + 2n_{x} n_{y} c_{x}^j \right) \phi_{y}^{j} = 0, \quad (i = 1, N) \]  

(31)
\[ w_{i} = 0; \quad X_{n}^{j} = \frac{1}{ij} (nxu_{j}^{y} - nyu_{j}^{x}) = 0, \quad \left( i = 1, N \right) \]

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**Fig. 5.** Normalized deflection distribution of a simply supported composite plate. (\( E_{2}/E_{1} = 2; \ G_{2} = G_{1}; \ v_{2} = v_{1} \)).

**Fig. 6.** Variation of the normalized deflection with Young’s modulus for composite plates: (a) Simply supported, (b) Clamped–clamped, (\( G_{2} = G_{1}; \ v_{2} = v_{1} \)).

**Fig. 7.** Variation of the normalized deflection with shear modulus for composite plates: (a) Simply supported, (b) Clamped–clamped, (\( E_{2} = E_{1}; \ v_{2} = v_{1} \)).
### 4. Numerical results

For the present results, Gaussian weight function with a circular influence domain is adopted for the MLS approximation because of its partial derivatives with respect to $x$ and $y$ coordinates exist to any desired order. It takes the form of \[28–31\]:

\[
W_i(x) = \begin{cases} 
\frac{\exp(-d_i/r^3) - \exp(-(r/c)^3)}{1 - \exp(-(r/c)^3)} & d_i \leq r \\
0 & d_i > r 
\end{cases}
\]

where $d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}$, is the distance from a nodal $x_i$ to a field $x$ in the influence domain of $x_i$, $r$ is the radius of support domain and $c$ is the dilation parameter. In the present work, the dilation parameter is selected such as: $c = r/4$. 

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**Fig. 8.** Variation of the normalized deflection with aspect ratio for composite plates: (a) simply supported, (b) clamped–clamped, $(E_2/E_1 = 2; G_2 = G_1; v_2 = v_1)$.

**Fig. 9.** Normalized deflection distribution through different locations for FG clamped plate: (a) $\gamma = 1$, (b) $\gamma = 10$. 

(c) **Free**  
\[ \begin{align*}
  kGh \sum_{j=1}^{n} [(n_x c_{ij}^x + n_y c_{ij}^y)] w^j - n_x \phi_y y^j - n_y \phi_x x^j = 0, \\
 - D \sum_{j=1}^{n} \left[ (n_x^2 + n_y^2) c_{ij}^x + (1 - \nu)n_x n_y c_{ij}^y \right] \phi_x^j + \left[ (n_x^2 + n_y^2) c_{ij}^x + (1 - \nu)n_x n_y c_{ij}^y \right] \phi_y^j = 0, \\
 - \frac{1}{2} D \sum_{j=1}^{n} \left[ (n_x^2 - n_y^2) c_{ij}^x - 2n_x n_y c_{ij}^y \right] \phi_x^j + \left[ (n_x^2 - n_y^2) c_{ij}^x + 2n_x n_y c_{ij}^y \right] \phi_y^j = 0, \quad (i = 1, N) 
\end{align*} \]  

Along the interface, the following algebraic equation must also be considered:

\[ w(x_i, y_i) = w^j(x_i, y_i), \quad \phi_x(x_i, y_i) = \phi_x^j(x_i, y_i), \quad \phi_y(x_i, y_i) = \phi_y^j(x_i, y_i), \quad (i = 1, N) \]
Also, a scaling factor $d_{\text{max}}$ is defined as:

$$d_{\text{max}} = \frac{r}{h_m}$$

where $h_m$ is the grid size, which can be regarded as the distance between the nodal point $x_i$ and the 2nd nearest neighboring field nodes. For regular node arrangement spaced by $\beta$, $h_m$ can be taken as: $h_m = \sqrt{2}\beta$.

To examine the validity of the obtained results, the problem a squared isotropic plate is considered. Then, the obtained numerical deflection of this plate is compared with the previous analytical ones [3,39]. For practical purposes the numerical results are normalized such as:

$$w = 100wD/(qa^4), \quad M_{ij} = 10M_{ij}/(qa^2), \quad \sigma_{ij} = \sigma_{ij}/E, \quad (i,j = x,y).$$

where $w, M_{ij}, \sigma_{ij}$ are the normalized deflection, moments, and stresses, respectively. $a$ is the width of the concerned squared plate. $D^*$ is the flexural rigidity of the isotropic plate.

A parametric study is introduced to investigate the performance of MLSDQM for solving bending plate problems. The convergence and numerical accuracy of the present method are carefully examined by considering the influence of the support domain, various completeness orders of the assumed basis functions and sensitivity of irregular grid points.

On suitable direct substitution from Eqs. (31)-(35), boundary conditions, into equilibrium ones (28)-(30), the reduced linear algebraic system is solved using MATLAB. Eqs. (28)-(35) are solved over a regular grid with $N_i = \{7, 25\}$. For different boundary conditions, Tables 1–3 show that the results for $N_i = 11$ are nearly the same as those corresponding to $N_i = \{13, 25\}$. Therefore, the parametric study is introduced over a grid $11 \times 11$ nodes. Also, the completeness order $N_i$ of the interpolation basis ranges from 2 to 5 with various scaling factors $d_{\text{max}}$ from 2 to 10, as shown Figs. 2–4. It is found that $d_{\text{max}} \geq N_i + 0.5$ is required for reasonable numerical solutions. This was previously recorded in [30]. For irregular discretizations, the discrete nodes inside the plate are randomly generated while the boundary nodes are still equally spaced. For regular and irregular discretizations, Figs. 2–4 show that the accuracy of the obtained results increases with increasing both of $N_i$ and the radius of support domain while it decreases with increasing of the grid size $h_m$.

For regular discretizations, Figs. 2 and 3 show that one can select $N_i = 4$ and $d_{\text{max}} \geq 5$ to obtain accurate results. Fig. 4 shows that, for irregular discretizations, one can select $N_i \geq 5$ and $d_{\text{max}} \geq 7$ to obtain accurate results.

Tables 4 and 5 show a very good agreement between the obtained results and previous analytical ones [3,39]. For simply supported plates, the error between obtained results and analytical ones [3,39] is $\leq 10^{-5}$. While this error records $\leq 10^{-6}$ for clamped plates.

As well as, the parametric study is extended to investigate behavior of the composite due to Young’s modulus gradation ratio, $(E_2/E_1)$, shear modulus gradation ratio $(G_2/G_1)$, Poisson’s ratio $(\nu_2/\nu_1)$, graduation factor $(\gamma)$, aspect ratio $a/b$ and the interface location $(a_1/a_2)$, where $a$ and $b$ are the width and length of the rectangular plate. $a = a_1 + a_2$.

Figs. 5–7 and 9 show that the values of normalized deflection decrease with increasing the Young’s modulus, the shear modulus and the graduation factor $\gamma$. While, these values increase with increasing of the aspect ratio $(a/b)$, as shown in Fig. 8. Fig. 10 shows stress distribution, $\sigma_{yy}$, through the composite. This may be investigated through the stiffness concepts. Also, the computations declare that the results do not affect significantly by the Poisson’s ratio $\nu_1, \nu_2$. Further, Fig. 9 insist the advantages of FG composites that treat the interfacial discontinuity problems.
Table 4
Comparison between the obtained deflection and the previous analytical ones [3,39] for simply supported plates.

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Comparison between the obtained results and the previous analytical ones, ($\omega a^4/d$) for clamped plates at the centre.

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5. Conclusion

This work extends the applications of MLSDQM for solving discontinuity problems of composite plate materials. The obtained results agreed with the previous analytical ones. Over a grid 11X11 nodes, $N_c = 4$ and $d_{max} \geq 5$ MLSDQM provides rapid and convergent solutions with a good accuracy. For irregular discretizations, MLSDQM leads to accurate results with $N_c \geq 5$ and $d_{max} \geq 7$. Further a parametric study is introduced to investigate the effects of computational, geometric and elastic characteristics of the problem on the values of the obtained results.

References


